2022

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Funder: Higher Education Authority, Ireland; TU Dublin Tallaght Campus

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Astronomy and Computing 38 (2022) 100537

Contents lists available at ScienceDirect

Astronomy and Computing

journal homepage: www.elsevier.com/locate/ascom

Full length article

The Locus Algorithm: A novel technique for identifying optimised pointings for differential photometry

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ARTICLE INFO

Article history:
Received 27 April 2021
Accepted 8 December 2021
Available online 15 December 2021

Keywords:
Differential photometry
Transit method
Exoplanets
Quasars
Optimisation
Algorithms

ABSTRACT

Studies of the photometric variability of astronomical sources from ground-based telescopes must overcome atmospheric extinction effects. Differential photometry by reference to an ensemble of reference stars which closely match the target in terms of magnitude and colour can mitigate these effects. This Paper describes the design, implementation, and operation of a novel algorithm – The Locus Algorithm – which enables optimised differential photometry. The Algorithm is intended to identify, for a given target and observational parameters, the Field of View (FoV) which includes the target and the maximum number of reference stars similar to the target. A collection of objects from a catalogue (e.g. SDSS) is filtered to identify candidate reference stars and determine a rating for each which quantifies its similarity to the target. The algorithm works by defining a locus of points around each candidate reference star, upon which the FoV can be centred and include the reference at the edge of the FoV. The Points of Intersection (PoI) between these loci are identified and a score for each PoI is calculated. The PoI with the highest score is output as the optimum pointing. The steps of the algorithm are precisely defined in this paper. The application of The Locus Algorithm to a sample target, SDSS1237680117417115655, from the Sloan Digital Sky Survey is described in detail. The algorithm has been defined here and implemented in software which is available online. The algorithm has also been used to generate catalogues of pointings to optimise Quasar variability studies and to generate catalogues of optimised pointings in the search for Exoplanets via the transit method.

1. Introduction

Photometric variability studies involve identifying variations in the brightness of celestial objects as a function of time. Intrinsic variability can occur on timescales from milliseconds to years and in some instances is a critical tool in allowing competing theoretical models to be assessed. For example, time-resolved precision photometry has the potential to infer very small scale structures in astrophysical jets at a scale which is not possible with direct imaging (Smith et al., 2008). Alternatively, precision photometry can detect Earth-sized planets around M-type stars via the transit method (Giltinan et al., 2011; Everett and Howell, 2001). Variability which is non-repeatable in nature, such as in astrophysical jets, is the most difficult to quantify and places the greatest requirements for the reliability of the photometry to be as high as possible.

The Earth's atmosphere causes incoherent wavelength-dependent variations in the flux detected from a source on timescales from 10 ms upwards. If uncorrected, this can appear as intrinsic variability, leading to erroneous conclusions about the structure of a source and the underlying astrophysical drivers (Smith et al., 2008). Differential photometry, in which a target is compared to a number of reference stars, attempts to minimise extrinsic effects atmospheric by comparing the brightness of a target to reference stars in the same Field of View (FoV). The assumption is that stars which are spatially close to the target undergo the same atmospheric influences as the target itself (Burdanov et al., 2014). If the stars are of similar brightness and colour as the target, then any effects of the atmosphere distributed across very short spatial scales should equally affect all objects in the FoV (Young et al., 1991; Howell, 2006). As per Milone and Pel (2011), the more similar the objects in the FoV used to determine the photometry, the less likely it is that the atmosphere will play a major role in determining the precision. However, photon-limited precisions are rarely achieved in practice, indicating that the removal of atmospheric effects even amongst objects which

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https://doi.org/10.1016/j.ascom.2021.100537
2213-1337/

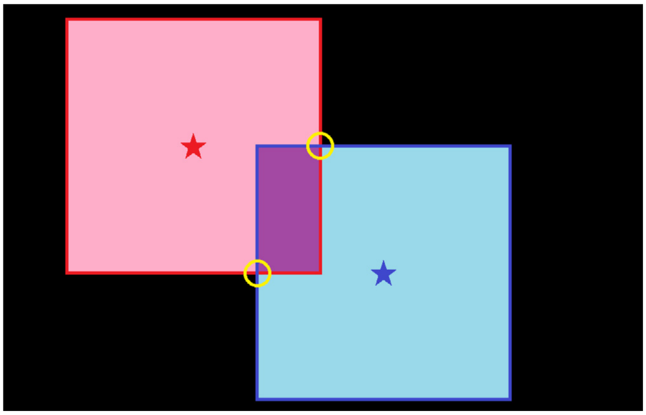

Fig. 1. Diagrammatic representation of two stars with loci (red and blue perimeter lines), which intersect and produce two Points of Intersection (PoI) circled in yellow.
Source: Modified from Creaner (2016).

are spatially close, is limited by the atmospheric conditions at the time of the observations in a complicated way (Everett and Howell, 2001; Howell and Everett, 2002). It has been shown that the general approach to differential photometry can be improved upon by taking very short integrations and selecting those time periods in which the atmosphere is most stable (Giltinan et al., 2011).

Despite the knowledge that reference stars which are similar to the source provide the best approach, there has been no systematic attempt to define the optimum FoV around a target source in which parameters such as colour, magnitude, field-crowding and field orientation have been used to determine the optimum pointings. Here we describe a novel algorithm – the Locus Algorithm – which identifies the pointing for the which the resultant observational FoV includes the target and the most photometrically appropriate reference stars available.

2. Conceptual basis to The Locus Algorithm

A locus can be defined around any star such that a FoV centred on any point on the locus will include the star at the edge of the FoV. For fields containing stars close to one another, if one locus intersects with another, they produce Points of Intersection (PoI) as shown in Fig. 1.

A FoV centred on any such PoI will include both stars associated with creating it. At PoI the set of stars that can be included in a FoV changes.

The Locus Algorithm considers candidate reference stars in what is termed a Candidate Zone (CZ) - the zone of sky centred on the target within which a FoV can be selected which includes both the reference star and the target. For Candidate Reference Stars within the CZ, loci are determined, and all relevant PoI are identified. Each PoI is assigned a score derived from the number and similarity of reference stars included in a FoV centred on that PoI. The PoI with the highest score becomes the pointing for the target.

3. Locus Algorithm design

Based on the conceptual outline above, this section provides a mathematical definition of the Locus Algorithm and an explanation of the terms used in it. Section 4 describes a worked example of this algorithm applied to a sample star, *SDSS1237680117417115655*.

3.1. Definition of coordinate system and locus

For computational efficiency, The Locus Algorithm considers a Field of View to be a rectangular area on the sky orientated such that the edges are aligned with the primary x and y axes of the Cartesian coordinate system. Movement of the field is restricted to x or y translations.

However, the Celestial coordinate system is defined by the Equatorial coordinate system, with coordinates specified by Right Ascension (RA) and Declination (Dec). Because this is a spherical coordinate system, unit angle in RA is foreshortened, with the degree of foreshortening defined in Expression (1)

$$angle\ in\ RA = \frac{True\ Angle}{cos(Dec)} \tag{1}$$

Expression 1: Right Ascension foreshortening with Declination

By using this conversion, it is possible to approximate to a high degree of accuracy a Cartesian coordinate system using RA and Dec; with a small FoV of East–West size R and North–South size S about a target located at point RA_t and Dec_t. Expression (2) defines a corrected angular size in RA direction (R')

$$R' = \frac{R}{cos(Dec_t)} \tag{2}$$

Expression 2: Definition of a corrected angular size along the RA direction (R')

Given these terms, Expression (3) defines the FoV. This definition is accurate to approximately 1% for a FoV of area 15 $'$ square outside celestial polar regions.

$$\begin{aligned} RA_t - \frac{R'}{2} \le RA \le RA_t + \frac{R'}{2} \\ Dec_t - \frac{S}{2} \le Dec \le Dec_t + \frac{S}{2} \end{aligned} \tag{3}$$

Expression 3: Definition of a FoV of size R x S centred on a target at RA_t, Dec_t)

We can therefore define the locus about any star on the sky located at RA_t and Dec_t as the values of Right Ascension and Declination as defined in Expression (3).

3.2. Candidate Zone

A Candidate Zone is defined as a region centred on the target. It is equal to four times the area of the FoV, within which any reference star can be included in a FoV with the target. Any such star can therefore be considered as a candidate reference star in identifying the optimum pointing. Conversely, stars outside the candidate zone cannot be included in a Field of View with the target and cannot therefore be considered as candidates reference stars. Hence the Candidate Zone is the maximum region of sky centred on the target from which to choose candidate reference stars when identifying an optimum pointing for a given target. For a target positioned at coordinates RA_t and Dec_t the resulting Candidate Zone is defined by Expression (4).

$$\begin{aligned} RA_t - R' \le RA_r \le RA_t + R' \\ Dec_t - S \le Dec_r \le Dec_t + S \end{aligned} \tag{4}$$

Expression 4: Definition of a Candidate Zone of size 2R x 2S centred on a target with coordinates (RA_t, Dec_t), in which zone reference stars with coordinates (RA_r, Dec_r) can be found.

3.3. Identification and filtering of reference stars

For each target, a list of candidate reference stars in its Candidate Zone is produced based on the following criteria:

Position: the reference star must be in the Candidate Zone as defined in Expression (4).

Magnitude: the magnitude of the reference star (mag_r) must be within a user-defined limit (Δmag) of the target's magnitude (mag_t) as shown in Expression (5).

Colour: the colour index (e.g. $g - r$) of the reference star (col_r) must match the colour of the target (col_t) to within a user-specified limit(Δcol) as shown in Expression (6).

Resolvability: the reference star must be resolvable, i.e. no other star that would impact brightness measurements within a user-specified resolution limit.

All stars in the Candidate Zone which pass these initial filters become the list of candidate reference stars for which loci will be identified.

$$mag_t - \Delta mag < mag_r < mag_t + \Delta mag \tag{5}$$

Expression 5: Definition of the limits of mag difference between the target and references.

$$col_t - \Delta col < col_r < col_t + \Delta col \tag{6}$$

Expression 6: Definition of the limits of colour difference between the target and references.

For each star which passes these filters, a *rating* is calculated. This calculation is a modular element of the algorithm, and can be modified to suit the needs of a given observer. The rating system gives a measure of how close, spectrally, each reference star is to the target as shown in Expression (7). These ratings are calculated by first calculating the colour indices for the target and the reference using the next longer-wavelength filter (col_l) and the next shorter-wavelength filter (col_s). The difference between each of these colour indices for the target and the reference is calculated (Δcol) and compared with the limit (Δcol_{max}). The ratio between these values is subtracted from one to get a normalised rating between 0 and 1 (note that as defined in Expression (6), $\Delta col < \Delta col_{max}$, preventing negative ratings). The ratings from each of the pairs of colour indices are multiplied together to give the final rating for a Candidate Reference Star.

$$\begin{aligned}
col_l &= r - i & col_s &= g - r \\
\Delta col_l &= col_{l,t} - col_{l,r} & \Delta col_s &= col_{s,t} - col_{s,r} \\
Rating_l &= 1 - \left|\frac{\Delta col_l}{\Delta col_{max}}\right| & Rating_s &= 1 - \left|\frac{\Delta col_s}{\Delta col_{max}}\right| \\
Rating &= Rating_l \times Rating_s
\end{aligned} \tag{7}$$

Expression 7: Definition of the scoring system as used in the generation of the Quasar Catalogue. g, r and i are SDSS magnitudes. *col* refers to colour indices. Subscript l and s refer to long- and short-wavelength colour indices respectively. Subscript t refers to the target, while subscript r refers to a reference.

3.4. Identifying the effective locus for each candidate reference star

The locus associated with each candidate reference star must be identified based on Expression (3). For the purposes of identifying PoI, only the side surrounding a given candidate reference star closest to the target need be considered. Hence, we can define the effective locus for such a candidate reference star as a single line of constant RA and a single line of constant Dec nearest the target star as shown in Fig. 2.

Specifically, the effective locus can be defined as a corner point of the locus and two lines: one of constant RA and the other of constant Dec emanating from the corner point.

Using the Equatorial Coordinate System discussed in Section 3.1, with coordinates of the target specified by (RA_t, Dec_t) and coordinates of the candidate reference star defined by (RA_r, Dec_r) and a size of FoV of horizontal length R and vertical length S, the coordinates of the corner point (RA_c, Dec_c) are defined as shown in Expression (8).

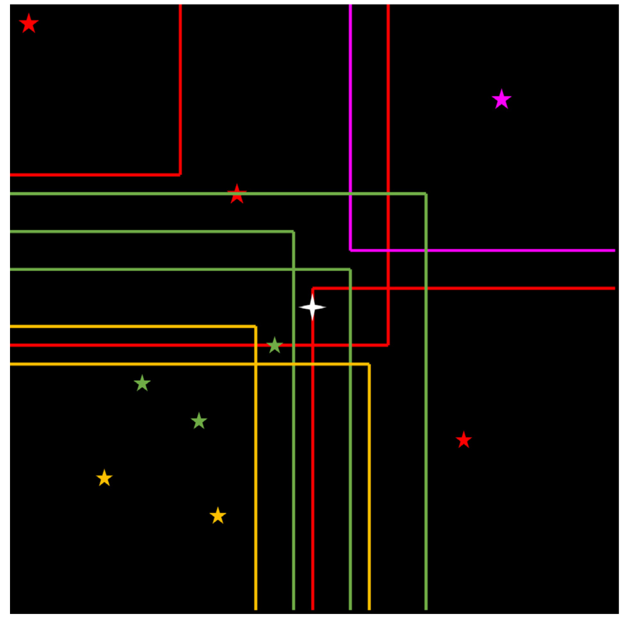

Fig. 2. Each effective locus is defined by assigning a pair of RA and Dec coordinates for a corner point and a pair of lines North or South and East or West from the corner point. For example, the magenta reference star lies to the North-East of the target. Its corner point is thus to the South and West of it, and its locus (also in magenta) drawn to the North and East from its cornerpoint.
Source: Modified from Creaner (2016).

$$\begin{aligned}
RA_t \le RA_r &\Rightarrow RA_c = RA_r - \frac{R'}{2} \\
RA_t > RA_r &\Rightarrow RA_c = RA_r + \frac{R'}{2} \\
Dec_t \le Dec_r &\Rightarrow Dec_c = Dec_r - \frac{S}{2} \\
Dec_t > Dec_r &\Rightarrow Dec_c = Dec_r + \frac{S}{2}
\end{aligned} \tag{8}$$

Expression 8: Definition of the corner point (RA_C, Dec_C) of the effective locus for a FoV of size R x S for a candidate reference star at (RA_r, Dec_r) and a target at (RA_t, Dec_t)

The directions *DirRA* (the direction of the line of constant RA) and *DirDec* (the direction of the line of constant Dec) of the lines is determined by the RA and Dec of the candidate reference star relative to that of the target are given in Expression (9) and as described below.

- If the RA of the candidate is greater than the target, the line of constant Dec is defined to be in the direction of increasing RA
- If the RA of the candidate is less than the target, the line of constant Dec is defined to be in the direction of decreasing RA
- If the Dec of the candidate is greater than the target, the line of constant RA is defined to be in the direction of increasing Dec
- If the Dec of the candidate is less than the target, the line of constant RA is defined to be in the direction of decreasing Dec

$$\begin{aligned}
RA_t \le RA_r &\Rightarrow DirDec = +ive \\
RA_t > RA_r &\Rightarrow DirDec = -ive \\
Dec_t \le Dec_r &\Rightarrow DirRA = +ive \\
Dec_t > Dec_r &\Rightarrow DirRA = -ive
\end{aligned} \tag{9}$$

Expression 9: Definition of the directions ($DirRA$, $DirDec$) of the lines from the corner point of that define the effective locus for a FoV of size R x S for a candidate reference star at (RA_r, Dec_r) and given a target at (RA_t, Dec_t). In current implementations, these values are encoded as a binary switch, with 1 representing increasing ($+ive$) direction and 0 representing decreasing ($-ive$) direction.

3.5. Identifying and scoring points of intersection

The points where lines from any two loci intersect are identified and defined as PoI. This involves comparing the corner point RA and Dec and direction of lines for one locus with the corner point RA and Dec and direction of lines for a second locus. In total eight variables associated with each pair of loci are checked:

- For locus 1: RA_{c1}, Dec_{c1}, $DirRA_1$, $DirDec_1$
- For locus 2: RA_{c2}, Dec_{c2}, $DirRA_2$, $DirDec_2$

Using these parameters, a check as to whether an intersection between the two loci occurs is achieved as follows:

- A line of constant Dec in the positive RA direction from the corner point of locus 1 will intersect with a line of constant RA in the positive Dec direction from the corner point of locus 2 if locus 1 has a lower RA than locus 2 and locus 1 has a higher Dec than locus 2.
- A line of constant RA in the positive Dec direction from the corner point of locus 1 will intersect with a line of constant Dec in the positive RA direction from the corner point of locus 2 if locus 1 has a lower Dec than locus 2 and locus 1 has a higher RA than locus 2.

By checking all such possible combinations, all pairs of loci in the field which result in a PoI are identified and their RA and Dec noted. The above cases are mathematically expressed in Expression (10).

$$
\begin{aligned}
&if : DirDec_1 = +ive, DirRA_2 = +ive\\
&and : RA_{r1} < RA_{r2}\\
&and : Dec_{r1} > Dec_{r2}\\
&PoI\ exists\ at\ RA_p = RA_{c2}, Dec_p = Dec_{c1}\\
&if : DirRA_1 = +ive, DirDec_2 = +ive \qquad (10)\\
&and : RA_{r1} > RA_{r2}\\
&and : Dec_{r1} < Dec_{r2}\\
&PoI\ exists\ at\ RA_p = RA_{c1}, Dec_p = Dec_{c2}\\
&\dots
\end{aligned}
$$

Expression 10: Definition of a PoI (RA_p, Dec_p) given several sample cases.

Subsequent to identification, each Point of Intersection is then scored. This is achieved as follows:

- All of the reference stars which can be included in the FoV centred on the PoI are identified according to Expression (3).
- Each reference star has been assigned a *rating* between 0 and 1 based on its similarity in colour to the target according to Expression (7).
- The ratings from all counted reference stars in the Field of View are added together to give an overall *score* for the pointing (see Expression (11) and Fig. 3).
- The Point of Intersection with the highest score becomes the pointing for the target (Fig. 3).

$$Score = \sum_{ref}^{FoV} Ratings \qquad (11)$$

Expression 11: Definition of the scoring system. Score is calculated as the sum of all the Ratings for reference stars in the FoV.

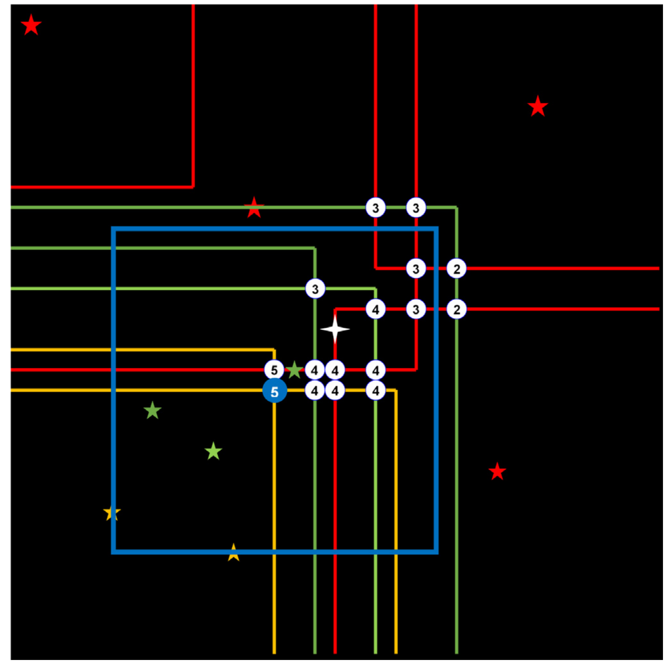

Fig. 3. Points of Intersection (PoI), and their associated score. In this diagram each star has a *rating* of 1, hence the score associated with each PoI is equal to the number of reference stars within a FoV centred at that PoI. The final Pointing & FoV are shown blue.
Source: Modified from Creaner (2016).

Scenarios can arise which result in an inability to identify an optimum pointing for a given target for example if there are no, or a maximum of one reference stars in the candidate zone; and if no points of intersection arise — a scenario which can arise if two (or more) references fall in one quadrant of the candidate zone resulting in concentric loci, or where reference stars are too far apart in different quadrants of the candidate zone in order for their loci to intersect. All four of these scenarios are considered in practical implementations of the Locus Algorithm aimed at identifying the optimum pointings for a set of targets in a catalogue or list of targets.

In summary, the Locus Algorithm successfully identifies the RA and Dec coordinates of the optimum pointing for a given target, where optimum means a field of view with the maximum number of reference stars which are similar in magnitude and colour to the target.

4. Example implementation of the Locus Algorithm

To illustrate the operations of the Locus Algorithm, a worked example is given here. The process described here in producing an optimal pointing for a given star follows the same sequence of steps described in the first part of this paper. The process is implemented in the R programming language and is geared for reproducible research. The code is available on Hickey et al. (2017) It can be trivially adapted for different target stars and telescope parameters.

4.1. Target

The star *SDSS1237680117417115655*, henceforth called the target, ($RA = 346.6500°$, $DEC = -5.0393°$) is used as the example. This star, in the constellation Aquarius, has SDSS magnitudes as given in Table 1

Table 1
SDSS *ugriz* Magnitudes for *SDSS1237680117417115655*, the target used in the sample implementation of the Locus Algorithm.

Band	SDSS Magnitude
u	17.20
g	15.38
r	14.65
i	14.40
z	14.28

Table 2
Observational parameters for the example use of the Locus Algorithm.

Parameters	Values
Field of View in degrees	0.1667
Resolution Limit in degrees	0.0030
Dynamic Range in magnitudes	2.0000
Colour Match Limit	0.1000

The observational parameters are taken from the telescope at Blackrock Castle Observatory.[1] This telescope has parameters given in Table 2

4.2. Candidate Zone

The size of the FoV when corrected for shortening by declination is given in expression (12), by substituting into Expression (2).

$$R' = \frac{R}{cos(Dec_c)}$$
$$R' = \frac{0.1667^\circ}{cos(-5.0393^\circ)} \quad (12)$$
$$R' = 0.16731^\circ$$

Expression 12: Definition of R' for the target.

The locus of the target is given in Expression (13) by substituting into Expression (3).

$$346.5664^\circ \leq RA \leq 346.7337^\circ$$
$$-5.1226^\circ \leq Dec \leq -4.9560^\circ \quad (13)$$

Expression 13: Definition of the locus centred on the target.

The candidate zone as defined above is the area of sky within which reference stars can possibly be included in the same field of view as the target. This is four times the size of the FoV and is given in Expression (14) by substituting into Expression (4),

$$346.4827^\circ \leq RA \leq 346.8173^\circ$$
$$-5.2060^\circ \leq Dec \leq -4.8726^\circ \quad (14)$$

Expression 14: Definition of the Candidate Zone (CZ) centred on the target.

4.3. Identification and filtering of reference stars

The potential reference stars are selected as follows:

- Position: In the CZ defined in Expression (14), SDSS records 1345 objects with clean photometry (Aguado et al., 2019).
- Magnitude: A reference star must be in the range 12.648 $\leq r \leq$ 16.648.
- Colour: A reference star must match the colour of the target in the range $0.634 \leq (g - r) \leq 0.834$ and $0.149 \leq (r - i) \leq 0.349$.

[1] MTU Blackrock Castle Observatory, Castle Road, Blackrock, Cork, T12 YW52, Ireland.

Table 3
A summary of the number of candidate reference stars remaining at each stage.

Filters	Numbers
Position, in Field of View	1345
Correct Magnitude	41
Correct Colour	15
Resolvable	14
In Final Field of View	7

- Resolvability: Any object within 11" (0.003°) to a potential reference star and with an r-band magnitude brighter than 1% that of the reference will pollute the light from the potential reference star.

These numbers are presented in Table 3, accessed using Lang and the CRAN team (2019).

Table 4 gives the 14 stars in the candidate zone which pass the filters. 7 of those 14 are used in the calculation of score for the pointing that is ultimately selected. Also shown in this table is the target itself, highlighted in green. Table 4 also includes ratings for each star calculated as per Expression (7).

4.4. Identifying the effective locus and points of intersection for each candidate reference star

Expression (8) defines how the corner points for a given reference can be calculated. Substituting in for Stars #6 & 8, the resulting corner point is calculated as shown in Table 5. From Expression (9), the direction the locus must be drawn from the corner point to generate the locus can be determined. Applying this to Stars #6 & 8 gives the directions of the loci shown on Table 5.

We can compare these two effective loci to see if they intersect to form a valid PoI. Applying Expression (10) to Stars #6 and #8 gives a PoI at (346.6463°, −5.1153°). From Expression (3), the FoV centred on the PoI can be calculated to be $346.5626^\circ \leq RA \leq 346.7299^\circ$, $-5.1987^\circ \leq Dec \leq -5.0320^\circ$.

The locus associated with each of these 14 potential references is identified and the Points of Intersection associated with these loci are calculated. This leads to the situation shown in Fig. 4.

All these 38 points of intersection are checked in turn as potential pointings. For each one, a field of view is constructed and all the potential reference stars within each one are identified. This is used to calculate a score for each point of intersection. For the star, *SDSS1237680117417115655*, an optimised pointing was thus discovered with $RA = 346.6463^\circ$, $Dec = -5.1153^\circ$ The field of view centred on this pointing included both the target and 7 reference stars (shown in red in Table 4). Following Expression (11), this pointing is calculated to have a score of 3.87 and is illustrated in Fig. 5. By contrast, a naïve pointing aimed directly at the target would include just 3 reference stars (stars #1, 3 & 11) and would have a score of 2.48.

5. Applications of the algorithm

The pointings generated in this algorithm are the optimum pointing for each target given the observational parameters and scoring system used. This is of use to any observer aiming to carry out differential photometry observations as it automates the selection of pointing and identification of reference stars. Two main use-cases are envisaged for this system: targeted use (where an observer wishes to identify the optimum pointing for a pre-determined target or set of targets) and catalogue generation (where many targets are submitted to the system and a set of scores and pointings are generated for each).

Table 4
Details of the candidate reference stars from SDSS. The seven stars highlighted in red are found in the final field of view. The target star itself is shown highlighted in green. The numbers given in the first column "ref" are used to identify the candidate reference stars in the worked examples shown in SubSection 4.4. Data accessed using RCurl by Lang and the CRAN team (2019), table generated with knitr by Xie (2019, 2015, 2014) and kableExtra by Zhu (2019).

ref.	objID	ra	dec	u	g	r	i	z	ratings
1	**1237680117417115692**	**346.7237**	**−5.0473**	**18.36**	**16.58**	**15.84**	**15.57**	**15.46**	**0.7345**
2	1237680065885241391	346.4833	−4.9462	18.97	17.14	16.42	16.16	15.99	0.8080
3	**1237680117417115683**	**346.7128**	**−5.0499**	**17.59**	**15.78**	**15.11**	**14.87**	**14.80**	**0.3610**
4	1237680117417050117	346.5378	−5.0185	18.40	16.52	15.77	15.53	15.39	0.7006
5	1237680117417181202	346.8104	−4.9749	17.46	15.62	14.85	14.58	14.44	0.5353
6	**1237680065348435996**	**346.7072**	**−5.1987**	**16.70**	**14.70**	**13.90**	**13.68**	**13.52**	**0.3220**
7	1237680065348501526	346.8037	−5.2019	15.88	14.15	13.39	13.15	12.99	0.6440
8	**1237680117417050120**	**346.5626**	**−5.1530**	**18.46**	**16.50**	**15.77**	**15.53**	**15.40**	**0.8298**
10	**1237680117417115655**	**346.6500**	**−5.0393**	**17.20**	**15.38**	**14.65**	**14.40**	**14.28**	**1.0000**
11	**1237680117417115762**	**346.6755**	**−5.1195**	**18.92**	**17.02**	**16.28**	**15.97**	**15.85**	**0.3804**
12	**1237680117417050133**	**346.5944**	**−5.1611**	**16.70**	**14.83**	**14.07**	**13.89**	**13.65**	**0.2410**
13	1237680065885241371	346.5598	−4.9312	17.68	15.93	15.23	15.01	14.90	0.4846
14	1237680065885306903	346.6695	−4.9149	17.72	15.65	14.90	14.69	14.45	0.4292
15	1237680117417115701	346.7433	−5.0054	18.46	16.41	15.58	15.32	15.16	0.0683

Table 5
A summary of the defining parameters of the effective loci for example Stars #6 and #8.

quantity	Ref #6	Ref #8
RA_c	346.6235°	346.6463°
Dec_c	−5.1153°	−5.0697°
$DirRA$	$-ive$	$-ive$
$DirDec$	$+ive$	$-ive$

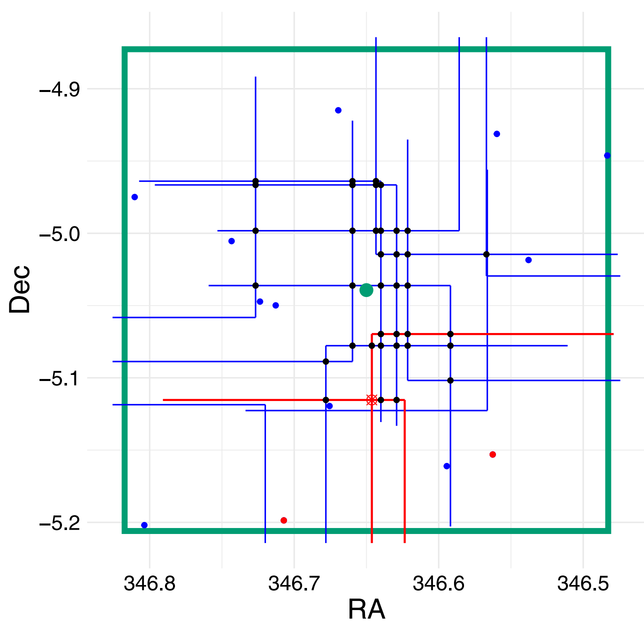

Fig. 4. A plot of the Candidate Zone (green box), the target (green dot), the candidate reference stars (blue dots), the effective loci for each candidate reference star (blue lines) the Points of Intersection between those loci (black dots). Highlighted in red are the candidate reference stars, loci and PoI used to identify the final pointing. Plot generated using tidyverse by Wickham (2017).

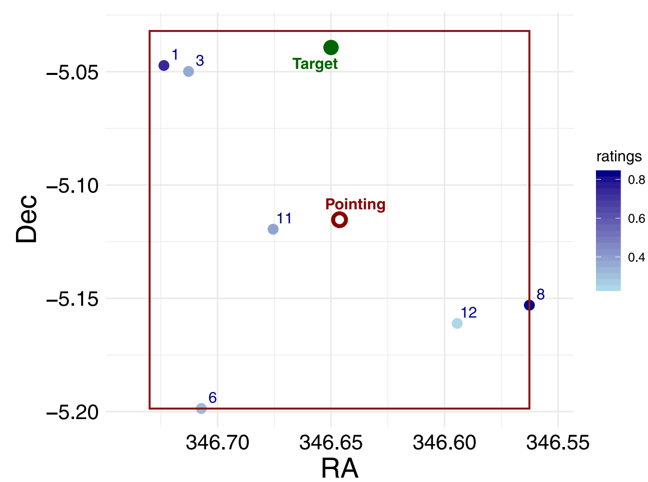

Fig. 5. A plot illustrating the final pointing. On this plot, the target is shown in green, the pointing and the resulting final Field of View are shown in red, and the final reference stars are shown in shades of blue that vary from light to dark based on the rating of the reference star. Plot generated using tidyverse by Wickham (2017).

5.1. Targeted use

This scenario considers an observer who wishes to perform differential photometry observations of a pre-determined target(s). The observer may use the algorithm to identify the optimum pointing for their target(s). As illustrated for the target above, this pointing may be offset from the target but will always include the target and the reference stars with the maximum combined rating. Software to identify this optimum pointing and select the reference stars centred on that pointing is available online at Creaner et al. (2007). A web interface to this software is planned.

5.2. Improvements in field of view

An analysis of a sample of 10,000 stars from the exoplanet candidate catalogue available from Creaner et al. (2019a) shows a mean improvement in score for the generated pointing compared with the naïve pointing of 60% and an increase of 66% in the mean number of reference stars in the FoV.

5.3. Catalogue generation

By submitting many targets at once, the optimum pointing for each can be determined and scores calculated for each. These are output together and can be collated into a catalogue as demonstrated in Creaner et al. (2020a). The catalogues can then be used by an observer to select targets suitable for their needs by filtering the catalogue. For example, by using the scores for each target, targets with a higher score (and thus a better set of reference stars) can be selected for observation over targets with worse scores. The catalogue production software generates lists of targets, their pointings and the scores associated with those pointings. These catalogues are available from Creaner et al. (2019b,a). Users who select targets from the catalogues can then follow up their choice by identifying the set of reference stars with the SQL queries found at Creaner et al. (2007).

6. Conclusions

This paper presents the Locus Algorithm, a novel system for the identification of optimal pointings for differential photometry given a set of parameters of the planned observation provided by the user. The algorithm is presented in two stages. In the first stage, the concept of the algorithm is laid out and the steps of the algorithm are defined. In the second stage, a fully-worked example is shown applying the algorithm to the star *SDSS1237680117417115655* for a 10′ FoV. The premise of the algorithm is that a locus can be defined about any target or reference star, upon which a Field of View (FoV) can be centred and include that object at the edge of the FoV. At the Points of Intersection (PoI) between these loci, the set of targets which can be included in a FoV changes if the FoV moves in any direction. Therefore, these PoI are the essential points to compare when determining the maximum number and quality of reference stars which can be included in a FoV of a given size.

6.1. Summary of algorithm

The algorithm can be summarised in the following nine steps. For demonstration purposes, this algorithm has been applied to *SDSS1237680117417115655* (referred to as the target).

Identify the Target: The target is identified by its coordinates in the RA/Dec coordinate system. The example target is found at ($RA = 346.6500°$, $Dec = -5.0393°$).

Provide Observational Parameters: The algorithm requires a FoV size, magnitude and colour difference limits and a resolution parameter. The FoV for the demonstration is 10 ′ (0.1667°). The magnitude difference limit is ± 2.0 mag. The colour difference limit is ±0.1 mag. The resolution selected is 11" (0.003°).

Define a Candidate Zone: identify all stars which could be included in a FoV with the target by translating the position of the FoV in accordance with Expression (4), which defines the CZ. The CZ around the target is defined by ($346.4827° \leq RA \leq 346.8173°$, $-5.2060° \leq Dec \leq -4.8726°$) and contains 1345 objects.

Filter Candidate References: for each star in the CZ, apply the filtering criteria magnitude, colour and resolution (Expressions (5) and (6)) to identify the candidate reference stars. A candidate reference must meet the following filtering criteria ($12.648 \leq r \leq 16.648$), ($0.634 \leq g - r \leq 0.834$) and ($0.149 \leq r - i \leq 0.349$) and have no polluting stars within 0.003° of it. Applying the filtering criteria leaves 14 Candidate References.

Calculate Rating: for each candidate reference star, a rating is calculated to indicate how closely its colour matches that of the target in accordance with Expression (7). For example, the rating of Star #8 is 0.83.

Calculate Loci: The effective locus around each candidate reference star (i.e. the path upon which the FoV may be centred and include the candidate and the target) is calculated as per Expressions (8) and (9). For example for Star #8, the locus is defined by $RA_c = 346.6463°$ $Dec_c = 5.0697°$, $DirRA = -ive$ and $DirDec = -ive$.

Identify Points of Intersection: The points where the effective loci for two candidate reference stars intersect with one another are identified by combining their coordinates as shown in Expression (10). Given 14 candidate reference stars and their corresponding Loci, there are 364 possible combinations each of which could be a PoI, which are generated by combining the coordinates of the corner-points of the Loci in pairs and checking the directions of the lines from each to determine whether a PoI actually exists. PoI actually exist in 38 cases. For example, a PoI exists between the loci for Stars #6 and #8 at ($RA = 346.6463°$, $Dec = -5.1153°$).

Calculate Score: For each PoI, a score is calculated by combining the ratings for each candidate reference star which can be included in a FoV centred on that target in accordance with (11). For example, for the PoI between stars #6 and #8 above, seven candidate reference stars can be included in the FoV and their ratings sum to 3.87.

Output Optimum Pointing: The PoI with the best score is then selected as the optimised pointing for that target. The pointing used in this example at ($RA = 346.6463°$, $Dec = -5.1153°$) and $Score = 3.87$.

7. Further work: Scalable software implementation of the Locus Algorithm

The Locus Algorithm as defined in this paper is independent of implementation details such as input catalogue or software language. The implementation of this algorithm beyond the single example given here requires a software implementation. The requirements for this software are to:

- Read data in from a source catalogue.
- Identify potential pointings from within that data for a set of targets.
- Compare those pointings to determine the optimal pointing for each target.
- Output each target and its pointing.

These requirements have been implemented in a software system as detailed in Nolan et al. (2020), and available online from Creaner et al. (2007). While the algorithm is defined without reference to a specific implementation, the implementation described in Nolan et al. (2020) uses data from SDSS. This imposes a number of constraints in addition to the requirements specified above. For example, the SDSS catalogue provides data in FITS format, thus mandating use of a FITSIO library compatible with the language to be used. Scaling that software to allow for large-scale use required the use a Grid Computing solution, described in Creaner et al. (2020b). By supplying a set of quasars from the 4th Quasar Catalogue of the Sloan Digital Sky Survey (SDSS) as targets and the remainder of SDSS as potential reference stars, it was possible to generate a catalogue of optimised pointings for 23,779 quasars as per Creaner et al. (2020a, 2019b). Using all of the stars in SDSS as targets allowed for the optimum pointing to be determined for all such stars. These pointings are of use, for example, in the search for extrasolar planets by the transit method, where high-precision differential photometry is required (Creaner et al., 2019a). A paper describing the latter catalogue is in preparation.

CRediT authorship contribution statement

O. Creaner: Methodology, Software, Formal analysis, Data curation, Writing – original draft, Writing – review & editing, Visualization. **K. Nolan:** Methodology, Software, Writing – review & editing, Supervision, Project administration, Funding acquisition. **E. Hickey:** Methodology, Software, Writing – review & editing, Supervision, Project administration, Funding acquisition. **N. Smith:** Conceptualization, Writing – review & editing, Supervision, Funding acquisition.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Oisin Creaner reports financial support was provided by Higher Education authority. Oisin Creaner reports financial support from the ITTD Continuation Fund.

Acknowledgements

Funding for this work

This publication has received funding from Higher Education Authority Technological Sector Research Fund and the Institute of Technology, Tallaght, Dublin Continuation Fund (now Technological University Dublin, Tallaght Campus).

SDSS Acknowledgement: This paper makes use of data from the Sloan Digital Sky Survey (SDSS). Funding for the SDSS and SDSS-II has been provided by the Alfred P. Sloan Foundation, the Participating Institutions, the National Science Foundation, the U.S. Department of Energy, the National Aeronautics and Space Administration, the Japanese Monbukagakusho, the Max Planck Society, and the Higher Education Funding Council for England. The SDSS Web Site is http://www.sdss.org/.

The SDSS is managed by the Astrophysical Research Consortium for the Participating Institutions. The Participating Institutions are the American Museum of Natural History, Astrophysical Institute Potsdam, University of Basel, University of Cambridge, Case Western Reserve University, University of Chicago, Drexel University, Fermilab, the Institute for Advanced Study, the Japan Participation Group, Johns Hopkins University, the Joint Institute for Nuclear Astrophysics, the Kavli Institute for Particle Astrophysics and Cosmology, the Korean Scientist Group, the Chinese Academy of Sciences (LAMOST), Los Alamos National Laboratory, the Max-Planck-Institute for Astronomy (MPIA), the Max-Planck-Institute for Astrophysics (MPA), New Mexico State University, Ohio State University, University of Pittsburgh, University of Portsmouth, Princeton University, the United States Naval Observatory, and the University of Washington.

This paper makes use of the following software packages:

RCurl: General Network (HTTP/FTP/...) Client Interface for R (Lang and the CRAN team, 2019).

tidyverse: R packages for data science (Wickham, 2017).

knitr: A General-Purpose Package for Dynamic Report Generation in R (Xie, 2019, 2015, 2014)

kableExtra: Construct Complex Table with 'kable' and Pipe Syntax (Zhu, 2019)

Appendix. Supplementary expressions

As an example, the derivation of the rating, corner-point and locus for star *SDSS1237680117417050120* (Star #8 on Table 4) is shown in detail below in Expressions (15), (16) & (17). The derivation of the PoI between this locus and the locus for star *SDSS1237680065348435996* (Star #6 on Table 4) is given in Expression (18) and the final FoV is defined in Expression (19).

$$\begin{aligned}
col_{l,r} &= r_r - i_r = 15.771 - 15.533 = 0.238\\
col_{l,t} &= r_t - i_t = 14.648 - 14.399 = 0.249\\
\Delta col_l &= col_{l,r} - col_{l,t} = 0.249 - 0.238 = 0.011\\
Rating_l &= 1 - \left|\frac{\Delta col_l}{\Delta col_{max}}\right| = 1 - \left|\frac{0.011}{0.1}\right| = 0.89\\
col_{s,r} &= g_r - r_r = 16.498 - 15.771 = 0.727\\
col_{s,t} &= r_r - i_r = 115.382 - 14.648 = 0.734\\
\Delta col_s &= col_{s,r} - col_{s,t} = 0.727 - 0.734 = -0.007\\
Rating_s &= 1 - \left|\frac{\Delta col_s}{\Delta col_{max}}\right| = 1 - \left|\frac{-0.007}{0.1}\right| = 0.93\\
Rating &= Rating_l \times Rating_s\\
&= 0.89 \times 0.93 = 0.83
\end{aligned} \tag{15}$$

Expression 15: Definition of the rating of Star #8

$$\begin{aligned}
&if\ :RA_r < RA_t\\
&\implies RA_c = RA_r + \frac{R'}{2}\\
&because\ :346.5626^\circ < 346.6500^\circ\\
&\implies RA_{c,8} = 346.5626^\circ + 0.0837^\circ\\
&RA_{c,8} = 346.6463^\circ\\
&and\\
&if\ :Dec_r < Dec_t\\
&\implies Dec_c = Dec_r + \frac{S}{2}\\
&because\ : -5.1530^\circ < -5.0393^\circ\\
&\implies Dec_{c,8} = -5.1530^\circ + 0.0833^\circ\\
&Dec_{c,8} = -5.0697^\circ
\end{aligned} \tag{16}$$

Expression 16: Definition of the corner-point of the effective locus for Reference Star 8

$$\begin{aligned}
&if\ : RA_r < RA_t \implies DirDec = -ive\\
&because\ : 346.5626^\circ < 346.6500^\circ \implies DirDec_8 = -ive\\
&and\\
&if\ : Dec_r < Dec_t \implies DirRA = -ive\\
&because\ : -5.1530^\circ < -5.0393^\circ \implies DirRA_8 = -ive
\end{aligned} \tag{17}$$

Expression 17: Definition of the directions of the lines drawn from the corner-point of the effective locus for Star #8

$$\begin{aligned}
&DirRA_8 = +ive,\ DirDec_6 = -ive\\
&RA_{r,8} < RA_{r,6}\ and\ Dec_{r,8} > Dec_{r,6}\\
&\implies\\
&PoI\ exist\ at\ :\\
&(RA_p, Dec_p) = (RA_{c,6}, Dec_{c,8})\\
&(RA_p, Dec_p) = (346.6463^\circ, -5.1153^\circ)
\end{aligned} \tag{18}$$

Expression 18: Definition of the PoI between the effective Loci of Stars #6 and #8

$$\begin{aligned}
&RA_p - \frac{R'}{2} \le RA \le RA_p + \frac{R'}{2}\\
&\implies 346.6463^\circ - 0.0837^\circ \le RA \le 346.6463^\circ + 0.0837^\circ\\
&346.5626^\circ \le RA \le 346.7299^\circ\\
&and\ :\\
&Dec_p - \frac{S}{2} \le Dec \le Dec_p + \frac{S}{2}\\
&-5.1153^\circ - 0.0833^\circ \le Dec \le -5.1153^\circ + 0.0833^\circ\\
&-5.1987^\circ \le Dec \le -5.0320^\circ
\end{aligned} \tag{19}$$

Expression 19: Definition of the FoV centred on the PoI between the effective Loci of Stars #6 and #8

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