Comparison of Non-Coherent Linear Detection Algorithms Applied to a 2-D Numerical Breast Model

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Comparison of Non-Coherent Linear Breast Cancer Detection Algorithms Applied to a 2-D Numerical Breast Model

Giuseppe Ruvio, Raffaele Solimene, Antonio Cuccaro, and Max J. Ammann

Abstract—A comparative analysis of an imaging method based on a multi-frequency Multiple Signal Classification (MUSIC) approach against two common linear detection algorithms based on non-coherent migration is made. The different techniques are tested using synthetic data generated through CST Microwave Studio and a phantom developed from MRI scans of a mostly fat breast. The multi-frequency MUSIC approach shows an overall superior performance compared to the non-coherent techniques. This paper reports that this highly performing algorithm does not require any antenna calibration or phase response estimation and allows the use of efficient and complex antenna geometries without difficult algorithm redefinitions.

Index Terms—Microwave detection algorithms, Breast cancer detection.

I. INTRODUCTION

X-ray mammography is the golden standard in breast cancer imaging. Notwithstanding this, it has a number of drawbacks that stimulate research towards different diagnostic methods to provide mammography. In particular, microwave-based diagnostic methods have received much attention and many imaging techniques have been tailored for breast diagnostics [1]. Microwave breast tomography implies solving a non-linear ill-posed inverse problem, since diffraction effects cannot be ignored as in X-ray tomography. When the main objective is to detect and localize tumors, linear scattering models can be exploited. Currently, most of focusing algorithms in the literature are founded on simplified linear scattering models. Nevertheless, the problem is challenging. On one side, benign breast tissues are the main source of clutter, which the signal coming from the tumor must compete against. In addition, it was shown ([2], [3]) that the benign/malign dielectric contrast is not as high as previously believed. In [2], [3] ex vivo breast tissues were electrically characterized across a large part of the microwave spectrum although a more recent study [4] indicated that the measurements performed ex vivo did not consider variations in the range of 20% to 39% for the dielectric properties of in vivo breast tissue. Moreover, the use of coherent methods can be strongly limited by the large number of uncertainties. In terms of frequency dispersion and spatial distribution, breast tissues are at best known with a considerable degree of uncertainty and variations from patient to patient. Finally, while the antenna response is known in free-space, when in close proximity to breast tissue, this becomes hard to predict, unless stringent approximations and simple antenna geometries are adopted. In particular, except for properly devised cases, generally it is not possible to run a characterization stage before imaging in order to estimate the antenna behavior in the presence of the breast. Therefore, imaging methods should be developed by accounting for this inconvenience. Those methods requiring coherence between data acquired at different frequencies will not be considered in this paper. In order to limit the dependence on the antenna' response, a subspace projection method was developed in [5]. There, a non-characterized printed Vivaldi antenna was used to scan a very simplified breast model. The numerical analysis showed that the detection capabilities of the method does not suffer from unknown antenna responses. Here, the investigation advances towards a more realistic scenario and a more sophisticated anti-clutter rejection procedure. Hence, a Vivaldi antenna is used to probe a realistic numerical phantom captured by MRI-scans and data are generated by CST Microwave Studio. Finally, the performance of such MUSIC-Inspired (M-I) algorithm is compared to two common linear non-coherent detection algorithms, non-coherent migration (N-M) [6] and standard MUSIC (S-M) [7].

II. DETECTION SYSTEM

The proposed detection system consists of a Vivaldi antenna scanning a 2-D MRI-based phantom and a signal subspace projection method as detection algorithm. In particular, the antenna touches the skin layer and rotates around the phantom in order to synthesize a multi-monostatic configuration (i.e. TX and RX are co-located) in correspondence to N measurement positions. Both phantom and antenna are immersed in a coupling medium with a relative dielectric permittivity $\epsilon_r = 10$. This medium can be easily obtained as a solution of water and paraffin oil and it fully covers the antenna and the interface between radiator and tissues.

A. Operating environment

The antenna used for this numerical investigation is the antipodal planar Vivaldi antenna shown in Fig. 1 a. Its characteristic parameters $L$ and $W$ are chosen to compromise compactness, directive behavior and broad impedance matching properties. The antenna is designed by using a computationally efficient multiobjective evolutionary algorithm ParEGO to
work with the antenna immersed in the coupling medium. While this solution enables more energy to be coupled from the antenna to the phantom, a strong degree of miniaturization can be also achieved. A 2-mm substrate extension was added from the top edge of the tapered arms to mitigate mismatch when the antenna touches the skin layer. Adopting 2-D phantoms (if not 1-D) is a convenient choice for a faster but still fair algorithm assessment and is very common in the literature [8], [9], [10]. Phantom simplification enables a more detailed numerical investigation when non-ideal antennas are used [11], whereas more complex 3-D models are generally considered when using ideal sources [12]. For this investigation 3 different incoherent detection algorithms are compared by considering the 2-D breast model displayed in Fig. 1 b. This cylindrical phantom was constructed on the basis of the MRIs documented in [13] and labelled with the code ID071904. The MRIs were taken from patients in a prone position and different tissues were expressed in terms of a 3-D voxel representation. Each cubic voxel measures 0.5 mm x 0.5 mm x 0.5 mm for high resolution and is assigned to a certain tissue type. The 2-D model considered in this paper was built starting from an azimuthal slice which was extracted in such a way that all the tissues are represented. The different tissue contours were then converted into polygons that can be easily imported into the CST simulation tool and processed with its Finite Integral Technique (FIT). Finally, each polygon representing morphologic features was converted into a surface and extruded into an infinite cylindric structure with the corresponding electric characterization. This model presents several advantages when compared to voxel structures as it enables editing and convenient mesh refinement. Breast tissues were electrically characterized starting from the realistic dispersive dielectric properties reported by Lazebnik et al. [2], [3]. However, a simplified model based on constant permittivity and effective conductivity was adopted. Specifically, tissue permittivity was calculated as the average value within percentile regions in [2], [3] at the center frequency (2 GHz), whereas the effective conductivity was calculated taking into account the dispersive behavior due to the imaginary part of the permittivity. This simplified electric characterization is justified for two reasons:

1) the detection algorithms under investigation operate on single-frequency samples;
2) frequency samples are uniformly taken just in the spectrum from 1 to 3 GHz.

Finally, electric parameters for the 5-mm diameter tumor inclusion were calculated from the curve corresponding to the electromagnetically denser material in [3]. The overall benign/malignant dielectric contrast is then of the order of < 10%, which is consistent with recent literature ([8], [9], [11]). A relaxation time of 13 ps was assumed for each tissue model. Table I summarizes permittivity and effective conductivity values corresponding to each tissue in the breast model under test.

![Fig. 1. a) Antenna geometry (L= 145 mm, W= 50 mm); b) Breast phantom cross-section.](image)

<table>
<thead>
<tr>
<th>Tissue</th>
<th>$\epsilon_r$</th>
<th>$\sigma_{eff}$ [S/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fat 1</td>
<td>2.42</td>
<td>0.0518</td>
</tr>
<tr>
<td>Fat 2</td>
<td>4.025</td>
<td>0.04786</td>
</tr>
<tr>
<td>Fat 3</td>
<td>6.13</td>
<td>0.07142</td>
</tr>
<tr>
<td>Fibroconnective 1</td>
<td>42.02</td>
<td>0.8674</td>
</tr>
<tr>
<td>Fibroconnective 2</td>
<td>50.5</td>
<td>1.365</td>
</tr>
<tr>
<td>Fibroconnective 3</td>
<td>59.43</td>
<td>1.544</td>
</tr>
<tr>
<td>Transitional</td>
<td>21.76</td>
<td>0.1565</td>
</tr>
<tr>
<td>Skin</td>
<td>39.25</td>
<td>1.247</td>
</tr>
<tr>
<td>Tumor</td>
<td>65.89</td>
<td>2.158</td>
</tr>
</tbody>
</table>

**TABLE I**

**ELECTRIC PROPERTIES OF BREAST TISSUES**

**B. Clutter rejection and MUSIC-Inspired algorithm**

$N$ is the number of TX/RX positions ($\Omega_{o1}, \Omega_{o2}, \cdots, \Omega_{oN}$) and $N_f$ the number of adopted frequencies. Accordingly, the data can be arranged in the $N_f \times N$ scattering matrix

$$S = [S^1 \cdots S^n \cdots S^N]$$

where $S^n$ is the column vector of data collected at the $n$-th position as the frequency varies. This data consists of contributions coming from antenna internal reflection, breast tissues and embedded malignancies. The first two contributions are a source of clutter that tends to mask tumor signal. In order to reduce clutter, a subspace based technique [14] is applied. Accordingly, $S$ is expressed through its Singular Value Decomposition (SVD) as

$$S = UV\Lambda V^H$$

where $U$ and $V$ are unitary matrices containing the left and right singular values, respectively, and $\Lambda$ is a diagonal matrix containing the singular values $\lambda_1, \lambda_2, \cdots, \lambda_p$ in decreasing order, with $P = \min\{N_f, N\}$. Considering that clutter contributions are much stronger than the tumor signal, it can be mitigated by disregarding the projection of the scattering matrix on the singular functions corresponding to the highest singular values. Determining how many projections to discard
requires a priori information on the clutter. A conservative choice to remove the dominant clutter contribution coming from internal antenna reflections and skin interface is adopted here with the first two projections being removed. Accordingly, the de-cluttered data matrix is

\[ S_d = \sum_{k=3}^{P} \lambda_k u^k v^{kH}. \]  

(3)

with \( H \) representing the Hermitian operator.

Once clutter has been rejected the scattering data vectors (the rows of \( S_d \)) are expressed as

\[ S_n^l = H_R(f_n)A(f_n)H_T(f_n)b(f_n) \]  

(4)

where \( f_n \) is the \( n \)-th working frequency, \( H_T(f_n) \) and \( H_R(f_n) \) are the antenna responses, \( b(f_n) \) is the vector of objects' scattering coefficients and \( A(f_n) \) is the propagator which maps \( M \) (with \( N > M \)) scatterers located at \((\xi_1, \xi_2, \ldots, \xi_M)\) to the scattered field. In particular, its \( l \)-th column has the form

\[ A^l(\xi_l; f_n) = [G^2(\xi_1, \xi_l; f_n), \ldots, G^2(\xi_{N-1}, \xi_l; f_n)]^T \]  

(5)

\( G(\cdot) \) being the relevant Green’s function. Note that the square factor in (5) is due to the adopted multimonomostatic configuration. Green’s function computation requires spatial and dispersion information on breast tissues properties. But those are actually not a priori known. Therefore, an equivalent homogeneous breast medium is considered to construct the Green’s function. This significantly simplifies the algorithm implementation but it also is justified by an inevitably unknown breast tissues’ distribution at a diagnostic stage. In particular, in this analysis the dielectric permittivity of the coupling medium is assigned to the equivalent breast medium. However, data can be separately processed at each frequency and the corresponding reconstructions can be then suitably combined. Moreover, since the information concerning scatterers’ locations is embodied within the columns of \( A(f_n) \), a subspace projection method like MUSIC [15] is applied for their detection. To this end, the eigenspectrum of the correlation matrix

\[ R(f_n) = S_n^H S_n \]  

(6)

can be adopted. Now, as the scattering coefficient vector is deterministic, hence \( R(f_n) \) is rank-deficient with rank one. Therefore, the adoption of the only significant singular vector \( u_1(f_n) \) of \( R(f_n) \) to implement is proposed. Accordingly, the following scheme to identify the scatterers’ locations is used

\[ P(\xi_k; f_n) = 1 - |<A^k(f_n), u_1(f_n)>|^2 \]  

(7)

where \( A^k \) is the steering vector evaluated at the trial position \( \xi_k \) with \( \langle \cdot, \cdot \rangle \) denoting the Hermitian scalar product. Detection is achieved where \( P(\xi_k; f_n) \) peaks.

Due to rank-deficiency of \( R(f_n) \) it is not expected that (7) performs optimally. To restore the rank of the correlation matrix some methods can be employed [16], which take advantage of multifrequency illumination or a multistatic configurations. However, as they require coherence between different data acquisitions, here the detection scheme is modified to take advantage of multifrequency data as

\[ P_{mf}(\xi_k) = \prod_{n=1}^{N_f} P(\xi_k; f_n) \]  

(8)

A detailed theoretical study of the algorithm can be found in [18].

III. NUMERICAL COMPARISON AND CONCLUSIONS

In order to test the detection system a 5-mm diameter tumor is inserted in the phantom (see Fig. 1 b). In particular, the tumor was located within the fibrograndular region. This allows the assessment of the detection system for the critical situation of reduced dielectric contrast between tumor and surrounding tissues. Synthetic data were generated through CST by collecting the scattered field over 18 different positions taken uniformly around the breast phantom at multiples of 20°, whereas 30 beating frequencies within the band 1 – 3 GHz are adopted. Finally, results returned by eq. (8) are compared to those obtained by non-coherent migration, i.e.,

\[ M(\xi_k) = \sum_{n=1}^{N_f} \sum_{m=1}^{N_f} |S_d^{m*}(f_m)G^2(\xi_m, \xi_k; f_m)|^2 \]  

(9)

where * means conjugation, and a multi-frequency MUSIC implementation already present in literature [7], that is

\[ P_f(\xi_k) = 1 / \left[ \sum_{n=1}^{N_f} 1 - |<A^k(f_n), u_1(f_n)>|^2 \right] \]  

(10)

**TABLE II RECONSTRUCTION METRICS**

<table>
<thead>
<tr>
<th>Method</th>
<th>Signal-to-Clutter Ratio[dB]</th>
<th>Signal-to-Mean Ratio[dB]</th>
<th>Spatial Displacement[mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>M-I</td>
<td>1.0215</td>
<td>28.4676</td>
<td>1.5</td>
</tr>
<tr>
<td>N-M</td>
<td>1.0051</td>
<td>1.99427</td>
<td>13.1</td>
</tr>
<tr>
<td>S-M</td>
<td>1.0058</td>
<td>1.1257</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Corresponding reconstructions are shown in Fig. 2. All cases presented above were obtained by corrupting the scattered field with an additive white complex Gaussian noise with signal-to-noise ratio \( SNR = 10 \) dB. As can be seen, those reconstructions show that under the same situation the
MUSIC-inspired method presented here has superior performance when compared to the other non-coherent methods. Together with visual reconstruction of the detection algorithm results, its performance was measured by suitable metrics such as Signal-to-Clutter Ratio (SCR), Signal-to-Mean Ratio (SMR) and Spatial Displacement (SD) as defined in [17].

The MUSIC-inspired method offers better focusing capabilities and greater dynamic range between clutter and tumor levels when compared to standard MUSIC and non-coherent migration. Moreover, considering that the limited dielectric contrast between tumor and fibroconnective tissues and an \textit{a priori} antenna response has not been used, the proposed MUSIC-inspired system presents promising features for early-stage breast cancer diagnostics.

**ACKNOWLEDGMENT**

This work was supported by Italian Ministry of University and Research through the FIRB initiative under the project MICENEA (RBFR12A7CD), POR Campania FSE 2007/2013-"MASTRI" and the COST Action IC1102 VISTA.

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