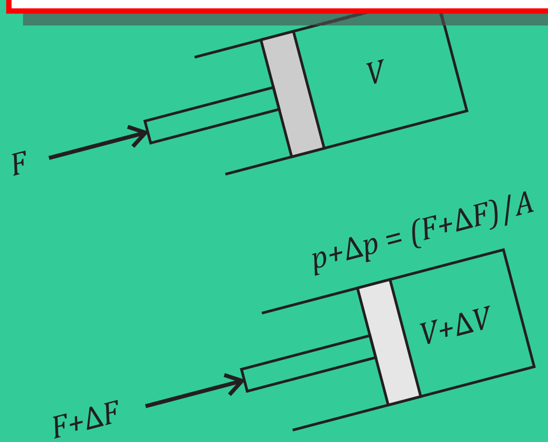
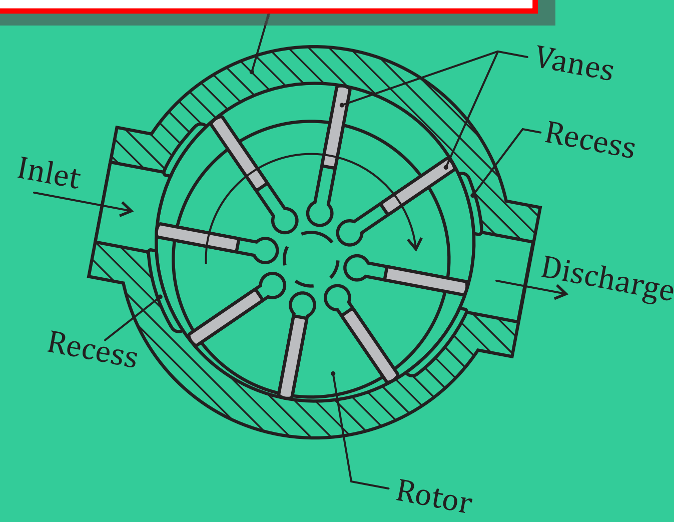
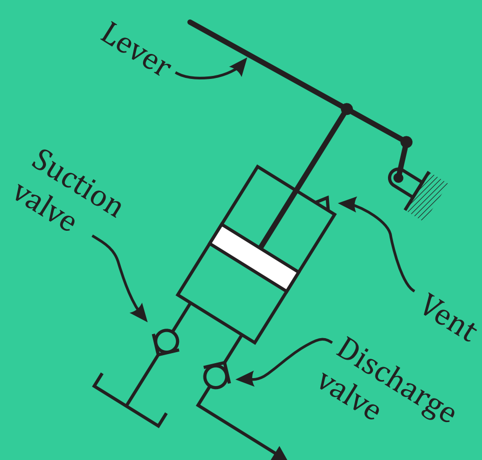
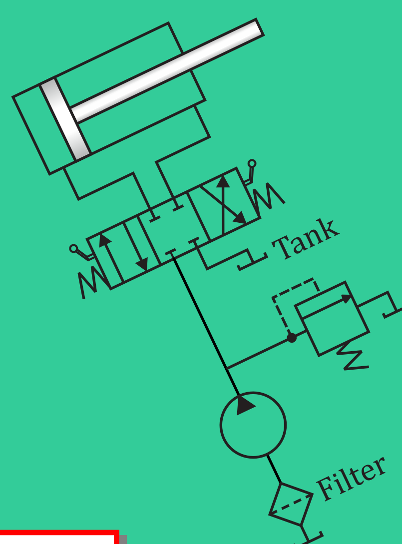
Applied Energy Systems — Rudimentary Fluid Mechanics Solutions Manual

Jim McGovern









SI Units 2014 Edition

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Preface

This document contains complete worked solutions for all of the end-of-chapter problems in the associated textbook. Some instructors might wonder why these solutions have been made publicly available, rather than making them available to instructors only. Some learners may mistakenly assume that if they have read the solutions they will have acquired the ability to solve such problems. Nonetheless, for a textbook that is made freely available on a Creative Commons license, the author felt this was the best approach. There are caveats for instructors and for learners.

In general, it would not be appropriate for instructors to use the problems in an unmodified form for grading learners. However, they are free to modify and adapt them, as appropriate, to meet their assessment needs and to offer learners problems for which worked solutions are not available.

The caveat for learners is that they should resist any temptation to read the solutions before they have attempted the problems themselves. There are sufficient worked examples in the textbook itself for the purpose of studying how solutions can be set out. Learning must be an active process and the way to learn how to solve problems is to solve problems ourselves. Effective learners have tenacity and should manage to obtain answers themselves to all, or almost all, of the problems they attempt. Having worked through a problem and obtained a solution or answers, the learner can get confirmation of the correctness of their approach by referring to the solutions in this manual. This is effective learning.

Chapter 1. Units and Dimensions

1-1 Write the fundamental SI units (symbol only) for the quantities in the table below. Also write the fundamental dimensions of each quantity using only M for mass, L for distance, T for time and Θ for temperature.

Solution

Quantity	**Fundamental SI Units**	**Fundamental Dimensions**
Mass	kg	M
Length	m	L
Time	s	T
Temperature	K	Θ
Angle	rad	dimensionless
Area	m^2	L^2
Volume	m^3	L^3
Velocity	$m\,s^{-2}$	$L\,T^{-1}$
Angular velocity	rad s^{-1}	T^{-1}
Acceleration	$m\,s^{-2}$	$L\,T^{-2}$
Angular acceleration	rad s^{-2}	T^{-2}
Volume flow rate	$m^3\,s^{-1}$	$L^3\,T^{-1}$
Kinematic viscosity	$m^2\,s^{-1}$	$L^2\,T^{-1}$
Force	N	$M\,L\,T^{-2}$
Energy	J	$M\,L^2\,T^{-2}$
Moment or torque	N m	$M\,L^2\,T^{-2}$

Quantity	Fundamental SI Units	Fundamental Dimensions
Power	W	ML^2T^{-3}
Density	kg m^{-3}	$M L^{-3}$
Pressure	Pa	$ML^{-1}T^{-2}$
Shear stress	N m^{-2}	$M L^{-1} T^{-2}$
Dynamic viscosity	Pas	$ML^{-1}T^{-1}$

Chapter 2. Fluids, Pressure and Compressibility

2-1 What is a fluid (in 20 words or less)?

Solution

A fluid is a substance that cannot sustain a shear stress without undergoing relative movement. (15 words)

or, for example:

A fluid is a substance that cannot remain at rest while a shear force acts within it. (17 words)

2-2 A mineral oil is placed in a rigid cylindrical container that has a flat base and a flat leak-tight piston at the top in contact with the mineral oil. A force is applied to the piston, causing the pressure to increase by 15.8 MPa. As a result, the mineral oil undergoes a decrease in its volume and the volumetric strain, $-\Delta V/V$, is 0.00913. Calculate the compressibility and the bulk modulus of the mineral oil.

Solution

The compressibility is given by

$$\beta = \frac{\left(-\frac{\Delta V}{V}\right)}{\Delta p} = \frac{0.00913}{15.8\ [\mathrm{MPa}]} = 0.578 \times 10^{-3}\ \mathrm{MPa^{-1}}$$

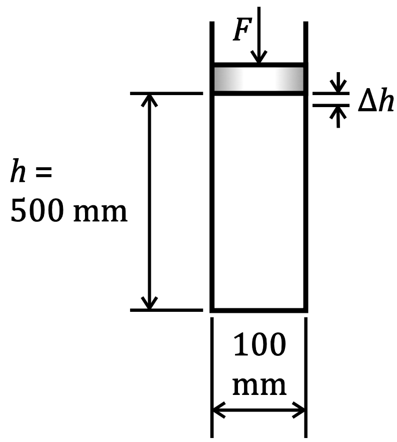
$$= 0.578\ \mathrm{GPa^{-1}}$$

The bulk modulus is equal to the inverse of the compressibility:

$$K = \frac{1}{\beta} = \frac{1}{0.578\ \mathrm{GPa^{-1}}} = 1.73\ \mathrm{GPa}$$

2-3 Hydraulic oil at a pressure of 2 MPa and having a bulk modulus of elasticity of 1.6 GPa fills a rigid cylinder of diameter 100 mm and height 500 mm. The ends of the cylinder are flat and the top end is a frictionless and leak-tight piston. If the force on the piston is increased so that the oil pressure rises to 8 MPa, calculate the distance that the piston would move.

Solution



$$p_1 = 2 \text{ MPa}$$

$$p_2 = 8 \text{ MPa}$$

$$\Delta p = (8\text{-}2)\text{MPa} = 6 \text{ MPa} = 6 \times 10^6 \text{Pa}$$

$$K = 1.6 \text{ GPa} = 1.6 \times 10^9 \text{Pa}$$

$$K = -\frac{\Delta p}{\left(\frac{\Delta V}{V}\right)} = -\frac{\Delta p\ V}{\Delta V}$$

However,

$$\frac{\Delta h}{h} = \frac{\Delta V}{V}$$

Hence,

$$K = -\frac{\Delta p}{\left(\frac{\Delta h}{h}\right)} = -\frac{\Delta p\ h}{\Delta h}$$

Therefore

$$\Delta h = -\frac{\Delta p\ h}{K}$$

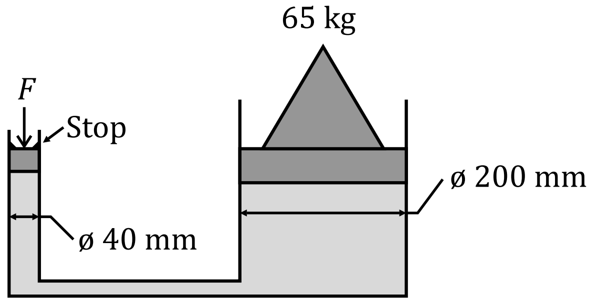
$$= -\frac{6 \times 10^6 [\mathrm{Pa}]\ 0.5\ [\mathrm{m}]}{1.6 \times 10^9 [\mathrm{Pa}]}$$

$$= -1.875 \times 10^{-3}\ \mathrm{m} = -1.875\ \mathrm{mm}$$

Therefore, the piston moves about 1.88 mm downwards.

Chapter 3. Hydrostatic Work and Power Transmission

3-1



In the arrangement shown above, a mass of 65 kg is to be raised through a distance of 30 mm by the application of a force, *F*, to the small piston on the left. The masses of the two pistons are such that they apply the same pressure to the hydraulic fluid owing to their weight (the two pistons have the same mass or weight per unit area). Calculate the force that must be applied to the left hand piston and the distance through which it must travel.

Solution

$$F_1 = pA_1 \text{ and } F_2 = pA_2$$

Hence

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$F_1 = F_2 \frac{A_1}{A_2} = F_2 \frac{\frac{\pi D_1^2}{4}}{\frac{\pi D_2^2}{4}} = F_2 \frac{D_1^2}{D_2^2} = F_2 \left(\frac{D_1}{D_2}\right)^2$$

$$F_2 = 65\ [\text{kg}] \times 9.81\ \left[\frac{\text{m}}{\text{s}^2}\right]$$

$$= 637.7\ \text{N}$$

Therefore,

$$F_1 = 637.7\ [\text{N}] \left(\frac{40}{200}\right)^2 = 25.51\ \text{N}$$

$$A_1 \Delta s_1 = A_2 \Delta s_2$$

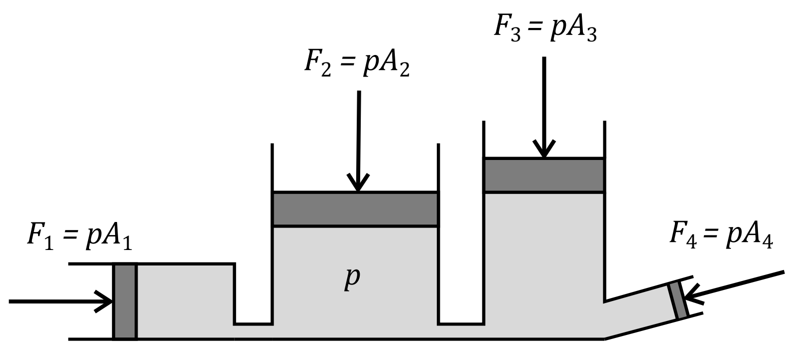
$$\Delta s_1 = \Delta s_2 \frac{A_2}{A_1} = \Delta s_2 \frac{\frac{\pi D_2^2}{4}}{\frac{\pi D_1^2}{4}}$$

$$= \Delta s_2 \frac{D_2^2}{D_1^2} = \Delta s_2 \left(\frac{D_2}{D_1}\right)^2$$

$$= 30\ [\text{mm}] \left(\frac{200}{40}\right)^2 = 30\ [\text{mm}] \times 5^2$$

$$= 750\ \text{mm}$$

3-2



In the figure above, take it that the four pistons shown are massless, frictionless and leak tight and enclose liquid water in the system shown. The pressure within the system is 2.35 MPa gauge and all four pistons are held at rest by the application of the four forces shown. Area A_3 is 0.0303 m^2 and the diameter of piston 1 equals 75% of the diameter of piston 3. The diameter of piston 4 is 64.2 mm and $F_2 = 2.31\ F_3$. Calculate the value of each of the four forces shown and the diameter of piston 2.

Solution

$$F_3 = pA_3 = 2.35\ [\text{MNm}^{-2}]\ 0.0303\ [\text{m}^2]$$

$$= 0.07121\ \text{MN} = 71.21\ \text{kN}$$

$$F_2 = 2.31 F_3 = 2.31 \times 71.21\ \text{kN} = 164.5\ \text{kN}$$

$$A_4 = \pi D_4^2/4 = \pi\ 0.0642^2\ [\text{m}^2]/4 = 3.237 \times 10^{-3}\ \text{m}^2$$

$$F_4 = pA_4 = p\,\pi D_4^2/4$$

$$= 2.35\ [\text{MNm}^{-2}]\ 3.237 \times 10^{-3}\ [\text{m}^2]$$

$$= 0.007607\ \text{MN} = 7.607\ \text{kN}$$

$$F_1 = pA_1 = pA_3\ \frac{A_1}{A_3}$$

$$= F_3\ \left(\frac{D_1}{D_3}\right)^2$$

$$= 71.21\ [\text{kN}]\ 0.75^2$$

$$= 40.05\ \text{kN}$$

$$A_2 = \frac{F_2}{p} = \frac{164.5\ [\text{kN}]}{2.35 \times 10^3\ [\text{kNm}^{-2}]}$$

$$= \frac{164.5\ [\text{kN}]}{2.35 \times 10^3\ [\text{kNm}^{-2}]}$$

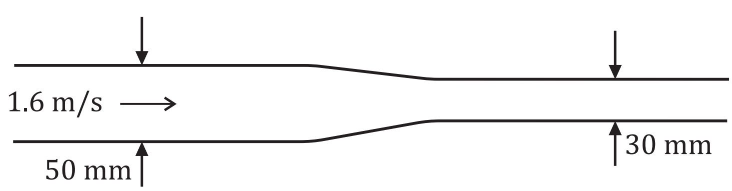
$$= 0.07000\ [\text{m}^2]$$

$$D_2 = \sqrt{\frac{4\ A}{\pi}} = \sqrt{\frac{4\ \times 0.07000\ [\text{m}^2]}{\pi}}$$

$$= 0.2985\ \text{m} = 299\ \text{mm}$$

3-3 Water flows in a pipe that has an internal diameter of 50 mm at a mean velocity of 1.6 m/s. Calculate the volume flow rate through the pipe. If the internal diameter of the pipe reduces to 30 mm further downstream, calculate the mean velocity at that position for the same flow rate through the pipe.

Solution



$$A_1 = \frac{\pi d_1^2}{4} = \frac{\pi\ 0.05^2\ [\text{m}^2]}{4} = 1.963 \times 10^{-3}\ \text{m}^2$$

$$Q = v_1 A_1 = 1.6\ [\text{ms}^{-1}] \times 1.963 \times 10^{-3}\ \text{m}^2$$

$$= 3.141 \times 10^{-3}\ \text{m}^3/\text{s}$$

$$v_1 A_1 = v_2 A_2$$

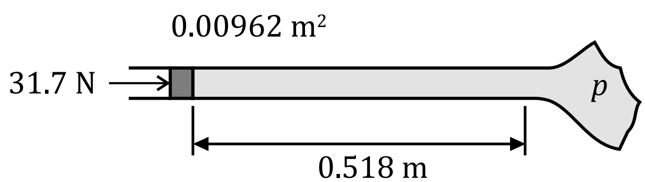
Hence,

$$v_2 = v_1 \frac{A_1}{A_2} = v_1 \left(\frac{d_1}{d_2}\right)^2$$

$$= 1.6\ [\text{ms}^{-1}] \left(\frac{50}{30}\right)^2 = 1.6\ [\text{ms}^{-1}] \left(\frac{5}{3}\right)^2$$

$$= 4.44\ \text{m/s}$$

3-4



The diagram above shows a piston that applies a constant force of 31.7 N to hydraulic fluid as it moves through a distance of 0.518 m in 1.62 seconds. The area of the piston is 0.00962 m^2. Calculate the pressure of the hydraulic fluid that resists the motion, the work done and the power required to push the piston.

Solution

$$p = \frac{F}{A} = \frac{31.7\ [\text{N}]}{0.00962\ [\text{m}^2]} = 3{,}295\ \text{N/m}^2$$

$$W = p\Delta V = 3{,}295\ \left[\frac{\text{N}}{\text{m}^2}\right] \times 0.00962\ [\text{m}^2] \times 0.518\ [\text{m}]$$

$$= 16.42\ \text{Nm} = 16.42\ \text{J}$$

Power is given by

$$P = \frac{W}{\Delta t} = \frac{16.42\ [\mathrm{J}]}{1.62\ [\mathrm{s}]} = 10.14\ \mathrm{W}$$

Chapter 4. Positive Displacement Pumps

4-1 An ideal hydraulic gear pump has a displacement of 25 mL per revolution and operates at 3,000 r.p.m. Calculate the flow rate produced by the pump.

Solution

The displacement per revolution is given by

$$V_{\text{displ}} = 25\left[\frac{\text{mL}}{\text{rev}}\right] = 25 \times 10^{-6}\left[\frac{\text{m}^3}{\text{rev}}\right]$$

The speed of rotation is given by

$$N = 3000 \text{ rpm} = \frac{3000}{60}\ \frac{\text{rev}}{\text{s}} = 50\ \frac{\text{rev}}{\text{s}}$$

Hence, the volume flow rate is given by

$$Q = V_{\text{displ}}N = 25 \times 10^{-6}\left[\frac{\text{m}^3}{\text{rev}}\right] \times 50\left[\frac{\text{rev}}{\text{s}}\right]$$

$$= 1250 \times 10^{-6}\frac{\text{m}^3}{\text{s}} = 1.25 \times 10^{-3}\frac{\text{m}^3}{\text{s}}$$

4-2 An ideal swash plate pump operates at 1,500 r.p.m. and has eight pistons. Each piston has a diameter of 1 cm and a stroke of 2 cm. Calculate the volume flow rate of the pump.

Solution

The displacement of one piston is given by

$$V_{\text{displ, piston}} = \frac{\pi D^2}{4}L = \frac{\pi\ 0.01^2}{4}0.02 \text{ m}^3 = 1.571 \times 10^{-6} \text{ m}^3$$

As there are eight pistons, the displacement per revolution is given by

$$V_{\text{displ}} = 8 \times 1.571 \times 10^{-6}\ \frac{\text{m}^3}{\text{rev}} = 12.57 \times 10^{-6}\ \frac{\text{m}^3}{\text{rev}}$$

The speed of rotation is given by

$$N = 1500 \text{ rpm} = \frac{1500}{60} \frac{\text{rev}}{\text{s}} = 25 \frac{\text{rev}}{\text{s}}$$

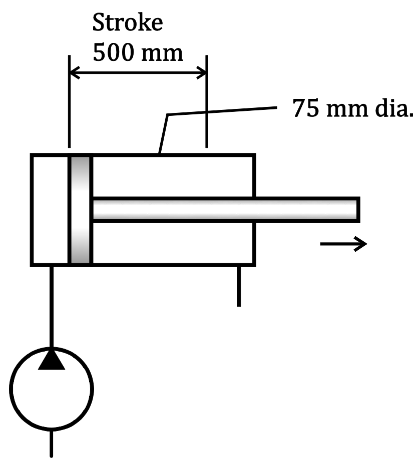
Hence, the volume flow rate is given by

$$Q = V_{\text{displ}} N = 12.57 \times 10^{-6} \left[\frac{\text{m}^3}{\text{rev}}\right] \times 25 \left[\frac{\text{rev}}{\text{s}}\right]$$

$$= 314.3 \times 10^{-6} \frac{\text{m}^3}{\text{s}}$$

4-3 Suppose each of the pumps in questions 1 and 2 above is connected to a hydraulic cylinder with a bore of 75 mm and a stroke of 500 mm: how long would it take for the cylinder to extend fully in each case? The volume of fluid required to extend the cylinder equals the product of the cross-sectional area and the stroke, $V = \frac{\pi d^2}{4} L_{\text{stroke}}$.

Solution



The swept volume of the cylinder is given by

$$V_{\text{cyl}} = \frac{\pi D^2}{4} L_{\text{stroke}} = \frac{\pi \, 0.075^2}{4} [\text{m}^2] \; 0.5 \; [\text{m}]$$

$$= 2.209 \times 10^{-3} \text{ m}^3$$

The time taken for the piston stroke with pump 1 is given by

$$t = \frac{V_{\text{cyl}}}{Q_{\text{pump}}} = \frac{2.209 \times 10^{-3}\,[\text{m}^3]}{1.25 \times 10^{-3}\left[\frac{\text{m}^3}{\text{s}}\right]}$$

$$= 1.77\text{ s}$$

The time taken for the piston stroke with pump 2 is given by

$$t = \frac{V_{\text{cyl}}}{Q_{\text{pump}}} = \frac{2.209 \times 10^{-3}\,[\text{m}^3]}{314.3 \times 10^{-6}\left[\frac{\text{m}^3}{\text{s}}\right]}$$

$$= \frac{2209}{314.3}\,[\text{s}] = 7.03\text{ s}$$

4-4 The following equations relating to the efficiency of pumps are given:

$$E_{\text{pump, mech}} = \frac{Q_{\text{ideal}}\,\Delta p}{P_{\text{actual}}}$$

$$E_{\text{pump, vol}} = \frac{Q_{\text{actual}}}{Q_{\text{ideal}}}$$

$$E_{\text{pump, overall}} = E_{\text{pump, mech}} \times E_{\text{pump, vol}}$$

$$= \frac{Q_{\text{actual}}\,\Delta p}{P_{\text{actual}}}.$$

A vane pump has a displacement of 3.61 mL per revolution and operates at 1,450 r.p.m. with a pressure difference of 791 kPa. The measured flow rate is 4.79 L/min. The measured mechanical power input to drive the pump is 77.4 W. Calculate the mechanical efficiency, the volumetric efficiency and the overall efficiency of the pump.

Solution

The ideal flow rate of the pump is given by

$$Q_{\text{ideal}} = 3.61 \times 10^{-6}[\text{m}^3] \times \frac{1450}{60}\,[\text{s}^{-1}]$$

$$= 87.24 \times 10^{-6}\,\frac{\mathrm{m^3}}{\mathrm{s}}$$

$$E_{\text{pump, mech}} = \frac{Q_{\text{ideal}}\,\Delta p}{P_{\text{actual}}}$$

$$= \frac{87.24 \times 10^{-6}\left[\frac{\mathrm{m^3}}{\mathrm{s}}\right] \times 791 \times 10^3\ \left[\frac{\mathrm{N}}{\mathrm{m^2}}\right]}{77.4\ [\mathrm{W}]}$$

$$= 0.8916 = 89.2\%$$

$$Q_{\text{actual}} = \frac{4.79 \times 10^{-3}}{60}\left[\frac{\mathrm{m^3}}{\mathrm{s}}\right]$$

$$= 79.83 \times 10^{-6}\,\frac{\mathrm{m^3}}{\mathrm{s}}$$

$$E_{\text{pump, vol}} = \frac{Q_{\text{actual}}}{Q_{\text{ideal}}} = \frac{79.83}{87.24} = 0.9150 = 91.5\%$$

$$E_{\text{pump, overall}} = E_{\text{pump, mech}} \times E_{\text{pump, vol}}$$

$$= 0.8916 \times 0.9150 = 0.8158 = 81.6\%$$

Chapter 5. Hydraulic System Components

5-1 Size a cylinder to lift 5 tonne through 250 mm. This involves specifying the diameter of the cylinder and specifying its stroke. The force developed is related to the pressure by the expression $p = F/A$. Hence, $A = F/p$. One of the following standard bores should be used: 25, 32, 40, 50, 63, 80, 100, 125, 160, 200, 250, 320 mm.

For our purposes, anything in the range of 70 to 210 bar is acceptable.

Solution

Start with A as the unknown, and no fixed value for p:

$$A = F/p$$

where F = the weight of 5 tonne = 5,000 [kg] × 9.81 [m/s^2] = 49,050 N = 49.05 kN.

In this example, we will try 100 bar.

$$A = \frac{49.05 \times 10^3 \text{ [N]}}{100 \times 10^5 \text{ [Nm}^{-2}\text{]}}$$

$$= 4.905 \times 10^{-3} \text{ m}^2 = 4{,}905 \text{ mm}^2$$

$$D = \sqrt{\frac{4\,A}{\pi}} = \sqrt{\frac{4 \times 4.905 \times 10^{-3} \text{ [m}^2\text{]}}{\pi}}$$

$$= 7.903 \times 10^{-2} \text{ m} = 79.03 \text{ mm}$$

The nearest standard size is 80 mm.

5-2 Size a pump to drive an 80 mm bore cylinder at 100 mm/s.

Solution

The solution is based on $Q = v \times A$.

$$Q = 0.1\ [\mathrm{m/s}] \times \frac{\pi\ 0.08^2}{4} [\mathrm{m}^2] = 5.03 \times 10^{-4}\ [\mathrm{m}^3/\mathrm{s}]$$
$$= 0.503\ \text{litre/s} = 30.2\ \text{litre/min}$$

5-3 What hydraulic power is required for the previous example if the operating pressure is 100 bar?

Solution

The hydraulic power needed is:

$$\text{Power} = p \times Q$$

$$= 100 \times 10^5\ [\mathrm{Nm}^{-2}] \times 5.03 \times 10^{-4}\ [\mathrm{m}^3/\mathrm{s}]$$

$$= 5.03 \times 10^3\ \mathrm{W} = 5\ \mathrm{kW}$$

5-4 In a double-acting single-rod hydraulic cylinder the inside diameter of the cylinder is 120 mm and the rod diameter is 40 mm. Calculate the maximum force that can be developed by the cylinder on the outstroke and on the return stroke if the hydraulic pump can provide a maximum pressure of 8.5 MPa. Note: in each case the force is the product of the supply pressure and the effective area of the piston.

Solution

$$A_{\mathrm{cyl}} = \frac{\pi d_{\mathrm{cyl}}^2}{4} = \frac{\pi \times 0.12^2\ [\mathrm{m}^2]}{4} = 11.31 \times 10^{-3}\ [\mathrm{m}^2]$$

$$A_{\mathrm{rod}} = \frac{\pi d_{\mathrm{rod}}^2}{4} = \frac{\pi \times 0.04^2\ [\mathrm{m}^2]}{4} = 1.257 \times 10^{-3}\ [\mathrm{m}^2]$$

$$F_{\mathrm{outstroke}} = pA_{\mathrm{cyl}}$$

$$= 8.5 \left[\frac{\mathrm{MN}}{\mathrm{m}^2}\right] \times 11.31 \times 10^{-3}\ [\mathrm{m}^2] = 96.14 \times 10^{-3}\ [\mathrm{MN}]$$

$$= 96.14\ \mathrm{kN}$$

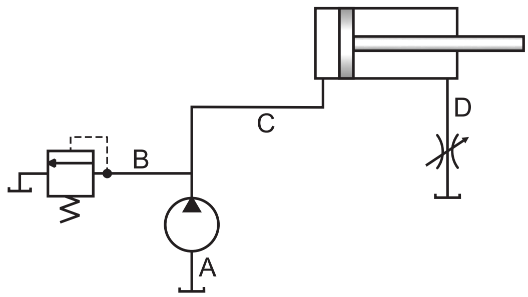
$$F_{\text{return stroke}} = p(A_{\mathrm{cyl}} - \mathrm{A}_{\mathrm{rod}})$$

$$= 8.5 \left[\frac{\mathrm{MN}}{\mathrm{m}^2}\right] \times (11.31 - 1.257) \times 10^{-3}\ [\mathrm{m}^2]$$

$$= 85.45 \times 10^{-3}\ [\text{MN}] = 85.5\ \text{kN}$$

Chapter 6. Design of Hydraulic Systems

6-1



The diagram above shows a hydraulic power transmission system with meter-out flow control. The hydraulic cylinder has a bore of 75 mm, while the diameter of the cylinder rod is 30 mm. The hydraulic pump produces a flow rate of 0.754 m^3/hour. For a given load and setting of the flow restrictor, the volume flow rate out of the cylinder at D is 0.452 m^3/hour. Calculate the volume flow rate at positions A, B and C assuming the pump is ideal. Also, calculate the velocity of the piston.

Solution

The effective area of the piston at the rod end of the cylinder is given by

$$A_{\text{rod end}} = \frac{\pi}{4}\left(d_{\text{cyl}}^2 - d_{\text{rod}}^2\right)$$

$$= \frac{\pi}{4}(0.075^2 - 0.03^2)\ [\text{m}^2]$$

$$= 3.711 \times 10^{-3}\ \text{m}^2$$

The area of the piston at the cap end of the cylinder is given by

$$A_{\text{cap end}} = \frac{\pi}{4} d_{\text{cyl}}^2$$

$$= \frac{\pi}{4} 0.075^2\ [\text{m}^2]$$

$$= 4.418 \times 10^{-3}\ \text{m}^2.$$

The volume flow rate at each end of the cylinder is given by the product of the effective area and the velocity of the piston. Therefore

$$Q \propto A_{\text{effective}}.$$

Hence,

$$\frac{Q_\text{C}}{Q_\text{D}} = \frac{A_{\text{cap end}}}{A_{\text{rod end}}}.$$

Therefore

$$Q_\text{C} = 0.452 \left[\frac{\text{m}^3}{\text{hour}}\right] \times \frac{4.418}{3.711} = 0.538\ \frac{\text{m}^3}{\text{hour}}$$

As the pump is assumed ideal, the suction flow rate equals the discharge flow rate. Therefore

$$Q_\text{A} = 0.754\ \frac{\text{m}^3}{\text{hour}}.$$

$$Q_\text{B} = Q_\text{A} - Q_\text{C} = (0.754 - 0.538)\frac{\text{m}^3}{\text{hour}}$$

$$= 0.216\ \frac{\text{m}^3}{\text{hour}}.$$

The velocity of the piston can be calculated from the flow and area information at either end of the cylinder.

$$v_{\text{piston}} = \frac{Q_\text{C}}{A_{\text{cap end}}} = \frac{0.538\ \left[\frac{\text{m}^3}{\text{hour}}\right]}{4.418 \times 10^{-3}\ [\text{m}^2]}$$

$$= 121.8\ \left[\frac{\text{m}}{\text{hour}}\right] = \frac{121.8}{3600}\ \left[\frac{\text{m}}{\text{s}}\right] = 0.0338\frac{\text{m}}{\text{s}}$$

$$= 33.8\ \text{mm/s}.$$

6-2 A single-acting single-rod hydraulic cylinder is required for a pallet-lifter. The hydraulic ram is required to exert a force of 165 kN and must stroke through a distance of 1.4 m in 3 seconds. One of the following standard bores should be used: 25, 32, 40, 50, 63, 80, 100, 125, 160, 200, 250, 320 mm. The pressure relief valve should be set to a pressure that is 10% higher than that required by the design load. Select an appropriate cylinder bore. Specify the setting of the pressure relief valve and calculate the flow rate required from the hydraulic pump. Start with a trial pressure of 12 MPa. The running pressure should not exceed this value.

Solution

First, to find the required cylinder bore, use $A = F/p$, and the trial pressure of 12 MPa.

$$A = \frac{165 \times 10^3 \text{ [N]}}{12 \times 10^6 \text{ [N m}^{-2}]} = 13.75 \times 10^{-3} \text{ m}^2$$

and

$$D = \sqrt{4\frac{A}{\pi}} = \sqrt{4 \times 13.75 \times \frac{10^{-3}}{\pi}} \text{ [m]}$$

$$= 132.3 \times 10^{-3} \text{ m} = 132.3 \text{ mm.}$$

We must choose a standard size, the nearest ones being 125 and 160 mm.

In order to keep the running pressure below 12 MPa, we choose the larger (160 mm) bore.

Now calculate the running pressure using the standard bore.

$$A_{\text{std}} = \frac{\pi \times D_{\text{std}}^2}{4} = 20.11 \times 10^{-3} \text{ m}^2$$

$$p_{\text{running}} = \frac{F}{A_{\text{std}}} = \frac{165 \times 10^3 \text{ [N]}}{20.11 \times 10^{-3} \text{ [m}^2]}$$

$$= 8.206 \times 10^6 \frac{\text{N}}{\text{m}^2} = 8.21 \text{ MPa}$$

Set the pressure relief valve at a pressure that is 10% higher than the pressure required by the maximum load.

$$p_{\mathrm{rv}} = 8.21\ [\mathrm{MPa}] \times 1.1\ =\ 9.03\ \mathrm{MPa}.$$

Pump size:

$$Q\ =\ A_{\mathrm{std}} \times v\ =\ 20.11 \times 10^{-3}[\mathrm{m}^2]\left(\frac{1.4}{3}\right)[\mathrm{m\ s}^{-1}]$$

$$=\ 9.383 \times 10^{-3}\ \frac{\mathrm{m}^3}{\mathrm{s}} =\ 9.383\ \frac{\mathrm{Iitre}}{\mathrm{s}} =\ 563\ \frac{\mathrm{litre}}{\mathrm{min}}$$

Chapter 7. Viscosity

7-1 Calculate the force required to cause relative motion at a rate of 2 m/s of a flat plate of area 3.6 m^2 with respect to a parallel flat surface where the plate is separated by a distance of 1 mm and where the space between the plate and the surface is filled with oil having an absolute viscosity of 70 centipoise. (1 centipoise = 10^{-2} poise = 10^{-3} Pas). The following formula may be used

$$\tau = \frac{F}{A} = \mu \frac{\Delta v}{\Delta y}$$

Solution

The shear force is given by

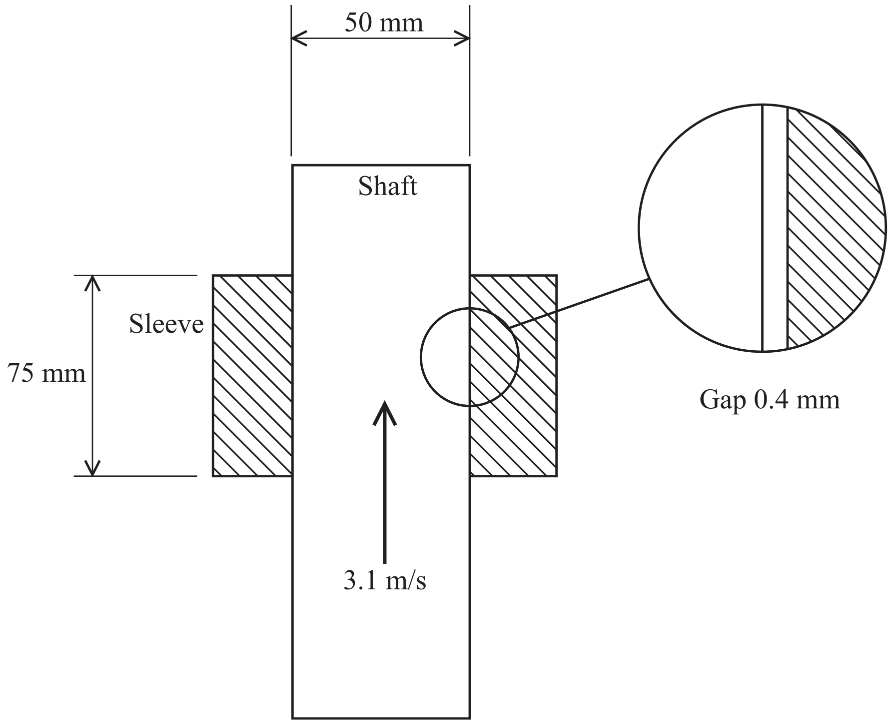
$$F = \tau\, A = \mu \frac{\Delta v}{\Delta y}\, A$$

$$= 70 \times 10^{-3} [\mathrm{N\ m^{-2}\ s}] \frac{2\ [\mathrm{m\ s^{-1}}]}{0.001\ [\mathrm{m}]}\, 3.6\ [\mathrm{m^2}]$$

$$= 504\ \mathrm{N}$$

7-2 A steel shaft of diameter 50 mm moves up and down within a sleeve of height 75 mm. The all-around clearance of 0.4 mm between the shaft and the sleeve is filled with oil having an absolute viscosity of 3.9×10^{-2} Pas. Determine the force exerted on the shaft by the oil in the clearance gap and its direction at an instant when the shaft is moving upwards at the rate of 3.1 m/s.

Solution



Newton's viscosity equation can be written in the form:

$$\tau = \mu \frac{\Delta v}{\Delta y}$$

Hence the shear force is given by

$$F = \tau\, A = \mu \frac{\Delta v}{\Delta y}\, A$$

The area involved is the surface of a cylinder and is calculated as

$$A = \pi\, D\, h = \pi \times 0.05\ [\mathrm{m}] \times 0.075\ [\mathrm{m}] = 0.01178\ \mathrm{m}^2$$

Hence the shear force is given by

$$F = 3.9 \times 10^{-2} [\mathrm{N\ m^{-2}\ s}] \frac{3.1\ [\mathrm{m\ s^{-1}}]}{0.0004\ [\mathrm{m}]} 0.01178\ \ [\mathrm{m}^2]$$

$$= 3.561\ \mathrm{N}$$

The direction in which the force exerted by the oil on the shaft acts is downwards, i.e. opposing the movement of the shaft.

7-3 Engine oil has an absolute viscosity of 4.86×10^{-3} Ns/m^2 and a density of 884.1 kg/m^3 when it is at a temperature of 300 K. Calculate its kinematic viscosity at the same temperature.

Solution

$$\nu = \frac{\mu}{\rho} = \frac{4.86 \times 10^{-3}\ [\mathrm{Ns/m^2}]}{884.1\ [\mathrm{kg/m^3}]}$$

$$= 5.497 \times 10^{-6} \left[\frac{\mathrm{kg\ m\ s^{-2}\ s\ m^{-2}}}{\mathrm{kg\ m^{-3}}}\right]$$

$$= 5.497 \times 10^{-6}\ \mathrm{m^2/s}$$

7-4 Air at 300 °C and at atmospheric pressure has a kinematic viscosity of 15.89×10^{-3} m^2/s and a density of 1.1614 kg/m^3. Calculate its absolute viscosity.

Solution

$$\mu = \nu\rho = 15.89 \times 10^{-3} \left[\frac{\mathrm{m^2}}{\mathrm{s}}\right] \times 1.1614 \left[\frac{\mathrm{kg}}{\mathrm{m^3}}\right]$$

$$= 18.45 \times 10^{-3}\ \mathrm{m^2\ s^{-1}\ kg\ m^{-3}}$$

$$= 18.45 \times 10^{-3}\ \mathrm{kg\ m^{-1}\ s^{-1}}$$

$$= 18.45 \times 10^{-3}\ \mathrm{Pa\ s}$$

Chapter 8. Pressure Losses in Pipes and Fittings

8-1 If water flows in a pipe of 12 mm diameter at the rate of 8 litres per minute what is the Reynolds number?

Solution

$$\rho = 1000\,\frac{\text{kg}}{\text{m}^3}$$

$$d = 0.012\ \text{m}$$

$$\mu = 0.93 \times 10^{-3}\ \text{Pas} = 0.93 \times 10^{-3}\ \text{kg m}^{-1}\ \text{s}^{-1}$$

$$Q = 8\,\frac{\text{L}}{\text{min}} = 8 \times \frac{10^{-3}}{60}\left[\frac{\text{m}^3}{\text{s}}\right] = 1.333 \times 10^{-4}\ \frac{\text{m}^3}{\text{s}}$$

$$A = \frac{\pi d^2}{4} = \frac{\pi \times 0.012^2}{4}\,[\text{m}^2] = 1.131 \times 10^{-4}\ \text{m}^2$$

$$v = \frac{Q}{A}$$

$$= \frac{1.333 \times 10^{-4}\ \left[\frac{\text{m}^3}{\text{s}}\right]}{1.131 \times 10^{-4}\ [\text{m}^2]} = 1.179\ \text{m/s}$$

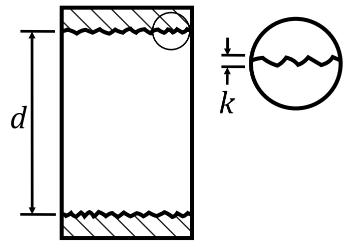
$$\text{Re} = \frac{\rho v d}{\mu}$$

$$= \frac{1000\ \left[\frac{\text{kg}}{\text{m}^3}\right] \times 1.179\left[\frac{\text{m}}{\text{s}}\right] \times 0.012\ [\text{m}]}{0.93 \times 10^{-3}\ [\text{kg m}^{-1}\ \text{s}^{-1}]}$$

$$= 15{,}213$$

8-2 What is the relative roughness of a concrete pipe of 1 m bore if the surface roughness is 1.2 mm?

Solution



$$\frac{k}{d} = \frac{1.2 \times 10^{-3}\ [\text{m}]}{1\ [\text{m}]} = 1.2 \times 10^{-3} = 0.0012$$

8-3 Calculate the friction factor for water flow in a pipe where

a) k/d = 0.01 and Re = 3,740,000

b) the pipe is smooth and Re = 10^6

c) Re = 55,000 and k/d = 0.00006

d) Re = 3,500 and k/d = 0.02

e) Re = 850 and k/d = 0.016

Solution

a) From the friction factor diagram $f = 0.0095$

b) From the friction factor diagram $f = 0.0029$

c) From the friction factor diagram $f = 0.0052$

d) From the friction factor diagram, $f = 0.0145$ if turbulent
or $f = \frac{16}{3{,}500} = 0.0046$ if laminar (unlikely)

e) Flow is laminar and $f = \frac{16}{850} = 0.0188$

8-4 Oil having a density of 840 kg/m^3 and a kinematic viscosity of 118 centistoke flows with an average velocity of 0.9 m/s in a 10 mm pipe with a length of 0.317 m. What is the pressure loss over the length of the pipe and what theoretical pumping power is required? (1 centistoke = 10^{-6} m^2/s).

Solution

$$\mathrm{Re} = \frac{\rho\, v\, d}{\mu} = \frac{v\, d}{\nu}$$

$$= \frac{0.9 \times 0.01}{118 \times 10^{-6}} \left[\frac{\mathrm{m\ s^{-1}\ m}}{\mathrm{m^2\ s^{-1}}}\right] = 76.3$$

As $\mathrm{Re} < 2{,}000$ the flow is laminar and

$$f = \frac{16}{\mathrm{Re}} = \frac{16}{76.3} = 0.2098$$

From the Darcy-Weisbach equation

$$\Delta p = \rho\ g\ h_{\mathrm{f}} = \frac{4fL}{d}\frac{\rho v^2}{2}$$

$$= \frac{4 \times 0.2098 \times 0.317}{0.01} \times \frac{840 \times 0.9^2}{2} \left[\frac{\mathrm{m\,kg\,m^{-3}\ m^2 s^{-2}}}{\mathrm{m}}\right]$$

$$= 9{,}046\ [\mathrm{kg\ m\ s^{-2}\ m^{-2}}] = 9{,}049\ \mathrm{Pa}$$

The theoretical pumping power is given by

$$P = Q\Delta p$$

$$Q = vA = 0.9\ [\mathrm{ms^{-1}}] \times \frac{\pi\ 0.01^2}{4}\ [\mathrm{m^2}]$$

$$= \ 70.69 \times 10^{-6}\ \mathrm{m^3 s^{-1}}$$

Hence

$$P = Q\Delta p = \ 70.69 \times 10^{-6}\ [\mathrm{m^3 s^{-1}}] \times 9{,}049\ [\mathrm{Nm^{-2}}]$$

$$= 0.640\ \mathrm{W}$$

Chapter 9. Valves and Heat Exchanger Flow Paths

9-1 Water is required to flow at a mean velocity of 1.2 m/s in a horizontal cast iron pipe that has a bore of 1.25 m and a length of 0.8 km. What pressure difference is required to provide this flow rate?

Solution

From the friction factor diagram, k = 0.26 mm

$$\frac{k}{d} = \frac{0.26 \times 10^{-3}\ \text{m}}{1.25\ \text{m}} = 0.0002$$

$$\text{Re} = \frac{\rho\, v\, d}{\mu} = \frac{1{,}000 \times 1.2 \times 1.25}{0.93 \times 10^{-3}} \left[\frac{\text{kg m}^{-3}\ \text{m s}^{-1}\ \text{m}}{\text{kg m}^{-1}\text{s}^{-1}}\right]$$

$$= 1.61 \times 10^{6}$$

As Re > 4,000 the flow is turbulent.

The friction factor is found from the friction factor diagram for Re = 1.61×10^6 and k/d = 0.0002 to be 0.0036.

The head loss due to friction in a pipe (Darcy-Weisbach equation) is

$$h_{\text{f}} = \frac{4fL}{d}\frac{v^2}{2g}.$$

The corresponding pressure difference is

$$\Delta p = \rho\ g\ h_{\text{f}} = \frac{4fL}{d}\ \frac{\rho v^2}{2}.$$

Hence

$$\Delta p = \frac{4 \times 0.0036 \times 800}{1.25}\ \frac{1{,}000 \times 1.2^2}{2} \left[\frac{\text{m kg m}^{-3}\ \text{m}^2\ \text{s}^{-2}}{\text{m}}\right]$$

$$= 6{,}636\ [\text{kg m}^{-1}\text{s}^{-2}] = 6{,}636\ [\text{N m}^{-1}\text{s}^{2}\text{m}^{-1}\text{s}^{-2}] = 6{,}636\ \text{Pa}.$$

9-2 Water flows in a 50 mm bore pipe with an average velocity of 1.63 m/s. The pipeline includes a globe valve and the pressure drop across the valve only is measured as 5.94 kPa when it is fully open. What is the head loss coefficient of the fully open valve? The expression for pressure loss in terms of the head loss coefficient is

$$p_{\text{loss}} = \rho g h_{\text{loss}} = \rho g K \frac{v^2}{2g} = K \frac{\rho v^2}{2}.$$

Solution

$$p_{\text{loss}} = K \frac{\rho v^2}{2}$$

Hence,

$$K = \frac{2p_{\text{loss}}}{\rho v^2} = \frac{2 \times 5940\ [\text{Nm}^{-2}]}{1000\ [\text{kg m}^{-3}] \times 1.63^2\ [\text{m}^2\text{s}^{-2}]}.$$

$$= 4.47 \frac{[\text{kg m s}^{-2}\ \text{m}^{-2}]}{[\text{kg m}^{-3}\text{m}^2\text{s}^{-2}]} = 4.47$$

9-3 Water flows at the rate of 31.3 L/min in a pipe (drawn tubing) that has an internal diameter of 12.7 mm. The total length of straight pipe is 15 m and the pipe run includes a heat exchanger with a head loss coefficient of 11.2 and six right-angle bends that each have a head loss coefficient of 0.51. Calculate the overall head loss of the pipe run. Take the density of water as 1,000 kg/m^3 and take its absolute viscosity as 1.002×10^{-3} kg/ms.

$$h_{\text{f}} = \frac{4fL}{d}\frac{v^2}{2g}$$

$$h_{\text{loss}} = K \frac{v^2}{2g}$$

Solution

From the friction factor diagram, k = 0.0015 mm

$$A = \frac{\pi d^2}{4} = \frac{\pi \times 0.0127^2}{4}[\text{m}^2] = 126.7 \times 10^{-6}\ \text{m}^2$$

$$Q = \frac{31.3}{60 \times 1000} \left[\frac{\mathrm{m}^3}{\mathrm{s}}\right] = 521.7 \times 10^{-6} \ \frac{\mathrm{m}^3}{\mathrm{s}}$$

$$v = \frac{Q}{A} = \frac{521.7 \times 10^{-6} \ \left[\frac{\mathrm{m}^3}{\mathrm{s}}\right]}{126.7 \times 10^{-6} \ [\mathrm{m}^2]} = 4.119 \ \frac{\mathrm{m}}{\mathrm{s}}$$

$$\mathrm{Re} = \frac{\rho \, v \, d}{\mu} = \frac{1{,}000 \times 4.119 \times 0.0127}{1.002 \times 10^{-3}} \left[\frac{\mathrm{kg \ m^{-3} \ m \ s^{-1} \ m}}{\mathrm{kg \ m^{-1} s^{-1}}}\right]$$

$$= 52{,}195 = 5.22 \times \ 10^4$$

$$\frac{k}{d} = \frac{0.0015 \ \times 10^{-3} \ \mathrm{m}}{0.0127 \ \mathrm{m}} = 0.000118$$

From the friction factor chart $f = 0.00528$

$$h_\mathrm{f} = \frac{4fL}{d} \frac{v^2}{2g} = \frac{4 \times 0.00528 \times 15}{0.0127} \ \frac{4.119^2}{2 \times 9.81} \ [\mathrm{m}]$$

$$= 21.56 \ \mathrm{m}$$

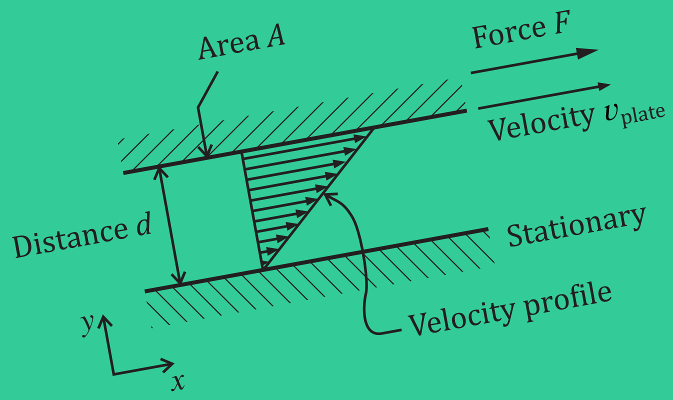
Calculate the total head loss coefficient for minor losses

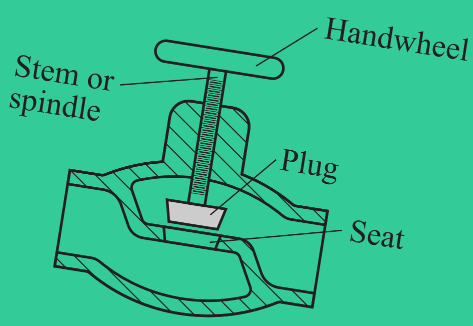
$$K_\mathrm{total} = 11.2 + 6 \times 0.51 = 14.26$$

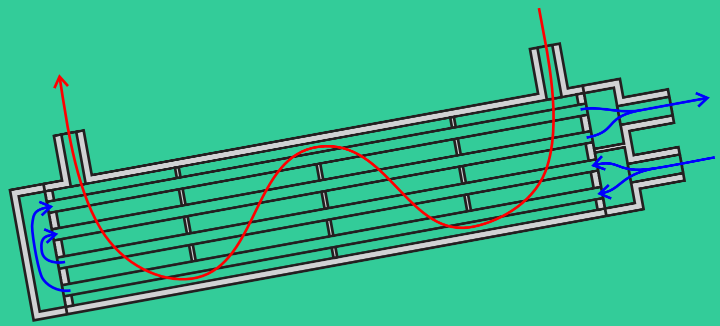
$$h_\mathrm{loss} = K_\mathrm{total} \frac{v^2}{2\mathrm{g}} = 14.26 \times \frac{4.119^2}{2 \times 9.81} \ [\mathrm{m}] = 12.33 \ \mathrm{m}$$

Calculate the total head loss:

$$h_\mathrm{loss,\,overall} = h_\mathrm{f} + h_\mathrm{loss} = (21.56 + 12.33) = 33.9 \ \mathrm{m}$$







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