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# PI and PID controller tuning rules for time delay processes: a summary. Part 2: PID controller tuning rules

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## Abstract

A summary of tuning rules for the PID control of single input, single output (SISO) processes with time delay, modeled in stable first order lag plus time delay (FOLPD) form, is provided in this part of the paper.

**Keywords:** PID, tuning rules, time delay.

## 1. Introduction

This part of the paper summarises the most directly applicable tuning rules for PID controllers that have been developed to compensate stable SISO processes with time delay, modeled in FOLPD form. The structure of this part of the paper is similar to that of Part 1.

The ideal continuous time domain PID controller for a SISO process is expressed in the Laplace domain as follows:

$$U(s) = G_c(s)E(s) \quad (1)$$

with 
$$G_c(s) = K_c \left( 1 + \frac{1}{T_i s} + T_d s \right) \quad (2)$$

and with  $K_c$  = proportional gain,  $T_i$  = integral time constant and  $T_d$  = derivative time constant. A number of other PID controller structures may be described. One typical example of such a structure is the classical form of the PID controller:

$$G_c(s) = K_c \left( 1 + \frac{1}{T_i s} \right) \left( \frac{1 + T_d s}{1 + \frac{T_d s}{N}} \right) \quad (3)$$

Tuning rules for these and other such PID controller structures are explicitly indicated in the table. A list of symbols used in the paper is provided in Appendix 1.

## 2. PID tuning rules - FOLPD model

Rule	$K_c$	$T_i$	$T_d$	Comment
<u>Ideal controller</u>	$G_c(s) = K_c \left( 1 + \frac{1}{T_i s} + T_d s \right)$			
<b>Process reaction</b>				
Ziegler and Nichols [1]	$\left[ \frac{1.2T_m}{K_m \tau_m}, \frac{2T_m}{K_m \tau_m} \right]$	$2\tau_m$	$0.5\tau_m$	Quarter decay ratio
Astrom and Hagglund [2]	$\frac{0.94T_m}{K_m \tau_m}$	$2\tau_m$	$0.5\tau_m$	ultimate cycle Ziegler-Nichols equivalent

Parr [3]	$\frac{1.25T_m}{K_m \tau_m}$	$2.5\tau_m$	$0.4\tau_m$	
Rule	$K_c$	$T_i$	$T_d$	Comment
Chien <i>et al.</i> [4] - regulator	$\frac{0.95T_m}{K_m \tau_m}$	$2.38\tau_m$	$0.42\tau_m$	0% overshoot; $0.11 < \frac{\tau_m}{T_m} < 1$
Chien <i>et al.</i> [4] - regulator	$\frac{1.2T_m}{K_m \tau_m}$	$2\tau_m$	$0.42\tau_m$	20% overshoot; $0.11 < \frac{\tau_m}{T_m} < 1$
Chien <i>et al.</i> [4] - servo	$\frac{0.6T_m}{K_m \tau_m}$	$T_m$	$0.5\tau_m$	0% overshoot; $0.11 < \frac{\tau_m}{T_m} < 1$
Chien <i>et al.</i> [4] - servo	$\frac{0.95T_m}{K_m \tau_m}$	$1.36T_m$	$0.47\tau_m$	20% overshoot; $0.11 < \frac{\tau_m}{T_m} < 1$
Three constraints method - Murrill [5] $0.1 \leq \frac{\tau_m}{T_m} \leq 1$	$\frac{1.370}{K_m} \left( \frac{T_m}{\tau_m} \right)^{0.950}$	$\frac{T_m}{1.351} \left( \frac{\tau_m}{T_m} \right)^{0.738}$	$0.365T_m \left( \frac{\tau_m}{T_m} \right)^{0.950}$	Quarter decay ratio; minimum integral error (servo mode); $\frac{K_c K_m T_d}{T_m} = 0.5$
Cohen and Coon [6]	$\frac{1}{K_m} \left( 1.35 \frac{T_m}{\tau_m} + 0.25 \right) T_m$	$\frac{\left( 2.5 \frac{\tau_m}{T_m} + 0.46 \left( \frac{\tau_m}{T_m} \right)^2 \right)}{1 + 0.61 \frac{\tau_m}{T_m}}$	$\frac{0.37\tau_m}{1 + 0.2 \frac{\tau_m}{T_m}}$	Quarter decay ratio
Sain and Ozgen [7]	$\frac{1}{K_m} \left( 0.6939 \frac{T_m}{\tau_m} + 0.1814 \right)$	$\frac{0.8647T_m + 0.226\tau_m}{\tau_m + 0.8647}$	$\frac{0.0565T_m}{0.8647 \frac{T_m}{\tau_m} + 0.226}$	
<b>Regulator</b>				
Minimum IAE - Murrill [5]	$\frac{1.435}{K_m} \left( \frac{T_m}{\tau_m} \right)^{0.921}$	$\frac{T_m}{0.878} \left( \frac{T_m}{\tau_m} \right)^{0.749}$	$0.482T_m \left( \frac{\tau_m}{T_m} \right)^{1.137}$	$0.1 < \frac{\tau_m}{T_m} \leq 1$
Modified minimum IAE - Cheng and Hung [8]	$\frac{3}{K_m} \left( \frac{T_m}{\tau_m} \right)^{0.921}$	$\frac{T_m}{0.878} \left( \frac{T_m}{\tau_m} \right)^{0.749}$	$0.482T_m \left( \frac{\tau_m}{T_m} \right)^{1.137}$	
Minimum ISE - Murrill [5]	$\frac{1.495}{K_m} \left( \frac{T_m}{\tau_m} \right)^{0.945}$	$\frac{T_m}{1.101} \left( \frac{T_m}{\tau_m} \right)^{0.771}$	$0.56T_m \left( \frac{\tau_m}{T_m} \right)^{1.006}$	$0.1 < \frac{\tau_m}{T_m} \leq 1$
Minimum ISE - Zhuang and Atherton [9]	$\frac{1.473}{K_m} \left( \frac{T_m}{\tau_m} \right)^{0.970}$	$\frac{T_m}{1.115} \left( \frac{T_m}{\tau_m} \right)^{0.753}$	$0.55T_m \left( \frac{\tau_m}{T_m} \right)^{0.948}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1.0$
	$\frac{1.524}{K_m} \left( \frac{T_m}{\tau_m} \right)^{0.735}$	$\frac{T_m}{1.130} \left( \frac{T_m}{\tau_m} \right)^{0.641}$	$0.552T_m \left( \frac{\tau_m}{T_m} \right)^{0.851}$	$1.1 \leq \frac{\tau_m}{T_m} \leq 2.0$
Minimum ITAE - Murrill [5]	$\frac{1.357}{K_m} \left( \frac{T_m}{\tau_m} \right)^{0.947}$	$\frac{T_m}{0.842} \left( \frac{T_m}{\tau_m} \right)^{0.738}$	$0.381T_m \left( \frac{\tau_m}{T_m} \right)^{0.995}$	$0.1 < \frac{\tau_m}{T_m} \leq 1$
Minimum ISTSE - Zhuang and Atherton [9]	$\frac{1.468}{K_m} \left( \frac{T_m}{\tau_m} \right)^{0.970}$	$\frac{T_m}{0.942} \left( \frac{T_m}{\tau_m} \right)^{0.725}$	$0.443T_m \left( \frac{\tau_m}{T_m} \right)^{0.939}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1.0$
	$\frac{1.515}{K_m} \left( \frac{T_m}{\tau_m} \right)^{0.730}$	$\frac{T_m}{0.957} \left( \frac{T_m}{\tau_m} \right)^{0.598}$	$0.444T_m \left( \frac{\tau_m}{T_m} \right)^{0.847}$	$1.1 \leq \frac{\tau_m}{T_m} \leq 2.0$

Minimum ISTES - Zhuang and Atherton [9]	$\frac{1531}{K_m} \left( \frac{T_m}{\tau_m} \right)^{0.960}$	$\frac{T_m}{0.971} \left( \frac{T_m}{\tau_m} \right)^{0.746}$	$0.413 T_m \left( \frac{\tau_m}{T_m} \right)^{0.933}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1.0$
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Rule	$K_c$	$T_i$	$T_d$	Comment
Minimum ISTES - Zhuang and Atherton [9] (continued)	$\frac{1592}{K_m} \left( \frac{T_m}{\tau_m} \right)^{0.705}$	$\frac{T_m}{0.957} \left( \frac{T_m}{\tau_m} \right)^{0.597}$	$0.414 T_m \left( \frac{\tau_m}{T_m} \right)^{0.850}$	$1.1 \leq \frac{\tau_m}{T_m} \leq 2.0$
<b>Servo</b>				
Minimum IAE - Rovira <i>et al.</i> [10]	$\frac{1.086}{K_m} \left( \frac{T_m}{\tau_m} \right)^{0.869}$	$\frac{T_m}{0.740 - 0.13 \frac{\tau_m}{T_m}}$	$0.348 T_m \left( \frac{\tau_m}{T_m} \right)^{0.914}$	$0.1 < \frac{\tau_m}{T_m} \leq 1$
Minimum IAE - Wang <i>et al.</i> [11]	$\frac{\left( 0.7645 + \frac{0.6032}{\tau_m/T_m} \right) (T_m + 0.5\tau_m)}{K_m (T_m + \tau_m)}$	$T_m + 0.5\tau_m$	$\frac{0.5 T_m \tau_m}{T_m + 0.5\tau_m}$	$0.05 < \frac{\tau_m}{T_m} < 6$
Minimum ISE - Wang <i>et al.</i> [11]	$\frac{\left( 0.9155 + \frac{0.7524}{\tau_m/T_m} \right) (T_m + 0.5\tau_m)}{K_m (T_m + \tau_m)}$	$T_m + 0.5\tau_m$	$\frac{0.5 T_m \tau_m}{T_m + 0.5\tau_m}$	$0.05 < \frac{\tau_m}{T_m} < 6$
Minimum ISE - Zhuang and Atherton [9]	$\frac{1.048}{K_m} \left( \frac{T_m}{\tau_m} \right)^{0.897}$	$\frac{T_m}{1.195 - 0.368 \frac{\tau_m}{T_m}}$	$0.489 T_m \left( \frac{\tau_m}{T_m} \right)^{0.888}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1.0$
	$\frac{1.154}{K_m} \left( \frac{T_m}{\tau_m} \right)^{0.567}$	$\frac{T_m}{1.047 - 0.220 \frac{\tau_m}{T_m}}$	$0.490 T_m \left( \frac{\tau_m}{T_m} \right)^{0.708}$	$1.1 \leq \frac{\tau_m}{T_m} \leq 2.0$
Minimum ITAE - Rovira <i>et al.</i> [10]	$\frac{0.965}{K_m} \left( \frac{T_m}{\tau_m} \right)^{0.85}$	$\frac{T_m}{0.796 - 0.1465 \frac{\tau_m}{T_m}}$	$0.308 T_m \left( \frac{\tau_m}{T_m} \right)^{0.929}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1$
Modified minimum ITAE - Cheng and Hung [8]	$\frac{1.2}{K_m} \left( \frac{T_m}{\tau_m} \right)^{0.855}$	$\frac{T_m}{0.796 - 0.147 \frac{\tau_m}{T_m}}$	$0.308 T_m \left( \frac{\tau_m}{T_m} \right)^{0.929}$	Damping factor of closed loop system = 0.707.
Minimum ITAE - Wang <i>et al.</i> [11]	$\frac{\left( 0.7303 + \frac{0.5307}{\tau_m/T_m} \right) (T_m + 0.5\tau_m)}{K_m (T_m + \tau_m)}$	$T_m + 0.5\tau_m$	$\frac{0.5 T_m \tau_m}{T_m + 0.5\tau_m}$	$0.05 < \frac{\tau_m}{T_m} < 6$
Minimum ISTSE - Zhuang and Atherton [9]	$\frac{1.042}{K_m} \left( \frac{T_m}{\tau_m} \right)^{0.897}$	$\frac{T_m}{0.987 - 0.238 \frac{\tau_m}{T_m}}$	$0.385 T_m \left( \frac{\tau_m}{T_m} \right)^{0.906}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1.0$
	$\frac{1.142}{K_m} \left( \frac{T_m}{\tau_m} \right)^{0.579}$	$\frac{T_m}{0.919 - 0.172 \frac{\tau_m}{T_m}}$	$0.384 T_m \left( \frac{\tau_m}{T_m} \right)^{0.839}$	$1.1 \leq \frac{\tau_m}{T_m} \leq 2.0$
Minimum ISTES - Zhuang and Atherton [9]	$\frac{0.968}{K_m} \left( \frac{T_m}{\tau_m} \right)^{0.904}$	$\frac{T_m}{0.977 - 0.253 \frac{\tau_m}{T_m}}$	$0.316 T_m \left( \frac{\tau_m}{T_m} \right)^{0.892}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1.0$
	$\frac{1.061}{K_m} \left( \frac{T_m}{\tau_m} \right)^{0.583}$	$\frac{T_m}{0.892 - 0.165 \frac{\tau_m}{T_m}}$	$0.315 T_m \left( \frac{\tau_m}{T_m} \right)^{0.832}$	$1.1 \leq \frac{\tau_m}{T_m} \leq 2.0$
<b>Direct synthesis</b>				
Regulator - min. IAE - Smith and Corripio [12]	$\frac{T_m}{K_m \tau_m}$	$T_m$	$0.5\tau_m$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1.5$

Servo - min. IAE - Smith and Corripio [12]	$\frac{5T_m}{6K_m\tau_m}$	$T_m$	$0.5\tau_m$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1.5$
Servo - 5% overshoot - Smith and Corripio [12]	$\frac{T_m}{2K_m\tau_m}$	$T_m$	$0.5\tau_m$	

Rule	$K_c$	$T_i$	$T_d$	Comment
Suyama [13]	$\frac{1}{K_m} \left[ 0.7236 \frac{T_m}{\tau_m} + 0.2236 \right]$	$T_m + 0.309\tau_m$	$\frac{2.236T_m\tau_m}{7.236T_m + 2.236\tau_m}$	
Gain and phase margin - Zhuang and Atherton [9]	$mK_u \cos(\phi_m)$ , $m = 0.614(1 - 0.233e^{-0.347K_mK_u})$ $\phi_m = 33.8^\circ(1 - 0.97e^{-0.45K_mK_u})$	$\frac{\tan(\phi_m) + \sqrt{\frac{4}{\alpha} + \tan^2(\phi_m)}}{2\omega_u}$ , $\alpha = 0.413(3.302K_mK_u + 1)$	$\frac{\tan(\phi_m) + \sqrt{\frac{4}{\alpha} + \tan^2(\phi_m)}}{2\omega_u}$	Gain margin = 2, phase margin = $60^\circ$ $0.1 \leq \frac{\tau_m}{T_m} \leq 2.0$
Abbas [14]	$\frac{0.177 + 0.348 \left( \frac{\tau_m}{T_m} \right)^{-1.002}}{0.531 - 0.359V^{0.713}}$	$T_m + 0.5\tau_m$	$\frac{T_m\tau_m}{2T_m + \tau_m}$	V=fractional overshoot $0 \leq V \leq 0.2$ $0.1 \leq \frac{\tau_m}{T_m} \leq 5.0$
Servo - min. ISE - Ho <i>et al.</i> [15]	$\frac{1.8578}{K_m} \frac{\phi_m^{0.0821}}{A_m^{0.9087}} \left( \frac{\tau_m}{T_m} \right)^{-0.9471}$	$T_i^{(1)}$	$\frac{0.4899T_m\phi_m^{0.1457}}{A_m^{0.0845}} \left( \frac{\tau_m}{T_m} \right)^{1.0264}$	$A_m \in [2, 5]$ , $\phi_m \in [30^\circ, 60^\circ]$ , $0.1 \leq \frac{\tau_m}{T_m} \leq 1.0$
Regulator - min. ISE - Ho <i>et al.</i> [15]	$\frac{1.0722}{K_m} \frac{\phi_m^{-0.116}}{A_m^{0.8432}} \left( \frac{\tau_m}{T_m} \right)^{-0.908}$	$\frac{1.2497T_m\phi_m^{1.0082}}{A_m^{0.2099}} \left( \frac{\tau_m}{T_m} \right)^{0.3678}$	$\frac{0.4763T_m\phi_m^{-0.328}}{A_m^{0.0961}} \left( \frac{\tau_m}{T_m} \right)^{1.0317}$	$A_m \in [2, 5]$ , $\phi_m \in [30^\circ, 60^\circ]$ , $0.1 \leq \frac{\tau_m}{T_m} \leq 1.0$
<b>Robust</b>				
Brambilla <i>et al.</i> [16]	$\frac{1}{K_m} \left( \frac{T_m + 0.5\tau_m}{\lambda\tau_m} \right)$	$T_m + 0.5\tau_m$	$\frac{T_m\tau_m}{2T_m + \tau_m}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 10$ ; $I \approx 0.35$
Rivera <i>et al.</i> [17]	$\frac{1}{K_m} \left( \frac{T_m + 0.5\tau_m}{\lambda + 0.5\tau_m} \right)$	$T_m + 0.5\tau_m$	$\frac{T_m\tau_m}{2T_m + \tau_m}$	$\lambda > 0.1T_m$ , $\lambda \geq 0.8\tau_m$ .
Fruehauf <i>et al.</i> [18]	$\frac{5T_m}{9\tau_mK_m}$	$5\tau_m$	$\leq 0.5\tau_m$	$\frac{\tau_m}{T_m} < 0.33$
	$\frac{T_m}{2\tau_mK_m}$	$T_m$	$\leq 0.5\tau_m$	$\frac{\tau_m}{T_m} \geq 0.33$
Lee <i>et al.</i> [19]	$\frac{T_i}{K_m(\lambda + \tau_m)}$	$T_m + \frac{\tau_m^2}{2(\lambda + \tau_m)}$	$\frac{\tau_m^2}{2(\lambda + \tau_m)} \left( 1 - \frac{\tau_m}{3T_i} \right)$	$\lambda = \frac{\tau_m}{3}$
<b>Ultimate cycle</b>				
Regulator - minimum IAE - Pessen [20]	$0.7K_u$	$0.4T_u$	$0.149T_u$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1.0$
Servo - minimum ISTSE - Zhuang and	$0.509K_u$	$0.051(3.302K_mK_u + 1)T_u$	$0.125T_u$	$0.1 \leq \frac{\tau_m}{T_m} \leq 2.0$

$${}^1 T_i^{(1)} = \frac{0.0211T_m \left[ 1 + 0.3289A_m + 6.4572\phi_m + 25.1914 \left( \frac{\tau_m}{T_m} \right) \right]}{1 + 0.0625A_m - 0.8079\phi_m + 0.347 \left( \frac{\tau_m}{T_m} \right)}$$

Atherton [9]				
Regulator - minimum ISTSE - Zhuang and Atherton [9]	$\frac{4.434K_m K_u - 0.966}{5.12K_m K_u + 1.734} K_u$	$\frac{1.751K_m K_u - 0.612}{3.776K_m K_u + 1.388} T_u$	$0.144T_u$	$0.1 \leq \frac{\tau_m}{T_m} \leq 2.0$

Rule	$K_c$	$T_i$	$T_d$	Comment
Servo - nearly minimum IAE, ITAE - Hwang and Fang [21]	$\left[ c_1 + c_2 \frac{\tau_m}{T_m} + c_3 \left( \frac{\tau_m}{T_m} \right)^2 \right] K_u$ $c_1 = 0.537, c_2 = -0.0165$ $c_3 = 0.00173$	$\frac{K_c}{K_u \omega_u \left[ c_4 + c_5 \frac{\tau_m}{T_m} + c_6 \left( \frac{\tau_m}{T_m} \right)^2 \right]}$ $c_4 = 0.0503, c_5 = 0.163$ $c_6 = -0.0389$	$\frac{\left[ c_7 + c_8 \frac{\tau_m}{T_m} + c_9 \left( \frac{\tau_m}{T_m} \right)^2 \right] \frac{K_u}{\omega_u K_c}}{\omega_u K_c}$ $c_7 = 0.350, c_8 = -0.0344$ $c_9 = 0.00644$	$0.1 \leq \frac{\tau_m}{T_m} \leq 2.0$ - decay ratio = 0.03
Regulator - nearly minimum IAE, ITAE - Hwang and Fang [21]	$\left[ c_1 + c_2 \frac{\tau_m}{T_m} + c_3 \left( \frac{\tau_m}{T_m} \right)^2 \right] K_u$ $c_1 = 0.802, c_2 = -0.154$ $c_3 = 0.0460$	$\frac{K_c}{K_u \omega_u \left[ c_4 + c_5 \frac{\tau_m}{T_m} + c_6 \left( \frac{\tau_m}{T_m} \right)^2 \right]}$ $c_4 = 0.190, c_5 = 0.0532$ $c_6 = -0.00509$	$\frac{\left[ c_7 + c_8 \frac{\tau_m}{T_m} + c_9 \left( \frac{\tau_m}{T_m} \right)^2 \right] \frac{K_u}{\omega_u K_c}}{\omega_u K_c}$ $c_7 = 0.421, c_8 = 0.00915$ $c_9 = -0.00152$	$0.1 \leq \frac{\tau_m}{T_m} \leq 2.0$ - decay ratio = 0.12
Simultaneous Servo/regulator - nearly minimum IAE, ITAE - Hwang and Fang [21]	$\left[ c_1 + c_2 \frac{\tau_m}{T_m} + c_3 \left( \frac{\tau_m}{T_m} \right)^2 \right] K_u$ $c_1 = 0.713, c_2 = -0.176$ $c_3 = 0.0513$	$\frac{K_c}{K_u \omega_u \left[ c_4 + c_5 \frac{\tau_m}{T_m} + c_6 \left( \frac{\tau_m}{T_m} \right)^2 \right]}$ $c_4 = 0.149, c_5 = 0.0556$ $c_6 = -0.00566$	$\frac{\left[ c_7 + c_8 \frac{\tau_m}{T_m} + c_9 \left( \frac{\tau_m}{T_m} \right)^2 \right] \frac{K_u}{\omega_u K_c}}{\omega_u K_c}$ $c_7 = 0.371, c_8 = -0.0274$ $c_9 = 0.00557$	$0.1 \leq \frac{\tau_m}{T_m} \leq 2.0$
McMillan [22]	$\frac{1.415 T_m}{K_m \tau_m} \left\{ \frac{1}{1 + \left( \frac{T_m}{T_m + \tau_m} \right)^{0.65}} \right\}$	$\tau_m \left\{ 1 + \left( \frac{T_m}{T_m + \tau_m} \right)^{0.65} \right\}$	$0.25 \tau_m \left\{ 1 + \left( \frac{T_m}{T_m + \tau_m} \right)^{0.65} \right\}$	Tuning rules developed from $K_u, T_u$
Tan <i>et al.</i> [23]	$\frac{K_u}{A_m} \cos \phi_m$	$\alpha T_d$	$T_u \frac{\tan \phi_m + \sqrt{\frac{4}{\alpha} + \tan^2 \phi_m}}{4\pi}$	$A_m = 2, \phi_m = 45^\circ$ ; $\alpha$ chosen arbitrarily
	$\frac{K_\phi}{A_m}$	$\frac{rK_\phi (\omega_u^2 - \omega_\phi^2)}{\omega_u \omega_\phi^2 \sqrt{K_u^2 - r^2 K_\phi^2}}$	$\frac{1}{\omega_\phi^2 T_i}$	Arbitrary $A_m, \phi_m$ at $\omega_\phi$ ; $r = 0.1 + 0.9(K_u/K_\phi)$
<u>Ideal controller with set-point weighting</u>	$G_c(s) = K_c \left( b + \frac{1}{T_i s} + T_d s \right)$			
<b>Maximum sensitivity</b>				
Astrom and Hagglund [2] - maximum sensitivity = 1.4	$\frac{3.8e^{-8.4\tau+7.3\tau^2} T_m}{K_m \tau_m},$ $\tau = \tau_m / (\tau_m + T_m)$	$5.2\tau_m e^{-2.5\tau-1.4\tau^2}$ or $0.46T_m e^{2.8\tau-2.1\tau^2}$	$0.89\tau_m e^{-0.37\tau-4.1\tau^2}$ or $0.077T_m e^{5.0\tau-4.8\tau^2}$	$b = 0.40e^{0.18\tau+2.8\tau^2}$ $0.14 \leq \frac{\tau_m}{T_m} \leq 5.5$
Astrom and Hagglund [2] - maximum sensitivity = 2.0	$\frac{8.4e^{-9.6\tau+9.8\tau^2} T_m}{K_m \tau_m}$	$3.2\tau_m e^{-1.5\tau-0.93\tau^2}$ or $0.28T_m e^{3.8\tau-1.6\tau^2}$	$0.86\tau_m e^{-1.9\tau-0.44\tau^2}$ or $0.076T_m e^{3.4\tau-1.1\tau^2}$	$b = 0.22e^{0.65\tau+0.051\tau^2}$ $0.14 \leq \frac{\tau_m}{T_m} \leq 5.5$
<u>Classical controller</u>	$G_c(s) = K_c \left( 1 + \frac{1}{T_i s} \right) \left( \frac{1 + T_d s}{1 + \frac{T_d s}{N}} \right)$			
<b>Process reaction</b>				
Hang <i>et al.</i> [24]	$\frac{0.83T_m}{K_m \tau_m}$	$1.5\tau_m$	$0.25\tau_m$	Foxboro EXACT controller 'pretune'; N=10

Witt and Waggoner [25]	$[\frac{0.6T_m}{K_m \tau_m}, \frac{T_m}{K_m \tau_m}]$	$\tau_m$	$\tau_m$	Equivalent to Ziegler and Nichols [1]; $N = [10,20]$
St. Clair [26]	$\frac{T_m}{K_m \tau_m}$	$5\tau_m$	$0.5\tau_m$	'aggressive' tuning
	$\frac{0.5T_m}{K_m \tau_m}$	$5\tau_m$	$0.5\tau_m$	'conservative' tuning

Rule	$K_c$	$T_i$	$T_d$	Comment								
<b>Regulator</b>												
Minimum IAE - Kaya and Scheib [27]	$\frac{0.98089}{K_m} \left( \frac{T_m}{\tau_m} \right)^{0.76167}$	$\frac{T_m}{0.91032} \left( \frac{T_m}{\tau_m} \right)^{1.05221}$	$0.59974 T_m \left( \frac{\tau_m}{T_m} \right)^{0.89819}$	$0 < \frac{\tau_m}{T_m} \leq 1$ ; $N=10$								
Minimum ISE - Kaya and Scheib [27]	$\frac{1.11907}{K_m} \left( \frac{T_m}{\tau_m} \right)^{0.89711}$	$\frac{T_m}{0.7987} \left( \frac{T_m}{\tau_m} \right)^{0.9548}$	$0.54766 T_m \left( \frac{\tau_m}{T_m} \right)^{0.87798}$	$0 < \frac{\tau_m}{T_m} \leq 1$ ; $N=10$								
Minimum ITAE - Kaya and Scheib [27]	$\frac{0.77902}{K_m} \left( \frac{T_m}{\tau_m} \right)^{1.06401}$	$\frac{T_m}{1.14311} \left( \frac{T_m}{\tau_m} \right)^{0.70949}$	$0.57137 T_m \left( \frac{\tau_m}{T_m} \right)^{1.03826}$	$0 < \frac{\tau_m}{T_m} \leq 1$ ; $N=10$								
<b>Servo</b>												
Minimum IAE - Kaya and Scheib [27]	$\frac{0.65}{K_m} \left( \frac{T_m}{\tau_m} \right)^{1.04432}$	$\frac{T_m}{0.9895 + 0.09539 \frac{\tau_m}{T_m}}$	$0.50814 T_m \left( \frac{\tau_m}{T_m} \right)^{1.08433}$	$0 < \frac{\tau_m}{T_m} \leq 1$ ; $N=10$								
Minimum ISE - Kaya and Scheib [27]	$\frac{0.71959}{K_m} \left( \frac{T_m}{\tau_m} \right)^{1.03092}$	$\frac{T_m}{1.12666 - 0.18145 \frac{\tau_m}{T_m}}$	$0.54568 T_m \left( \frac{\tau_m}{T_m} \right)^{0.86411}$	$0 < \frac{\tau_m}{T_m} \leq 1$ ; $N=10$								
Minimum ITAE - Kaya and Scheib [27]	$\frac{1.12762}{K_m} \left( \frac{T_m}{\tau_m} \right)^{0.80368}$	$\frac{T_m}{0.99783 + 0.02860 \frac{\tau_m}{T_m}}$	$0.42844 T_m \left( \frac{\tau_m}{T_m} \right)^{1.0081}$	$0 < \frac{\tau_m}{T_m} \leq 1$ ; $N=10$								
<b>Direct synthesis</b>												
Tsang and Rad [28]	$\frac{0.809 T_m}{K_m \tau_m}$	$T_m$	0.	$N=5$								
Tsang . [29]	$\frac{1}{K_m \tau_m}$	$\tau_m$	$0.25\tau_m$	$N = 2.5$								
	a	$\xi$	$\zeta$	a	$\xi$	a	$\xi$	a	$\xi$			
	1.682	0.0	1.161	0.2	0.859	0.4	0.669	0.6	0.543	0.8	0.457	1.0
	1.383	0.1	0.992	0.3	0.754	0.5	0.600	0.7	0.496	0.9		
<b>Robust</b>												
Chien [30]	$\frac{1}{K_m} \left( \frac{T_m}{\lambda + 0.5\tau_m} \right)$	$T_m$	$0.5\tau_m$	$\lambda = [\tau_m, T_m]$ ; $N=10$								
Chien [30]	$\frac{1}{K_m} \left( \frac{0.5\tau_m}{\lambda + 0.5\tau_m} \right)$	$0.5\tau_m$	$T_m$	$\lambda = [\tau_m, T_m]$ ; $N=10$								
<b>Ultimate cycle</b>												
Minimum IAE regulator - Shinsky [31]	$\frac{K_m \tau_m}{0.3\tau_m - 0.32T_u}$	$T_u \left( 0.15 \frac{T_u}{\tau_m} - 0.05 \right)$	$0.14T_u$									
Minimum IAE regulator - Shinsky [32]	$0.95T_m/K_m \tau_m$ $0.95T_m/K_m \tau_m$ $1.14T_m/K_m \tau_m$ $1.39T_m/K_m \tau_m$	$1.43\tau_m$ $1.17\tau_m$ $1.03\tau_m$ $0.77\tau_m$	$0.52\tau_m$ $0.48\tau_m$ $0.40\tau_m$ $0.35\tau_m$	$\tau_m/T_m = 0.2$ $\tau_m/T_m = 0.5$ $\tau_m/T_m = 1$ $\tau_m/T_m = 2$								

<u>Industrial controller</u>	$U(s) = K_c \left( 1 + \frac{1}{T_i s} \right) \left( R(s) - \frac{1 + T_d s}{1 + \frac{T_d s}{N}} Y(s) \right)$			
<b>Regulator</b>				
Minimum IAE - Kaya and Scheib [27]	$\frac{0.91}{K_m} \left( \frac{T_m}{\tau_m} \right)^{0.7938}$	$\frac{T_m}{1.01495} \left( \frac{T_m}{\tau_m} \right)^{1.00403}$	$0.5414 T_m \left( \frac{\tau_m}{T_m} \right)^{0.7848}$	$0 < \frac{\tau_m}{T_m} \leq 1 ; N=10$



Rule	$K_c$	$T_i$	$T_d$	Comment
Minimum ISE - Kaya and Scheib [27]	$\frac{1.1147}{K_m} \left( \frac{T_m}{\tau_m} \right)^{0.8992}$	$\frac{T_m}{0.9324} \left( \frac{T_m}{\tau_m} \right)^{0.8753}$	$0.56508 T_m \left( \frac{\tau_m}{T_m} \right)^{0.91107}$	$0 < \frac{\tau_m}{T_m} \leq 1; N=10$
Minimum ITAE - Kaya and Scheib [27]	$\frac{0.7058}{K_m} \left( \frac{T_m}{\tau_m} \right)^{0.8872}$	$\frac{T_m}{1.03326} \left( \frac{T_m}{\tau_m} \right)^{0.99138}$	$0.60006 T_m \left( \frac{\tau_m}{T_m} \right)^{0.971}$	$0 < \frac{\tau_m}{T_m} \leq 1; N=10$
<b>Servo</b>				
Minimum IAE - Kaya and Scheib [27]	$\frac{0.81699}{K_m} \left( \frac{T_m}{\tau_m} \right)^{1.004}$	$\frac{T_m}{1.09112 - 0.22387 \frac{\tau_m}{T_m}}$	$0.44278 T_m \left( \frac{\tau_m}{T_m} \right)^{0.97186}$	$0 < \frac{\tau_m}{T_m} \leq 1; N=10$
Minimum ISE - Kaya and Scheib [27]	$\frac{1.1427}{K_m} \left( \frac{T_m}{\tau_m} \right)^{0.9365}$	$\frac{T_m}{0.99223 - 0.35269 \frac{\tau_m}{T_m}}$	$0.35308 T_m \left( \frac{\tau_m}{T_m} \right)^{0.78088}$	$0 < \frac{\tau_m}{T_m} \leq 1; N=10$
Minimum ITAE - Kaya and Scheib [27]	$\frac{0.8326}{K_m} \left( \frac{T_m}{\tau_m} \right)^{0.7607}$	$\frac{T_m}{1.00268 + 0.00854 \frac{\tau_m}{T_m}}$	$0.44243 T_m \left( \frac{\tau_m}{T_m} \right)^{1.11499}$	$0 < \frac{\tau_m}{T_m} \leq 1; N=10$

### 3. Conclusions

This part of the paper has presented a useable summary of the tuning rules for PID controllers that have been developed to compensate SISO processes with time delay, modeled in FOLPD form.

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**Appendix 1:** List of symbols used (more than once) in the paper.

$A_m$  = gain margin

$b$  = setpoint weighting factor

$E(s)$  = Desired variable,  $R(s)$ , minus controlled variable,  $Y(s)$

$G_c(s)$  = PID controller transfer function

$G_p(j\omega)$  = process transfer function at frequency  $\omega$ ,  $|G_p(j\omega)|$  = magnitude of  $G_p(j\omega)$

IAE = integral of absolute error

ISE = integral of squared error

ISTES = integral of squared time multiplied by error, all to be squared

ISTSE = integral of squared time multiplied by squared error

ITAE = integral of time multiplied by absolute error

$K_c$  = Proportional gain of the controller

$K_m$  = Gain of the FOLPD process model

$K_\phi$  = Magnitude of the FOLPD process model at a phase lag of  $\phi$

$K_u$  = Ultimate gain

$M_s$  = Closed loop sensitivity

$N$  = Indication of the amount of filtering on the derivative term

$R(s)$  = Desired variable

$T_d$  = Derivative time of the controller

$T_i$  = Integral time of the controller

$T_m$  = Time constant of the FOLPD process model

$T_u$  = Ultimate time constant

$U(s)$  = manipulated variable

$Y(s)$  = controlled variable

$\lambda$  = Parameter that determines robustness of compensated system.

$\xi$  = damping factor of the compensated system

$\phi_m$  = phase margin

$\tau_m$  = time delay of the FOLPD process model,  $\tau = \tau_m / (\tau_m + T_m)$

$\omega_u$  = ultimate (angular) frequency,  $\omega_\phi$  = angular frequency at a phase lag of  $\phi$