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## System identification using the delta operator

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# **System identification using the delta operator**

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## **Structure of presentation**

1. Definition of the delta operator
2. Advantages of the delta domain representation
3. System identification using the delta operator
4. Time delay estimation using the delta operator
5. Conclusions

# 1. Definition of the delta operator

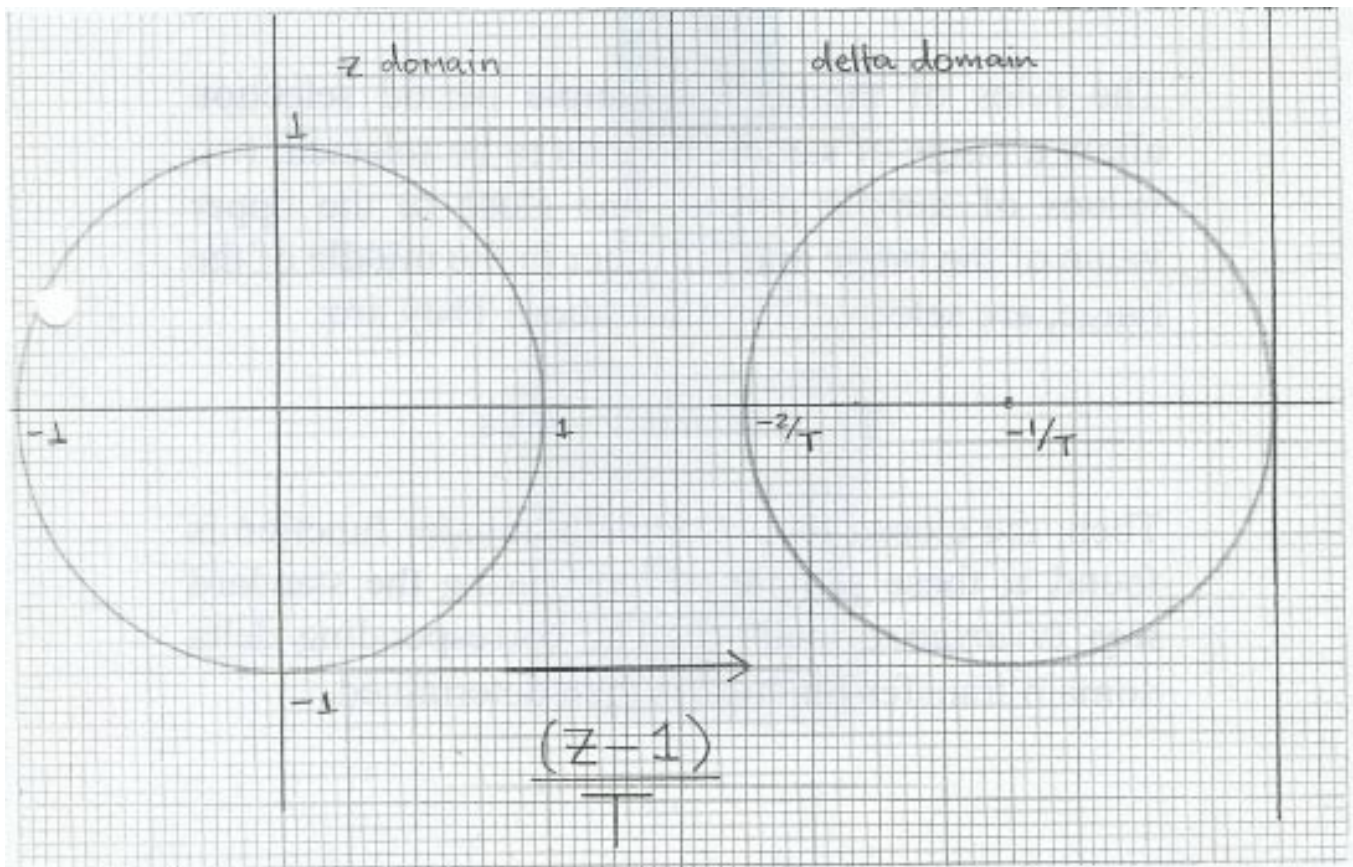
The **DELTA** or **EULER** operator is defined as:

$$\delta = \frac{z-1}{T}, \quad T = \text{sample time}$$

If  $z.X_k = X_{k+1}$ ,

then  $\delta.X_k = \frac{z-1}{T} X_k = \frac{X_{k+1} - X_k}{T}$

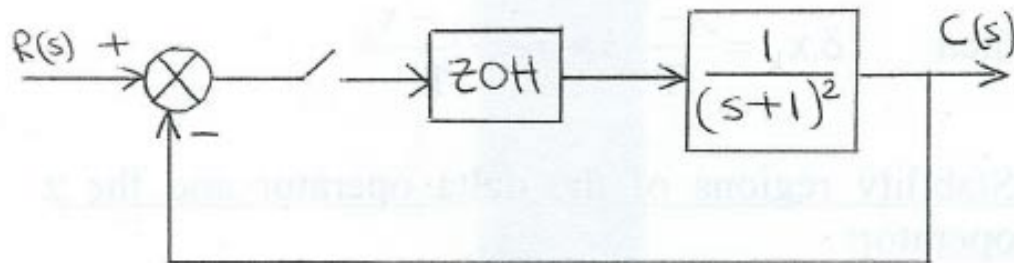
Stability regions of the delta operator and the z operator:



## 2. Advantages of delta domain representation:

- a. The sensitivity of process pole locations to small changes in the identified parameters is *much less* in the delta domain than it is in the z domain.

Example:



The rules of thumb for sample period selection allow a range of sample periods to be selected; two values of sample period, one close to the upper value of the range and the other in the middle of the range, are selected, for illustration. They are:

$$T = 0.5s \text{ and } T = 0.2s.$$

Then, the closed loop z-domain transfer function is found at these sample periods (using the standard *step invariance* method). The corresponding delta domain transfer function is then determined.

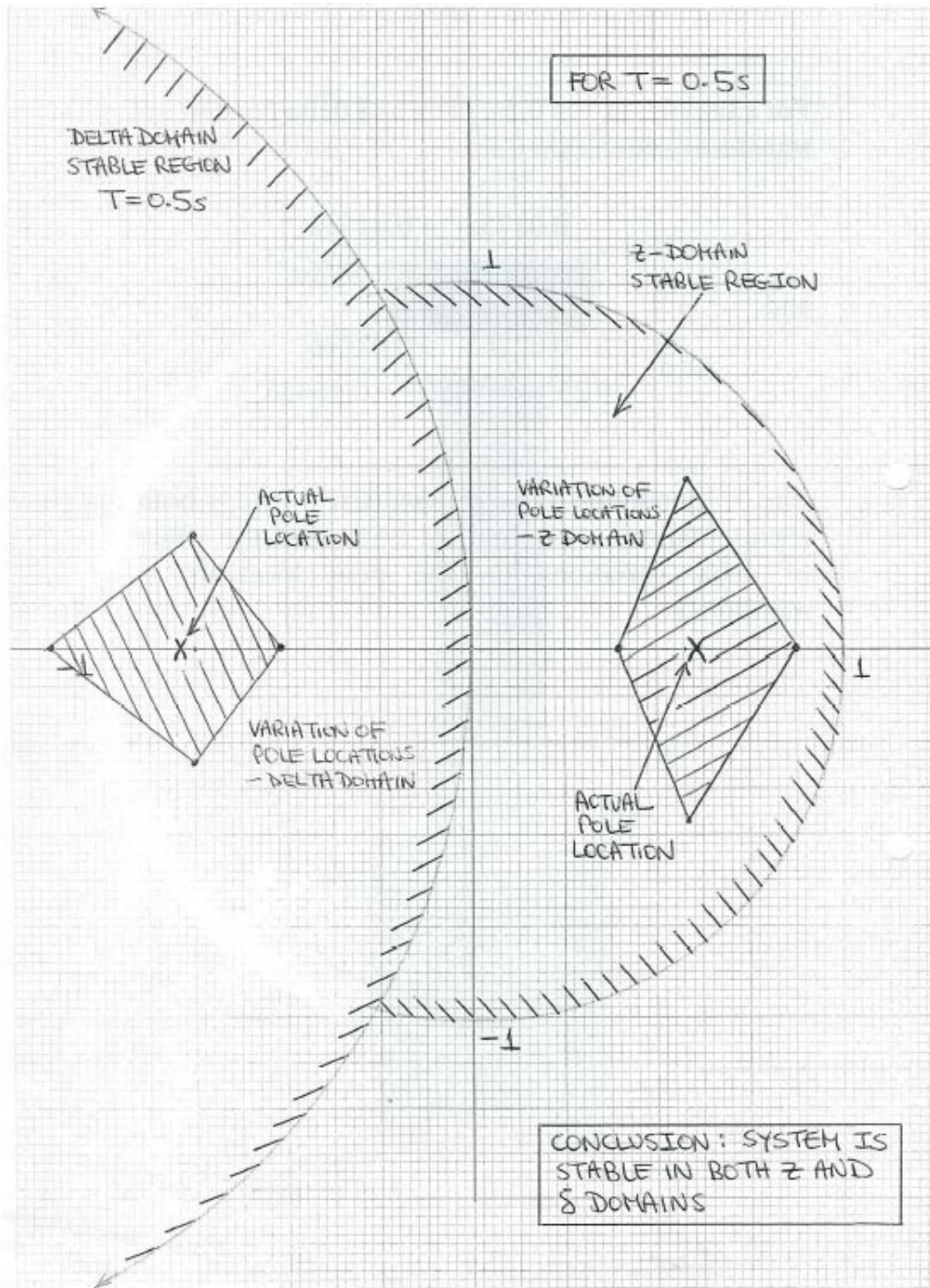
For example, at  $T = 0.5\text{s}$ , the  $z$  and delta domain transfer functions are

$$G(z) = \frac{0.0175z^{-1} + 0.0153z^{-2}}{1 - 1.6199z^{-1} + 0.6856z^{-2}}$$

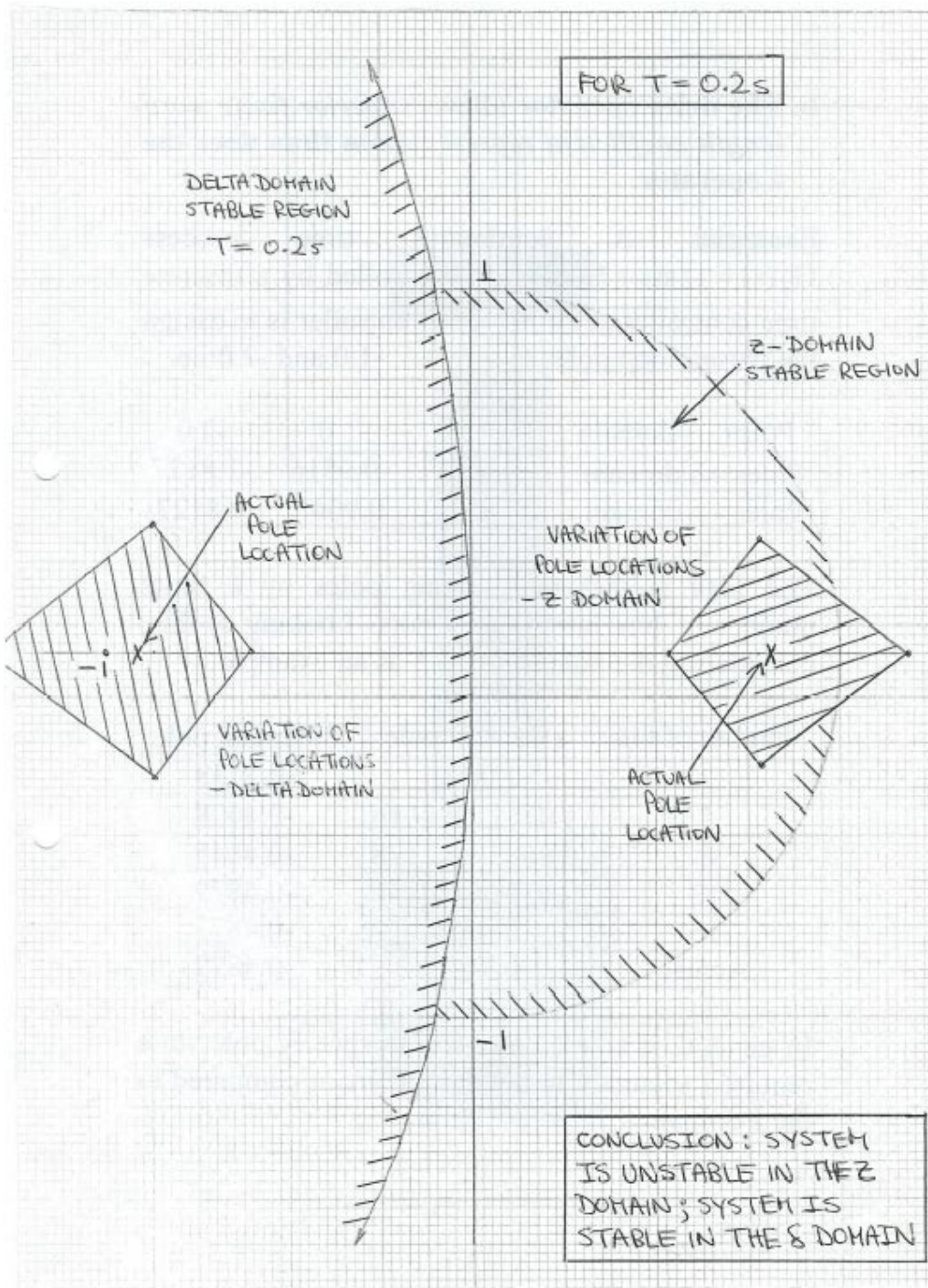
and

$$G(\delta) = \frac{0.0226\delta^{-1} + 0.0774\delta^{-2}}{0.25 + 0.3935\delta^{-1} + 0.1548\delta^{-2}}$$

The denominator parameters of both representations are varied by  $\pm 5\%$ , and the effect of such variations on the pole locations is monitored.









**b. The delta operator allows *superior finite word length coefficient representation* than does the z operator.**

Example: A continuous time process  $G_p(s) = 1/s(s+1)$  is to be controlled by a discrete time controller at 10 Hz. The controller is to assign the closed loop poles to  $s = -0.5, -1.0$  and  $-2.0$  i.e.

Poles - s domain	-0.5	-1.0	-2.0
Poles - $\delta$ domain	-0.4877	-0.9516	-1.8127
Poles - z domain	0.9516	0.9048	0.8127

The idea is that representing poles by a finite number of bits introduces errors. Looking at this, considering the pole at  $s = -0.5$  as an example:

Number of bits	z domain pole	$\delta$ domain pole
2	1.0000	-0.3738
5	1.0161+j0.2616	-0.4408
8	0.9682+j0.1178	-0.4845
12	0.9329+j0.0251	-0.4879
16	0.9516	-0.4877
Actual	0.9516	-0.4877

In general, it takes **7 bits less** to specify poles to a required accuracy in the delta domain compared to the z domain.

- c. **The delta operator almost always has less *roundoff noise* associated with it than does the *z* operator.** Roundoff noise is the error introduced due to the finite number of bits used to store and calculate intermediate quantities.

### **3. System identification using the delta operator**

The recursive least squares (RLS) algorithm has a long tradition of use in the area of system identification and modelling. The RLS algorithm may be based on the *z* operator or the  $\delta$  operator; both implementations have been detailed.<sup>1</sup>

It is interesting to consider the results obtained by Terrett and Downing; the process whose parameters are to be estimated is a 2<sup>nd</sup> order underdamped active RC network with a bandwidth of 2.4 kHz.

The input to the process is a square wave, of period 200 samples and of  $\pm 1$  amplitude.

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<sup>1</sup> Terrett, P. and Downing, C.J. (1993). "System identification and modelling on the TMS320C25 fixed-point processor using the  $\delta$ -operator", *Proceedings of the Irish Colloquium on DSP and Control*, University College, Dublin, pp. 239-247.

Figures 2a and 2b show the estimation error using the z operator and  $\delta$  operator, respectively.

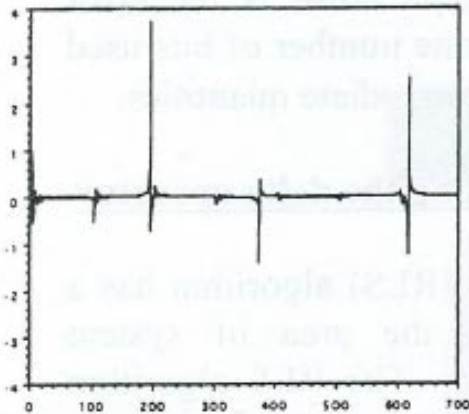


FIGURE 2a

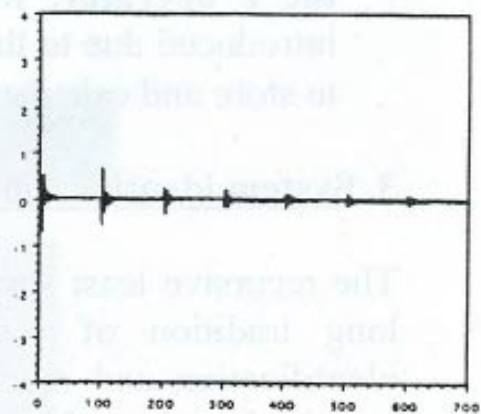


FIGURE 2b

The estimation error is significantly smaller, and less erratic, when the  $\delta$  operator is used.

Figures 3a-6a and 3b-6b show the estimates of the four model parameters using the z operator and  $\delta$  operator, respectively.

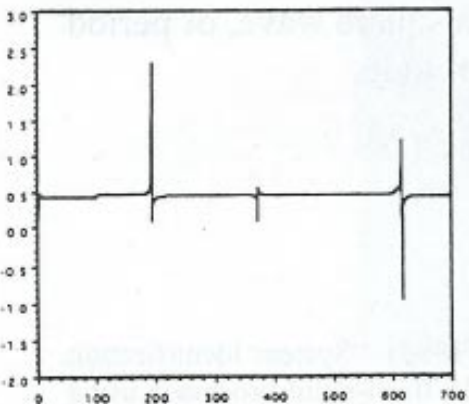


FIGURE 3a

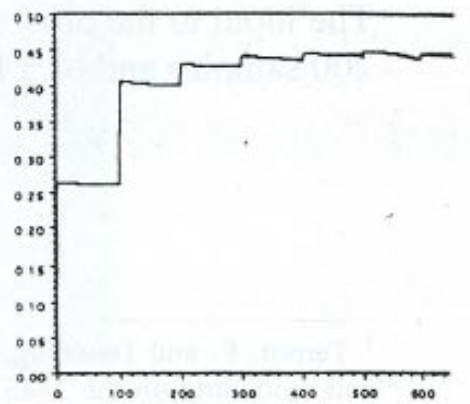
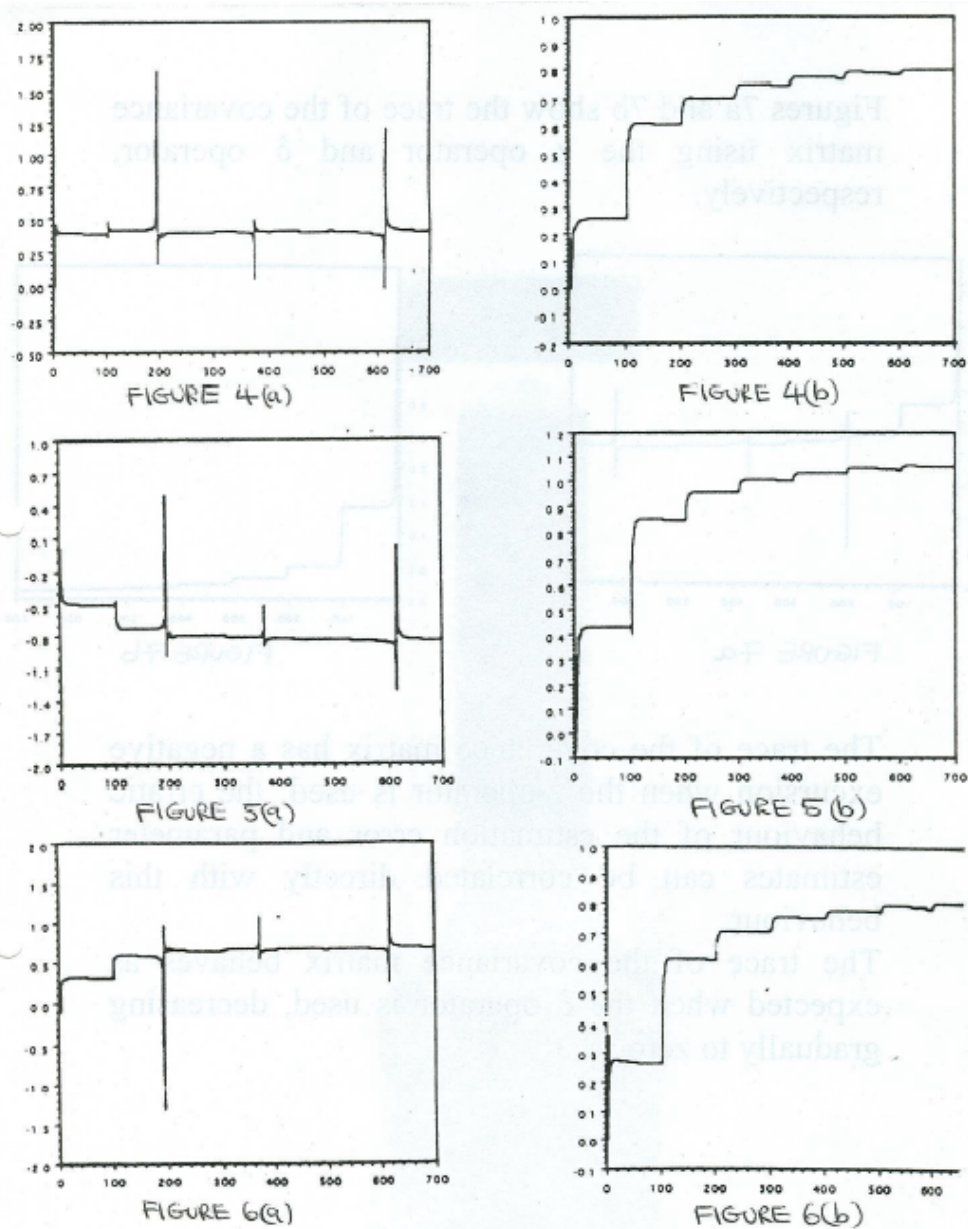
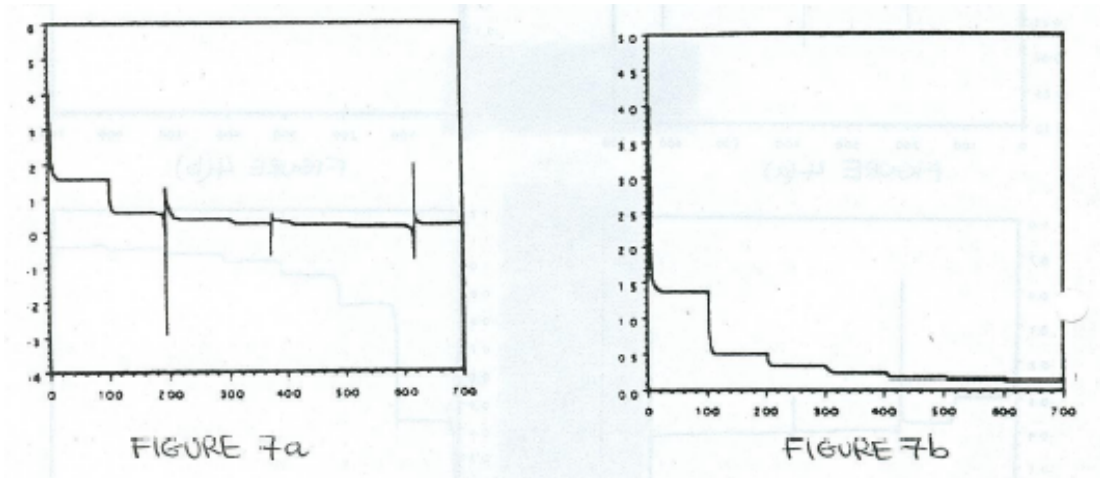


FIGURE 3b



The parameter estimates are inconsistent when the  $z$ -operator is used; in contrast, *the estimates using the  $\delta$  operator all converge to the correct values.*

Figures 7a and 7b show the trace of the covariance matrix using the  $z$  operator and  $\delta$  operator, respectively.



The trace of the covariance matrix has a negative excursion when the  $z$ -operator is used; the erratic behaviour of the estimation error and parameter estimates can be correlated directly with this behaviour.

The trace of the covariance matrix behaves as expected when the  $\delta$  operator is used, decreasing gradually to zero.

#### 4. Time delay estimation using the delta operator:

Time delays arise in many signal processing applications, where they are also known as a time difference of arrival between two signals; such a measurement arises, for example, in underwater tracking applications, biomedicine, geophysics, astronomy, acoustics and seismology.

In telecommunications, time delays also arise in *networked computer control*, as it takes time for information to transfer. Time delays can be

- Deterministic (e.g. with Profibus)
- Uncertain (e.g. with Ethernet)<sup>2</sup>

One technique of time delay estimation that has been well described in the z-domain is the method of *overparameterisation*.

**Example:** Suppose the process in the s domain is described by the transfer function

$$G_p(s) = \frac{e^{-s}}{1+s} \text{ i.e. time delay} = 1 \text{ second}$$

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<sup>2</sup> Professor George Irwin, of Queens University Belfast, explored these issues in a plenary presentation given at the Irish Signals and Systems Conference, held at Queens University Belfast, in July 2004.



The z-domain equivalent ( $T = 0.5$  seconds)

$$G_p(z) = \frac{0.39z^{-4}}{1 - 0.61z^{-1}}$$

The numerator polynomial is overparameterised as follows:

$$G_p(z) = \frac{0z^{-1} + 0z^{-2} + 0z^{-3} + 0.39z^{-4}}{1 - 0.61z^{-1}}$$

The recursive least squares algorithm may be used to estimate the resulting parameters, and the time delay is calculated based on the parameters identified (the time delay corresponds to the first non-zero numerator parameter identified).

The  $\delta$  domain equivalent is

$$G_p(\delta) = \frac{3.1\delta^{-4}}{1 + 6.78\delta^{-1} + 16.68\delta^{-2} + 17.36\delta^{-3} + 6.24\delta^{-4}}$$

Three points can be made:

- (a) The numerator retains the same form in the z domain and the  $\delta$  domain; the same overparameterisation algorithms can be used.
- (b) The parameters are larger in magnitude in the  $\delta$  domain, which means that the identification algorithm works more efficiently.

- (c) However, there is a greater number of parameters to be estimated in the  $\delta$  domain, increasing the computational burden of the identification algorithm. This can be eased, in some cases, by estimating the denominator parameters off-line (if the system time constant did not change significantly, for example).

## **5. Conclusions:**

1. Representation of digital systems in the delta domain has numerical advantages over the representation in the z domain.
2. The method of overparameterisation may be used to estimate the time delay in the delta domain, though a greater number of parameters need to be estimated in the delta domain than in the corresponding z domain.
3. Because the region of stability of the delta domain approaches that of the s domain as the sample time reduces, greater use may be made of continuous time intuition in discrete time design in the delta domain.

## Some further reading:

### *Coefficient sensitivity issues*

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### *Word length coefficient representation/roundoff noise issues*

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Soh, C.B. (1991a). Robust stability of discrete-time systems using delta operators, *IEEE Transactions on Automatic Control*, **36**, 377-380.

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