2022

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Recommended Citation

Xu, J., Davis, M. and de Fréin, R. (2022), Dominant frequency component tracking of noisy time-varying signals using the linear predictive coding pole processing method. Electron. Lett., 58: 79-81. DOI: 10.1049/ell2.12362

Funder: Technological University Dublin; Science Foundation Ireland

Dominant frequency component tracking of noisy time-varying signals using the linear predictive coding pole processing method

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The linear predictive coding pole processing (LPCPP) method proposed in our previous work overcomes the shortcomings of the LPC method, especially its sensitivity to noise and the filter order. The LPCPP method is a parameterised method that involves processing the LPC poles to produce a series of reduced-order filter transfer functions to estimate the dominant frequency components of a signal. This paper analyses the ability of the LPCPP method to track the frequency changes of noisy, time-varying signals in real-time. Linear chirped frequency modulation signals are used in a series of experiments to simulate signals with different rates of frequency change. The results show that the LPCPP method can achieve real-time tracking of the dominant frequency in the signal and outperforms the LPC method under different frequency change rates and different noise levels. Specifically, the valid estimate percentage of LPCPP is up to 41.3% higher than that of LPC which indicates that the LPCPP method significantly improves the validity of frequency estimates.

Introduction: The objective of frequency tracking is to estimate the dominant frequency of a signal within a short time interval. This technique is a useful tool in many practical applications, such as recognising users' emotions by analysing the frequency characteristics of EEG signals in real-time [1], diagnosing engine failures by analysing the engine sounds [2], controlling and protecting power equipment by tracking the frequency changes of the power system [3]. Time-frequency analysis methods provide a way to estimate the signal frequency components and reveal their time-varying features. Many time-frequency methods are waveform methods, such as the short-time Fourier transform and the continuous wavelet transform [4]. These methods are excellent at demonstrating whether a certain frequency component exists or not by showing how the energy of the signal is distributed across the time-frequency domain. The linear predictive coding (LPC) method provides a parameterised time-frequency method that can give us frequency estimates. The LPC method has been widely used for encoding signals and it has been influential in the field of speech coding for the past 40 years [5]. Recently, the LPC method has been applied to EEG signal analysis [6] and data transmission [7]. In our previous work [8], a new parameterised time-frequency method called linear predictive coding pole processing (LPCPP) was proposed which is based on a z-plane analysis of the LPC poles. It uses the LPC poles to produce a series of reduced-order filter transform functions which can overcome the shortcomings of LPC, specifically its sensitivity to noise and its filter order [6, 8–10]. The LPCPP method can identify the dominant frequency of single-component and multi-component signals. A detailed description of the LPCPP method can be found in our previous work [8].

In this paper, the ability of the LPCPP method to track the dominant frequency changes in real-time is further analysed. The term "real-time" in this paper means that the LPC-based methods (i.e. LPCPP method and LPC method) can estimate the frequency at every sampling instant. Linear chirped frequency modulation (LCFM) signals are used to facilitate the analysis of the frequency tracking performance of the LPCPP method under different rates of frequency change. Furthermore, four performance metrics are proposed to analyse the frequency tracking performance and the LPC method is used as a benchmark method. The experimental results show that: (1) The LPCPP method is a new parameterised time-frequency method that can achieve real-time dominant frequency tracking in a noisy, time-varying signal; (2) The LPCPP method has a greater ability to track the dominant frequency changes than LPC under different frequency change rates; (3) The LPCPP method tracks the dominant frequency changes, under different noise levels, in a robust way.

Method Introduction: LPC is a method for encoding a sampled signal at time n using a linear function of the past samples of the signal, i.e. $\hat{s}(n) = \sum_{i=1}^{P} a_i s(n-i)$ where P is the LPC filter order. The coefficients, a_i, are determined by minimising the sum of squared differences between the actual signal $s(n)$ and the linearly predicted ones $\hat{s}(n)$. The LPC synthesis filter $H(z)$ given by

$$H(z) = \frac{1}{1 - \sum_{i=1}^{P} a_i z^{-i}} \tag{1}$$

The fundamental theorem of algebra tells us that $H(z)$ has P complex poles which are the values of z for which $H(z) = \infty$. The poles occur in complex conjugate pairs which are mirrored in the z-plane. Here, the poles with non-negative imaginary parts $\mathrm{Im}(z_i) \geq 0$ are considered as the outputs of LPC method [11]. The frequencies estimated by LPC are $\{\hat{f}_1, \hat{f}_2, \ldots\}$.

The LPCPP method is based on a z-plane analysis which processes the LPC poles to produce a series of reduced-order transfer functions in order to obtain a more accurate and more robust frequency estimate. The LPCPP method is comprised of three steps.

In the first step, the LPC poles are categorised into dominant poles and non-dominant poles based upon an analysis of the spectral peaks in the LPC spectrum. A threshold value γ is used to identify the dominant peaks in the LPC spectrum. The LPC poles closest to the dominant peaks are marked as the dominant poles d_k where $k = 1, 2, \ldots, M$ and M is the number of the dominant poles. The rest of the poles are marked as non-dominant poles. It is to be noted that the LPC poles here still only consider the poles with non-negative imaginary parts $\mathrm{Im}(z_i) \geq 0$.

The second step is to identify the local poles associated with each dominant pole. The non-dominant poles located around the dominant poles will determine the location of the spectrum peaks and these poles are called local poles. A parameter α is defined. When a non-dominant pole is less than α Hz away from a dominant pole, it is considered to be an associated local pole $\tilde{d}_{kj}$, $j = 1, 2, \ldots, L_k$ of the kth dominant pole d_k. The number of local poles for the kth dominant pole is L_k. It should be noted that the different dominant poles may share common local poles.

In the third step, the dominant poles and their corresponding local pole(s) are used to form a series of reduced order transform functions $\tilde{H}_k$ given by

$$\tilde{H}_k(z) = (d_k - z^{-1}) \times \prod_{j=1}^{L_k} (\tilde{d}_{kj} - z^{-1}) \tag{2}$$

As the new filter transfer function $\tilde{H}_k$ has a lower order and a single dominant pole, it is easier to determine the spectral peak. The peak $\tilde{f}_k$ of $\tilde{H}_k$ is the kth estimation frequency of the LPCPP method. So the frequency estimation results of the LPCPP method are $\{\tilde{f}_1, \tilde{f}_2, \ldots\}$.

Experimental set-up: The linear chirped frequency modulation (LCFM) signal has an instantaneous frequency sweep $f(t)$ given by $f(t) = f(0) + \delta t$. The ratio $\delta = \Delta f / \Delta t$ represents the frequency change rate where Δf represents the frequency change over the interval Δt. The frequency of the LCFM signal increases linearly with time. additive white Gaussian noise (AWGN) is added to the signal. The spectral components of AWGN are uniformly distributed and have an equal intensity at all frequencies.

The instantaneous frequency at a time t is estimated from a narrow sample window of the signal which is composed of w samples on either side of the time instant t and the window size for each instantaneous frequency estimation is $(2w + 1)$ samples. A diagram of the sample window is shown in Figure 1(a). As the instantaneous frequency $f(t)$ of the LCFM signal is known, the frequency error e can be defined as the absolute value of the difference between the instantaneous frequency and the estimated frequency of the LPC-based method. A threshold value β is defined, when the frequency error e is less than β, we consider the estimated value to be valid and the real frequency is considered to

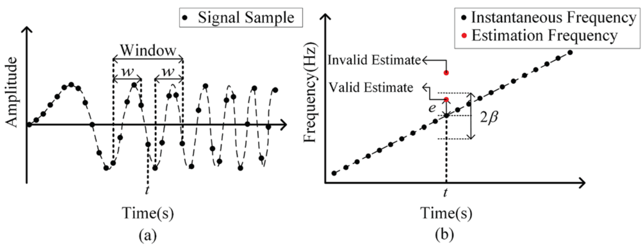

Fig. 1 *The sampling window and the frequency estimation acceptance criterion*

Table 1. The measurement parameters

Parameters	**Symbol**
The number of all instantaneous frequencies	N_χ
The number of identified frequencies	N_ψ
The number of all frequency estimates	N_ϕ
The number of valid frequency estimates	N_φ
The number of ideal experiments	N_τ

Table 2. The four metrics for LPC and LPCPP methods

Method	**AEP (100%)**	**VEP (100%)**	**IFP (100%)**	**IEP (100%)**
LPC	0.77	45.37	85.76	7.87
LPCPP	0.76	84.63	85.91	84.60

be correctly identified as shown in Figure 1(b). In attempting to analyse the frequency estimation performance of the LPCPP method there are some questions that need to be considered. (1) What is the average error between the estimated frequency and the instantaneous frequency? (2) How many estimated frequencies are valid? (3) How many instantaneous frequencies are identified? To answer these questions four measurement parameters are proposed as shown in Table 1.

The average error percentage (AEP) is the average of the relative errors across all the identified frequencies and their corresponding valid estimates. The relative error at time t is defined as $\mathrm{ADP}_i = e_i/f(t)$ and the AEP is expressed as the percentage $\mathrm{AEP} = \sum_{i=1}^{N_\psi} \mathrm{ADP}_i/N_\chi \times 100\%$. The valid estimate percentage (VEP) is the proportion of valid estimates from all the estimates and is defined as the percentage $\mathrm{VEP} = N_\varphi/N_\phi \times 100\%$. The identification frequency percentage (IFP) is the percentage of the number of identified frequencies from all the instantaneous frequencies of the LCFM signal. The IFP is defined as the percentage $\mathrm{IFP} = N_\psi/N_\chi \times 100\%$. The ideal experiment percentage (IEP) is the percentage of experiments that have no invalid estimates and is defined as the percentage $\mathrm{IEP} = N_\tau/N_\chi \times 100\%$.

Results: The first experiment demonstrates the simple scenario of an LCFM signal with a signal-to-noise ratio (SNR) of 10 dB where the frequency of the signal changes from 100 to 400 Hz and the duration of the signal is 2 s (i.e. $\delta = 150$ Hz s^{-1}). The sampling frequency of the LCFM signal is Fs =1000 Hz and the window size is 21 samples. The filter order is P=5 for both LPC and LPCPP methods. For the LPCPP method, the parameters are $\gamma = 0.55$ and $\alpha = 10$ Hz. The threshold value is set to β=4 Hz. The real-time frequency estimation of the LCFM signal by the LPC-based methods is shown in Figure 2 and the values of the four metrics are shown in Table 2. The VEP and IEP values of LPCPP are significantly improved compared to LPC and there is little difference between the IFP and AEP values of the two methods. In other words, LPCPP can produce more accurate and fewer invalid dominant frequency estimates over time. Not all the estimates from the LPC method correspond to the dominant frequency. The LPCPP method achieves real-time tracking of the dominant frequency changes and it can significantly reduce the generation of invalid estimates.

The second experiment demonstrates the performance of the LPC and LPCPP methods for LCFM signals with different frequency change rates (i.e. δ values). The duration of all the LCFM signals is 2 s, the sampling frequency is Fs =1000 Hz and the signals are corrupted by AWGN where SNR = 10 dB. The start and end frequencies corresponding to the different rates δ are detailed in Table 3. The other experimental parameters are the same as the above experiment. Figure 3 presents the results of the four metrics. The AEP value of LPCPP is slightly smaller than that of LPC at the same δ, so the LPCPP method can produce more accurate frequency estimates. The VEP values of both methods decrease when the rate of frequency changes δ increases. However, the VEP of LPCPP is always much higher than LPC under the same δ value. Specifically, the VEP value of LPCPP is up to 41.3% higher than that of the LPC method which indicates that the LPCPP method improves the validity of estimates. In the IFP analysis, the values of the two methods decrease with an increase in δ and the IFP of the two methods are similar under different rates. The VEP value of LPCPP is greatly improved compared with LPC and there is little difference between the IFP of LPCPP and LPC. The IEP of LPCPP is up to 69.52% higher than LPC and the highest IEP is 84.10%. This shows that LPCPP has a greater ability to identify and

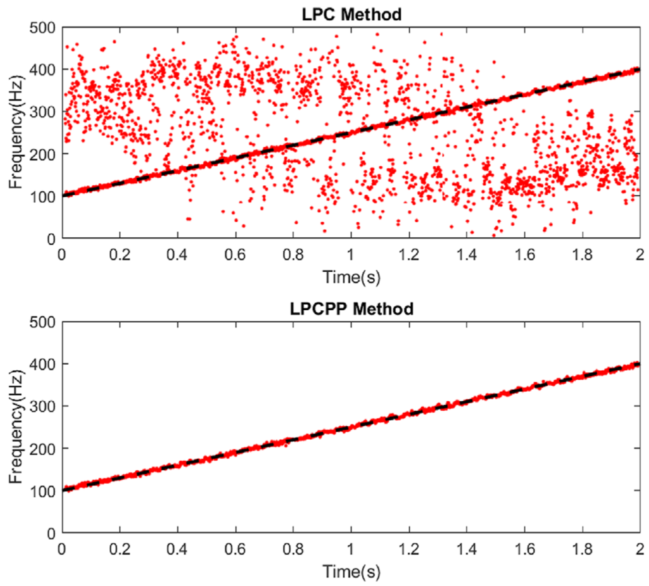

Fig. 2 *The estimated frequency results from LPC and LPCPP method for a LCFM signal. The red points are the estimates from the LPC method and the black trace is the instantaneous frequency $f(t)$ which is a reference trace*

Table 3. The starting frequencies of the different δ LCFM signals

δ	50	100	150	200	250
$f(0)$ (Hz)	200	150	100	50	0

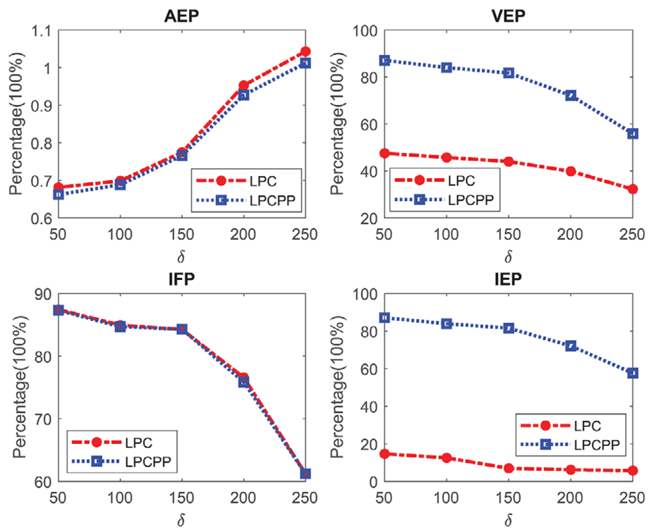

Fig. 3 *Performance analysis of the LPC and LPCPP method for LCFM signals with different rates of frequency change*

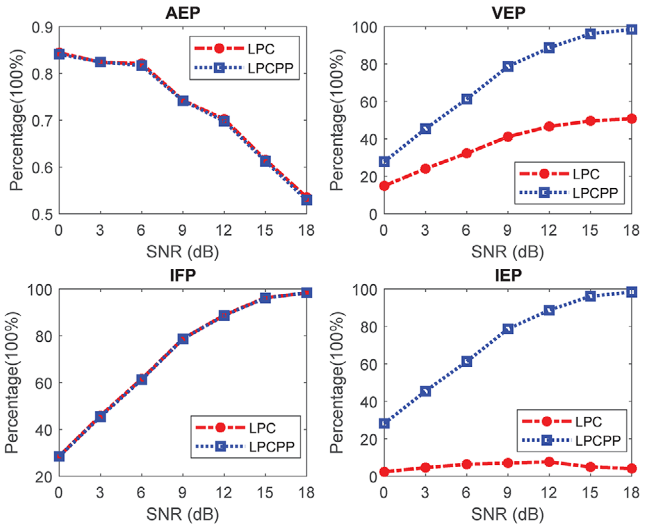

Fig. 4 *Performance analysis of the LPC and LPCPP method under various SNRs*

track the frequency of the LCFM signal than LPC under the different frequency change rates.

The third experiment shows the performance of the two LPC-based methods under different noise levels. The rate of frequency change is δ =150, the frequency changes from 100 to 400 Hz and the duration of the signal is 2 s. The SNR changes from 0 to 18 dB and the step is 3 dB. All other experimental parameters are the same as those above. Figure 4 shows the results of this experiment. As we can see, the AEP value of the two methods decreases with increasing SNR. However, the VEP and IFP of the two LPC-based methods increase when the value of SNR increases. But the VEP value of LPCPP is always greater than the VEP value of LPC for the same SNR value. In addition, the IEP value of the LPCPP method increases as the SNR value increases. The IEP of LPCPP performs better as the noise intensity decreases. However, the number of experiments with all-valid estimates of LPC does not change significantly with the change of noise intensity which shows that the non-dominant poles of LPC will not reduce with a reduction in the noise level. In summary, these experiments demonstrate that the LPCPP method can successfully track the frequencies of a time-varying signal in high noise environments.

The LCFM signals used here have only a single frequency component which facilitates the analysis of the ability of the LPCPP method to track the frequency variations of a signal. The LPCPP method is also applicable to multi-component signals and its parameters need to be adjusted accordingly. For example, when the number of frequency components in the signal increases, the filter order should also increase so that the LPC method can provide the required spectral separation of frequencies. When the filter order is increased, a larger value of γ is required to filter out the non-dominant poles. If the filter order is decreased, a smaller value of γ is required to find the dominant poles. In other words, the value of γ can be used to achieve an acceptable trade-off between VEP and IFP in our previous work [8]. The VEP value decreases when γ increases. Conversely, the IFP value increases with an increase in γ.

Conclusion: The LPCPP method further processes the LPC poles to produce a series of reduced-order filter transfer functions to estimate the frequency of the dominant spectral feature in a signal. This paper focuses on analysing the ability of LPCPP to track the dominant frequency changes of a noisy time-varying signal in real-time. Four performance metrics are proposed to measure the performance of the LPC-based methods on the LCFM signals. The experiments show that LPCPP achieves the real-time dominant frequency tracking and it significantly reduces the redundant frequency estimates of LPC. The LPCPP method outperforms LPC under fast frequency changes and has a high tolerance to noise. In future work, we will apply the LPCPP method to a practical bioelectric signal application, i.e. EEG dominant frequency tracking, as the EEG signal is typically a time-varying, multi-component, and noisy signal. In summary, LPCPP is a parameterised time-frequency method that allows us to track the dominant frequency changes in real-time.

Acknowledgements: This work has been supported through the Graduate School of Technological University Dublin and this publication has emanated from research conducted with the financial support of Science Foundation Ireland (SFI) under the Grant Number 15/SIRG/3459.

Conflict of Interest: The author(s) declare no competing interests.

Data availability statement: Data sharing is not applicable to this article as no new data were created or analysed in this study.

Received: *20 July 2021* Accepted: *20 October 2021*
doi: 10.1049/ell2.12362

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