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An LPC pole processing method for enhancing the identification of dominant spectral features

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An LPC pole processing method for enhancing the identification of dominant spectral features

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This paper proposes a new time-resolved spectral analysis method based on a modification to the linear predictive coding (LPC) method for enhancing the identification of the dominant frequencies of a signal. The method described here is based on a z-plane analysis of the LPC poles. These poles are used to produce a series of reduced order filter transfer functions which can accurately identify and estimate the frequency of the dominant spectral features. The standard LPC method has been shown to suffer from a sensitivity to noise and its performance is dependent on the filter order. The proposed method can accurately identify the dominant frequency components over a range of filter orders and is shown to be robust in the presence of noise. Compared with traditional time-resolved methods, it is a parameterised method where the identification of the dominant frequency changes can be directly obtained in the form of frequency measurements. In a series of 10,000 Monte Carlo experiments on single component and multiple component signals, this LPC pole processing method outperforms the standard LPC method by accurately identifying the dominant frequency components in the signals.

Introduction: Time-resolved spectral methods enable us to study the time-frequency characteristics of signals which exhibit transient oscillatory behaviour. Many spectral analysis methods, such as the short-time Fourier transform and the continuous wavelet transform [1], are waveform methods which identify the dominant frequencies by estimating the complete spectrum at discrete time intervals. These waveform methods are excellent at demonstrating whether a certain frequency component exits or not by showing how the energy of the signal is distributed across the time-frequency domain. LPC is a parameterised spectral analysis method which can directly estimate the dominant frequencies in a signal. To date, researchers have used the LPC poles to estimate the frequencies of the spectral peaks [2–4]. However, not all of the LPC poles correspond to the dominant frequencies in the signal. Furthermore, the standard LPC method suffers from a sensitivity to the filter order used and exhibits a poor tolerance of high noise environments [2, 3]. Specifically, a LPC model with too low a filter order tends to provide a poor spectral separation of frequencies in the frequency domain, while a model with too high an order causes a deterioration in the noise immunity of the spectral estimator by producing a profusion of candidate spectral peaks. Here we propose a LPC-PP method which is based upon a modification to the LPC method which overcomes these short-comings of the LPC method.

The LPC-PP method implements a further processing of the LPC poles estimated by LPC to generate a series of reduced order filter transform functions which can more easily identify and estimate the dominant frequencies of a signal. In a series of Monte Carlo experiments, a pseudo-randomly varying frequency signal is used to analyse the performance of the method where the standard LPC method is used as a comparison method. The experimental results show that: (1) The LPC-PP method can significantly reduce the number of invalid frequency estimates and increase the percentage of the valid frequency estimates; (2) The performance of the LPC-PP method is less sensitive to the filter order; (3) The LPC-PP method has a more robust performance in high noise environments; (4) The LPC-PP method can give more accurate estimates than the LPC method. In summary, the LPC-PP method provides a new parameterisation spectral analysis method for identifying the dominant frequency components in a signal.

LPC method: LPC estimates the parameters that characterise a linear time-varying system [3, 5]. It is based on the assumption that the current signal sample $s(n)$ can be closely approximated as a linear combination of past P samples as follows:

$$\hat{s}(n) = \sum_{i=1}^{P} a_i s(n-i), \tag{1}$$

where the predictor coefficient a_i is determined by minimising the sum of the squared differences between the actual signal samples $s(n)$ and the linearly predicted ones $\hat{s}(n)$, and P is the LPC filter order. In the z-transform domain, a Pth order linear predictor is a system of the form

$$Q(z) = \sum_{i=1}^{P} a_i z^{-i} = \frac{\hat{S}(z)}{S(z)}, \tag{2}$$

where $\hat{S}(z)$ is the output of the filter. The z-transform of the prediction error is written as

$$E(z) = S(z) - \sum_{i=1}^{P} a_i S(z) z^{-i}. \tag{3}$$

The prediction error is the output of a system with the transfer function

$$A(z) = \frac{E(z)}{S(z)} = 1 - Q(z) = 1 - \sum_{i=1}^{P} a_i z^{-i}, \tag{4}$$

where $A(z)$ is an inverse filter of the LPC synthesis filter $H(z)$ and is given by $H(z) = 1/A(z)$. The fundamental theorem of algebra tells us that $A(z)$ has P roots which are the values of z for which $H(z) = \infty$. The roots of $A(z)$ are called the poles of $H(z)$. The poles of $H(z)$ are expressed as $z_i = \gamma_i e^{j\omega_i}$, where $\omega_i = \tan^{-1}(\mathrm{Im}(z_i)/\mathrm{Re}(z_i))$ is the angle associated with the ith pole. The magnitude of a pole is $m_i = |z_i|$ and the corresponding pole frequency is $p_i = \frac{\omega_i}{2\pi T_s}$, where T_s is the sample period. The roots occur as complex conjugate pole pairs which are mirrored in the real axis of the z-plane. The number of poles generated equals P. Here, we consider those poles with non-negative imaginary parts $\mathrm{Im}(z_i) \geq 0$ as the estimation results of the LPC method.

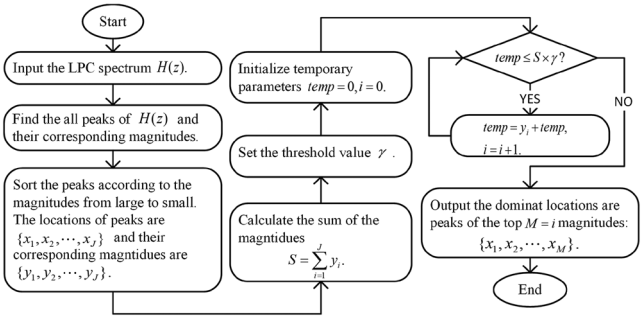
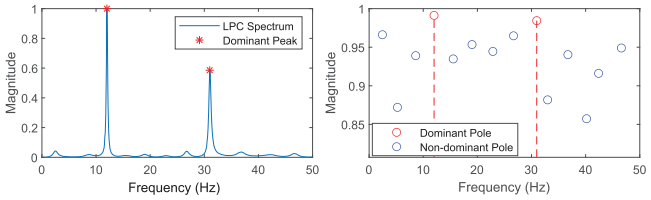


Fig. 1 *Flow chart of the method used to identify the dominant peaks*

Pole processing method: The LPC poles of the filter transfer function $H(z)$ are first categorised into dominant poles and non-dominant poles. The method for identifying the dominant and non-dominant poles is shown in Figure 1.

The parameter γ is a threshold value which has a range from 0 to 1.0, the details for choosing γ will be discussed later. The LPC poles closest to the dominant peaks are marked as the dominant poles $\{d_1, d_2, \ldots, d_M\}$, while the other poles are marked as non-dominant poles $\{\bar{d}_1, \bar{d}_2, \ldots, \bar{d}_W\}$ where M and W are the number of dominant poles and non-dominant poles respectively. Figure 2(a) shows the two dominant peaks that were found and Figure 2(b) shows the classification of the dominant poles and non-dominant poles. The input signal in Figure 2 is composed of two sinusoidal signals with frequencies $f_1 = 12$ Hz, $f_2 = 31$ Hz, the sampling frequency is $Fs = 100$ Hz, the window size is $N = 100$ samples and the signal is corrupted with additive white Gaussian noise (AWGN) where the SNR = 3 dB. The filter order is $P = 30$ and $\gamma = 0.5$.



(a) LPC spectrum. (b) LPC poles.

Fig. 2 *Identifying dominant poles and non-dominant poles*

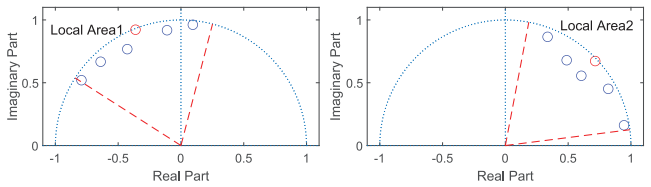


Fig. 3 *Dominant poles and local poles on the z-plane. The red circles represent the dominant poles and the blue circles represent the local poles of each dominant pole within a range of* $\alpha = 10$ *Hz*

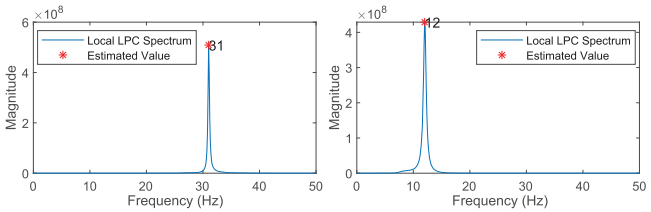


Fig. 4 *Local spectrum. The spectral responses of each of the individual (reduced order) transfer functions* $\tilde{H}(d)$ *and the estimated spectral peaks are* $\hat{p}_1 = 12$ *Hz,* $\hat{p}_2 = 31$ *Hz*

The second step is to use the local pole(s) in conjunction with the dominant pole to determine the final position of the spectral peak. The local poles are selected from the non-dominant poles and they depend on the distance (frequency separation) Δf between the non-dominant poles and the dominant pole. A distance threshold value α is defined to identify the local poles. If the distance $\Delta f < \alpha$, we consider them to be the local poles $\{\widehat{d}_{i1}, \widehat{d}_{i2}, \ldots, \widehat{d}_{iL}\}$ of the ith dominant pole d_i and L is the number of local poles. When the sampling frequency and the filter order are the same, the larger the value of α, the more local poles will be selected. In Figure 3, we chose $\alpha = 10$ Hz where the red lines represent the frequency range 2α around each dominant pole where we can identify the local poles associated with this dominant pole.

In the last step, the dominant poles and their corresponding local pole(s) are used to form a series of reduced order filter transfer functions $\{\tilde{H}_1(d), \tilde{H}_2(d), \ldots, \tilde{H}_M(d)\}$ and each $\tilde{H}_i(d)$ is given by

$$\tilde{H}_i(d) = \frac{1}{(1 - d_i^{-1})} \times \prod_{j=1}^{L} \frac{1}{(1 - \hat{d}_{ij}^{-1})}, \tag{5}$$

where the d_i is the ith dominant pole, the d_{ij} are the associated local pole(s). As the new filter transfer function $\tilde{H}_i(d)$ has a lower order, it has fewer local maxima which makes it easier to find the peaks. The maximum peak $\hat{p}_i$ of the $\tilde{H}_i(d)$ is one estimate of the dominant frequency, estimated by the LPC-PP method. The results of this process are shown in Figure 4.

Experimental design: The simulation experiment uses sinusoidal signals whose frequencies are uniformly distributed in the range 0 to $Fs/2$. The signals are corrupted by the AWGN noise and they are evaluated using 10,000 Monte Carlo trials. In correctly identifying and estimating the frequency components of these simulation signals, there are three questions that need to be considered: (1) How many dominant components in the signal are identified? (2) How many of the frequency estimates of dominant components are valid? (3) How accurate are the frequency estimates of the dominant components? To answer these questions, three performance metrics are proposed. Before defining the three metrics,

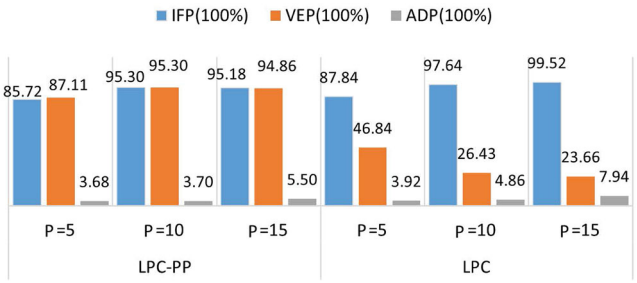


Fig. 5 *The simulation results for a single component test signal*

we first need a threshold value β to determine whether the frequency component of the signal has been correctly identified and whether the estimate generated by the method is valid. The frequency error between the true frequency and the estimated frequency is expressed as Δe. When $\Delta e < \beta$, the frequency of the signal component is considered to be identified and the estimate is valid. The performance metrics are defined as follows: The identification frequency percentage (IFP) is used to indicate the proportion of the identified dominant components. It is defined as

$$\text{IFP} = \frac{\text{total number of identified frequencies}}{\text{total number of all signal frequencies}} \times 100(\%). \tag{6}$$

The valid estimate percentage (VEP) is used to measure the proportion of valid estimates. It is defined as

$$\text{VEP} = \frac{\text{the number of valid estimates}}{\text{the number of all estimates}} \times 100(\%). \tag{7}$$

The relative deviation percentage (RDP) measures the relative error between an identified frequency and its corresponding valid estimate. It is defined as $\text{RDP}_i = \Delta e_i / f_i$, where f_i is the signal frequency. The average deviation percentage (ADP) is defined as the average of the RDP values which is defined as

$$\text{ADP} = \frac{\sum_{i=1}^{C} \text{RDP}_i}{C} \times 100(\%), \tag{8}$$

where C is the number of the identified signal frequency.

Numerical evaluation: The first experiment demonstrates a simple scenario of a single frequency component signal in a high noise environment where the SNR = 3 dB. The sampling frequency $Fs = 100$ Hz and the window size is N =20 samples. The parameters of the LPC-PP method are $\gamma = 0.3$ and $\alpha = 10$ Hz. The value of the threshold parameter $\beta = 2.5$ Hz. Figure 5 shows the results of the three metrics. For the different filter orders considered, the IFP values of the LPC-PP method and the LPC method are all above 85%. Consequently both methods can identify the dominant frequency component in most of the experiments. The VEP values of the LPC-PP method are greater than 87% and are much greater than the VEP values of the LPC method for different filter orders. Specifically, when the filter order $P = 15$, the LPC method can identify 99.52% of the signal frequencies, but the VEP value of the LPC method is only 23.66%. This is because the LPC method achieves high IFP values while also producing many invalid estimates. For the same filter order, the ADP value of the LPC-PP method is less than that of the LPC method, so the LPC-PP method is more accurate than the LPC method in this experiment.

The second simulation experiment is an analysis of the performance using a multiple components signal where the signal comprises three frequencies which are produced from a random uniform distribution in the range of 0 to 50 Hz. The threshold value of the LPC-PP method is $\gamma = 0.85$. The other parameters are the same as for the last experiment. The results for the three metrics are shown in Figure 6. For the same filter order, the IFP value of the LPC-PP is slightly lower than that of the LPC, but the VEP value of LPC-PP is still much higher than for LPC. When the filter order is 10 and 15, the VEP value of the LPC-PP method is 27% greater than the VEP value of the LPC. This feature of the LPC-PP method is particularly attractive for scenarios where the number

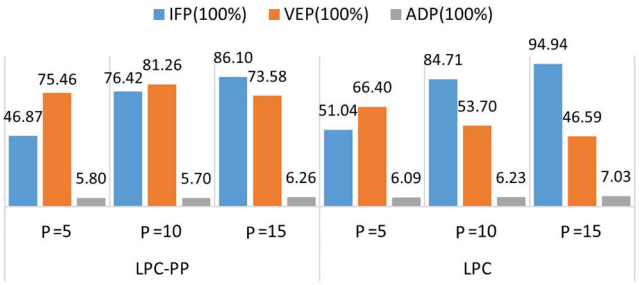


Fig. 6 *The simulation results for a multiple component test signal*

Table 1. The number of experiments with all-valid estimates

Signal type	Single-component signal		Multi-components signal	
Filter order	LPC-PP	LPC	LPC-PP	LPC
$P = 5$	8572	670	36	43
$P = 10$	9530	0	4671	330
$P = 15$	9509	0	4178	9

of frequency components in the signal is unknown. Similarly, the ADP values of the LPC-PP method are less than that of the LPC method. The LPC-PP method produces more accurate frequency estimates.

Although the IFP value of the LPC-PP method is slightly lower than that of the LPC method in the above experiments, the LPC-PP method improves the VEP value and reduces the number of invalid estimates. For different filter orders, LPC-PP is less sensitive to the filter order than LPC and it has a more robust performance. For the two experiments above, we also analysed the number of experiments which had all-valid estimates in the Monte Carlo experiments, i.e. where all the signal frequencies were identified without additional estimates generated, in the Table 1. In the single component signal analysis, the number of experiments with all-valid estimates for the LPC-PP method was much greater than that of LPC for the same filter order. This result shows LPC is adversely affected by the filter order. When the filter order is increased, the LPC method will produce invalid estimates. This is the reason why the LPC method produces no experiment results with all-valid estimates when the filter order is 10 and 15. In the multiple component signal analysis, when the filter order is $P = 5$, the number of experiments with all-valid estimates for both methods is low. This is because the filter order is too small to produce sufficient numbers of poles to estimate the three frequency components. It is worth noting that the LPC has more experiments with all-valid estimates when the filter order is 10 than when the filter order is 5 or 15. This is because the LPC method is sensitive to the filter order. When LPC has an appropriate filter order, it exhibits a better performance. But its performance is still worse than the LPC-PP method.

The selection of the threshold γ value in the LPC-PP is analysed in the Figure 7. The IFP value increases with an increase in γ. Conversely, the VEP value decreases when γ decreases. There is a trade-off between the IFP and the VEP in the LPC-PP method. The γ value is used to achieve an acceptable trade-off between the two metrics. The ADP value shows little change when γ increases. When the filter order is 10 and 15, the number of experiments with all-valid estimates has a maximum at the intersection of the IFP and VEP curves. The γ value corresponding to the intersection point of the IFP and VEP curves is optimal for LPC-PP in order to produce the maximum number of experiments with all-valid estimates. The IFP and VEP curves do not intersect in Figure 7(a), the reason is that the filter order is too low (P=5) to produce sufficient poles to estimate the components. Therefore, the greater the intensity of the noise and the filter order, the smaller the value of γ required to filter out the peaks caused by the noise and the redundant poles. Similarly, when the number of frequencies that is needed to be identified increases, the value of γ needs to increase in order to find more dominant peaks. Accordingly, the γ value used is different for the single component signal analysis and the multiple component signal case.

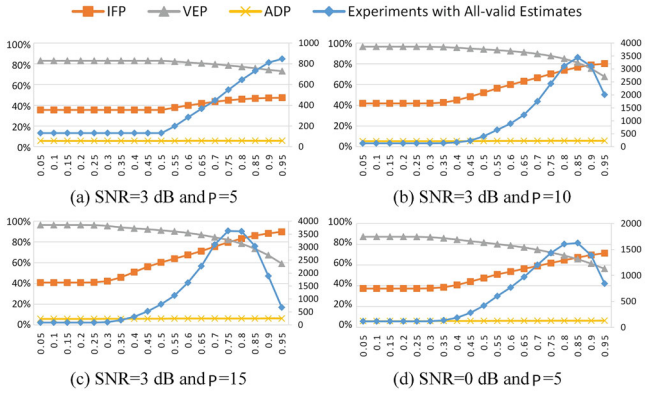


Fig. 7 *Performance analysis of LPC-PP method under different threshold values γ. The x-axis is the threshold γ. The left* y*-axis is for IFP, VEP and ADP. The right* y*-axis represents the number of experiments with all-valid estimates*

Conclusion: This paper has proposed a modification to the standard LPC method that overcomes a number of its short-comings, specifically its sensitivity to the filter order and its poor performance in noisy environments. The LPC-PP method is based upon a further processing of the poles generated by LPC method in order to produce a series of reduced order filter transfer functions that enhances the identification of the dominant spectral features. The majority of the frequency estimates of the LPC-PP method are valid and are more accurate than the estimates of the LPC method. This is particularly useful for applications where it is required to identify the dominant frequencies of an unknown signal. The LPC-PP method can identify the dominant frequencies without being adversely affected by the filter order and has a high tolerance of noise on the signal. Furthermore, the LPC-PP method is a parameterised method that can identify the dominant frequencies and produce numerical frequency estimates. As the LPC-PP method requires further processing of the LPC poles, it has a slightly higher computational cost compared to the standard LPC method. The extra computational complexity of LPC-PP is $O(P \times N \log N)$. We believe that the additional computational effort is worthwhile compared to the significant improvement in the accuracy of the frequency estimates.

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