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# Progressive Transmission of Spatial Data Over the WWW: Challenges

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# Trasmissione progressiva di dati spaziali su WWW: problemi

## Progressive transmission of spatial data over the WWW: challenges

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### Abstract

*La trasmissione progressiva di immagini su WWW è stata utilizzata con successo per fornire agli utenti versioni meno dettagliate dei dati prima di ottenere un'immagine completa. Per dati spaziali in formato vettoriale, la trasmissione progressiva rimane un punto critico. Aumentare il livello di dettaglio di dataset vettoriali è un processo complesso che non coinvolge semplicemente l'aggiunta di pixel. In questo lavoro presentiamo una rassegna dei modelli per la trasmissione progressiva di dati spaziali e discutiamo i principali problemi aperti relativi alla trasmissione progressiva di mappe geografiche in formato vettoriale. Infine, descriviamo un modello basato su un'architettura client-server e sulla pre-computazione di rappresentazioni multiple di mappe che può essere utilizzato per trasmettere mappe geografiche in modo progressivo su WWW.*

*Progressive transmission of raster images over the WWW has been successfully applied to provide users with coarser versions of the data before downloading a complete image. In the vector domain, progressive transmission remains a challenging topic. Increasing the level of detail of a vector dataset is a complex process that does not simply imply adding pixels to it. In this paper we present a survey of existing models for progressive transmission of spatial data and discuss open problems and challenges related to the progressive transmission of geographic maps in vector format. Finally, we propose a model based on a client-server architecture and on the pre-computation of multiple map representations that can be used to transmit geographic maps progressively over the WWW.*

# 1. Introduction

The transmission of geospatial data over the WWW still presents some technical impediments. Indeed, many available geospatial datasets have reached such a fine level of resolution that their downloading through slow communication links and mobile devices has become prohibitive. In these cases, progressive transmission techniques can be very useful. Progressive transmission of raster images over the WWW has been successfully applied to provide the user with temporary versions of the data before downloading a complete image. For example, interleaving techniques transmit a sequence of versions of an image. A coarser representation obtained by subsampling the pixels of the image to be displayed on the receiver's screen is sent so that the user can start working without having to wait for the whole image to be downloaded. The initial version is then progressively completed by adding new pixels. Compression mechanisms are also employed.

So far, implementations have focused on progressive transmission of raster data through the Web. The transmission of vector geospatial data[1], with the exception of the particular case of data in the form of triangular meshes, is generally done by means of a one-step long process. The user attempting to download a vector file needs to wait for the complete version without having the possibility to start working with a coarser version of the data stored in a smaller file. In particular, progressive transmission of vector map data is still a challenging topic due to the intrinsic complexity of map generalization. In this paper, we present existing methods for progressive transmission of geospatial vector data over the WWW and we discuss major challenges and open problems.

The remainder of the paper is organized as follows. In Section 2 we review existing methods for progressive transmission of geospatial data over the WWW. In Section 3 we outline the challenging issues related to the progressive transmission of vector map data. In Section 4 we propose a possible solution to progressive vector data transmission based on a distributed architecture, and on the pre-computation and storage of multiple map representations on the server site. Finally, Section 5 draws some conclusions and discusses future research directions.

# 2. Related work

This section reviews existing methods for progressive transmission of geospatial data both in raster (Section 2.1) and in vector format (Section 2.2).

## 2.1 Raster Data Transmission

The simplest method for progressive raster transmission extracts subsets of pixels randomly from the image that is being downloaded and incrementally completes the image by adding pixels. The implicit row/column ordering of images can be exploited to choose the subset of pixels more uniformly (e.g., one every five pixels in each row/column, etc.). If hierarchical structures such as quadtrees [Sam90a, Sam90b] are used to partition the image, more pixels can be extracted from those parts of the image that

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[1] Geospatial data in vector format refers to a set of spatial entities in the form of points, lines, and polygons that are related through spatial relations.

contain greater concentration of detail. More sophisticated methods are based on image compression techniques (see, for example, [Rau99, Sri99]). Probably the most used image compression method is the JPEG (Joint Photographic Experts Group) format (see [Gon93] for a detailed description). The JPEG method is a transform-based compression method that decomposes the original data before compression. As images in JPEG format are segmented into rectangular sub-blocks and each block is transformed independently, at high compression ratios they take on unnatural blocky artifacts.

Newer techniques are based on wavelet decompositions [Mor84]. Wavelet methods are also transform-based but they act on the entire image. The result is a very robust digital representation of a picture, which maintains its natural look even at (relatively) high compression ratios (see [Dav98a] for an overview). The mathematics behind wavelets is closely related to Fourier transforms [Tit48, Coo65]. As Fourier transforms are based on trigonometric functions (i.e., non-local smooth curves which stretch into infinity), they do not well represent sharp spikes and chopped signals. Fractal theory [Bar89] has also been exploited for image compression. A fractal is a geometrical figure whose local features resemble its global characteristics (self-similar property). The difficulty in compressing an image using fractals is in finding a small number of affine transformations to generate the image and in finding subparts of the input image that have self-similarity properties [Fis95; Dav96]. More recently, hybrid compression methods, e.g., combining fractals and wavelets, have been defined [Zha96; Dav98b] to improve the performance of fractal-based compression mechanisms.

When transform-based methods are used to compress an image, during transmission over the WWW, instead of the image, the function coefficients are transmitted, and the image is subsequently reconstructed by inverting the transformation. Both transmission and reconstruction are very efficient, as they can be performed simply by means of a single pass through each color band of the image. The purpose of raster data exchanging over the WWW is very often only visual. As visual meaning can be extracted from images at very low resolution, techniques for progressive raster transmission are usually very efficient. Even when measurements need be performed on the images (e.g., for photogrammetric purposes), common compression mechanisms such as JPEG have proved to be effective. However, sometimes objects need to be directly handled and manipulated. In this case a vector representation is required and preservation of topological and metric properties is a major issue. This topic is discussed in the next section.

## 2.2 Vector Data Transmission

Part of the vector geospatial data exchanged over the WWW comes in the form of triangular meshes. For example, triangulations are used for digital terrain modeling and for real objects surface reconstruction and rendering [Baj99b; DeF98].

Several compression methods for triangular meshes have been defined in the literature (see [DeF95] for a survey). Among them, the progressive mesh scheme proposed in [Hop96], represents a fundamental milestone for progressive transmission. However, this method does not have a satisfactory performance as, like many other traditional methods [Eck95; Zor97], it is topology preserving. This limits the level of compression. More recent methods achieve higher compression by slightly modifying the topology of the input mesh [Baj99a; Baj99b].

In [Jun98a; Jun98b] a parallel point decimation technique for TIN simplification has been defined that allows for progressive transmission and rendering of TIN data (check http://www.cs.ubc.ca/spider/snoeyink/terrain/Demo.html for a demo). Finally, De Floriani and Puppo [DeF95] proposed a general framework for multiresolution hierarchical mesh representation. Such a model has a very high storage cost. A more efficient encoding structure has been described in [DeF98] to allow for progressive transmission. Although these progressive transmission techniques are effective and they have already produced several prototypes [Baj99b; Jun98], they can only be applied to data represented in the form of triangular meshes.

Progressive transmission of vector map data is more problematic. In [But99] a model has been defined for progressive transmission of data in the form of sets of line features (e.g., files storing the hydrography of a given geographic area). Such a model is based on the pre-computation of multiple coarser representations. The building procedure consists of the following three steps.

1. *Separation of themes*: different thematic layers (e.g., hydrography, and transportation) are stored in separate files.
2. *Determination of an ordering of features within a theme*: an ordering of the features is established for transmission purposes.
3. *Hierarchical subdivision of vectors within each feature*: each line (represented as a set of arcs) is iteratively subdivided using the Douglas-Peucker algorithm [Dou73] and stored in a hierarchical strip tree [Bal81].

A pointer-based encoding structure for the tree is provided and used to visit the tree during progressive transmission and for reconstruction of each level.

Although this method presents the drawback of requiring pre-processing, it has the advantage of allowing for real-time progressive analysis of generalized data that might not be possible if on-line simplification was to be applied on demand. However, a major drawback of this method relates to the fact that the Douglas-Peucker algorithm, although adopted in many commercial GIS packages, does not guarantee preservation of topological consistency. This problem must be solved if the purpose of progressive transmission is actual manipulation and querying of the exchanged file. A solution consists of performing *a posteriori* checks on the model (see also [Saa99]). This step, of course, increases pre-processing time.

Finally, this method is applicable only for linear features, while it does not handle intrinsically 2-dimensional entities. Therefore, only line simplification operations can be performed on the transmitted vector files, while other basic operations (e.g., selection, aggregation, etc.) commonly used in cartographic generalization [McM92] cannot be applied.

A different strategy, based on the application of on-line generalization algorithms, is being investigated by Cecconi and Weibel and briefly described in [Cec00]. However, this work is still at a very early stage and will involve both the use of existing generalization algorithms and the definition of new ones to compensate for currently missing automated solutions. The problems involved with the automation of the map generalization problems are discussed in Section 3.

## 3. Challenges and open problems

Progressive transmission of raster data is relatively simple. For example, if the purpose of the transmission is just visualization, adding pixels to an incomplete image generates a more refined version of it. Very effective compression mechanisms are also available (as described in Section 2.1).

When the purpose of the transmission is object manipulation, a vector representation is required. Except for the particular case of triangular meshes for which effective compression methods have been defined (see previous section), progressive vector transmission is still challenging. Decreasing the level of detail of a vector map is a complex and time-consuming process called *map generalization* whose automation and on-line application are currently open problems. In fact, it relies on cartographic principles whose complete formalization is still lacking (see [Wei99] for a survey). Automated solutions have only been proposed for sub-problems and are based on heuristic methods (e.g., the Douglas-Peucker algorithm [Dou73]). Therefore, the process still requires interaction between semi-automatic solutions and expert cartographers.

A critical issue in map generalization is preservation of consistency (e.g., constraints on overlappings) while decreasing the level of detail [Rob84; McM92; Mül95; Wei99]. For example, random subsampling of objects to be eliminated or to be simplified does not usually generate a consistent representation at coarser detail. In Figure 1, an example is shown where random subsampling generates topological inconsistencies across different representations.

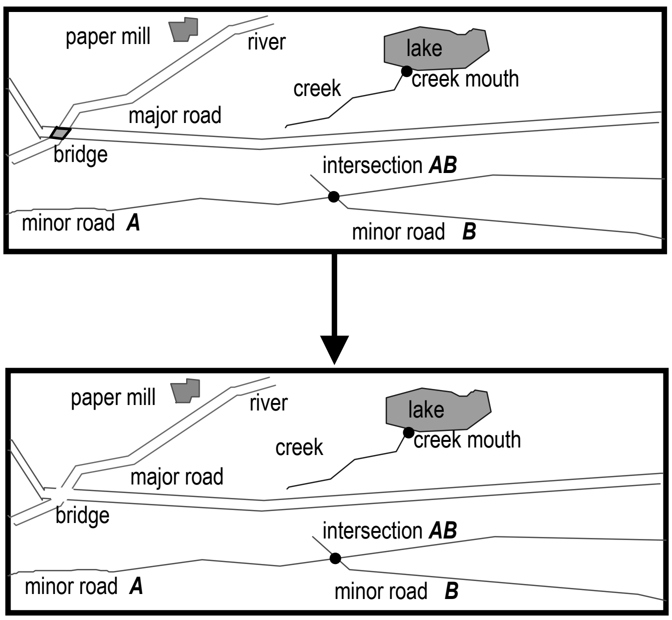


**Figure 1**: Random elimination of some entities (e.g., *bridge*) results in topologically incorrect representations: the two branches of *major road* and *river* have incomplete boundaries.

Consistency is an essential property for the usability of generalized data (e.g., for analysis and querying purposes). In the literature, a set of basic operations applied in the generalization process has been detected [Rob84]. Although some attempts have been made to provide formal frameworks for such operations, a few major problems are still far from being solved. Unfortunately, the majority of available automatic generalization techniques do not intrinsically provide consistency and thus *a posteriori* checks are required in order to adjust the result when some inconsistency has been introduced.

During progressive transmission, consistent datasets need to be transmitted to the client site, where there is no way to discover inconsistencies. Thus, consistency must be checked for on the server site either implicitly by adopting generalization operators that preserve it or by means of *a posteriori checks* on the generalized datasets. As on-line map generalization is still not feasible, in the following section we propose an alternative solution to enable progressive vector map data transmission.

## 4. An alternative solution

Our alternative solution is based on a client-server architecture (described in Section 4.1) and on the pre-computation of a sequence of consistent multiple map representations stored on the server site (described in Section 4.2).

### 4.1 Client-server architecture

In this section we describe a distributed client-server architecture that enables progressive transmission of vector map data [Ber99]. A sequence of consistent representations at lower levels of detail is pre-computed from a fully detailed map and stored on the server site. Such representations are transmitted in order of increasing detail to the client upon request (Figure 2).

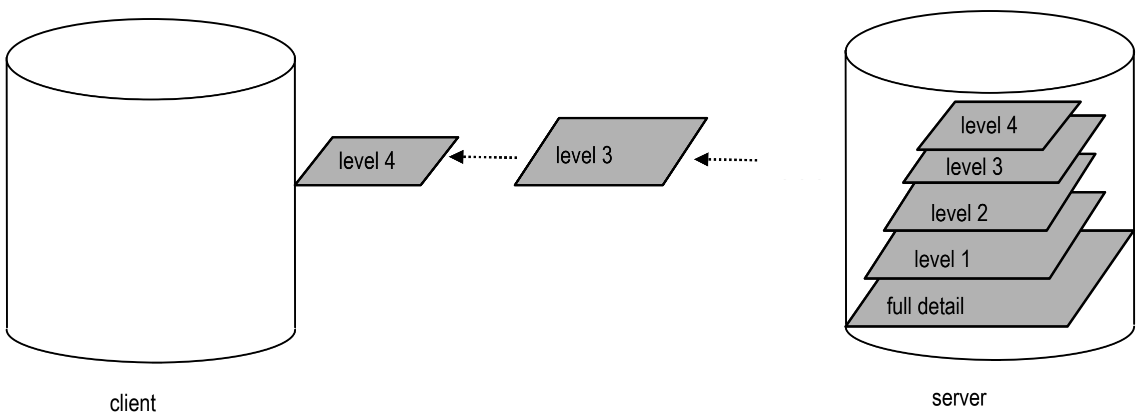


**Figure 2:** Representations at different levels of detail are maintained on the server and transmitted to the client in order of increasing detail.

Besides providing temporary versions of the data on which preliminary operations can be performed, in this way, users can realize during the process that the detail of the currently displayed representation is good enough for their purposes and so they can decide to interrupt the downloading of more detailed representations (stored in larger files). Therefore, both time and disk space can be saved.

An interesting property of some techniques for progressive transmission of raster data (e.g., interleaving techniques) is the fact that they only transmit increments (i.e., sets of new pixels) at each step. Such increments are added to the currently displayed image to improve its resolution without requiring the transmission or downloading of another complete image file.

Since vector files can be very large, it would be useful to speed up the transmission in a similar way to reduce network traffic. However, increments between two consecutive levels of detail of a vector dataset can be complex sets of entities that are added to the level at finer detail to refine the representation of a set of entities at the lower level. The integration of such increments with the currently downloaded or displayed representation is a non-trivial task if consistency between different representations must be preserved.

This solution to progressive vector map transmission relies on a distributed client-server architecture, with a server provided with methods for building, manipulating, and transmitting a sequence of map representations at different levels of detail; and a thin client provided with a set of operations for visualizing as well as updating and integrating the transmitted levels.

In the following section we describe a method for building a model for consistent multiple map representations initially presented in [Pup95] and then further formalized in [Ber98]. Such model can be applied for progressive transmission over the WWW [Ber99].

## 4.2 Computing consistent multiple representations

The model we describe in this section is based on a set of generalization operators that perform changes of detail on a vector map. Before describing the operators we introduce some preliminary definitions.

We define a *map* in vector format as a pair-wise disjoint set of points, simple lines and (possibly multiply connected) regions in the Euclidean plane related through spatial relations (e.g., topological, metric and direction relations [Ege91]). The mathematical model adopted for the formalization of the topological structure of a map is the (2-dimensional) *abstract cell complex* [Vic73]. A 2-dimensional abstract cell complex is a collection of pair-wise disjoint 0-, 1-, and 2-cells representing the points, lines and regions of the map. Topological relations are intrinsically modeled by this mathematical structure. This model also allows for semantic information related to the spatial entities in a map to be attached to the representation in the form of attributes of the cells.

Atomic operators for changing the level of detail of a map (i.e., changing the representation of a given entity either in terms of its dimension – e.g., from line to point – or in terms of its complexity – e.g., from a group of entities to a single entity) have been formalized as functions between the abstract cell complexes describing the original and the resulting maps [Ber98].

The list of atomic operators follows:

- **Line contraction**: contraction of an open line (including its endpoints) to a point (Figure 3a).
- **Region contraction**: contraction of a simply connected region (with its boundary) to a point (Figure 3b).
- **Region thinning**: a region (and its bounding lines) is reduced to a line (Figure 3c).
- **Line merge**: fusion of two lines sharing an endpoint into a single line (Figure 3d).
- **Region merge**: fusion of two regions sharing a boundary line into a single region (Figure 3e).
- **Point abstraction**: elimination of an isolated point inside a region (Figure 3f).
- **Line abstraction**: elimination of a line inside a region (Figure 3g).

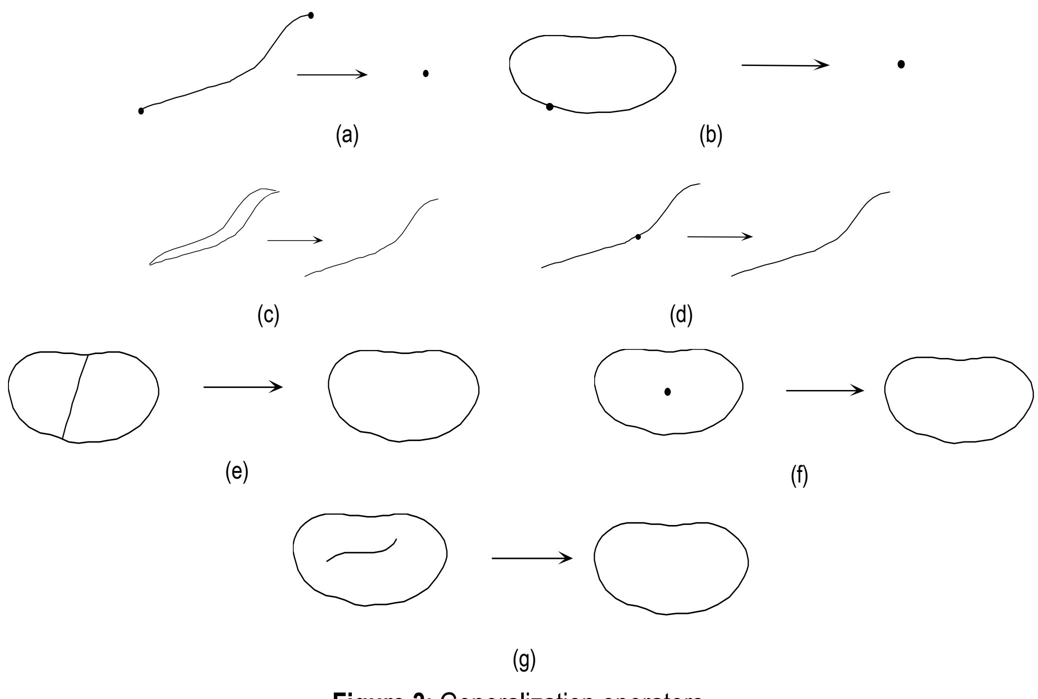


**Figure 3**: Generalization operators.

Contractions and thinning correspond to a decrease of dimension for a group of entities, merge processes group two entities of the same dimension into a single one, while abstractions correspond to the elimination of some lower-dimensional entity from a region. These operators are called atomic, because they perform minimal changes, i.e. they modify minimal sets of entities and preserve the others. By composing functions corresponding to such operators, complex map transformations can be defined.

As mentioned in Section 3, in the context of map generalization an important problem is guaranteeing that the resulting representation be consistent with the source fully detailed dataset. The majority of proposed methods do not intrinsically provide consistency and thus *a posteriori* checks are required in order to adjust the result when some inconsistency has been introduced. In the case of vector data transmission between two remote sites, the preservation of consistency is essential, because the client has no way of checking the consistency. This should be done on the server site, i.e., the datasets that are being sent over should be consistent (either implicitly by construction or by means of *a posteriori* checks).

The greatest advantage in using the set of generalization operators we have defined relies on the fact that they preserve topological consistency. Indeed, it has been shown that they represent a minimal and sufficient set of functions that generate by composition only consistent transformations for modifying the level of detail of a map [Ber98]. In particular, they generate all map transformations that preserve the boundary of entities and whose inverse image preserves connectivity (thus, they do not allow to model aggregation of non-connected entities). Spatial relations are consistently transformed. Therefore, intrinsically consistent multiple representation sequences are built on the basis of such operators.

## 5. Conclusions and future developments

In this paper, we have analyzed critical issues related to the transmission of geospatial datasets over the WWW. As these datasets can be very large, progressive techniques are a viable solution to overcome technical limitations related to slow, unreliable and wireless communication links. Progressive raster transmission is commonly used with success. In the vector domain, prototypes have been produced only to compress and transmit triangular meshes. For more general data, such as vector maps, the investigation is still at an early stage. A comprehensive overview of existing methods and a discussion of research challenges related to progressive transmission of geospatial data is presented in this paper.

Major problems with progressive transmission of geographic maps in vector format are related to the fact that on-line map generalization is still not feasible. Attempts to overcome these problems are currently being made [Cec00]. In the meantime, an alternative solution relies on the pre-computation of multiple map representations stored on a server and delivered progressively to the client upon request. These representations must be generated using sound cartographic principles. Two models have been proposed so far, one based on the Douglas-Peucker line simplification algorithm [But99] and one based on a set of operators that perform topological changes on a geographic map [Ber98, Ber99]. The major drawbacks of the first approach are: (1) it is only applicable to datasets composed exclusively of line features and (2) Douglas-Peucker algorithm does not guarantee preservation of consistency; therefore, *a posteriori* checks are required.

Two main advantages of the second approach are the fact that it also handles 2-dimensional entities and the fact that it inherently preserves topology. However, it has the defect of not taking into account metric and semantic aspects involved in a typical map generalization process. Only topological changes are possible by applying the atomic operators defined in [Ber98]. Therefore, this model should be extended to include also operations such as change in geometric shape, etc. We are currently investigating this issue.

Finally, as briefly noted in Section 4.1, efficient encoding and transmission strategies play an important role when delivering data over the WWW. An efficient data structure that avoids redundancies in multiple map representations has been proposed in [Ber98]. Only the coarsest level is completely stored and for each intermediate level just the increments (i.e., newly introduced entities and new representations of old entities) with respect to previous levels are explicitly maintained. While this structure saves both storage space and transmission time, it requires the implementation of techniques to reconstruct full intermediate levels on the client site by integrating previous levels with increments. The development of algorithms for solving this problem is part of our future work. A prototype implementation of the model and its application for progressive transmission is also under development.

## References

**[Baj99a]** C.L. Bajaj, V. Pascucci, and G. Zhuang, 1999a, "Single resolution compression of arbitrary triangular meshes with properties", *IEEE Data Compression Conference*, pp. 247-256.

**[Baj99b]** C.L. Bajaj, V. Pascucci, and G. Zhuang, 1999b, "Progressive Compression and Transmission of Arbitrary Triangular Meshes", *Technical Report*, Department of Computer Science, University of Texas at Austin, Austin, TX, http://king.ticam.utexas.edu/CCV/papers/cpm-all.pdf

**[Bal81]** D. H. Ballard, 1981, "Strip Trees: A Hierarchical Representation for Curves", *Communications of the ACM*, 24(5), pp. 310-321.

**[Bar89]** M. Barnsley, 1989, *Fractals Everywhere*, Academic Press, San Diego.

**[Ber98]** M. Bertolotto, 1998, "Geometric Modeling of Spatial Entities at Multiple Levels of Resolution", *Ph.D. Thesis*, Department of Computer and Information Sciences, University of Genova, Italy.

**[Ber99]** M. Bertolotto and M. Egenhofer, 1999, "Progressive Vector Transmission", *Proceedings ACMGIS99*, Kansas City, MO, pp. 152-157.

**[But99]** B. Buttenfield, 1999, "Progressive Transmission of Vector Data on the Internet: A Cartographic Solution", *Proceedings 18th International Cartographic Conference*, Ottawa, Canada.

**[Cec00]** A. Cecconi and R. Weibel, 2000, "Map Generalization for On-demand Web Mapping", *GIScience2000*, Savannah, GA.

**[Coo65]** J. W. Cooley and J. W. Tukey, 1965, "An Algorithm for the Machine Calculation of Complex Fourier Series", *Math. of Comput.*, 19, pp. 297-301.

**[Dav96]** G. Davis, 1996, "Implicit Image Models in Fractal Image Compression", *SPIE Conference on Wavelet Applications in Signal and Image Processing IV*, Denver, TX.

**[Dav98a]** G. Davis and A. Nosratinia, 1998, "Wavelet-Based Image Coding: An Overview", *Applied and Computational Control, Signals, and Circuits*, Vol. 1, No. 1.

**[Dav98b]** G. Davis, 1998, “A Wavelet-based Analysis of Fractal Image Compression”, *IEEE Transactions on Image Processing*, 7(2), pp. 141-154.

**[DeF95]** L. De Floriani and E. Puppo, 1995, “Hierarchical Triangulation for Multiresolution Surface Description”, *ACM Transaction on Graphics,* 14(4), pp. 363-411.

**[DeF98]** L. De Floriani P. Magillo and E. Puppo, 1998, “Efficient Implementation of Multi-Triangulations” *Proceedings IEEE Visualization ‘98*, Research Triangle Park, NC, USA, pp. 43-50.

**[Dou73]** D. H. Douglas and T.K. Peucker, 1973, “Algorithms for the reduction of the number of points required to represent a digitized line or its caricature”, *The Canadian Cartographer,* 10, 2, pp. 112-122.

**[Eck95]** M. Eck, T. DeRose, T. Duchamp, and H. Hoppe, 1995, “Multiresolution analysis of arbitrary meshes”, *Computer Graphics* (*Proceedings SIGGRAPH ’95)*, pp. 173-182.

**[Ege91]** M. Egenhofer and R. Franzosa, 1991, “Point-set topological spatial relations”, *International Journal of Geographic Information Systems*, 5(2), pp. 161-174.

**[Fis95]** Y. Fisher (ed.), 1995, *Fractal Image Compression: Theory and Application to Digital Images*, Springer Verlag, New York.

**[Gon93]** R. C. Gonzales and R. E. Woods, 1993, *Digital Image Processing,* Addison-Wesley.

**[Hop96]** H. Hoppe, 1996, “Progressive Meshes”, *Proceedings SIGGRAPH96*, pp. 99-108.

**[Jun98a]** B. Junger and J. Snoeyink, 1998a, “Importance Measures for TIN Simplification by parallel decimation”, *Proceedings Spatial Data Handling ‘98*, Vancouver, Canada, pp. 637-646.

**[Jun98b]** B. Junger and J. Snoeyink, 1998b, “Selecting Independent Vertices For Terrain Simplification”, *Proceedings WSCG '98*, Plzen, Czech Republic.

**[McM92]** R.B. McMaster and K.S. Shea, 1992, *Generalization in Digital Cartography*, Washington DC, Association of American Geographers.

**[Mor84]** J. Morlet and A. Grossman, 1984, “Decomposition of Hardy Functions into Square Integrable Wavelets of Constant Shape” - *Siam. J. Math, Anal.,* 15(4), pp. 723 –736.

**[Mül95]** J.C. Müller, R. Weibel, J.P. Lagrange, and F. Salgé, 1995, “Generalization: State of the Art and Issues”, in GIS and Generalization: Methodology and Practice, J.C. Müller, J.P. Lagrange, R. Weibel (eds.), Taylor and Francis, pp. 3-7.

**[Pup95]** E. Puppo and G. Dettori, 1995, “Towards a formal model for multiresolution spatial maps”, in M. J. Egenhofer, J. R. Herring (eds.), *Advances in Spatial Databases,* Lecture Notes in Computer Science N. 951, Springer Verlag, pp. 152-169.

**[Rau99]** U. Rauschenbach, and H. Schumann, 1999, “Demand-driven Image Transmission with Levels of Detail and Regions of Interest”, *Computers and Graphics*, 23(6), pp. 857-866, 1999.

**[Rob84]** A.H. Robinson, R.D. Sale, J.L. Morrison, and P.C. Muehrcke, 1984, *Elements of Cartography,* John Wiley, New York, 5th edition.

**[Saa99]** A. Saalfeld, 1999, “Topologically Consistent Line Simplification with the Douglas-Peucker Algorithm”, *Cartography and Geographic Information Science,* 26(1), pp. 7-18.

**[Sam90a]** H. Samet,1990a, *The Design and Analysis of Spatial Data Structures,* Addison-Wesley, Reading, MA.

**[Sam90b]** H. Samet,1990b, *Applications of Spatial Data Structures,* Addison-Wesley, Reading, MA.

**[Sri99]** B. S. Srinivas, R. Ladner, M. Azizoglu, and E. A. Riskin, 1999, "Progressive Transmission of Images using MAP Detection over Channels with Memory*", IEEE Transactions on Image Processing*, 8(4), pp. 462-75.

**[Tit48]** E. C. Titchmarsh, 1948, *Introduction to the Theory of Fourier Intervals,* Oxford University Press, New York.

**[Vic73]** J. M. Vick, 1973, *Homology Theory: an Introduction ot Algebraic Topology*, Pure and Applied Mathematics, Academic Press.

**[Wei99]** R. Weibel and G. Dutton, 1999, "Generalising spatial data and dealing with multiple representations", in *Geographical Information Systems: Principles, Techniques, Management and Applications,* P. Longley, M.F. Goodchild, D.J. Maguire and D.W.Rhind (eds.), Second Edition, Cambridge, GeoInformation International, pp. 125-155.

**[Zha96]** Y. Zhao and B. Yuan, 1996, "A hybrid image compression scheme combining block-based fractal coding and DCT", *Signal Processing: Image Communication,* 8(2), pp. 73-78.

**[Zor97]** D. Zorin, P. Shroder, and W. Sweldens, 1997, "Interactive multiresolution mesh editing", *Computer Graphics* (*Proceedings SIGGRAPH '97)*, pp. 259-268.