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Predicting Currency Pair Trends using the Fractal Market Hypothesis

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Abstract — **This paper reports on the results of a research and development programme concerned with the analysis of currency pair exchange time series for Forex trading in an intensive applications and services environment. In particular, we present some of the preliminary results obtained for Forex trading using MetaTrader 4 with a new set of trend indicators deigned using a mathematical model that is based on the Fractal Market Hypothesis. This includes examples of various currency pair exchange rates considered over different time intervals and use of the indicators in a live trading environment to place a buy/sell order.**

Keywords — **Financial time series, Currency pair trading, Forex markets, Fractal Market Hypothesis, Predictive analysis**

I Introduction

The principal aim of a financial trader is to attempt to obtain information that can provide some confidence in the immediate future of a stock. This is often based on repeating patterns from the past, patterns that are ultimately based on the interplay between greed and fear. One of the principal components of this aim is based on the observation that there are 'waves within waves' known as Elliot Waves after Ralph Elliot who was among the first to observe this phenomenon on a qualitative basis in 1938. Elliot Waves permeate financial signals when studied with sufficient detail and imagination. It is these repeating patterns that occupy both the financial investor and the financial systems modeler alike and it is clear that although economies have undergone many changes in the last one hundred years, ignoring scale, the dynamics of market behaviour does not appear to have changed significantly.

a) Financial Time Series Modelling

In modern economies, the distribution of stock returns and anomalies like market crashes emerge as a result of considerable complex interaction. In the analysis of financial time series it is inevitable that assumptions need to be made with regard to developing a suitable model. This is the most vulnerable stage of the process with regard to developing a financial risk management model as over simplistic assumptions lead to unrealistic solutions. However, by considering the global behaviour of the financial markets, they can be modeled statistically provided the 'macroeconomic system' is complex enough in terms of its network of interconnection and interacting components.

Market behaviour results from either a strong theoretical reasoning or from compelling experimental evidence or both. In econometrics, the processes that create time series have many component parts and the interaction of those components is so complex that a deterministic description is simply not possible. When creating models of complex systems, there is a trade-off between simplifying and deriving the statistics we want to compare with reality and simulation. Stochastic simulation allows us to investigate the effect of various traders' behaviour with regard to the global statistics of the market, an approach that provides

for a natural interpretation and an understanding of how the amalgamation of certain concepts leads to these statistics and correlations in time over different scales. One cause of correlations in market price changes (and volatility) is mimetic behaviour, known as herding. In general, market crashes happen when large numbers of agents place sell orders simultaneously creating an imbalance to the extent that market makers are unable to absorb the other side without lowering prices substantially. Most of these agents do not communicate with each other, nor do they take orders from a leader. In fact, most of the time they are in disagreement, and submit roughly the same amount of buy and sell orders. This provides a diffusive economy which underlies the Efficient Market Hypothesis (EMH) and financial portfolio rationalization. The EMH is the basis for the Black-Scholes model developed for the Pricing of Options and Corporate Liabilities for which Scholes won the Nobel Prize for economics in 1997. However, there is a fundamental flaw with this model which is that it is based on a hypothesis (the EMH) that assumes price movements, in particular, the log-derivate of a price, is normally distributed and this is simply not the case. Indeed, all economic time series are characterized by long tail distributions which do not conform to Gaussian statistics thereby making financial risk management models such as the Black-Scholes equation redundant.

b) What is the Fractal Market Hypothesis?

The Fractal Market Hypothesis (FMH) is compounded in a fractional dynamic model that is non-stationary and describes diffusive processes that have a directional bias leading to long tail distributions.

The economic basis for the FMH is as follows:

- The market is stable when it consists of investors covering a large number of investment horizons which ensures that there is ample liquidity for traders;
- information is more related to market sentiment and technical factors in the short term than in the long term - as investment horizons increase and longer term fundamental information dominates;
- if an event occurs that puts the validity of fundamental information in question, long-term investors either withdraw completely or invest on shorter terms (i.e. when the overall investment horizon of the market shrinks to a uniform level, the market becomes unstable);
- prices reflect a combination of short-term technical and long-term fundamental valuation and thus, short-term price movements are likely to be more volatile than long-term trades - they are more likely to be the result of crowd behaviour;
- if a security has no tie to the economic cycle, then there will be no long-term trend and short-term technical information will dominate.

II Fractal Time Series and Rescaled Range Analysis

A time series is fractal if the data exhibits statistical self-affinity and has no characteristic scale. The data has no characteristic scale if it has a PDF with an infinite second moment. The data may have an infinite first moment as well; in this case, the data would have no stable mean either. Time series of this type are example of Hurst processes; time series that scale according to the power law,

$$\langle u(t) \rangle_t \propto t^H$$

where H is the Hurst exponent and $\langle u(t) \rangle_t$ denotes the mean value of $u(t)$ at a time t.

H. E. Hurst (1900-1978) was an English civil engineer who built dams and worked on the Nile river dam project. He studied the Nile so extensively that some Egyptians reportedly nicknamed him 'the father of the Nile.' The Nile river posed an interesting problem for Hurst as a hydrologist. When designing a dam, hydrologists need to estimate the necessary storage capacity of the resulting reservoir. An influx of water occurs through various natural sources (rainfall, river overflows etc.) and a regulated amount needed to be released for primarily agricultural purposes. The storage capacity of a reservoir is based on the net water flow. Hydrologists usually begin by assuming that the water influx is random, a perfectly reasonable assumption when dealing with a complex ecosystem. Hurst, however, had studied the 847-year record that the Egyptians had kept of the Nile river overflows, from 622 to 1469. Hurst noticed that large overflows tended to be followed by large overflows until abruptly, the system would then change to low overflows, which also tended to be followed by low overflows. There seemed to be cycles, but with no predictable period. Standard statistical analysis revealed no significant correlations between observations, so Hurst developed his own methodology. Hurst was aware of Einstein's (1905) work on Brownian motion (the erratic path followed by a particle suspended in a fluid) who observed that the distance the particle covers increased with the square root of time, i.e.

$$R \propto \sqrt{t}$$

where R is the range covered, and t is time. This relationship results from the fact that increments

are identically and independently distributed random variables. Hurst's idea was to use this property to test the Nile River's overflows for randomness. In short, his method was as follows: Begin with a time series x_i (with $i = 1, 2, ..., n$) which in Hurst's case was annual discharges of the Nile River. (For markets it might be the daily changes in the price of a stock index.) Next, create the adjusted series, $y_i = x_i - \bar{x}$ (where $\bar{x}$ is the mean of x_i). Cumulate this time series to give

$$Y_i = \sum_{j=1}^{i} y_j$$

such that the start and end of the series are both zero and there is some curve in between. (The final value, Y_n has to be zero because the mean is zero.) Then, define the range to be the maximum minus the minimum value of this time series,

$$R_n = \max(Y) - \min(Y).$$

This adjusted range, R_n is the distance the systems travels for the time index n, i.e. the distance covered by a random walker if the data set y_i were the set of steps. If we set $n = t$ we can apply Einstein's equation provided that the time series x_i is independent for increasing values of n. However, Einstein's equation only applies to series that are in Brownian motion. Hurst's contribution was to generalize this equation to

$$(R/S)_n = cn^H$$

where S is the standard deviation for the same n observations and c is a constant. We define a Hurst process to be a process with a (fairly) constant H value and the R/S is referred to as the 'rescaled range' because it has zero mean and is expressed in terms of local standard deviations. In general, the R/S value increases according to a power law value equal to H known as the Hurst exponent. This scaling law behaviour is the first connection between Hurst processes and fractal geometry.

Rescaling the adjusted range was a major innovation. Hurst originally performed this operation to enable him to compare diverse phenomenon. Rescaling, fortunately, also allows us to compare time periods many years apart in financial time series. As discussed previously, it is the relative price change and not the change itself that is of interest. Due to inflationary growth, prices themselves are a significantly higher today than in the past, and although relative price changes may be similar, actual price changes and therefore volatility (standard deviation of returns) are significantly higher. Measuring in standard deviations (units of volatility) allows us to minimize this problem. Rescaled range analysis can also describe time series that have no characteristic scale, another characteristic of fractals. By considering the logarithmic version of Hurst's equation, i.e.

$$\log(R/S)_n = \log(c) + H\log(n)$$

it is clear that the Hurst exponent can be estimated by plotting $\log(R/S)$ against the $\log(n)$ and solving for the gradient with a least squares fit. If the system were independently distributed, then $H = 0.5$. Hurst found that the exponent for the Nile River was $H = 0.91$, i.e. the rescaled range increases at a faster rate than the square root of time. This meant that the system was covering more distance than a random process would, and therefore the annual discharges of the Nile had to be correlated.

It is important to appreciate that this method makes no prior assumptions about any underlying distributions, it simply tells us how the system is scaling with respect to time. So how do we interpret the Hurst exponent? We know that $H = 0.5$ is consistent with an independently distributed system. The range $0.5 < H \leq 1$, implies a persistent time series, and a persistent time series is characterized by positive correlations. Theoretically, what happens today will ultimately have a lasting effect on the future. The range $0 < H \leq 0.5$ indicates anti-persistence which means that the time series covers less ground than a random process. In other words, there are negative correlations. For a system to cover less distance, it must reverse itself more often than a random process.

III Lévy Processes

Lévy processes are random walks whose distribution has infinite moments and 'long tails'. The statistics of (conventional) physical systems are usually concerned with stochastic fields that have PDFs where (at least) the first two moments (the mean and variance) are well defined and finite. Lévy statistics is concerned with statistical systems where all the moments (starting with the mean) are infinite. Many distributions exist where the mean and variance are finite but are not representative of the process, e.g. the tail of the distribution is significant, where rare but extreme events occur. These distributions include Lévy distributions [1],[2]. Lévy's original approach to deriving such distributions is based on the following question: Under what circumstances does the distribution associated with a random walk of a few steps look the same as the distribution after many steps (except for scaling)? This question is effectively the same as asking under what circumstances do we obtain a random walk that is statistically self-affine. The characteristic function $P(k)$ of such a distribution $p(x)$ was first shown by Lévy to be

given by (for symmetric distributions only)

$$P(k) = \exp(-a \mid k \mid^{\gamma}), \quad 0 < \gamma \leq 2$$

where a is a constant and γ is the Lévy index. For $\gamma \geq 2$, the second moment of the Lévy distribution exists and the sums of large numbers of independent trials are Gaussian distributed. For example, if the result were a random walk with a step length distribution governed by $p(x), \quad \gamma \geq 2$, then the result would be normal (Gaussian) diffusion, i.e. a Brownian random walk process. For $\gamma < 2$ the second moment of this PDF (the mean square), diverges and the characteristic scale of the walk is lost. For values of γ between 0 and 2, Lévy's characteristic function corresponds to a PDF of the form

$$p(x) \sim \frac{1}{x^{1+\gamma}}, \quad x \to \infty$$

a) Long Tails

If we compare this PDF with a Gaussian distribution given by (ignoring scaling normalisation constants)

$$p(x) = \exp(-\beta x^2)$$

which is the case when $\gamma = 2$ then it is clear that a Lévy distribution has a longer tail. This is illustrated in Figure 1. The long tail Lévy distribution

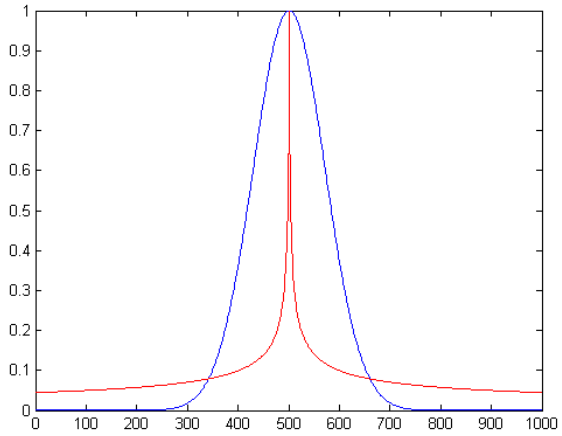

Fig. 1: Comparison between a Gaussian distribution (blue) for $\beta = 0.0001$ and a Lévy distribution (red) for $\gamma = 0.5$ and $p(0) = 1$.

represents a stochastic process in which extreme events are more likely when compared to a Gaussian process. This includes fast moving trends that occur in economic time series analysis. Moreover, the length of the tails of a Lévy distribution is determined by the value of the Lévy index such that the larger the value of the index the shorter the tail becomes. Unlike the Gaussian distribution which has finite statistical moments, the Lévy distribution has infinite moments and 'long tails'.

b) Lévy Processes and the Fractional Diffusion Equation

Lévy processes are consistent with a fractional diffusion equation [3].

$$\sigma \frac{\partial}{\partial t} u(x,t) = \frac{\partial^{\gamma}}{\partial x^{\gamma}} u(x,t), \quad \gamma \in (0,2]$$

where σ is the coefficient of diffusion. For unit coefficient of diffusion, we consider the equation

$$\left(\frac{\partial^{\gamma}}{\partial x^{\gamma}} - \frac{\partial}{\partial t} \right) u(x,t) = \delta(x) n(t), \quad q > 0, \quad x \to 0$$

where $n(t)$ is 'white noise' whose solution is, ignoring scaling constants, given by

$$u(t) = \frac{1}{t^{1-1/\gamma}} \otimes n(t)$$

This solution is consistent with the solution to the fractional diffusion equation

$$\left(\frac{\partial^2}{\partial x^2} - \frac{\partial^q}{\partial t^q} \right) u(x,t) = \delta(x) n(t),$$

where $\gamma^{-1} = q/2$ [4] and where q - the 'Fourier Dimension' - is related to the Hurst exponent by $q = 2H + 1$. Thus, the Lévy index γ, the Fourier Dimension q and the Hurst exponent H are all simply related to each other. Moreover, these parameters quantify stochastic processes that have long tails and thereby by transcend financial models based on normal distributions such as the Black-Scholes model discussed in Section II. In this paper, we study the behaviour of q focusing on its predictive power for indicating the likelihood of a future trend in Forex time series.

IV Forex Market

The Forex or Foreign Exchange market is the largest and most fluid of the global markets involving trades approaching 4 Trillion per day. The market is primarily concerned with trading currency pairs but includes currency futures and options markets. It is similar to other financial markets but the volume of trade is much higher which comes from the nature of the market in terms of its short term profitability. The market determines the relative values of different currencies and most banks contribute to the market as do financial companies, institutions, individual speculators and investors and even import/export companies. The high volume of the Forex market leads to high liquidity and thereby guarantees stable spreads during a working week and contract execution with relatively small slippages even in aggressive price movements. In a typical foreign exchange transaction, a party purchases a quantity of one currency by paying a quantity of another currency.

The Forex is a de-centralised 'over the counter market' meaning that there are no agreed centres or exchanges which an investor needs to be connected to in order to trade. It is the largest world wide network allowing customers trade 24 hours per day usually from Monday to Friday. Traders can trade on Forex without any limitations no matter where they live or the time chosen to enter a trade. The accessibility of the Forex market has made it particularly popular with traders and consequently, a range of Forex trading software has been developed for internet based trading. In this paper, we report on a new indicator based on the interpretation of q computed via the Hurst exponent H that has been designed to optimize Forex trading through integration into the MetaTrader 4 system.

V MetaTrader 4

MetaTrader 4 is a platform for e-trading that is used by online Forex traders [5] and provides the user with real time internet access to most of the major currency exchange rates over a range of sampling intervals including 1 min, 4 mins, 1 hour and 1 day. The system includes a built-in editor and compiler with access to a user contributed free library of software, articles and help. The software utilizes a proprietary scripting language, MQL4 [6] (based on C), which enables traders to develop Expert Advisors, custom indicators and scripts. MetaTrader's popularity largely stems from its support of algorithmic trading. This includes a range of indicators and the focus of the work reported in this paper, i.e. the incorporation of a new indicator based on the approach considered in Section III and Section IV.

a) Basic Algorithm - The 'q-Algorithm'

Given a stream of Forex data $u_n, \quad n = 1, 2, ..., N$ where N defines the 'look-back' window or 'period', we consider the Hurst model

$$u_n = cn^H$$

which is linearised by taking the logarithmic transform to give

$$\log(u_n) = \log(c) + H \log(n)$$

where c is a constant of proportionality

The basic algorithm is as follows:

1. For a moving window of length N (moved one element at a time) operating on an array of length L, compute $q_j = 1 + 2H_j, \quad j = 1, 2, ..., L-N$ using the Orthogonal Linear Regression Algorithm [7] and plot the result.

2. For a moving window of length M compute the moving average of q_j denoted by $\langle q_j \rangle_i$ and plot the result in the same window as the plot of q_j.

3. Compute the gradient of $\langle q_j \rangle_i$ using a different user defined moving average window of length K and a forward differencing scheme and plot the result.

4. Compute the second gradient of $\langle q_j \rangle_i$ after applying a moving average filter using a centre differencing scheme and plot the result in the same window.

b) Fundamental Observations

The second gradient is computed to provide an estimate of the acceleration associated with moving average characteristics of q_j. However, the gradient of $\langle q_j \rangle_i$ denoted by $\langle q_j \rangle_i'$ provides the most significant behaviour in terms of assessing the point in time at which a trend is likely to occur, in particular, the points in time at which $\langle q_j \rangle_i'$ crosses zero. The principal characteristic is compounded in the following observation:

$\langle q_j \rangle_i' > 0$ correlates with an upward trend
$\langle q_j \rangle_i' < 0$ correlates with a downward trend

where a change in the polarity of $\langle q_j \rangle_i' < 0$ indicates a change in the trend subject to a given tolerance T. A tolerance zone is therefore established $| \langle q_j \rangle_i' | \in T$ such that if the signal $\langle q_j \rangle_i' > 0$ enters the tolerance zone, then a bar is plotted indicating the end of an upward trend and if $\langle q_j \rangle_i' < 0$ enters the tolerance zone then a bar is plotted indicating the end of a downward trend. Figure 2 shows an example of the MetaTrader GUI with the new indicators included operating on the signal for the Euro-USD exchange rate with 1 min sampled data. The vertical bars clearly indicate the change in a trend for the window of data provided in this example. The parameters settings (N, M, K, T) for this example are $(512, 10, 300, 0.1)$. In each case, a change in the gradient correlates with a change in the trend of the time series in a way that is reproducible at all scales.

VI Benifits of the q-Algorithm

For FOREX data $q(t)$ varies between 1 and 2 as does γ for q in this range since $\gamma^{-1}(t) = q(t)/2$. As the value of q increases, the Lévy index decreases and the tail of the data therefore gets longer. Thus as $q(t)$ increases, so does the likelihood of a trend occurring. In this sense, $q(t)$ provides a measure on the behaviour of an economic time series in terms of a trend (up or down) or otherwise. By applying a moving average filter to $q(t)$ to smooth the data, we obtained a signal $\langle q(t) \rangle(\tau)$ that provides an indication of whether a trend is occurring in the data over a user defined window (the period).

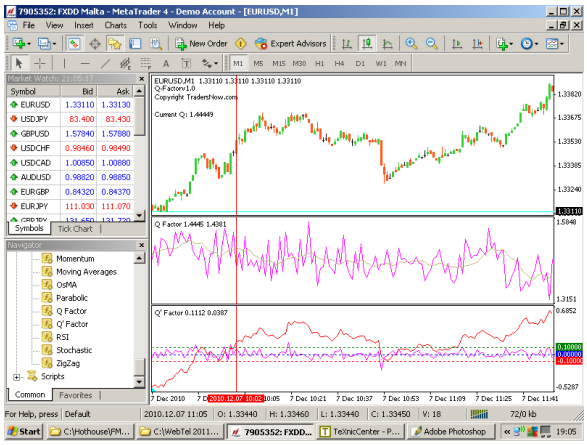

Fig. 2: MetaTrader 4 GUI for new indicators. Top window: Euro-USD exchange rate signal for 1 min sampled data using Japanese Candles (Green=up; Red=down); Center window: q_j (cyan) and moving average of q_j (Green); Bottom window: first (red) and second (cyan) gradients of the moving average for $(N, M, K, T) = (512, 10, 300, 0.1)$.

This observation reflects a result that is a fundamental kernel of the Fractal Market Hypothesis, namely, that a change in the Lévy index precedes a change in the financial signal from which the index has been computed (from past data). In order to observe this effect more clearly, the gradient $\langle q(t)\rangle'(\tau)$ is taken. This provides the user with a clear indication of a future trend based on the following observation: if $\langle q(t)\rangle'(\tau) > 0$, the trend is positive; if $\langle q(t)\rangle'(\tau) < 0$, the trend is negative; if $\langle q(t)\rangle'(\tau)$ passes through zero a change in the trend may occur. By establishing a tolerance zone associated with a polarity change in $\langle q(t)\rangle'(\tau)$, the importance of any indication of a change of trend can be regulated in order to optimise a buy or sell order. This is the principle basis and rationale for the 'q-algorithm.

VII Conclusion

The Fractal Market Hypothesis has many conceptual and quantitative advantages over the Efficient Market Hypothesis for modelling and analysing financial data. One of the most important points is that the Fractal Market Hypothesis is consistent with an economic time series that include long tails in which rare but extreme events may occur and, more commonly, trends evolve. In this paper we have focused on the use of the Hypothesis for modelling Forex data and have shown that by computing the Hurst exponent, an algorithm can be designed that appears to accurately predict the upward and downward trends in Forex data over a range of scales subject to appropriate parameter settings and tolerances. The optimisation of these parameters can be undertaken using a range of back-testing trials to develop a strategy for optimising the profitability of Forex trading. In the trials undertaken to date, the system can generate a profitable portfolio over a range of currency exchange rates involving hundreds of Pips[1] and over a range of scales providing the data is consistent and not subject to market shocks generated by entirely unpredictable effects that have a major impact on the markets. This result must be considered in the context that the Forex markets are noisy, especially over smaller time scales, and that the behaviour of these markets can, from time to time, yield a minimal change of Pips when $\langle q(t)\rangle'(\tau)$ is within the tolerance zone establish for a given currency pair exchange rate.

The use of the indicators discussed in this paper for Forex trading is an example of a number of intensive applications and services (RIAS) being developed for financial time series analysis and forecasting. MetaTrader 4 is just one of a range of financial risk management systems that are being used by the wider community for decentralised market trading, a trend that is set to increase throughout the financial services sector given the current economic environment. The current version of MetaTrader 4 described in this paper is undergoing continuous improvements and assessment, details of which can be obtained from TradersNow.com.

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[1] A Pip (Percentage in point) is the smallest price increment in Forex trading.