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Reply to "comment on 'Cyclic universe with an inflationary phase from a cosmological model with real gas quintessence'"

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Reply to "Comment on 'Cyclic universe with an inflationary phase from a cosmological model with real gas quintessence' " by John Miritzis

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In reply to the Comment [1] concerning our previous paper [2], the following applies.

1. An alternative proof of the stability of point Q is given in [3] whose arXiv version from 2016 precedes by far the submission and the publication of [1]. The proof is based on the fact that the system under consideration has a conserved quantity (first integral) $I(H, \rho)$. The function $I(H, \rho)$ has a minimum at Q and thus the curve $I(H, \rho) =$ constant represents closed orbits in the phase space. In fact, it has been shown in [3] that the dynamical system is Hamiltonian with a Hamiltonian given by the conserved quantity in proper variables — so that the theory of Hamiltonian dynamical systems is applicable in this case.

2. The words "attract trajectories" used for the explanation of Fig. 2 of [2] were not carefully chosen indeed; it should have been "deflect trajectories" instead. The introduction of the δ term gives, effectively, the correct explanation of the behaviour — the part of the dipole in the upper half plane attracts the trajectories from the upper half plane, while the one in the lower half plane is a repelor. Additional features of the behaviour around the origin are reported in [3]. There are special curves that can reach the origin (parabola $\rho = 3H^2$ and the line $\rho = 0$) representing the so called *second integrals.* Explicit solutions on these curves are provided — see (80) and (82) which show that the origin is reachable for an infinite time from the upper half plane (and for time $-\infty$ from the lower half plane).

The Van der Waals model has yet another interesting feature regarding the behaviour at the origin: closed homoclinic orbit passing through $(0, 0)$

— see Fig. 3 of [3] and the relevant explanations. The analysis again involves the conserved quantity (first integral) and Hamiltonian J.

3. The example described by system (3) of [1] is not completely adequate for the cosmological model of [2], since the system (3) of [1] is not Hamiltonian while the model in [2] is.

References

[1] John Miritzis, Phys. Rev. D 95, 128301 (2017)

[2] Rossen I. Ivanov and Emil M. Prodanov, Phys. Rev. D 86, 083536 (2012).

[3] Rossen I. Ivanov and Emil M. Prodanov, Hamiltonian Dynamics of Cosmological Quintessence Models, arXiv:1608.05732 [hep-th], Nonlinear Analysis: Real World Applications (2017) in press