2016

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Gavin Duffy
Technological University Dublin, gavin.duffy@tudublin.ie

Sheryl Sorby
Ohio State University

Steven Nozaki
Ohio State University

See next page for additional authors

Funder: National Science Foundation

Authors

Gavin Duffy, Sheryl Sorby, Steven Nozaki, and Brian Bowe

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Gavin Duffy
School of Electrical and Electronic Engineering
Dublin Institute of Technology
Dublin, Ireland
gavin.duffy@dit.ie

Steven Nozaki
Dept of Engineering Education
Ohio State University
Columbus, Ohio

Sheryl Sorby
Dept of Engineering Education
Ohio State University
Columbus, Ohio

Brian Bowe
College of Engineering & Built Environment
Dublin Institute of Technology
Dublin, Ireland

***Abstract*— While spatial aptitude is acknowledged as a key cognitive ability that accompanies success in STEM education, less is reported about the qualitative differences between weak and strong visualisers in how they approach and engage with assessments in STEM education. In this paper, we study one particular aspect of the STEM curriculum – solving convergent 'word' problems in mathematics – in an attempt to discern quantitative and qualitative differences between the approaches weak and strong visualisers adopt when solving these problems. The paper is a work-in-progress that started with a search for suitable convergent mathematics problems which were then presented to a small sample of engineering students using a think aloud protocol. Participants were asked to think aloud while they solved the problems and to write their answers using a LiveScribe pen to concurrently record spoken and written responses. They also completed a spatial skills test. The magnitude and significance of the correlation between the spatial and mathematics tests scores were measured to be $r = .79$, $p < .01$.**

Keywords—spatial skills, problem solving

I. Introduction

The prominence of spatial skills in the cognitive ability profile of the most successful Science, Technology, Engineering and Mathematics (STEM) students was revealed in a longitudinal study of a very large sample (n = 400,000) of high school students in the US [1]. Psychometric data, including spatial skills and mechanical reasoning, were collected while the participants were in grades 9 through 12 in the 1960s. Their education and career choices were checked 11 years later. Those who successfully completed STEM education came from the above average in spatial ability group while those who graduated from Humanities and Social Sciences (HSS) had lower than average spatial skills in high school with these two groups being separated by approximately 1 standard deviation on the spatial ability measure. The likelihood of earning an advanced degree in STEM education emerged as a function of spatial ability in this study, i.e. the average high school spatial test scores were higher for those gaining a Masters qualification than Bachelors, and higher for PhD than Masters. Those who have developed strong spatial skills before commencing STEM higher education courses are well positioned to perform well in their studies.

Not all freshman engineering students can be assumed to have good spatial skills, however. Up to 20 % of incoming first year engineering students were identified as 'weak visualizers' using the Purdue Spatial Visualization Test: Rotations (PSVT:R) in two studies at separate universities [2], [3]. Given the widely studied and reported gender gap in spatial skills [4], [5] it can be expected that women will be over represented in the group of first year engineering students with poor spatial skills. Indeed, [3] found consistently over a 13 year period that a significantly higher percentage of women 'fail' the PSVT:R than men. Compared to those with strong spatial skills, weak visualizers have been shown to have lower retention rates and lower Grade Point Average (GPA) [6].

Performance in several components of the STEM curriculum has been compared to spatial skills test scores. Reference [7] provides a meta-analysis of studies in which mathematical ability was compared with spatial ability and computed average correlations ranging from .35 to .47. The highest correlation found ($r = .67$) was between a mental rotation spatial test and a reasoning math test. In physics, low to moderate but significant correlations ($r \approx .3$) have been observed between scores on Newtonian mechanics concepts tests and spatial test scores e.g. [8], [9]. Reference [10] found that weak visualizers were more likely to interpret graphs of acceleration versus time incorrectly and to have difficulty resolving relative motion from two different frames of reference. The ability to comprehend open and closed electric circuits, as measured by an electric circuits concept test called DIRECT [11], has been shown to correlate moderately ($r \approx .5$)

and significantly with scores on several different spatial tests [12], [13]. Reference [14] found that high spatial ability chemistry students had an advantage when mentally manipulating 2-D representations of molecules and when problem solving skills were required. These were not mathematical problems. When rote memory or simple algorithms were required there was no difference in the groups. High spatial ability students were much more likely to sketch the structure of a molecule when answering questions whereas low spatial ability students often made errors when they did attempt to sketch a structure. Components of the STEM curriculum that appear to draw heavily on spatial thinking include reasoning tasks in mathematics, Newtonian mechanics, physical aspects of simple electric circuits and solving chemistry problems.

Therefore, the literature suggests that a common thread among these observations is the ability to solve novel problems, i.e. ones not practiced in the curriculum. A physics concepts test, such as the Force and Motion Conceptual Evaluation (FMCE) [15], contains questions that belong to the category of convergent problem solving, i.e. to the student participant taking the test, the question must first be comprehended and represented before proceeding to solve it. They are convergent in that there is only one correct answer to be selected from the multiple choices. Likewise, questions on DIRECT appear as challenging problems rather than routine computational activities with average scores on DIRECT measured to be approximately 40 % in samples of first year students [11], [16], i.e. it is not an easy test to take. Finally, in the context of chemistry education [17, p. 26] state that "*tests of spatial ability correlated best with the students' performance on novel problems, rather than routine exercises*".

Problem solving is a catch all phrase that can mean many things. A common approach to categorizing problem types is to place them on a spectrum from well- to ill-structured e.g. [18], [19]. References [19], [20] proposed two modes of thinking related to problem solving – convergent thinking which leads towards a single correct answer and divergent thinking in which one displays fluency, flexibility and originality to develop several alternative solution paths. Engineering education contains both convergent and divergent problems. Examples of the former include well structured text book, homework and in class problems, questions on closed book written examinations and so on. Design tasks are an example of the latter, particularly when they are open ended and have not been addressed before, e.g. if drawn from industry or the community. In this paper we discuss how we have started to examine the relationship between spatial skills and approaches to solving convergent problems.

There are several models of the problem solving process [24]. While these often vary in detail a theme that is common to many is that problem solving consists of a schema development stage in which the problem is represented which is then followed by the application of discipline, procedural knowledge such as core competencies related to the subject the problem is framed within. We intend to focus on how (if at all) schema development is related to spatial ability in solving convergent problems in STEM subjects.

II. Research Design

We developed a trial set of 'word' problems in mathematics by searching for short but challenging convergent problems that required a level of prior knowledge that any engineering student from freshman to senior level could be assumed to have. We assembled 15 problems to be presented sequentially to participants in order of increasing level of difficulty. To give an example, one problem asked how much the level of water in a barrel would increase by if it rained on a roof that drained into the barrel (dimensions are provided). Another required the participant to represent the words in the problem as a quadratic equation whose roots could then be obtained through factoring. The mathematical prior knowledge to solve the problems included equations for area of a circle, volume of a prism and a cylinder, solving two simultaneous linear equations and factoring a quadratic equation.

The 13 students who volunteered to participate in this study were all members of the School of XXX in University XXX and were remunerated for their efforts with a gift card of small value. Ethical approval for the study was obtained in advance. There were two interviewers who worked separately. Each individual interview lasted 90 minutes and started with an introduction to the project and completion of an informed consent form (10 mins). Next, each participant was asked to take the Mental Cutting Test (MCT) [22]. This is a paper and pencil test containing 25 multiple choice questions in which a 3-D object and cutting plan are presented and the participant must visualise how the shape is cut in two and select from the five options the cut face created. 20 minutes were allowed to complete this test. Finally, the participants were asked to attempt the series of mathematics problems and, while doing so, to articulate their thoughts and decisions as much as possible. They were also asked to use a LiveScribe pen to concurrently record spoken and written responses. A calculator was not allowed.

III. Results

The number of problems attempted by each participant ranged from 7 to 12. Expressed as a percentage of attempted problems, the average score was 56 % with a range of 11 to 100 %. The large variation in success rates on the problems matched our desire to have discriminating problems. Likewise, the average MCT score was 15.1 (60.4 %) with a range of 6 to 24 (24 to 96 %). A Pearson correlation coefficient for scores on the MCT and the mathematics test was calculated to be $r = .79$, $p < .01$ (2-tailed). While this is regarded as a 'large' correlation, the sample size is very small in this case and even though the significance value indicates a low probability of this occurring by chance, it would be important to collect more data from a larger sample before accepting this finding. In addition, there was variation in the selection and number of questions each student answered. We used 15 different problems but only 4 of these were answered by every participant with 5 more being answered by at least 11 of the 13 participants. As it stands, this result indicates that 63 % of the variation in math scores is shared with spatial ability.

The think aloud protocol using the LiveScribe pen produced concurrently recorded spoken and written responses. The workbooks were examined for the use of sketching and the quality of sketches produced. The number of sketches produced was moderately correlated (r = .42, N.S.) with the score on the math test and more highly correlated (r = .57, $p < .05$) with score on the MCT although the quality of the sketches did not seem to follow any pattern with respect to spatial ability or math score. The audio recording mostly contained a verbatim description of what was being written. At times of difficulty, a participant would often go silent. In one case, while solving a problem related to solution concentration that required the development of two simultaneous equations, a participant reached an impasse, went silent for a while, and then appeared to have a moment of insight before successfully completing the problem. When prompted to reveal what had come to her mind she described how she remembered doing similar problems in a chemistry course and how this illuminated a solution path.

IV. CONCLUSIONS & FUTURE WORK

The large and highly significant correlation we measured between spatial test and math scores is worthy of further investigation. It is higher than the highest correlation [7] found when doing her meta-analysis of correlations between math and spatial test scores. We intend to tidy up the math test by selecting a reduced set of six problems that can be completed in 30 mins so that an entire class group can be administered a spatial test and the math test in one hour. We are also considering the inclusion of simple mathematical questions to test prior knowledge of factoring a quadratic, solving simultaneous equations, equations for area of a circle and volume of a prism and a cylinder so we can identify participants who lack this knowledge.

Further work is also planned in qualitatively analyzing the think aloud data to search for qualitatively different approaches to problem solving. A phenomenographic framework is proposed in order to discern, within the sample, the outcome space describing the variation in approach to solving problems of this nature. Once this is established, participants can then be assigned to one or more categories of approach and it can then be checked if there is a pattern connecting approach and spatial ability.

Phenomenography has been used by others to examine approaches to problem solving. For example, in a study of physics students' approaches to solving problems related to Newtonian mechanics [23], several qualitatively different approaches to problem solving were identified ranging from 'no clear approach' at one end to a 'scientific approach' at the other. An analysis of the transcript excerpts provided in this paper shows that those taking the scientific approach demonstrated the development of a schema or representation of the problem which was then followed while those in the lower categories, e.g. 'plug and chug', failed to demonstrate such a schema. By using a similar research method, i.e. phenomenographic analysis of think aloud transcripts, we hope to develop a set of categories of approaches to solving problems but also collect spatial ability data so that we can then look for possible connections between these two measurements.

ACKNOWLEDGEMENT

The authors gratefully acknowledge the support of the National Science Foundation for this work through grant number DRL-1535356.

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