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Enhanced Fractional Adaptive Processing Paradigm for Power Signal Estimation

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RESEARCH ARTICLE

Enhanced fractional adaptive processing paradigm for power signal estimation

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Fractional calculus tools have been exploited to effectively model variety of engineering, physics and applied sciences problems. The concept of fractional derivative has been incorporated in the optimization process of least mean square (LMS) iterative adaptive method. This study exploits the recently introduced enhanced fractional derivative based LMS (EFDLMS) for parameter estimation of power signal formed by the combination of different sinusoids. The EFDLMS addresses the issue of fractional extreme points and provides faster convergence speed. The performance of EFDLMS is evaluated in detail by taking different levels of noise in the composite sinusoidal signal as well as considering various fractional orders in the EFDLMS. Simulation results reveal that the EDFLMS is faster in convergence speed than the conventional LMS (i.e., EFDLMS for unity fractional order).

KEYWORDS

fractional derivative, fractional methods, parameter estimation, sinusoidal signal

MSC CLASSIFICATION

26A33

1 | INTRODUCTION

1.1 | Literature review

Fractional calculus tools have been exploited to effectively solve various problems of natural/applied sciences, engineering and technology.[1–5] The application of fractional calculus was also explored in designing novel local and global search methods through incorporating fractional derivative concepts.[6–12] The fractional adaptive methods were successfully applied for solving different problems including identification of control autoregressive moving average systems,[13] parameter estimation of Hammerstein control autoregressive systems,[14] echo cancelation,[15] tracking of fading channels,[16] adaptive beamforming[17] recommender systems[18,19] and power signal modeling and estimation[20,21] and so on.

The parameter estimation of power signals is required in different fields including electrical networks for quality monitoring and reliability assessment.[22,23] Mehmood et al. proposed differential evolution and backtracking search based heuristics for power signal estimation.[24,25] Liu et al. proposed iterative estimation scheme by exploiting hierarchical principle,[26] Xu et al. developed iterative, recursive and multi innovation gradient based algorithms,[27–29] Malik et al. presented the fractional order swarm optimization algorithm for power system harmonics estimation[30] and Yao et al. presented an estimation approach for impulsive noise scenario.[31] The impulsive noise is difficult to handle in power signals and mitigation of impulse noise requires a more robust estimation algorithm.[32] Thus, the effective, accurate and robust parameter estimation of power harmonics is essential in order to mitigate the adverse effects of harmonics on power systems. In this regard, the current study investigates in exploiting the fractional derivative based estimation approach for effective parameter estimation.

The fractional derivatives are nonlocal in nature and hold a long memory effect. The research community exploited the properties of fractional derivatives in different perspective and proposed novel fractional adaptive methods through different approximations.[33–37] For instance, Raja and Qureshi proposed fractional adaptive method (i.e., FLMS) that incorporates the strength of both first and fractional derivatives to ensure convergence to actual extreme points.[38] Chaudhary, Raja and Machado extended the FLMS method by proposing different variants for faster convergence.[39,40] Cheng et al. introduced the concept of variable initial value to design a new fractional method called innovative FLMS (I-FLMS).[41] Chaudhary et al. compared the performance of the I-FLMS with the FLMS for power signal parameter estimation.[42] Wei et al. proposed a generalized fractional gradient method with fractional gradient direction (G-FGD) by expanding the fractional derivative through Taylor series and then truncating the resultant by retaining only first term,[43] Liu et al. extended the G-FGD and presented quasi FGD (Q-FGD) for multivariable functions.[44] Recently, Pu et al. introduced enhanced fractional derivative based fractional LMS, (EFDLMS) that computes the fractional derivative using Faa di Bruno formula.[45] The reported promising features of the EFDLMS motivated us to exploit it for effective parameter estimation of composite sinusoidal signals.

1.2 | Research contributions

The main features of the current study are

- An enhanced fractional derivative based LMS, that is, EFDLMS is presented for parameter estimation of composite sinusoidal signal.
- The EFDLMS computes the fractional derivative of composite cost function using Faa di Bruno formula and addresses the issue of fractional extreme points.
- The EFDLMS has faster convergence speed than the conventional LMS method for smaller fractional orders.
- The simulation studies verify the accurate, convergent and robust performance of the EFDLMS for power signal estimation.

1.3 | Paper outlines

Rest of the paper is organized in the following sections. The identification model is described in Section 2. The proposed methodology based on EFDLMS for power signal estimation is presented in Section 3. The results of numerical simulations in different tabular and graphical illustrations are provided in Section 4. Finally, the conclusions are presented in the Section 5 with some future research ideas.

2 | IDENTIFICATION MODEL

Consider a following composite sinusoidal signal with amplitude A_i, angular frequency w_i, phase p_i and noise μ[28,29]:

$$x(t) = \sum_{i=1}^{n} A_i \sin(w_i t + p_i) + \mu(t). \tag{1}$$

Using trigonometric identities to rewrite the (1) as

$$x(t) = \sum_{i=1}^{n} B_i \sin(w_i t) + C_i \cos(w_i t) + \mu(t), \tag{2}$$

where $B_i = A_i \cos p_i$ and $C_i = A_i \sin p_i$. Assuming sampling period s, then $t_j = js$ is the sampling time and the measured data can be written as $\{t_j, x(t_j)\}$. For simplicity, we write $x(j) = x(t_j)$. Then defining the parameter and information vector as

$$\boldsymbol{\upsilon} = -[B_1, C_1, B_2, C_2, \ldots, B_n, C_n] \in \mathbb{R}^{2n}, \tag{3}$$

$$\boldsymbol{\xi}(j) = \begin{bmatrix} \sin(w_1 js), \cos(w_1 js), \sin(w_2 js), \\ \cos(w_2 js), \ldots, \sin(w_n js), \cos(w_n js) \end{bmatrix} \in \mathbb{R}^{2n}, \tag{4}$$

using (3) and (4) in (2), the identification model for power signal estimation is given as

$$x(j) = \boldsymbol{\xi}^T(j)\boldsymbol{\upsilon} + \mu(j). \tag{5}$$

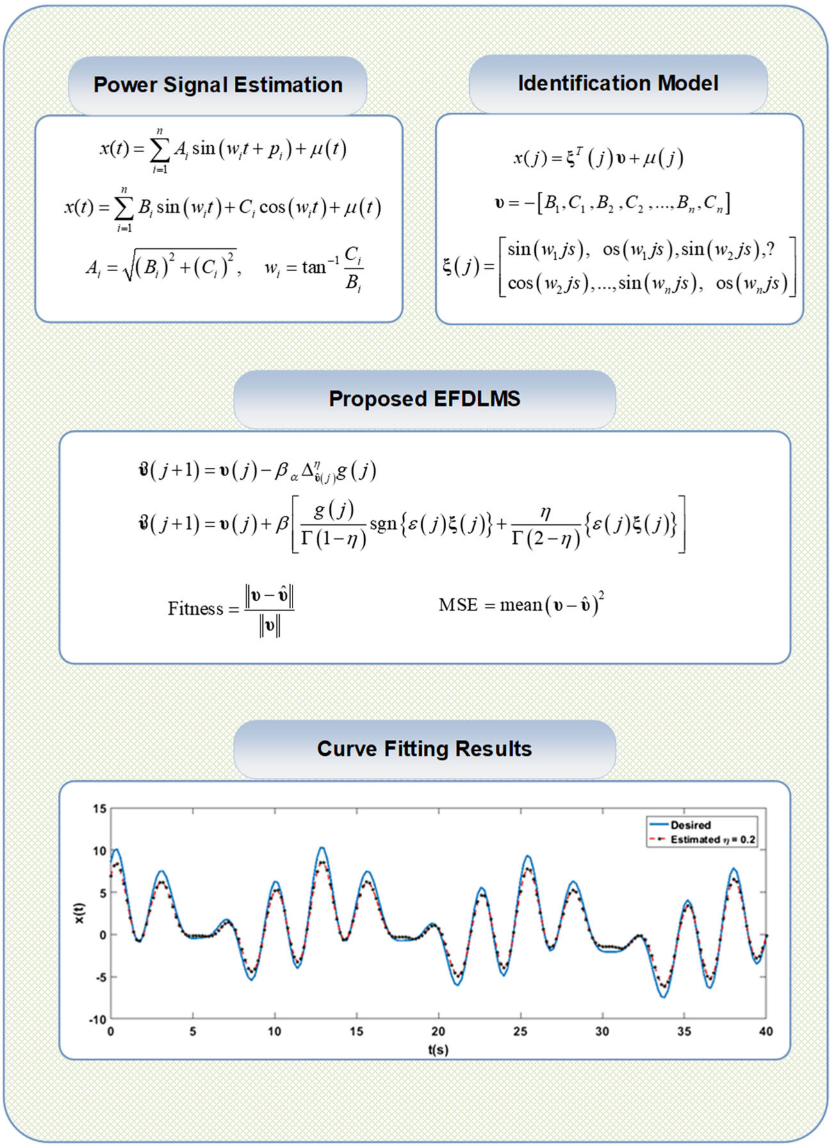


FIGURE 1 Graphical flow of the study [Colour figure can be viewed at wileyonlinelibrary.com]

The identification model given in Equation (5) is in the form of intermediate variables B_i, and C_i. The actual parameters amplitude A_i and angular frequency w_i can be calculated as

$$A_i = \sqrt{(B_i)^2 + (C_i)^2}, w_i = \tan^{-1}\frac{C_i}{B_i}. \quad (6)$$

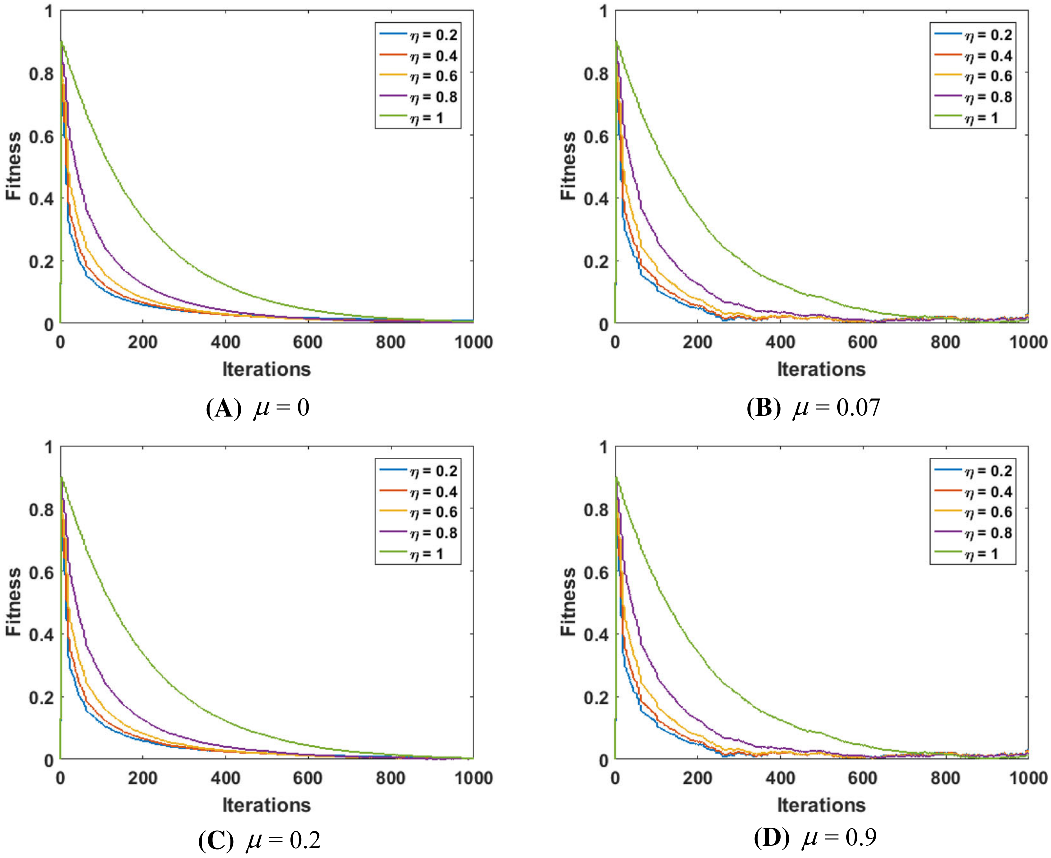


FIGURE 2 Iterative plots of EFDLMS scheme for different noise levels in Problem 1 [Colour figure can be viewed at wileyonlinelibrary.com]

TABLE 1 Fitness values against iterations for no noise scenario in Problem 1

t	$\eta = 0.2$	$\eta = 0.4$	$\eta = 0.6$	$\eta = 0.8$	$\eta = 1$
500	2.2255E-02	1.9978E-02	1.9758E-02	2.5211E-02	7.3709E-02
1000	8.5676E-03	5.0512E-03	3.1071E-03	2.5600E-03	5.7851E-03
1500	4.1824E-03	1.5284E-03	5.4952E-04	2.7904E-04	4.5784E-04
2000	2.2309E-03	4.8494E-04	9.9112E-05	3.0631E-05	3.6066E-05
2500	1.2405E-03	1.5633E-04	1.7988E-05	3.3798E-06	2.8663E-06
3000	7.0394E-04	5.0483E-05	3.2514E-06	3.7024E-07	2.2452E-07
3500	4.0430E-04	1.6341E-05	5.8883E-07	4.0679E-08	1.7680E-08
4000	2.3399E-04	5.3026E-06	1.0694E-07	4.4860E-09	1.4005E-09
4500	1.3588E-04	1.7192E-06	1.9393E-08	4.9353E-10	1.1029E-10
5000	7.9109E-05	5.5799E-07	3.5222E-09	5.4425E-11	8.7403E-12

3 | PROPOSED METHODOLOGY

The idea of enhanced fractional derivative based LMS is recently introduced[45] but not yet explored/exploited for any specific application. We call it as EFDLMS and exploring its application for effective parameter estimation of power signals comprising of multi frequency sine signal. Defining the cost function as

TABLE 2 Fitness values against iterations for 0.07 level noise scenario in Problem 1

t	$\eta = 0.2$	$\eta = 0.4$	$\eta = 0.6$	$\eta = 0.8$	$\eta = 1$
500	2.2533E-02	2.0361E-02	2.0300E-02	2.5916E-02	7.4456E-02
1000	7.1090E-03	3.9153E-03	2.2493E-03	1.8411E-03	4.8579E-03
1500	3.0641E-03	1.1469E-03	5.6481E-04	4.5738E-04	6.9777E-04
2000	1.4122E-03	5.1662E-04	4.3738E-04	4.9870E-04	5.6949E-04
2500	5.5820E-04	5.9034E-04	7.4983E-04	8.5214E-04	9.0997E-04
3000	5.5224E-04	5.0696E-04	5.5853E-04	5.8633E-04	5.9782E-04
3500	6.3497E-04	6.4259E-04	6.4760E-04	6.4331E-04	6.4059E-04
4000	6.4488E-04	7.1967E-04	8.1627E-04	8.9833E-04	9.4679E-04
4500	3.1914E-04	3.6815E-04	4.7333E-04	5.6198E-04	6.1836E-04
5000	3.8493E-04	5.6884E-04	6.8036E-04	7.4756E-04	7.8070E-04

TABLE 3 Fitness values against iterations for 0.2 level noise scenario in Problem 1

t	$\eta = 0.2$	$\eta = 0.4$	$\eta = 0.6$	$\eta = 0.8$	$\eta = 1$
500	2.0258E-02	1.9014E-02	1.9976E-02	2.6579E-02	7.5846E-02
1000	3.4050E-03	2.5920E-03	2.6533E-03	2.6979E-03	3.7357E-03
1500	1.2847E-03	9.3848E-04	8.6945E-04	9.1591E-04	1.1860E-03
2000	5.9868E-04	8.4755E-04	1.1489E-03	1.3965E-03	1.5654E-03
2500	1.7153E-03	2.0432E-03	2.3108E-03	2.5053E-03	2.6012E-03
3000	1.4397E-03	1.5605E-03	1.6421E-03	1.6865E-03	1.7082E-03
3500	1.8934E-03	1.9240E-03	1.9001E-03	1.8643E-03	1.8301E-03
4000	2.1490E-03	2.3370E-03	2.5195E-03	2.6538E-03	2.7045E-03
4500	8.5250E-04	9.5520E-04	1.2416E-03	1.5362E-03	1.7674E-03
5000	1.9856E-03	2.1392E-03	2.2276E-03	2.2598E-03	2.2313E-03

TABLE 4 Fitness values against iterations for 0.9 level noise scenario in Problem 1

t	$\eta = 0.2$	$\eta = 0.4$	$\eta = 0.6$	$\eta = 0.8$	$\eta = 1$
500	2.1491E-02	2.1163E-02	2.1899E-02	2.8236E-02	8.3409E-02
1000	2.5826E-02	2.4436E-02	2.2351E-02	1.9380E-02	1.2535E-02
1500	9.6965E-03	8.5647E-03	7.0863E-03	5.3217E-03	3.8779E-03
2000	1.0546E-02	9.4448E-03	8.1984E-03	7.2276E-03	6.8972E-03
2500	1.4565E-02	1.4363E-02	1.3810E-02	1.2894E-02	1.1696E-02
3000	1.1363E-02	1.0207E-02	9.0474E-03	8.1077E-03	7.6945E-03
3500	1.5605E-02	1.3887E-02	1.1805E-02	9.6689E-03	8.2327E-03
4000	1.6287E-02	1.5846E-02	1.5054E-02	1.3837E-02	1.2155E-02
4500	7.4461E-03	5.5667E-03	4.4702E-03	5.5217E-03	7.9682E-03
5000	2.0910E-02	1.8837E-02	1.6158E-02	1.3107E-02	1.0059E-02

$$g(j) = \varepsilon^2(j)$$
$$g(j) = [x(j) - \hat{x}(j)]^2 = \left[x(j) - \boldsymbol{\xi}^T(j)\hat{\boldsymbol{\upsilon}}\right]^2. \tag{7}$$

Calculating the fractional derivative of the cost function with respect to $\hat{\boldsymbol{\upsilon}}$ using Faa di Bruno formula and after doing some simplification as reported in Xie et al.,[45] we obtain:

$$_{\alpha}\Delta^{\eta}_{\hat{\upsilon}_k(j)}g(j) = \frac{[\hat{\upsilon}_k(j)-\alpha]^{-\eta}}{\Gamma(1-\eta)}g(j) + \frac{\Gamma(1+\eta)}{\Gamma(\eta)\Gamma(2)}1! \times \frac{[\hat{\upsilon}_k(j)-\alpha]^{1-\eta}}{\Gamma(2-\eta)}\,_{\alpha}\Delta^{1}_{\hat{x}(j)}g(j)\frac{1}{1!}\left[\frac{_{\alpha}\Delta^{1}_{\hat{\upsilon}_k(j)}\hat{x}(j)}{1!}\right]^1, \tag{8}$$

$_{\alpha}\Delta^{\eta}_{\hat{\upsilon}_k(j)}g(j)$ is the fractional derivative with lower bound α and order η. $\Gamma(\cdot)$ represents the standard Gamma function used in fractional calculus. Combine the Gamma function properties and the first derivative chain rule as did in Xie et al.,[45] we obtain:

$$_{\alpha}\Delta^{\eta}_{\hat{\upsilon}_k(j)}g(j) = \frac{[\hat{\upsilon}_k(j)-\alpha]^{-\eta}}{\Gamma(1-\eta)}g(j) + \frac{\eta[\hat{\upsilon}_k(j)-\alpha]^{1-\eta}}{\Gamma(2-\eta)}\,_{\alpha}\Delta^{1}_{\hat{\upsilon}_k(j)}g(j). \tag{9}$$

It is reported in Chaudhary et al.[40] that the gradient information can be destroyed through estimated values of the weights. Thus, applying this to (9) yields

TABLE 5 Final MSE values of EFDLMS in Problem 1

μ	η	$\widehat{\upsilon}_1$	$\widehat{\upsilon}_2$	$\widehat{\upsilon}_3$	$\widehat{\upsilon}_4$	$\widehat{\upsilon}_5$	$\widehat{\upsilon}_6$	**MSE**
0	0.2	3.0000	4.9998	9.9991	1.0200	0.7800	0.5200	1.42E-07
	0.4	3.0000	5.0000	10.0000	1.0200	0.7800	0.5200	7.05E-12
	0.6	3.0000	5.0000	10.0000	1.0200	0.7800	0.5200	2.81E-16
	0.8	3.0000	5.0000	10.0000	1.0200	0.7800	0.5200	6.71E-20
	1	3.0000	5.0000	10.0000	1.0200	0.7800	0.5200	1.73E-21
0.07	0.2	3.0021	5.0024	10.0032	1.0200	0.7803	0.5199	3.36E-06
	0.4	3.0019	5.0042	10.0048	1.0194	0.7804	0.5198	7.33E-06
	0.6	3.0016	5.0056	10.0053	1.0190	0.7806	0.5198	1.05E-05
	0.8	3.0013	5.0066	10.0054	1.0187	0.7807	0.5197	1.27E-05
	1	3.0010	5.0071	10.0053	1.0186	0.7807	0.5196	1.38E-05
0.2	0.2	3.0078	5.0174	10.0131	1.0210	0.7805	0.5195	8.93E-05
	0.4	3.0067	5.0190	10.0146	1.0192	0.7810	0.5193	1.04E-04
	0.6	3.0053	5.0202	10.0151	1.0178	0.7815	0.5191	1.12E-04
	0.8	3.0040	5.0208	10.0152	1.0167	0.7818	0.5190	1.16E-04
	1	3.0029	5.0204	10.0152	1.0159	0.7821	0.5190	1.13E-04
0.9	0.2	3.0627	5.2332	10.0287	1.0339	0.7800	0.5105	9.90E-03
	0.4	3.0507	5.2102	10.0368	1.0275	0.7817	0.5112	8.04E-03
	0.6	3.0376	5.1781	10.0478	1.0199	0.7838	0.5123	5.91E-03
	0.8	3.0246	5.1380	10.0595	1.0112	0.7864	0.5138	3.89E-03
	1	3.0135	5.0920	10.0683	1.0016	0.7894	0.5154	2.29E-03
$\boldsymbol{\upsilon}$		3.0000	5.0000	10.0000	1.0200	0.7800	0.5200	0

$$ {}_{\alpha}\Delta^{\eta}_{\hat{\upsilon}_k(j)}g(j) = \frac{1}{\Gamma(1-\eta)}g(j) + \frac{\eta}{\Gamma(2-\eta)}\,{}_{\alpha}\Delta^{1}_{\hat{\upsilon}(j)}g(j). \tag{10} $$

Introducing the sign function in (10) to avoid fractional extreme points and supporting the square error estimate for convergence improvement.[45]

$$ {}_{\alpha}\Delta^{\eta}_{\hat{\upsilon}_k(j)}g(j) = \frac{g(j)\,\mathrm{sgn}\left[{}_{\alpha}\Delta^{1}_{\hat{\upsilon}_k(j)}g(j)\right]}{\Gamma(1-\eta)} + \frac{\eta\left[{}_{\alpha}\Delta^{1}_{\hat{\upsilon}_k(j)}g(j)\right]}{\Gamma(2-\eta)}. \tag{11} $$

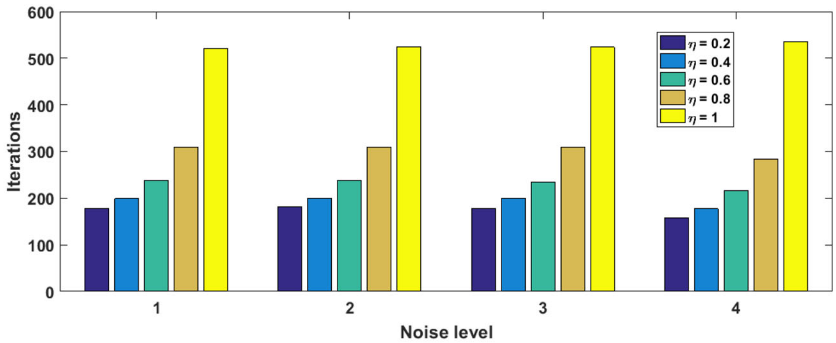
Then writing the enhanced fractional derivative of $g(j)$ with respect to the parameter vector as

$$ {}_{\alpha}\Delta^{\eta}_{\hat{\boldsymbol{\upsilon}}(j)}g(j) = \frac{g(j)\,\mathrm{sgn}\left[{}_{\alpha}\Delta^{1}_{\hat{\boldsymbol{\upsilon}}(j)}g(j)\right]}{\Gamma(1-\eta)} + \frac{\eta\left[{}_{\alpha}\Delta^{1}_{\hat{\boldsymbol{\upsilon}}(j)}g(j)\right]}{\Gamma(2-\eta)}. \tag{12} $$

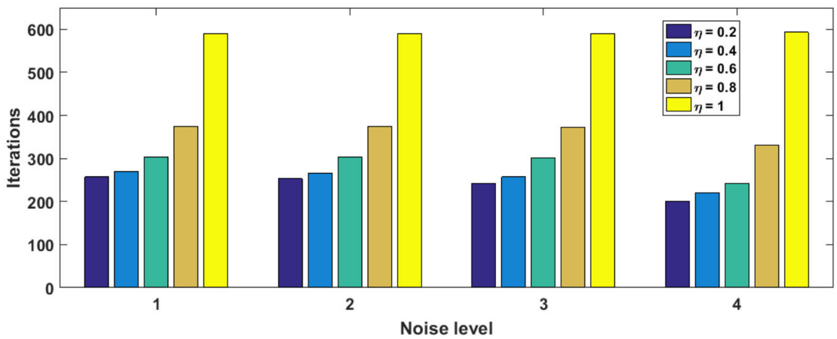
Thus, the iterative update rule of the EFDLMS for power signal parameter estimation is written as

$$ \begin{aligned} \hat{\boldsymbol{\upsilon}}(j+1) &= \hat{\boldsymbol{\upsilon}}(j) - \beta\,{}_{\alpha}\Delta^{\eta}_{\hat{\boldsymbol{\upsilon}}(j)}g(j) \\ \hat{\boldsymbol{\upsilon}}(j+1) &= \hat{\boldsymbol{\upsilon}}(j) + \beta\left[\frac{g(j)}{\Gamma(1-\eta)}\,\mathrm{sgn}\{\varepsilon(j)\boldsymbol{\xi}(j)\} + \frac{\eta}{\Gamma(2-\eta)}\{\varepsilon(j)\boldsymbol{\xi}(j)\}\right]. \end{aligned} \tag{13} $$

Putting $\eta = 1$ in (13) reduces the EFDLMS to standard LMS. The graphical interpretation of the study is given in Figure 1.



(A) Iterations for fitness 0.1



(B) Iterations for fitness 0.05

FIGURE 3 Plots representing the iterations taken in case of stopping criteria based on fitness value for Problem 1 [Colour figure can be viewed at wileyonlinelibrary.com]

4 | RESULTS AND DISCUSSION

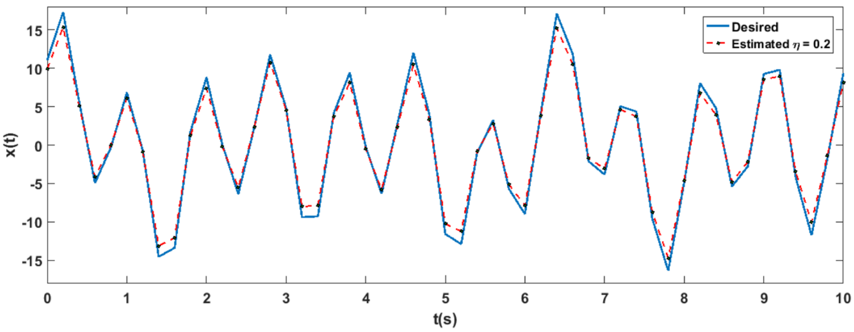
The results of numerical simulations for two case studies of the power signal estimation are given in this section with some necessary discussion. The evaluation metrics based on fitness and mean square error (MSE) are defined as

$$\text{Fitness} = \frac{\|\boldsymbol{\upsilon} - \hat{\boldsymbol{\upsilon}}\|}{\|\boldsymbol{\upsilon}\|}, \tag{14}$$

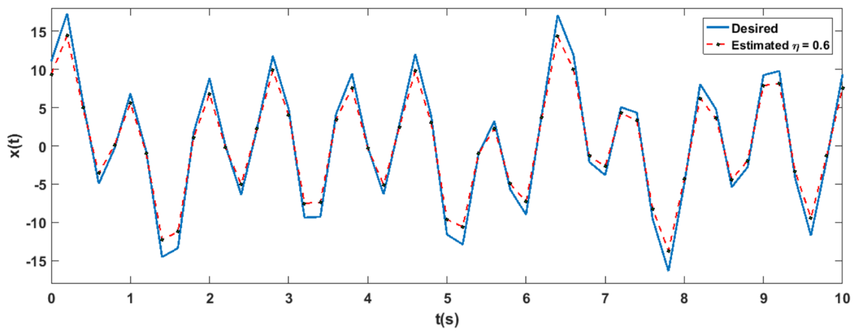
$$\text{MSE} = \text{mean}(\boldsymbol{\upsilon} - \hat{\boldsymbol{\upsilon}})^2. \tag{15}$$

4.1 | Problem 1

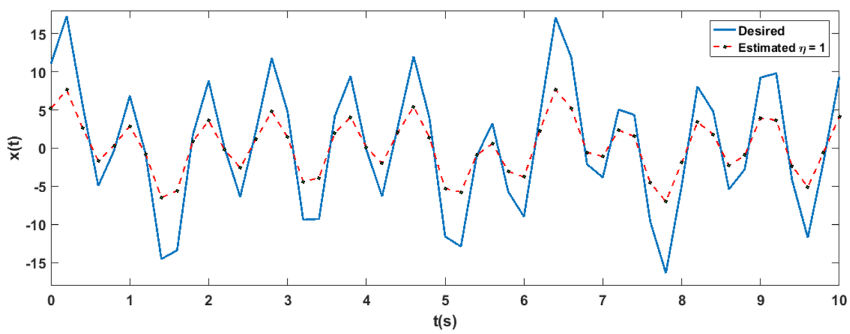
Consider a following multi frequency sine signal with known frequency.[27]



(A) $\eta = 0.2$



(B) $\eta = 0.6$



(C) $\eta = 1$

FIGURE 4 Curve fitting during initial 100 iterations in case of noise level 0.2 for Problem 1 [Colour figure can be viewed at wileyonlinelibrary.com]

$$x(t) = 3\sin(2t + 1.02) + 5\sin(3t + 0.78) + 10\sin(7t + 0.52), \tag{16}$$

then the characteristic parameters are amplitude and phased that need to be estimated

$$\boldsymbol{\upsilon} = [3,\ 5,\ 10,\ 1.02,\ 0.78,\ 0.52], \tag{17}$$

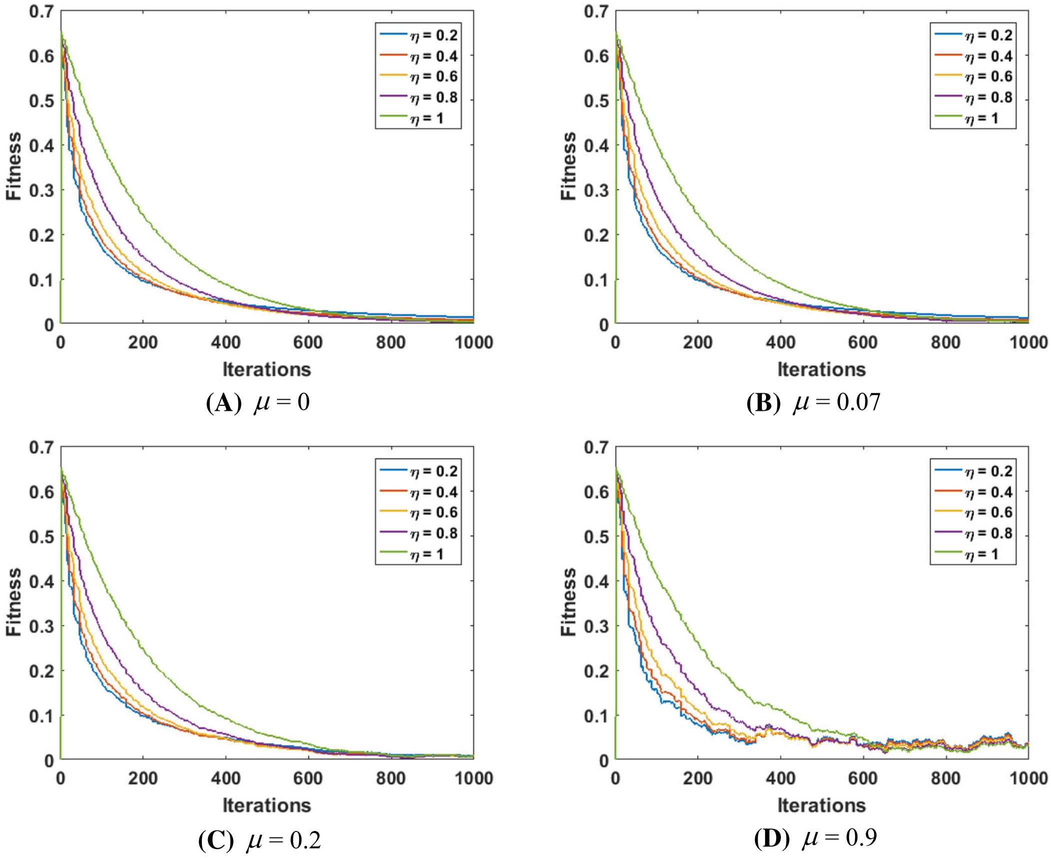


FIGURE 5 Iterative plots of EFDLMS scheme for different noise levels in Problem 2 [Colour figure can be viewed at wileyonlinelibrary.com]

TABLE 6 Fitness values against iterations for no noise scenario in Problem 2

t	$\eta = \mathbf{0.2}$	$\eta = \mathbf{0.4}$	$\eta = \mathbf{0.6}$	$\eta = \mathbf{0.8}$	$\eta = \mathbf{1}$
500	3.6859E-02	3.1561E-02	2.9100E-02	3.2191E-02	5.2789E-02
1000	1.4372E-02	8.0819E-03	4.6295E-03	3.3012E-03	4.1029E-03
1500	7.0299E-03	2.4447E-03	8.1483E-04	3.5575E-04	3.1778E-04
2000	3.7527E-03	7.7360E-04	1.4564E-04	3.8370E-05	2.4423E-05
2500	2.0905E-03	2.4969E-04	2.6374E-05	4.1946E-06	1.9044E-06
3000	1.1852E-03	8.0450E-05	4.7396E-06	4.5461E-07	1.4734E-07
3500	6.8181E-04	2.6071E-05	8.5692E-07	4.9565E-08	1.1457E-08
4000	3.9440E-04	8.4342E-06	1.5420E-07	5.3589E-09	8.7986E-10
4500	2.2830E-04	2.7106E-06	2.7469E-08	5.7236E-10	6.6732E-11
5000	1.3270E-04	8.7460E-07	4.9180E-09	6.1412E-11	5.0789E-12

The desired signal (16) with Gaussian distributed random noise having zero mean and constant variance μ^2 is generated in Matlab. Different levels of standard deviation in noise, that is, $\mu = 0$, 0.07, 0.2, 0.9, and fractional order, that is, $\eta = 0.2$, 0.4, 0.6, 0.8, 1, are considered to investigate the performance of the proposed EFDLMS scheme for power signal estimation. The learning rate δ is empirically selected as 0.01 and number of iterations considered are 5000.

TABLE 7 Fitness values against iterations for 0.07 level noise scenario in Problem 2

t	$\eta = 0.2$	$\eta = 0.4$	$\eta = 0.6$	$\eta = 0.8$	$\eta = 1$
500	3.7310E-02	3.2091E-02	2.9735E-02	3.2925E-02	5.3515E-02
1000	1.3124E-02	7.4770E-03	4.7112E-03	3.8880E-03	4.7295E-03
1500	5.4443E-03	1.9661E-03	1.2008E-03	1.2345E-03	1.2797E-03
2000	1.9016E-03	6.0872E-04	9.6000E-04	1.1330E-03	1.2066E-03
2500	6.6121E-04	1.3866E-03	1.7911E-03	2.0362E-03	2.1743E-03
3000	6.3120E-04	6.5435E-04	8.2075E-04	9.6181E-04	1.0469E-03
3500	6.6725E-04	8.0061E-04	9.6514E-04	1.0847E-03	1.1570E-03
4000	1.0986E-03	1.2843E-03	1.4367E-03	1.5493E-03	1.6211E-03
4500	9.9705E-04	1.3273E-03	1.7457E-03	2.1078E-03	2.3255E-03
5000	8.3899E-04	1.0697E-03	1.2664E-03	1.4185E-03	1.5015E-03

TABLE 8 Fitness values against iterations for 0.2 level noise scenario in Problem 2

t	$\eta = 0.2$	$\eta = 0.4$	$\eta = 0.6$	$\eta = 0.8$	$\eta = 1$
500	3.4199E-02	3.0353E-02	2.9300E-02	3.3581E-02	5.4999E-02
1000	8.7493E-03	7.0776E-03	6.8854E-03	7.1889E-03	8.0012E-03
1500	3.4207E-03	3.5187E-03	3.7497E-03	3.8639E-03	3.8072E-03
2000	2.8380E-03	3.1614E-03	3.3308E-03	3.4401E-03	3.4782E-03
2500	3.6271E-03	4.5670E-03	5.3351E-03	5.8987E-03	6.2164E-03
3000	1.9237E-03	2.2850E-03	2.6113E-03	2.8610E-03	2.9872E-03
3500	2.2132E-03	2.5750E-03	2.8959E-03	3.1517E-03	3.3008E-03
4000	3.5679E-03	3.7442E-03	4.0532E-03	4.3778E-03	4.6317E-03
4500	4.0211E-03	4.8625E-03	5.7236E-03	6.3710E-03	6.6430E-03
5000	3.6959E-03	3.9836E-03	4.2112E-03	4.3298E-03	4.2983E-03

TABLE 9 Fitness values against iterations for 0.9 level noise scenario

T	$\eta = 0.2$	$\eta = 0.4$	$\eta = 0.6$	$\eta = 0.8$	$\eta = 1$
500	4.8091E-02	4.5446E-02	4.2472E-02	4.2642E-02	6.5438E-02
1000	3.6358E-02	3.5526E-02	3.4415E-02	3.3009E-02	3.1468E-02
1500	3.0220E-02	2.8598E-02	2.6035E-02	2.2377E-02	1.7643E-02
2000	3.4524E-02	3.0512E-02	2.5520E-02	2.0124E-02	1.5886E-02
2500	3.0789E-02	3.0097E-02	2.9343E-02	2.8665E-02	2.8036E-02
3000	2.5662E-02	2.2890E-02	1.9575E-02	1.6133E-02	1.3348E-02
3500	1.7974E-02	1.7392E-02	1.6629E-02	1.5692E-02	1.4736E-02
4000	2.7660E-02	2.4209E-02	2.1132E-02	1.9700E-02	2.0841E-02
4500	4.4941E-02	4.3754E-02	4.1064E-02	3.6481E-02	2.9862E-02
5000	4.2731E-02	3.8551E-02	3.3021E-02	2.6422E-02	1.9540E-02

In order to evaluate the performance of the EFDLMS in terms of the initial convergence speed, the learning plots of fitness for all fractional order orders are given in Figure 2 in case of all noise scenarios. The results reveal that the $\eta = 0.2$ exhibits faster initial convergence with a decreasing trend as the fractional order increases.

In order to evaluate the performance of the EFDLMS in terms of the robustness and steady state dynamics, the iterative adaptive of fitness function in tabular form is presented in Tables 1–4 for μ= 0, 0.07, 0.2 and 0.9, respectively. The results reveal that the EFDLMS with $\eta = 1$ provides better steady state error with a decreasing trend as the fractional order decreases. Further, the results indicate that the EFDLMS is robust against noise and the accuracy level of the EFDLMS decreases with an increasing disturbance level. The performance of the EFDLMS is also assessed on MSE metric and results are presented in Table 5. The MSE results provide the same performance trend as seen in case of fitness function.

The EFDLMS is also evaluated by considering early stopping criteria based on achieved fitness and results are presented in Figure 3 for 0.1 and 0.05 fitness value. It is seen that the EFDLMS $\eta = 0.2$ attains the fitness value of 0.1 in around 190 iterations while the EFDLMS $\eta = 1$ (i.e., standard LMS) achieves the same fitness value in around 550 iterations. Similarly, EFDLMS $\eta = 0.2$ attains the fitness value of 0.05 in around 280 iterations while the EFDLMS $\eta = 1$ (i.e., standard LMS) achieves the same fitness value in around 580 iterations. This confirms the better performance of the EFDLMS over the standard LMS when using the early stopping criteria.

The curve fitting through estimated sinusoidal signal during initial 100 iterations are given in Figure 4 for $\eta = 0.2$, 0.6 and 1, in case of noise level 0.2. The results show that the better curve fitting is seen for $\eta = 0.2$ when compare with curve fitting in case of $\eta = 0.6$ and 1. This confirms the better performance of the EFDLMS over the conventional LMS for lower η values.

4.2 | Problem 2

Consider another multi frequency sine signal with known frequency.[29]

TABLE 10 Final MSE values of EFDLMS in Problem 1

μ	η	$\hat{\upsilon}_1$	$\hat{\upsilon}_2$	$\hat{\upsilon}_3$	$\hat{\upsilon}_4$	$\hat{\upsilon}_5$	$\hat{\upsilon}_6$	$\hat{\upsilon}_7$	$\hat{\upsilon}_8$	MSE
0	0.2	1.800	2.900	3.999	2.500	0.950	0.800	0.760	1.100	8.19E-08
	0.4	1.800	2.900	4.000	2.500	0.950	0.800	0.760	1.100	3.56E-12
	0.6	1.800	2.900	4.000	2.500	0.950	0.800	0.760	1.100	1.13E-16
	0.8	1.800	2.900	4.000	2.500	0.950	0.800	0.760	1.100	1.76E-20
	1	1.800	2.900	4.000	2.500	0.950	0.800	0.760	1.100	1.20E-22
0.07	0.2	1.804	2.899	4.002	2.500	0.948	0.799	0.759	1.099	3.28E-06
	0.4	1.804	2.899	4.003	2.500	0.947	0.799	0.758	1.098	5.33E-06
	0.6	1.805	2.899	4.004	2.500	0.946	0.799	0.758	1.099	7.46E-06
	0.8	1.805	2.899	4.005	2.500	0.946	0.800	0.758	1.099	9.36E-06
	1	1.805	2.899	4.005	2.500	0.945	0.800	0.758	1.099	1.05E-05
0.2	0.2	1.813	2.904	4.014	2.503	0.942	0.798	0.755	1.096	6.36E-05
	0.4	1.815	2.902	4.014	2.503	0.940	0.798	0.754	1.096	7.38E-05
	0.6	1.816	2.901	4.014	2.502	0.938	0.798	0.754	1.097	8.25E-05
	0.8	1.816	2.899	4.015	2.501	0.937	0.799	0.753	1.098	8.72E-05
	1	1.816	2.897	4.014	2.501	0.937	0.799	0.754	1.098	8.60E-05
0.9	0.2	1.960	2.998	4.149	2.528	0.869	0.795	0.709	1.118	8.50E-03
	0.4	1.944	2.978	4.137	2.528	0.871	0.796	0.713	1.115	6.92E-03
	0.6	1.923	2.953	4.121	2.526	0.876	0.797	0.718	1.109	5.07E-03
	0.8	1.899	2.922	4.098	2.518	0.883	0.797	0.724	1.101	3.25E-03
	1	1.873	2.888	4.066	2.504	0.892	0.795	0.731	1.091	1.78E-03
$\boldsymbol{\upsilon}$		1.800	2.900	4.000	2.500	0.950	0.800	0.760	1.100	0

$$x(t) = 1.8\sin(0.07t + 0.95) + 2.9\sin(0.5t + 0.8) + 4\sin(2t + 0.76) + 2.5\sin(1.6t + 1.1), \tag{18}$$

then the characteristic parameters are amplitude and phase that need to be estimated

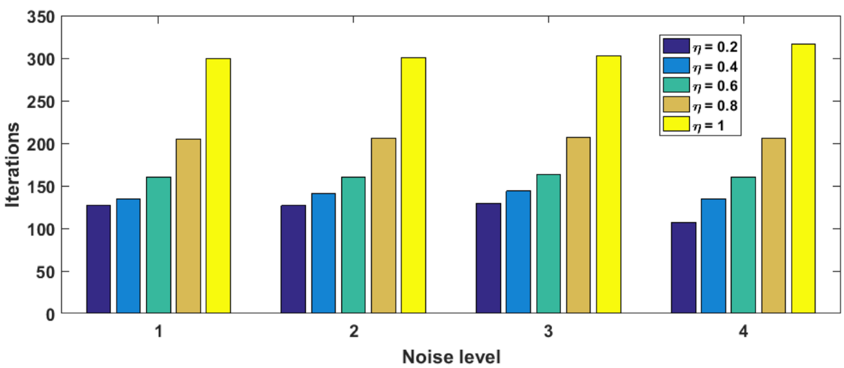
$$\mathbf{\upsilon} = [1.8, 2.9, 4, 2.5, 0.95, 0.8, 0.76, 1.1]. \tag{19}$$

The desired signal (18) of Problem 2 with Gaussian distributed random noise having zero mean and constant variance is generated in Matlab. The same learning rate, number of iterations, level of standard deviation in noise and fractional order variations are considered in this Problem as taken in Problem 1.

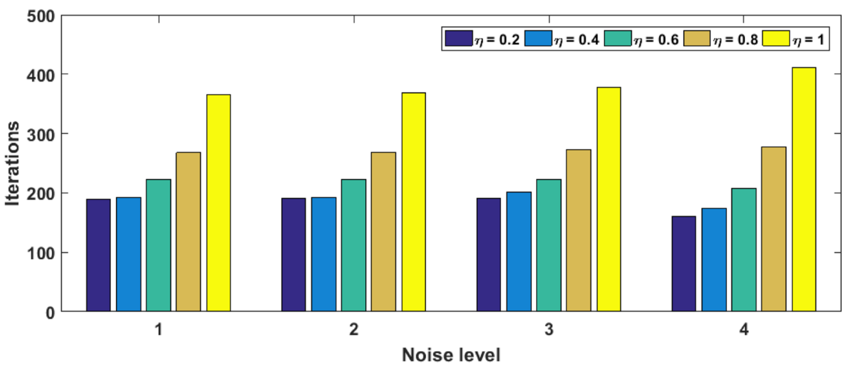
In order to evaluate convergence speed of the EFDLMS for power signal estimation of Problem 2, the learning plots of fitness for all fractional order orders are given in Figure 5 in case of all noise scenarios. The results provide the same inferences as in case of Problem 1 that $\eta = 0.2$ exhibits faster initial convergence with a decreasing trend as the fractional order increases.

In order to evaluate the steady state dynamics of the EFDLMS for Problem 2 of sinusoidal signal estimation, the iterative adaptation of fitness function in tabular form is presented in Tables 6–9 for $\mu = 0$, 0.07, 0.2 and 0.9, respectively. The results are of same trend as seen in Problem 1 that the EFDLMS with $\eta = 1$ provides better steady state error with a decreasing trend as the fractional order decreases. The results of Problem 2 based on MSE metric are presented in Table 10 for all μ and η variations. The MSE results of Table 10 further verify the robustness of the EFDLMS scheme for different disturbances.

The early stopping criteria is also applied to Problem 2 to show the effectiveness of the EFDLMS, and results are given in Figure 6 for 0.1 and 0.05 fitness value. It is seen that the EFDLMS $\eta = 0.2$ attains the fitness value of 0.1 in around 130 iterations while the EFDLMS $\eta = 1$ (i.e., standard LMS) achieves the same fitness value in around 300 iterations. Similarly, EFDLMS $\eta = 0.2$ attains the fitness value of 0.05 in around 190 iterations while the EFDLMS $\eta = 1$ (i.e.,



(A) Iterations for fitness 0.1



(B) Iterations for fitness 0.05

FIGURE 6 Plots representing the iterations taken in case of stopping criteria based on fitness value for Problem 2 [Colour figure can be viewed at wileyonlinelibrary.com] standard LMS) achieves the same fitness value in around 380 iterations. This further confirms the better convergence performance of the EFDLMS over the standard LMS in case of Problem 2.

The curve fitting is also performed in Problem 2 through estimated sinusoidal signal during initial 100 iterations and results are given in Figure 7 for $\eta = 0.2$, 0.6 and 1, in case of noise level 0.2. The results give the same inferences as in case of Problem 1 that the better curve fitting is seen for $\eta = 0.2$ when compare with curve fitting in case of $\eta = 0.6$ and 1. This shows the better performance of the EFDLMS over the conventional LMS for lower η values.

The comparison of the proposed EFDLMS for power signal estimation is also conducted with other fractional derivative based algorithms in the literature, that is, FLMS[21] and I-FLMS.[41] The fractional order range of FLMS and proposed EFDLMS ranges from 0 to 1 in contrast with the I-FLMS whose fractional order range is 0 to 1.5. The EFDLMS and I-FLMS performance is dependent on the value of fractional order, whereas the FLMS is not much affected with fractional order. The convergence speed of the EFDLMS is faster for lower fractional orders in contrast with the I-FLMS where fractional order greater than 1 gives fast convergence speed and fractional order less than 1 provides better final

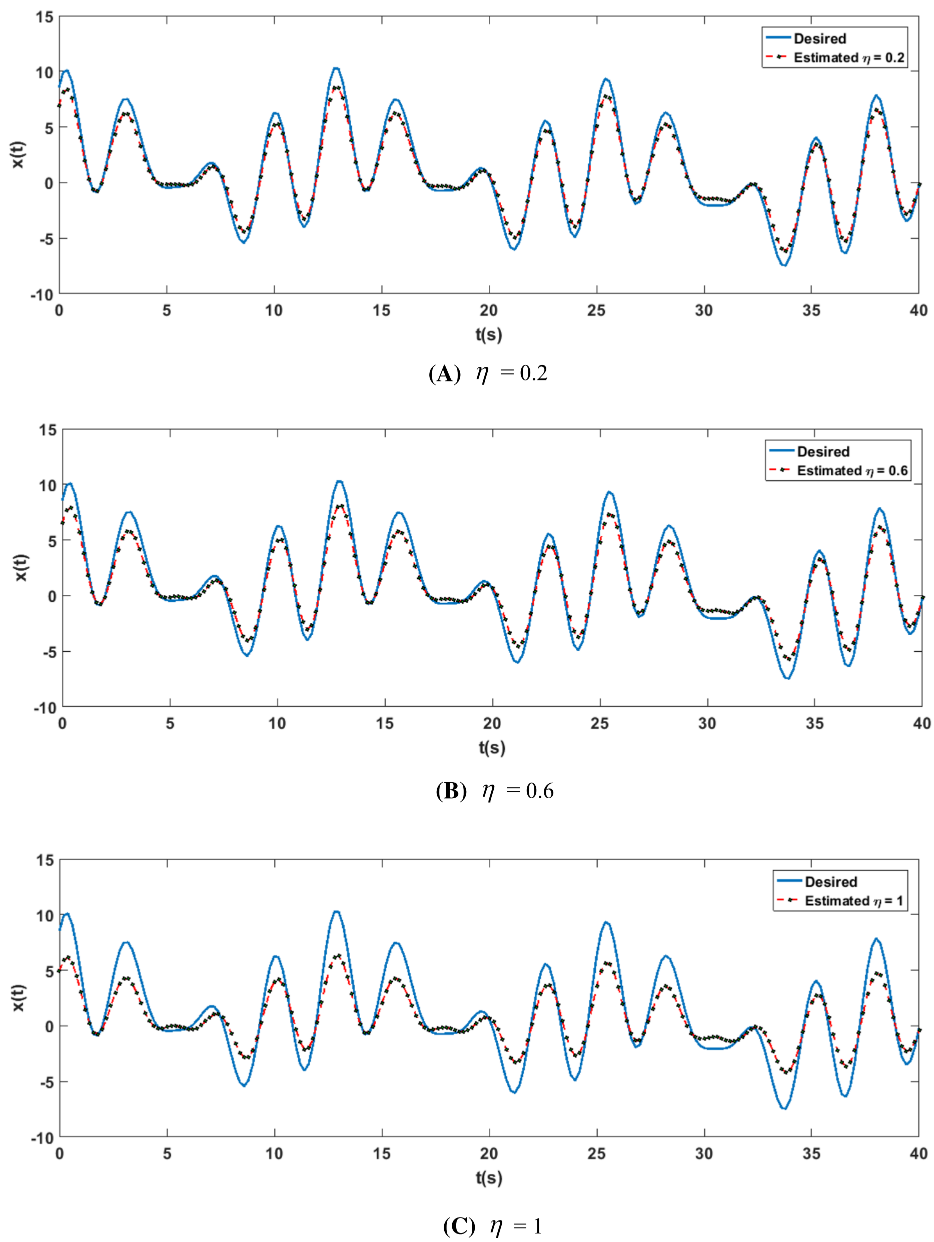


FIGURE 7 Curve fitting during initial 100 iteration in case of noise level 0.2 for Problem 2 [Colour figure can be viewed at wileyonlinelibrary.com]

estimates. Comparing the I-FLMS and FLMS for fractional order between 0 and 1, it is seen that I-FLMS performance becomes similar to the FLMS for middle fractional order values.[42]

5 | CONCLUSIONS

The main findings in the form of conclusions are given as follows.

An enhanced fractional derivative based least mean square, EFDLMS method is exploited for parameter estimation of multi frequency sinusoidal signals. The EFDLMS generalizes the conventional LMS to fractional order by using the strength of the Faa di Bruno formula to calculate the fractional derivative of the composite cost function and reduces to the standard LMS for unity fractional order. The EFDLMS involves the square of the error estimate in the update rule to improve the convergence speed. The EFDLMS is convergent and correctly estimate the characteristic parameters of the power signal based on amplitude and phase for all fractional orders. However, the convergence speed of the EFDLMS is faster for lower fractional orders. The numerical simulations verify the accurate and robust performance of the EFDLMS for various noise land fractional order variations.

In future, the proposed EFDLMS can be used for smart grid applications.[46,47] New fractional derivatives[48–50] can also be explored to design novel gradient based algorithms.

CONFLICTS OF INTEREST

This work does not have any conflicts of interest.

AUTHOR CONTRIBUTIONS

Naveed Ishtiaq Chaudhary: Investigation; methodology; software; writing-original draft. **Zeshan Aslam Khan:** Conceptualization; formal analysis; writing-review and editing. **Muhammad Asif Zahoor Raja:** Formal analysis; validation; visualization; writing-review and editing. **Iqra Ishtiaq Chaudhary:** Investigation; resources; validation; writing-review and editing.

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