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Modelling and Computer Simulation of Radar Screening using Plasma Clouds

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***Abstract*—Following a brief introduction on the principles of screening an aerospace vehicle using a plasma, we develop models for the Impulse Response Functions (IRFs) associated with microwave (Radar) back-scattering from a strong and weakly ionized plasma screen. In the latter case, it is shown that the strength of the return signal is determined by an IRF that is characterised by the simple negative exponential $\exp(-\sigma_0 t/\epsilon_0)$ where σ_0 is the average conductivity of the plasma, ϵ_0 is the permittivity of free space and t is the two-way travel time. For a weakly ionized plasma, the conductivity is determined by the number density of electrons. We develop a model for an electron beam induced plasma that includes the effect of cascade ionization and losses due to diffusion and recombination. Qualitative results are then derived for the number density of a plasma screen over a sub-sonic aerospace vehicle and a numerical simulation considered that is based on an iterative approach using a Green's function solution for a stationary and a moving vehicle. An example is provided for an idealised case relating to a sub-sonic missile such as a 'cruise missile'.**

***Index Terms*—Stealth Technology, Microwave Scattering, Radar, Weak Plasmas, Plasma Density Simulation**

I. INTRODUCTION

SINCE its original development in the late 1930s by Britain and Germany, Radio Detection and Ranging or Radar has been used for many years to detect airborne objects using ground and/or airborne platforms. The use of stealth technology for suppressing the detection of aerospace vehicles by Radar has been the subject of intensive research since the early 1970s following the development of radar guided surface-to-air missiles in the 1960s. One of the most notable current examples of the results of this research is the Lockhead-Martin F-117 stealth fighter and later the stealth bomber, first tested successfully under combat conditions in the Gulf war of 1991. Based on ideas first introduced by Denys Overholser in 1974 at Lockhead's advanced engineering laboratories, the technology is based on two principal aspects: (i) design features; (ii) radar absorbing materials and coatings. The geometry of the design is based on trying to minimize those features of an aerospace vehicle that are responsible for reflecting microwave radiation in such a way that the result can fly. Obvious features include embedding the gas turbine engines deep into the structure of the aircraft and introducing facets - diamond shaped flat surfaces - that reflect the microwave radiation away from the source. However, one of the principal factors for reducing the Radar Cross Section (RCS) is to minimize the profile of the aircraft while maximizing the 'smoothness' of the design. This effect was first noticed when a prototype 'flying wing' was developed in Germany by two Luftwaffe officers - the Horten brothers - and first tested in late 1944. This unique design was many years ahead of its time and was investigated further in the 1950s by the USA (the Northrop flying wing). However, limitations in control systems technology available at that time meant that the design was not practically viable due to an aerodynamic performance that was intrinsically unstable. The flying wing design only became of practical significance after the development of digital control processing (primarily in the 1970s), leading to the realisation of 'fly by wire'.

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The problem of designing stealthy aerospace vehicles can be posed as follows: given that the aircraft can be assumed to be a Born scatterer and that [2], [3]

$$(\nabla^2 + k^2)\mathbf{E}_s(\mathbf{r},\omega) = -k^2\gamma(\mathbf{r})\mathbf{E}_i(\mathbf{r},\omega)$$
$$+ikz_0\sigma\mathbf{E}_i(\mathbf{r},\omega) - \nabla[\mathbf{E}_i(\mathbf{r},\omega)\cdot\nabla\ln\epsilon_r(\mathbf{r})] \quad (1)$$

where

$$\gamma(\mathbf{r}) = \epsilon_r(\mathbf{r}) - 1,$$

find 'flying functions' γ and σ which are of compact support such that $\mathbf{E}_s = 0$. Here, $\mathbf{E}_s$ is the Fourier transform of the time-dependent scattered electric field vector $\mathbf{e}_s$ given by

$$\mathbf{E}_s(\mathbf{r},\omega) = \int_{-\infty}^{\infty} \mathbf{e}_s(\mathbf{r},t)\exp(i\omega t)dt,$$

$\mathbf{E}_i$ is the Fourier transform of the time-dependent incident electric field vector, i.e.

$$\mathbf{E}_i(\mathbf{r},\omega) = \int_{-\infty}^{\infty} \mathbf{e}_i(\mathbf{r},t)\exp(i\omega t)dt,$$

ϵ_r is the relative permittivity, σ is the conductivity, z_0 is the impedance of free space, $\mathbf{r}$ is the three-dimensional spatial vector and $k = \omega/c_0$ is the wavenumber where ω is the angular frequency and c_0 is the speed of light (in a vacuum).

In addition to investigating the RCS for different designs and materials, there is another approach to producing stealthy flying objects using a plasma. The reduction of the RCS of an aerospace vehicle through the generation of a plasma is an effect that has been known about for many years. The phenomenon has an obvious connection with the 'radio silence' phenomenon that occurs during re-entry of a spacecraft. This occurs when a plasma is formed around the spacecraft due to the 'friction' of the Earth's atmosphere.

A fundamental parameter of any plasma is the 'plasma (angular) frequency' ω_p given by

$$\omega_p = \left(\frac{4\pi n e^2}{m}\right)^{\frac{1}{2}}$$

where e is the charge of an electron (1.6×10^{-19} C), m is the mass of an electron (0.91×10^{-30} kg) and n is the number density of electrons in m^{-3}. For a plane (transverse) electromagnetic wave incident on a plasma [4]

$$k = \frac{1}{c_0}\sqrt{\omega^2 - \omega_p^2}.$$

A cut-off occurs when $\omega = \omega_p$, i.e. when there is a critical number density

$$n_c = \frac{m\omega^2}{4\pi e^2}.$$

Radio waves can only propagate through a plasma when $\omega > \omega_p$. For a typical laboratory plasma with $n = 10^{12}\ \text{cm}^{-3}$, a cut-off occurs when

$$f_p = \frac{\omega_p}{2\pi} \sim 10^4\sqrt{n} = 10\text{GHz}$$

which is in the microwave range. This effect is used as a method of measuring the density of laboratory plasmas.

The idea of screening an aerospace vehicle in a self-induced plasma with an appropriate critical number density is not a practical proposition. However, partial plasma screening of specific features which are good radar point-scatterers is possible, one example being the 'point' on the 'nose-cone' of a missile.

In this paper, we derive a model for radar signals generated by a conductor that is screened by a plasma. We develop an electromagnetic scattering model to investigate the effect that a plasma has on a conventional radar system. Expressions for the Impulse Response Function [5] generated by a scatterer with and without a plasma screen are studied. For a weakly ionized plasma, we derive a result that shows that the screening of the scatterer by the plasma is characterized by a simple negative exponential whose decay rate is determined by the conductivity which in turn, is proportional to the electron number density. A model for the distribution of the electron number density is then considered.

II. Microwave Scattering Model

Our aim is to develop a suitable model for the plasma screening effect by developing some relatively simple analytical results that explain why, under certain conditions, it provides a near-zero RCS. The basic reason for this effect is assumed to be due to the following: (i) a plasma is a (good) conductor and will therefore absorb (and disperse) electromagnetic (microwave) radiation before it is reflected by a scatterer; (ii) the air/plasma boundary is continuous (on the scale of the wavelength) and will therefore not generate a strong reflection compared with that generated by the surface of the scatterer which represents a sharp discontinuity on the scale of a wavelength (of a microwave field).

Let us model the problem using the scalar wave equation (under the Born approximation)

$$(\nabla^2 + k^2)E_s = -k^2\gamma(\mathbf{r})E_i + ikz_0\sigma(\mathbf{r})E_i, \quad \mathbf{r} \in V$$

where V is the volume of the scatterer. In order to obtain this equation, we are required to ignore the cross-polarisation term $\nabla[\mathbf{E}_i \cdot \nabla \ln \epsilon_r]$ in equation (1). A general solution to this equation can now be obtained using the Green's function method which, for homogeneous boundary conditions, gives

$$E_s = \int g(k^2\gamma - ikz_0\sigma)E_i d^3\mathbf{r}$$

where g is the 'out-going' Green's function given by [6]

$$g(\mathbf{r} \mid \mathbf{r}_0) = \frac{\exp(ik \mid \mathbf{r} - \mathbf{r}_0 \mid)}{4\pi \mid \mathbf{r} - \mathbf{r}_0 \mid}$$

and the integral is taken over the volume V of the scatterer. Here, $\mathbf{r}$ and $\mathbf{r}_0$ are the spatial coordinates of the scatterer and the position at which the scattered field is measured, respectively. The characteristics of the back-scattered field are dependent on ϵ_r, σ and their geometry (i.e. the shape of the scatterer over volume V). Note that, if $\epsilon_r = 1$ and $\sigma = 0$, then the scattered field is zero.

Let us assume that the scatterer is a good conductor, and that $\epsilon_r = 1$ so that $\gamma = 0$. This assumption is consistent with the application of a scalar wave equation since $\nabla(\mathbf{E}_i \cdot \nabla \ln \epsilon_r) = 0$ with $\epsilon_r = 1$. The scattered field is now determined by the conductivity alone. Let us also assume that the incident field is described by the Green's function g instead of a plane wave (the more usual case). This assumption helps to simplify slightly the analysis required in generating a model for the back-scattered field.

If the incident field propagates through a medium whose conductivity is effectively zero (i.e. air) then the solution for the back-scattered field will be given by

$$E_s = \int ikz_0\sigma g^2 d^3\mathbf{r}.$$

The volume over which scattering is effective will be determined by the skin depth

$$\delta = \left(\frac{2}{kz_0\sigma}\right)^{\frac{1}{2}}$$

which, although very small for a good conductor, will be considered to be finite. This allows us to adopt a volume scattering approach instead of one based on surface scattering. The reason for this is that we can then consider the volume scattering effects introduced by a plasma screen. Note that the homogeneous boundary conditions used to produce this Green's function solution yield a surface integral that is zero (i.e. E_s and ∇E_s are considered to be zero on the surface of V).

The solution for E_s in the far field (i.e. when $r/r_0 << 1$) is (ignoring numerical scaling factors)

$$E_s(\mathbf{r_0}, k) = \exp(2ikr_0)F$$

where F is the reflection coefficient given by

$$F(\mathbf{r}_0, k) = \int ikz_0\sigma \exp(-2ik\hat{\mathbf{n}} \cdot \mathbf{r})d^3\mathbf{r}$$

and $\hat{\mathbf{n}} = \mathbf{r}_0/r_0$. A relatively simple result can now be developed by considering the radar beam to be a narrow pencil-line beam oriented in the x-direction so that

$$r_0 \equiv \mid \mathbf{r}_0 \mid = x_0 \left(1 + \frac{y_0^2}{x_0^2} + \frac{z_0^2}{x_0^2}\right)^{\frac{1}{2}} \simeq x_0;$$
$$\frac{y_0}{x_0} << 1, \frac{z_0}{x_0} << 1.$$

This provides us with a solution for the reflection coefficient of the form

$$F(x_0, y_0, z_0, k) = \int ikz_0 \Omega \exp(-2ikx) dx$$

where

$$\Omega(x, x_0, y_0, z_0, k)$$
$$= \int \int \sigma(x, y, z) \exp(-2iky_0 y/x_0) \exp(-2ikz_0 z/x_0) dydz.$$

If we now consider the case when the back-scattered field is measured at a fixed point $(x_0, 0, 0)$, we obtain

$$F(k) = \int ikz_0 \Omega \exp(-ikx) dx$$

where

$$\Omega(x) = \int \int \sigma(x/2, y, z) dydz.$$

Note that this result has been obtained by replacing x by $x/2$ and then ignoring scaling. If we assume that σ is a constant as a function of y and z, then

$$\Omega(x) = A\sigma(x/2)$$

where A is the area of the scatterer. Here, we see that the back-scattered field (i.e. the reflection coefficient) is given by the Fourier transform of $ikz_0\Omega$. The time signature associated with the reflection coefficient (i.e. the temporal Impulse Response Function or IRF) can now be obtained by taking the inverse Fourier transform giving

$$f(t) = -Az_0 \frac{d\sigma}{dt}$$

where we have ignored scaling and where t is the 'two-way' travel time (i.e. $x = 2c_0 t$ where c_0 is the speed of an electromagnetic wave in a vacuum). This result illustrates that the strength of the return signal is determined by the following: (i) the area A of the scatterer that is illuminated by the radar beam; (ii) the gradient in the conductivity (from air to scatterer). Thus, assuming that the conductivity of air is zero, a scatterer, such as an aerospace vehicle composed of Aluminium alloy with a conductivity of approximately 2.5×10^7 siemens/metre, represents a huge change in the conductivity across the air/scatterer boundary and so produces a very strong reflection. It is useful to consider a scatterer with unit area, so that $\Omega = \sigma$ which is assumed from now on.

Let us now consider the case when the scatterer is embedded in a plasma which is taken to be a conductor with average conductivity σ_0. The screen contributes to the volume V over which the scattered field is to be computed and is assumed to have a conductivity profile with no distinct air/plasma boundary. The average conductivity σ_0 is taken to be the volume integral of the plasma conductivity profile divided by the volume of the screen over which the incident field is scattered. The effect of this is of course to introduce absorption (and frequency dispersion) of the electric field before it is incident upon the scatterer.

We consider the conductivity profile of the plasma together with the scatterer over volume V to be described by $\sigma_0 + \sigma(x, y, z)$. Our wave equation then becomes (for $\epsilon_r = 1$)

$$(\nabla^2 + k^2 - ikz_0\sigma_0)E_s = ikz_0\sigma E_i.$$

We can now repeat the calculation undertaken previously to obtain a far-field solution for the back-scattered field at $(x_0, 0, 0)$ produced by a narrow incident radar beam in the x-direction. In this case, the reflection coefficient is given by

$$F(k) = \int ikz_0 \sigma(x) \exp[-2i(k^2 - ikz_0\sigma_0)^{1/2} x] dx.$$

Note that an absorption effect has been introduced as a consequence of our model in which the electric field propagates through a plasma with an average conductivity σ_0 before incidence with the scatterer.

A. Impulse Response Function for a Good Conductive Plasma: $k << z_0\sigma_0$

We can simplify the equation for $F(k)$ by noting that for a good conductor $k^2 - ikz_0\sigma_0 \sim -ikz_0\sigma_0$. Using the result $\sqrt{-i} = (1 - i)/\sqrt{2}$ we can then write

$$F(k) = \int dxikz_0\sigma(x)...$$
$$... \exp\left[-2i\left(\frac{kz_0\sigma_0}{2}\right)^{\frac{1}{2}} x\right] \exp\left[-2\left(\frac{kz_0\sigma_0}{2}\right)^{\frac{1}{2}} x\right].$$

The form of this integral transform does not provide a simple Fourier-based relationship between σ and F. Nevertheless, the IRF $f(t)$ is given by the inverse Fourier transform of $F(k)$ and it is clear that a major feature of this integral transform is the negative exponential which characterizes the absorption of electromagnetic energy in the plasma. The plasma is in effect producing a conductive shield that screens the scatterer from incident radiation.

For a given wavelength, the skin depth δ depends on the average conductivity of the plasma; the more conductive the plasma, the shorter the skin depth (i.e. $\delta \propto \sigma_0^{-1/2}$). For a fixed average conductivity, there is less penetration of radiation at higher frequencies. Since radar relies on high frequency sweeping (i.e. the emission of chirped and other coded pulse) to obtain high resolution, the dispersion introduced through this integral transform will yield a spectrum at the receiver in which the frequency components are attenuated according to a $\exp(-\alpha\sqrt{k})$ power law, where $\alpha = \sqrt{z_0\sigma_0/2}$.

B. Impulse Response Function for a Weakly Conductive Plasma: $k >> z_0\sigma_0$

The equation for $F(k)$ given in the previous section is a consequence of considering the case when $k << z_0\sigma_0$, and it is not possible to Fourier invert to give an analytical expression for the IRF. However, if we consider the condition

$$k >> z_0\sigma_0$$

which is valid for the case when the plasma is weakly conductive, then we can consider the approximation $\sqrt{k^2 - ikz_0\sigma_0} \simeq k - iz_0\sigma_0/2$, giving

$$F(k) = \int ikz_0\sigma \exp(-2ikx)\exp(-z_0\sigma_0 x)dx.$$

Fourier inversion then allows us to establish the IRF, i.e.

$$f(t) = -z_0\frac{d}{dt}[\sigma(t)\exp(-\sigma_0 t/\epsilon_0)]$$

where, as before, t is the 'two-way' travel time and scaling has been ignored. Assuming that the variations in conductivity are smooth and that the boundary between the atmosphere and the plasma (in terms of variations in conductivity) is continuous, the effect of the plasma on the IRF is characterised by $\exp(-\sigma_0 t/\epsilon_0)$. On the other hand, if the air/plasma boundary is discontinuous, the IRF is dominated by the gradient in the conductivity across this boundary. In either case, there is no frequency dependence and the form of the negative exponential is the same as that describing the rate of decay of charge ρ in a conductor, i.e.

$$\rho = \rho_0 \exp(-\sigma_0 t/\epsilon_0).$$

Note that, since $\epsilon_0 \sim 10^{-11}$, only relatively low values of σ_0 are required to cause rapid decay in the IRF. For example, if we consider a 1cm wavelength radar, then the condition $k >> z_0\sigma_0$ that has been applied to achieve this simplification reduces to

$$\sigma_0 << 17.$$

The skin depth for this case is

$$\delta = \frac{10^{-3}}{\sqrt{\sigma_0}}$$

and, for a plasma with a very low conductivity of say 1 siemens/metre, the skin depth is 1 mm, i.e. the length over which the electric field strength has decayed by e^{-1} or by 63% .

The results obtained here are for the back-scattered field only; a special case has been considered where the field is measured at a fixed point $(x_0, 0, 0)$. For $k = k_0$ (i.e. the carrier wavenumber), the field strength as a function of $\theta \simeq y_0/x_0$ and $\phi \simeq z_0/x_0$ is determined by

$$\Omega(x, \theta, \phi)$$

$$= \int\int \sigma(x, y, z)\exp(-2ik_0\theta y)\exp(-2ik_0\phi z)dydz$$

and provides a Born estimate of the diffraction pattern produced by σ, i.e. a map of the back-scattered cross-section at small angles θ and ϕ.

C. The Radar Signal Equation

Assuming that the return has been demodulated with a carrier frequency ω_0, the radar signal $s(t)$ generated by a scatterer embedded in a weakly conductive plasma is (ignoring scaling)

$$s(t) = p(t) \otimes f(t) + r(t)$$

where

$$f(t) = \exp(-i\omega_0 t)\frac{d}{dt}[\sigma(t)\exp(-\sigma_0 t/\epsilon_0)],$$

$p(t)$ is the outgoing pulse (typically a linear FM pulse [7], [8]), $r(t)$ is the random noise associated with the whole system and $\otimes$ denotes the one-dimensional convolution integral [5].

The negative exponential component from which $f(t)$ is composed can be thought of as a Signal-to-Noise Ratio (SNR) control; as the conductivity increases, the SNR is reduced through negative exponential decay. In general, and in the practical application of using plasmas to screen aerospace vehicles, it is more likely that the plasma will be weakly ionized and weakly conductive. Hence, the equation above provides a useful initial model.

For a weakly ionized plasma, the electron number density determines its conductivity. In terms of this result, there are three principle factors affecting the performance of a practical radar plasma screening system: (i) maximizing the electron number density of the plasma; (ii) maximizing the thickness of the screen; (iii) maintaining continuity of the air/plasma interface. Points (i)-(iii) will depend on the power of the plasma generator, the stability of the plasma and its profile. Thus, a model is required for the electron number density profile of a plasma that is typically induced by application of an electron beam which is discharged through an appropriate feature on a moving aerospace vehicle. This is considered in the following sections.

III. Model for an Electron-beam Induced Plasma

The conductivity of a plasma depends upon whether we consider it to be weakly or strongly ionized. A weakly ionized plasma is one in which the frequency of collisions ν of electrons (e) and ions (i) with atoms (a) greatly exceeds that of collisions of these particles with one another, i.e.

$$\nu_{ea} >> \nu_{ee}, \nu_{ei}; \quad \nu_{ia} >> \nu_{ii}, \nu_{ie}.$$

A highly ionized plasma is described by the reverse of these conditions.

The conductivity of a weakly ionized plasma is given by [4]

$$\sigma = \frac{ne^2}{m_e\nu_{ea}} + \frac{2ne^2}{m_i\nu_{ia}}$$

where m_e and m_i are the masses of an electron and ion, respectively. This expression for the conductivity is dominated by the first term which describes the conductivity for the electron component of the plasma. The reason for this is that $m_i >> m_e$. Clearly, in this case, the conductivity is proportional to the electron number density n and the conductivity of a weakly ionized plasma can be approximated by

$$\sigma = \frac{ne^2}{m_e \nu_{ea}} \sim 10^{-9} \frac{n}{\nu_{ea}}$$

where ν_{ea} is the frequency of collisions between electrons and atoms. The ratio n/ν_{ea} will vary considerably from one regime (i.e. altitude and speed of flight) to another, although the values of n and ν_{ea} may tend to off-set each other. Assuming that the plasma is generated by e-beam breakdown of the atmosphere, at ambient atmospheric pressures, n will be large as will ν_{ea}. At higher altitudes, n will be less but so will ν_{ae}. Finally, above the atmosphere there will be relatively few atoms to break down and the collision frequency will be relatively small. However, if, for example, hydrogen gas could be generated prior to ionization, then it would be possible to generate large electron densities with low collision frequencies leading to high and sustainable plasma conductivities and, therefore, more effective plasma screening systems.

Since the conductivity of the plasma screen is linearly proportional to the electron number density, a principal problem is to determine the number density distribution for a given configuration (of source and aerospace vehicle). Thus, we are required to obtain a model that predicts the generation and transport of electrons subject to a variety of processes such ionization, recombination, diffusion, radiative losses, air flow, etc. This can be accomplished by considering the macroscopic properties of the plasma which are governed by the dynamics of the growth process, a process that involves avalanche electron multiplication (an exponential process), i.e. the ionization rate per initial electron. A limiting mechanism for the growth of the cascade is taken to be due to the (ambipolar) diffusion of electrons out the volume of the e-beam. Away from the plasma source, the electron number density is taken to be determined primarily by the recombination rate, radiative losses or bremsstrahlung radiation and flow regime. The ionization mechanism is taken to include inverse bremsstrahlung processes [4].

A. Ionization

The ionization of a neutral gas by an electron beam, for example, is determined by a cascade process that produces an exponential growth in the electron density. In the absence of diffusion processes, this electron density is determined by the equation

$$\frac{dn}{dt} = In$$

where I is the ionization rate per initial electron and is assumed to be a constant. The solution is trivial, represents exponential growth and is given by

$$n = n_0 \exp(It)$$

where n_0 is the initial electron density. Suppose that for a given volume, we require the e-beam to produce 10^{13} electrons say and that this number should be produced from an initial value of 10 electrons that have been ionized by electrons from the e-beam alone, then

$$\ln\left(\frac{n}{n_0}\right) = \int I dt \sim 40.$$

In other words, the cascade process requires 40 generations to produce 10^{13} electrons from just 10 of them. This number is not strongly dependent on the assumed value of n_0 within reasonable bounds. The electron density becomes large only near the end of the cascade process; 99% of the ionization is produced from the last 7 generations. Therefore, quantities such as the growth and losses from the cascade and the time to breakdown are determined by the conditions at times when the electron density is small.

The ionization rate will be determined by two principal processes: (i) the ionization rate I_b due to collisions of neutral atoms or molecules with electrons that have absorbed energy in the inverse bremsstrahlung process; (ii) the loss of potential ionizing electrons due to electron attachment with an ion which we denote by a rate coefficient I_a. Thus, in general

$$I = I_b - I_a.$$

The process of inverse bremsstrahlung involves raising a free electron to a higher energy state in the continuum of states available to it. The energy is a result of the absorption of a photon due to bremsstrahlung radiation which is itself produced by the acceleration of charged particles involved in elastic collisions. This absorption must occur with a simultaneous interaction with a heavy particle (atom, molecule or ion) in order that momentum is conserved.

B. Diffusion

The diffusion of electrons in a plasma is determined by the diffusion equation

$$\frac{\partial n}{\partial t} = D\nabla^2 n$$

where D is the (ambipolar) diffusion coefficient. In this equation, n represents the electron density of the plasma. With regard to ionization, the term In can be added to the diffusion equation to produce the inhomogeneous equation

$$\frac{\partial n}{\partial t} = D\nabla^2 n + In.$$

Note that, in general, I and D may be functions of both space and time. Another source term that is required is the multi-electron ionization rate due the e-beam alone which is responsible for the production of the initial electron density from which the cascade process develops. This ionization will also depend on both space and time and, in particular, on the distance of the beam away from the source. Thus, if we denote the e-beam ionization rate by B (for beam), then the diffusion equation becomes

$$\frac{\partial n}{\partial t} = D\nabla^2 n + In + B.$$

C. Recombination

Electron-ion collisions may lead to recombination, i.e. the production of a neutral atom as a result of the capture of an electron by an ion. The efficiency of the processes responsible for recombination is considerable at low electron energies at which the electron-ion interaction time is sufficiently large. Accordingly, at low electron temperatures (i.e. much less than the ionization energy) these processes strongly affect the balance of the charged plasma particles. The rate of charged particle removal due to recombination in a volume is determined by the total recombination cross section and depends of the number densities of both ions n_i and electrons n_e. Thus, the rate equation is given by

$$\frac{\partial n}{\partial t} = -Rn_i n_e = -Rn^2$$

where R is the recombination coefficient. The minus sign is introduced here because the process is lossy. This nonlinear equation has a simple analytical solution which can be obtained by inspection and is given by

$$\frac{1}{n} = \frac{1}{n_0} + Rt$$

where n_0 is the initial number density. After the density has fallen far below its initial value, it decays reciprocally with time, i.e.

$$n \propto \frac{1}{Rt}.$$

This is a fundamentally different behaviour from the exponential decay associated with diffusive processes and exponential growth associated with ionization processes. Since the recombination rate is proportional to n^2, for high values of n it can be expected to be the dominant process.

With regard to the diffusion equation, $-Rn^2$ is a source term and, thus, the diffusion equation must be modified again, this time to the nonlinear inhomogeneous form

$$\frac{\partial n}{\partial t} = D\nabla^2 n + In + B - Rn^2.$$

Note that, in general, it is expected that, like I, D and B, the recombination coefficient R may be a function of both space and time.

The rate equation above, has two source terms and two loss terms. The source terms are B and In which describe the initial population density of electrons produced by the e-beam alone and the population density generated by the cascade process. The loss terms $D\nabla^2 n$ and Rn^2 describe losses due to the processes of diffusion and recombination, respectively. Another effect that can be considered is loss through radiative processes. However, for weakly ionized plasmas, it reasonable to assume that this effect is relatively small compared to diffusion and recombination. These losses will also be proportional to n^2 since the total power P radiated per unit volume by a plasma is given by [4]

$$P \sim 1.5 \times 10^{-38} Z^2 n_e n_i T_e^{\frac{1}{2}} \qquad (\text{Watts/m}^3)$$

where n is in m^{-3} and T_e is in eV. Because the radiated power is proportional to the square of the atomic number Z, a low Z plasma (e.g. a hydrogen plasma) will last longer.

IV. Rate Equation Analysis

Analytical solutions to the rate equation

$$\frac{\partial n}{\partial t} = D\nabla^2 n + In + B - Rn^2$$

can be considered for different conditions compounded in the inclusion, or otherwise, of different terms.

In some practical cases, the diffusion loss will dominate over losses from recombination after initiation (when B can be ignored), and we can consider the electron density to be determined by the solution of

$$\frac{\partial n}{\partial t} = D\nabla^2 n + In.$$

For the characteristic diffusion length Λ of the breakdown, we may replace ∇^2 by $-1/\Lambda^2$ to obtain a solution of the form

$$n = n_0 \exp[(I - D/\Lambda^2)t].$$

This solution illustrates exponential growth of electrons, subject to exponential damping due to diffusion. Clearly, for a given coefficient of diffusion, the characteristic diffusion length should be large in order to achieve a high concentration of electrons.

Under conditions where, along with diffusion, the quadratic recombination term substantially affects the plasma decay, the rate equation takes the form

$$\frac{\partial n}{\partial t} = D\nabla^2 n + In - Rn^2$$

or, in terms of the characteristic length of diffusion,

$$\frac{dn}{dt} = -\left(\frac{D}{\Lambda^2} - I\right) n - Rn^2.$$

The solution to this equation is [4]

$$n(t) = \frac{\left(\frac{D}{\Lambda^2} - I\right) n_0 \exp\left(It - \frac{D}{\Lambda^2}t\right)}{\left(\frac{D}{\Lambda^2} - I\right) + Rn_0 \left[1 - \exp\left(It - \frac{D}{\Lambda^2}t\right)\right]}.$$

Note that, when $D/\Lambda^2 - I >> Rn$, this solution changes into an exponential form that is characteristic of ionization growth and diffusion decay. Alternatively, when $Rn >> D/\Lambda^2 - I$ the electron density is determined by the equation.

$$\frac{1}{n} = \frac{1}{n_0} + Rt.$$

V. Steady State Solutions

For steady state conditions

$$\frac{\partial n}{\partial t} = 0$$

and our rate equation reduces to

$$D\nabla^2 n + In + B - Rn^2 = 0.$$

Let us now consider some of the solutions available under different conditions.

A. Steady State Solution without Flow

If we consider the e-beam to produce ionization along the axis alone then the plasma source can be assumed to be axially symmetric. The electron density is then a function of the radius r and, using cylindrical coordinates, we have

$$\nabla^2 n = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial n}{\partial r}\right).$$

The simplest solution available to us in this case is obtained under the assumption that B, I and R are all zero. The plasma is therefore assumed to be a cylindrical plasma with losses due to diffusion alone. Except at $r = 0$, the density must satisfy

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial n}{\partial r}\right) = 0$$

which has the solution

$$n(r) = n_0 \ln r + c.$$

With the boundary condition $n(a) = 0$ (i.e. the electron density is zero some distance away from the source) we have $c = -n_0 \ln a$ and therefore

$$n(r) = n_0 \ln\left(\frac{a}{r}\right)$$

which is the fundamental solution to the 2D Laplace's equation.

Let us now consider the solution to the equation $D\nabla^2 n + In = 0$ in cylindrical coordinates. This requires that we solve the equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial n}{\partial r}\right) = -\frac{In}{D}$$

or

$$\frac{d^2 n}{dr^2} + \frac{1}{r}\frac{dn}{dr} + \frac{I}{D}n = 0$$

which is Bessel's equation of order zero. This has the solution

$$n(r) = n_0 \mathrm{J}_0\left(r\sqrt{\frac{I}{D}}\right)$$

where J_0 is the Bessel function of order zero. The boundary condition that must be applied is that $n = 0$ at $r = a$. The Bessel function is zero for multiple values of $x = r\sqrt{I/D}$. However, the first zero occurs when $x \simeq 2.4$ or when

$$r = a = 2.4\sqrt{\frac{D}{I}}.$$

This solution describes the lowest diffusion mode in which a can be taken to define the boundary between the plasma and air. Although it is possible for higher diffusion modes to occur, they tend to decay rapidly in most plasmas and may therefore be ignored. Note that the radial extent of the electron density is proportional to the square root of the coefficient of diffusion.

B. *Steady State Equation with Flow*

Suppose we consider the case when the plasma source is in a steady state condition (i.e. the e-beam is operating in the continuous mode) and that the radial distribution of the electron density is described by J_0. For the case when the plasma source is moving through the atmosphere, it will be expected that the plasma streams away from the source (down wind) producing a decay of the electron density due to: (i) air flow effects, e.g. boundary layer thickening; (ii) recombination. Let us assume that the plasma forms a boundary layer with thickness

$$\Delta \sim \frac{L}{\sqrt{R_e}}$$

where R_e is the Reynolds number given by

$$R_e = \frac{Lv}{\eta},$$

L is the characteristic length scale of the flow, v is the velocity of the flow and η is the kinematic viscosity of air. For a 10 m long aerospace vehicle travelling at 100 m/s, say, and with $\eta \sim 10^{-3}\mathrm{m}^2/\mathrm{s}$ for air,

$$\Delta = 1 \quad \mathrm{mm}.$$

For a 1mm thick plasma screen of 1 siemen/metre and considering the two-way travel path, the absorption of microwave radiation with a wavelength of 1cm (due to the skin depth effect) is 87%. Thus, relatively large absorption can occur over small boundary layers composed of low conductivity plasmas (i.e. plasmas with low electron number densities). As the plasma streams away from the source, the electron density will decrease due to an increase in the extent of the boundary layer (ignoring recombination). Since the initial radial extent of the plasma at source is given by a, we can expect the screen thickness to be of the order of $a+\Delta$. The decay of the electron density as a function of r and L can therefore be estimated by

$$n(r, L) = \frac{a}{a+\Delta} n_0 \mathrm{J}_0\left(r\sqrt{\frac{I}{D}}\right) = \frac{n_0 \mathrm{J}_0\left(r\sqrt{\frac{I}{D}}\right)}{1 + 0.4167\sqrt{\frac{L\eta I}{vD}}}.$$

This steady state estimate neglects the effects of recombination but provides a qualitative estimate of the electron density profile produced by a continuous on-axis e-beam.

C. *Numerical Simulation*

The rate equation for the electron density is given by

$$\frac{\partial n}{\partial t} = D\nabla^2 n + B + In - Rn^2.$$

If the plasma is generated in a flow of air then, to a good approximation, we can consider the electrons to flow with the air and thus conform to the conservation equation

$$\frac{\partial n}{\partial t} = \nabla \cdot (n\mathbf{v})$$

where $\mathbf{v}$ is the velocity of the flow. Hence, we are required to solve the equation

$$D\nabla^2 n + B + In - Rn^2 - \nabla \cdot (n\nabla u) = 0$$

where u is the velocity potential $\mathbf{v} = \nabla u$. Our problem is to find n given u which requires the velocity potential to be computed *a priori*. Suppose we compute the velocity potential for air (in the absence of a plasma). We can then consider a model in which the electron density is a characteristic of this potential. In other words, we consider the plasma to flow away from the source in a manner that is determined by the stream lines associated with the flow of air over the aerospace vehicle. For constant (air) density, the velocity potential is obtained by solving Laplace's equation

$$\nabla^2 u = 0$$

subject to appropriate boundary conditions. Noting that

$$\nabla u \cdot \nabla n = \nabla \cdot (u\nabla n) - u\nabla^2 n$$

we can write

$$(D+u)\nabla^2 n + B + In - Rn^2 - \nabla \cdot (u\nabla n) = 0.$$

This is the steady state equation for the electron density n subject to a flow regime characterized by velocity potential u. The 3D Green's function solution to this equation is [6]

$$n = \frac{1}{4\pi r} \otimes_3 \left(\frac{B}{u+D} + \frac{In}{u+D} - \frac{Rn^2}{u+D} - \nabla \cdot (u\nabla n) \right)$$

where $\otimes_3$ denotes the three-dimensional convolution integral. The order of iteration required to compute n can follow the order in which the physical mechanisms described by each of the terms occur. Thus:

Electron generation

$$n_1 = \frac{1}{4\pi r} \otimes \frac{B}{u+D}$$

Ionization

$$n_2 = n_1 + \frac{1}{4\pi r} \otimes \frac{In_1}{u+D}$$

Recombination

$$n_3 = n_1 + n_2 - \frac{1}{4\pi r} \otimes \frac{Rn_2^2}{u+D}$$

Flow

$$n_4 = n_1 + n_2 - n_3 - \frac{1}{4\pi r} \otimes \nabla \cdot (u\nabla n_3)$$

Figure 1 shows the effect of a plasma (specifically, the electron number density n_3) generated without ($u = 0$) and with ($\nabla^2 u = 0$) an air flow (from right to left) over a cone with a smooth point. Here, we assume that the screen is axially symmetric and undertake the computations in the plane $(x, y, 0)$. This is achieved by implementing the equations above on a two-dimensional uniform grid of size 700×300, applying the convolution theorem and using the result (where $\Longleftrightarrow$ denotes transformation from real space to Fourier space)

$$\frac{1}{\sqrt{x^2+y^2}} \Longleftrightarrow \frac{1}{\sqrt{k_x^2+k_y^2}}$$

with the boundary condition $n = 0$ (applied over the boundary and over the extent of the cone) and where k_x, k_y are the spatial frequency components in the $x-$ and $y-$ directions respectively. The e-beam is taken to be a 'pencil line beam' (one pixel wide) emitted from the point of the cone with uniform intensity along its extent. The coefficients B, I, R and D are assumed constant with values: $B = 4\pi$, $I = 4\pi$, $R = 4\pi$ and $D = 1$. The velocity potential u is computed using the Successive-over-Relaxation method [9] compounded in the following result (where $\omega = 1.1$ is the relaxation parameter)

$$u_{i,j}^{k+1} = u_{i,j}^k + \frac{\omega}{4}(u_{i+1,j}^k + u_{i-1,j}^{k+1} + u_{i,j+1}^k + u_{i,j-1}^{k+1} - 4u_{i,j}^k)$$

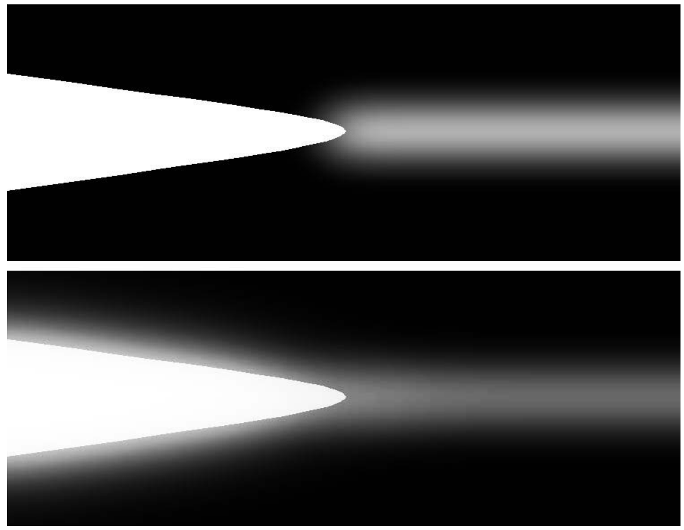

Fig. 1. Plasma density profile generated by an electron beam without airflow (above) and with an airflow (below) from right to left over a 'smoothed cone'. The beam is taken to be of uniform intensity and emitted from the 'point' of the cone 'travelling' to the right.

for $i = 1, 2, ..., N$ and $j = 1, 2, ..., M$ with conditions $u_{ij} = 0$ on the boundary and over the extent of the cone, $u_{1,j} = u_0 \;\; \forall j$, $u_{N,j} = u_0 \;\; \forall j$, $u_{i,M} = u_0 \;\; \forall i$, $u_{i,1} = u_0 \;\; \forall i \notin C$ where C is the extent of the cone at the extreme left-hand edge of the grid (with $u_0 = 1$).

The extent of the plasma screen that forms over the boundary of the cone to provide a radar screen is quite noticeable when air flow is present, an extent that is strongly determined by the magnitude of the recombination coefficient and air flow for a given beam energy and coefficient of ionization. Actual values for R along with I, D and the beam profile B (which will not be uniform as in the idealized simulation presented here) and the flow rate will depend on the operating conditions that apply. These include the vehicle velocity, the plasma medium, additives (readily ionizable or reactive species), the electron beam energy, its diameter and profile, details that remain classified. However, typical parameters include an electron beam energy of 100keV, a (Gaussian) beam diameter of less than 5mm with a loss of 1keV per cm for an aerospace vehicle travelling at up to 100ms^{-1} operating in a plasma medium of air (over a range of atmospheric pressures) and with additives such as water vapour. Applications include the plasma screening of in-coming missiles, for example, against close proximity anti-missile systems that use radar for targeting and control.

VI. Summary and Conclusions

The idea of using a weakly ionized plasma to screen an aerospace vehicle is not new but interest in this effect and appraisal of the applications to which it can be practically applied are likely to grow. This paper has developed a model for the radar signal generated with and without a plasma screen and illustrates that, for a weakly ionized plasma, the effect of such a screen is compounded in the function $\exp(-\sigma_0 t \epsilon_0)$ where t is the two-way travel time, σ_0 is the average conductivity of the plasma and ϵ_0 is the permittivity of free space. For a weakly ionized plasma, the conductivity is determined by the number density of electrons and qualitative results have been developed to estimate the number density of a plasma screen enveloping a moving vehicle. A numerical procedure to simulate the number density of a plasma has been developed and an example provided for the case when an e-beam induced plasma is generated from the front of a (sub-sonic) missile. This simulation is based on assuming cascade ionization with loss mechanisms due to diffusion and recombination. The simulator is not suitable for the super-sonic case when the airflow cannot by determined by the solution to Laplace's equation for the velocity potential. In this case, it may be expected that the plasma is partially distributed along the shock wave that is formed and thus, depending on the exact configuration of the aerospace vehicle, could provide a more extensive plasma screen. This will be the subject of a future publication.

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