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A Review of Probabilistic Methods of Assessment of Load Effects in Bridges

Eugene J. OBrien
University College Dublin

F Schmidt
French Institute of Science and Technology for Transport, Development and Networks (IFSTTAR), France

Donya Hajializadeh
University College Dublin

See next page for additional authors

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Authors
Eugene J. OBrien, F Schmidt, Donya Hajializadeh, X-Y Zhou, Bernard Enright, Colin C. Caprani, Simon Wilson, and Emma Sheils

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Abstract

This paper reviews a range of statistical approaches to illustrate the influence of data quality and quantity on the probabilistic modelling of traffic load effects. It also aims to demonstrate the importance of long-run simulations in calculating characteristic traffic load effects. The popular methods of Peaks Over Threshold and Generalized Extreme Value are considered but also other methods including the Box-Cox approach, fitting to a Normal distribution and the Rice formula. For these five methods, curves are fitted to the tails of the daily maximum data.

Bayesian Updating and Predictive Likelihood are also assessed, which require the entire data for fittings. The accuracy of each method in calculating 75-year characteristic values and probability of failure, using different quantities of data, is assessed. The nature of the problem is first introduced by a simple numerical example with a known theoretical answer. It is then extended to more realistic problems, where long-run simulations are used to provide benchmark results, against which each method is compared. Increasing the number of data in the sample results in higher accuracy of approximations but it is not able to completely eliminate the uncertainty associated with the extrapolation. Results also show that the accuracy of estimations of characteristic value and probabilities of failure are more a function of data quality than extrapolation technique. This highlights the importance of long-run simulations as a means of reducing the errors associated with the extrapolation process.

Keywords: Review, Bridge, Load, Traffic, Assessment, POT, Peaks-Over-Threshold, Extreme Value, GEV, Box-Cox, Rice, Predictive Likelihood.

1. Introduction

A necessary part of bridge management is assessment of the safety of bridge structures. In its simplest form, a bridge is safe when its capacity to resist load exceeds the load applied. More precisely, a bridge can be considered safe when there is an acceptably low probability that load exceeds capacity. A great deal of work has been carried out on methods of evaluating the load-carrying capacity of bridges and the associated uncertainties. Load-carrying capacity can be reduced by different forms of deterioration, depending on factors such as the structural material, the quality of workmanship during construction, the age of the structure, the environment and the loading history. To carry out a more accurate assessment of the load-carrying capacity, non-destructive and/or destructive tests can be carried out to get more detailed site specific information on these deterioration mechanisms to reduce uncertainty and associated conservatism (Al-Harthy et al., 2011, Frangopol & Liu, 2007, Richard et al., 2012, Rücker et al., 2005, Suo & Stewart, 2009). These inspection results can be incorporated into time-dependent reliability-based assessments to give up-to-date structure-specific deterioration rates. These in turn can be used to accurately predict the capacity of the structure and to schedule maintenance and repairs (Melchers, 1999, Orcesi & Cremona, 2009, Orcesi & Cremona, 2010, Sheils et al., 2010).

Traffic loading on bridges, one of the great sources of uncertainty, is the focus of this paper. In this study, historical developments in the field of traffic loading are reviewed. A wide range of statistical/probabilistic approaches have been applied to the problem, using different quantities of data, with no clear 'winner' emerging. Two Extreme Value examples are used here as benchmark tests, against which a range of approaches are compared. The first example is the problem of finding the maximum of numerous normally distributed random variables, a problem for which the exact theoretical solution is known. The nature of the problem is studied using a number of samples with different quantities of data.

The second example is based on a carefully calibrated traffic load simulation model. The simulation is run for 5000 years so that, while the exact solution is unknown, it can be estimated very well and there is a high degree of confidence in the lifetime maximum results. As for the first example, several methods of prediction, using modest quantities of data, are tested to demonstrate the importance of the quantity of data in probabilistic assessments.

In this study no allowance for growth in traffic loads is made. Vehicle traffic is a non-stationary phenomenon with variation in both vehicle proportions and weights experienced over time as a function of economic, legal and technological developments. Despite the recent economic downturn, the European Commission (2008) predicts a sustainable annual growth in road freight volume of between 1.5% and 2% per annum until 2030. O'Connor et al (2001) note a substantial increase in the number of 5-axle vehicles over a 10 year period. Sivakumar et al. (2011) recognise the need to allow for growth in truck weights and traffic intensities and propose an economic projection analysis. OBrien et al. (2014) consider growth in the numbers of heavy vehicles and provide a means of addressing the non-stationary nature of growing traffic. However, growth is considered to be beyond the scope of this paper.

2. Review of Literature

Load effects (LE's) – bending moments, shear forces, etc. – result from traffic passing over a bridge. The process varies in time with many periods of zero LE when there is no traffic on the bridge and peaks corresponding to heavy vehicle crossings or more complex vehicle meeting or overtaking scenarios. The majority of the local peaks in LE are due to cars which are relatively light and there have been many efforts to simplify the problem by excluding consideration of these data. The methods of statistical inference used in the literature to predict the extremes of traffic LE's are quite diverse.

Tail Fitting

In the context of this problem, many approaches fit a distribution to the tail of the Cumulative Distribution Function (CDF) of the LE's. This can be justified by the fact that the distribution is often made up of a mixture of load effect types – for example, LE's due to 2-axle trucks and those due to heavy low-loader vehicles. For bridge traffic loading, the heavier vehicles tend to dominate, with the lighter ones making very little contribution to the probability of exceedance at the extremes. The tail can be chosen by engineering judgement when the cumulative distribution is seen to change at a particular probability level. Alternatively, some authors have fitted to the top $2\sqrt{n}$ of a distribution of n data, based on theoretical considerations (Castillo, 1988). Others have fitted to the top 30% of data (Enright, 2010) based on sensitivity analyses.

Two of the tail fitting approaches are particularly popular – Peaks-Over-Threshold (POT) and Block Maximum. POT considers the extent by which the peaks of LE exceed a specified threshold. The POT LE's are fitted to a probability distribution such as the Generalized Pareto distribution. In the Block Maximum approach, only the maximum LE's in given blocks of time (days, years, etc.), are considered. This has the advantage of time referencing the data which is necessary when calculating lifetime maximum probabilities of exceedance. Block maximum LE's can be fitted to one of a range of distribution types such as Generalised Extreme Value (GEV) (incorporating Gumbel, Weibull and Fréchet), or Normal. Fitting block maximum values to GEV and Normal distributions will be considered here.

The Block Maximum approach has the disadvantage that only one LE in each block of time is considered, even if several very large LE's are recorded. The POT approach addresses this issue but the selection of the threshold, below which LE's are discarded, is subjective. The Box-Cox approach is more general and aims to address the disadvantages of both POT and GEV. The Rice formula is also investigated as it was used for the extrapolations in the background study supporting the development of the Eurocode for traffic loading on bridges. However, while the Rice formula is a fitting to tail data, it is applied to a histogram of 'upcrossings' past a threshold, not to a CDF, and assumes a normally distributed process.

Full Distribution Fitting

Bayesian Updating is another approach that can be applied to bridge traffic loading. A probability distribution is assumed for the block maximum LE's and is updated using available LE data. While only tail data could be used, in this work, the Bayesian approach is used to update the entire distribution, not just the tail. Predictive Likelihood also seeks to develop a probability distribution for all LE's but uses a frequentist likelihood approach, assigning likelihoods on the basis of the quality of the fit to the measured data.

2.1 Peaks Over Threshold (POT)

Block Maximum approaches use only the maximum LE in each block of time. There is therefore a risk that some important data is discarded: if two unrelated extreme loading events occur in the same block of time, only one of the resulting LE's is retained. In such a case, the POT approach would retain both LE's as valid data.

To find characteristic maximum values of LE, data above the threshold must be fitted to a probability distribution. Coles (2001) provides a brief outline proof that the Generalized Pareto (GP) distribution approximates the CDF of such POT data well. Crespo-Minguillón & Casas (1997) use the GP distribution to model the excesses of weekly maximum traffic LE's over a threshold. James (2003) applies the POT method to analyse load effects on railway bridges. Gindy & Nassif (2006) analyse load effects caused by combined data from over 33 Weigh-in-Motion sites over an 11-year measurement period, and compare extreme values as predicted by both GP and GEV distributions.

A significant drawback of the POT approach is the issue of selecting the threshold. There are many different kinds of loading scenario on a typical bridge. For example, there are usually many single-vehicle crossings of standard 5-axle trucks. The probability distribution of LE's due to such an event type may be quite different from that due to large cranes or that due to 2-truck meeting events (Caprani et al., 2007). If the threshold is too low, there may be an excessive mixing of extreme event types with other less critical types which can result in convergence to an incorrect characteristic LE.

On the other hand, if the threshold is too high, there will be too few peaks above the threshold, leading to high variance and unreliable results.

The basic principle in selecting a threshold is to adopt as low a threshold as possible, while maintaining a consistent trend in the data. The issue of threshold choice is analogous to the choice of block size in the block maxima approach, implying a balance between bias and variance. Two methods are available (Coles, 2001): one is an exploratory technique carried out prior to model estimation; the other is an assessment of the stability of parameter estimates, based on the fitting of models across a range of different thresholds. Crespo-Minguillón & Casas (1997) apply the latter method and select the optimal threshold based on a weighted least squares fit.

Having selected the threshold, the next step is to estimate the parameters of the GP (or other) distribution. Bermudez & Kotz (2010) consider several methods of estimating these parameters including the method of moments, the probability weighted method, the maximum likelihood method, and Bayesian updating. Crespo-Minguillón & Casas (1997) adopt a methodology that is based on the minimization of the weighted sum of squared errors. James (2003) and Gindy & Nassif (2006) use maximum likelihood estimation.

2.2 Block Maximum – Extreme Value Distributions

Extreme value theory is based around the extreme value theorem, proved by Gnedenko (1943) and based on initial work by Fisher & Tippett (1928) and Gumbel (1935). For a sequence of independent random variables X_1, X_2,..., with distribution function $F(x)$ = Prob($X \leq x$), the distribution of max(X_1,...,X_n) is $F(x)^n$. As n gets large, this degenerates to 0 if $F(x) < 1$, as is usual. The Fisher-Tippett theorem shows that a non-degenerate distribution can be found using a linear function of x, say $a_n + b_n x$. Then, there is a non-trivial limit to $F(a_n + b_n x)$ and this limit must be in the form of the Generalised Extreme Value distribution (GEV), also known as the Fisher-Tippett distribution (Jenkinson, 1955, Von Mises, 1936):

$$F_{GEV}(x) = \begin{cases} exp\left(-\left(1+\xi\left(\frac{x-\mu}{\sigma}\right)\right)^{-\frac{1}{\xi}}\right) & \text{, if } \xi \neq 0 \\ exp\left(-exp\left(-\frac{x-\mu}{\sigma}\right)\right) & \text{, if } \xi = 0 \end{cases} \qquad \text{Equation 1}$$

defined in terms of parameters μ, σ and ξ where $\mu \in R$ is the location parameter, $\sigma > 0$ the scale parameter and $\xi \in R$ the shape parameter, such that $1+\xi(x-\mu)/\sigma > 0$. Hence, for an appropriately large n, the exact distribution, $F(x)^n$, converges asymptotically to $F_{\text{GEV}}(x)$. For the Normal distribution, the theorem holds and it is well known that its limiting distribution is the Gumbel, the $\xi = 0$ case of the GEV. However, convergence is slow (Cramér, 1946).

Each block maximum LE is the maximum of many traffic loading scenarios. As convergence may be slow, Caprani (2005) and OBrien et al. (2010) have fitted block maximum LE data with a 'Normal to the power of n', i.e., a Normal distribution raised to some power, n, whose value is found by fitting to the data. This has merit for smaller data samples. Ghosn et al. (2003) determine the distribution of lifetime maximum LE by raising the parent distribution of LE to an appropriate power. In this way they determine the mean and coefficient of variation of the maximum LE. Caprani (2005) describes a probabilistic convolution method to obtain bending moments for single truck loading events and obtains the distribution of lifetime maximum LE by raising the parent distribution to an appropriate power. Other authors attempt to calculate the exact distribution of extreme load effect, based on a fit to the parent distribution (Bailey, 1996, Bailey & Bez, 1994, Cooper, 1995, Getachew, 2005, Ghosn & Moses, 1985, Nowak & Hong, 1991, Nowak et al., 1993). This is done by raising the initial distribution to an appropriate power.

Most researchers fit block maximum LE data to one of the extreme value distributions described by the GEV equation: Gumbel, Fréchet or Weibull (also known as Types I, II or III). The three types of distribution have distinct forms of behaviour, corresponding to the different forms of the tail in the original distribution function (Fisher & Tippett, 1928). Coles (2001) establishes the conditions under which the Gumbel, Fréchet and Weibull distributions are the limiting forms for various parent distributions (Gumbel, 1958).

In early applications of Extreme Value theory, it was usual to adopt one of the three distributions, and then to estimate the relevant parameters. There are two weaknesses with this: first, a technique is required to choose which of the three distributions is most appropriate for the data at hand; second, once such a decision is made, subsequent inferences presume this choice to be correct, and do not allow for the uncertainty such a selection involves, even though this uncertainty may be substantial (Coles, 2001). Nevertheless, many studies (Caprani & OBrien, 2006, Caprani et al., 2008, Kanda & Ellingwood, 1991, O'Connor & OBrien, 2005) indicate that LE data is either Weibull or Gumbel and, given that Gumbel is a special case of Weibull (with shape parameter, $\xi = 0$), an assumption that LE is always of the form of Equation 1, with $\xi \leq 0$, seems reasonable.

Grave (2001) uses a weighted least-squares approach to fit Weibull distributions to critical LE's. O'Connor (2001) fits Gumbel and Weibull distributions to a population of 'extreme' LE's. OBrien et al. (2003) plot hourly maximum strain values on Gumbel probability paper. A least-squares, straight-line fit is made to the upper $2\sqrt{n}$ data points in a similar manner to Grave (2001) and O'Connor (2001). González et al. (2003) also use the Gumbel and Weibull distributions to extrapolate bridge load effect. Getachew (2005) fits the Generalized Extreme Value distribution to the LE's from simulated 2-truck meeting events representing two weeks of traffic. Bailey (1996) describes the use of plots of the mean and standard deviation of load effects, to estimate the appropriate extreme value distribution. Bailey (1996), Bailey & Bez (1994) and Bailey & Bez (1999) describe a qualitative analysis of 500 simulated upper tails of mean maximum load effects plotted against the number of events that contribute. They determine that the Weibull distribution is most appropriate to model these tails and use maximum likelihood estimation. Cooper (1997) presents a traffic model of about 81 000 measured truck events, and uses it to determine the distribution of LE's due to a 'single event'. He raises this distribution to powers to determine the distribution of LE for 1, 4, 16, 256 and 1024 such events. A Gumbel distribution is then fitted to this 1024-event distribution and used to extrapolate to a 2400-year return period. Cooper (1997) converts histograms of two-week traffic LE's into CDF's, which he then raises to a power equal to the number of trucks per day, to give the distribution of daily block maxima.

Moyo et al. (2002) plot daily maximum strain values on Gumbel probability paper and use a least-squares fit to determine the parameters of the distribution. Buckland et al. (1980) use a Gumbel distribution to fit the 3-monthly maximum LE's and extrapolate to find characteristic values. Getachew (2005) uses the GEV distribution to model the parent distribution of load effect, but not as an asymptotic approximation to the distribution of extreme values. Sivakumar et al. (2011) adopt the Gumbel distribution to project the statistics of the maximum LE's for different return periods.

2.3 Box-Cox Approach

Researchers commonly debate the merits of the POT method relative to the Block Maximum approach. The Box-Cox transform (Box & Cox, 1964) is used by Bali (2003) to introduce a more general extreme value distribution that encompasses the Generalised Pareto and Generalised Extreme Value distributions (Caprani & OBrien, 2009, Rocco, 2010). In the Box-Cox transformation, transformed values are a monotonic function of the observations over some admissible range and are indexed by:

$$y_i^{(\lambda)} = \begin{cases} y_i^{\lambda} & \lambda \neq 0 \\ log\ (y_i) & \lambda = 0 \end{cases} \qquad \text{Equation 2}$$

where λ is the transform parameter and y_i is the observation (Box & Cox, 1964).

This transformation offers the possibility of improving the rate of convergence to the limiting extreme value form, since different distributions converge at different rates. This approach restricts the methodology to cases where the extreme data are strictly positive (Wadsworth et al., 2010) but still encompasses a wide variety of practical problems including traffic loading on bridges. The use of the Box-Cox transformation in extreme value analysis was considered before in an entirely different context by Eastoe & Tawn (2009).

The Box-Cox-GEV extreme value distributions are given by Bali (2003) as:

$$H(x) = \left(\frac{1}{\lambda}\right)\left(\left[exp\left\{-[h(x)]_+^{1/\xi}\right\}\right]^{\lambda} - 1\right) + 1 \qquad \text{Equation 3}$$

in which

$$h(x) = 1 - \xi\left(\frac{x-\mu}{\sigma}\right) \qquad \text{Equation 4}$$

The parameters of this distribution are those of the GEV (μ, σ, ξ) plus a 'model parameter', λ. As $\lambda \rightarrow 1$, Box-Cox converges to the GEV distribution. Conversely, as $\lambda \rightarrow 0$, by L'Hôpital's Rule, it converges to the GP distribution. To apply this model, a high threshold is set on the parent distribution (Caprani & OBrien, 2009, Rocco, 2010). Bali (2003) uses a threshold of two standard deviations about the sample mean. Caprani & OBrien's thresholds (2009) are taken in steps of 0.5 standard deviations in the range from −2.5 to +2.5 standard deviations about the sample mean. Tötterman (2010) suggests that the additional parameter should increase the accuracy for Box-Cox, compared with GEV and GP.

Bali & Theodossiou (2008) evaluate the performance of three extreme value distributions including the GP, GEV and Box-Cox. The empirical results show that the asymptotic distribution of the maximal and minimal returns fits the Box-Cox-GEV distribution in this case. A likelihood ratio test between the GEV and Box-Cox results in a rejection of the former (Bali & Theodossiou, 2008, Caprani & OBrien, 2009).

2.4 Block Maximum – Normal Distribution

Block maximum data is often fitted with extreme value distributions as each data point represents the maximum of a number of parent values. However, block maximum data is also sometimes fitted to a Normal distribution. Nowak (1999) uses a form of Normal (Gaussian) probability paper, i.e., he fits the data to a Normal distribution and extrapolates to find the characteristic maximum. In an earlier study, Nowak (1993) uses 2.4 hours as the block size and fits the maximum-per-block data to a Normal distribution. This distribution is then raised to an appropriate power to obtain the 75-year maximum LE distribution.

To calibrate the traffic load model for the AASHTO load and resistance factor design (LRFD) approach, Nowak and others use Normal probability paper to extrapolate the maximum LE's for time periods from 1 day to 75 years, based on a set of 9250 heavy vehicles representing about two weeks of heavy traffic measured on a highway in Ontario (Kulicki et al., 2007, Moses, 2001, Nowak, 1994, Nowak, 1995, Nowak, 1999, Nowak & Hong, 1991, Nowak et al., 1993, Sivakumar et al., 2011). The expected values of the lifetime maximum LE's are found by fitting a straight line to the tails of the data on Normal probability paper.

In the background studies for Eurocode 1, Flint & Jacob (1996) fit half-normal curves to the ends of the histograms of LE. They adopt a least-squares best fit method to estimate the distribution parameters. Multimodal (bimodal or trimodal) Gumbel and Normal distributions are also used.

2.5 Rice Formula

The Rice formula, introduced by Rice (1945) and described more recently by Leadbetter et al. (1983), can be used to find a parametric fit to statistical data. Ditlevsen (1994) suggests that a load effect created by the traffic on a long span bridge can be modelled as a Gaussian random process. Under that hypothesis, the mean rate $v(x; \sigma, m, \dot{\sigma})$ of up-crossings for a threshold level, $x > 0$ during a reference period T_{ref}, can be expressed by the Rice formula:

$$v(x) = \frac{\sigma'}{2\pi\sigma} exp\left[-\frac{(x-m)^2}{2\sigma^2}\right] \qquad \text{Equation 5}$$

where m is the mean value, σ is the standard deviation and σ' is the standard deviation of the stochastic process derivative $\dot{x}$. The CDF can be found from the definition of return period which is the mean period between two occurrences, or the value with an expectation of being crossed one time during the return period, R (Cremona, 2001):

$$F(x) = ex\,p\left[-Rv_0 \times ex\,p\left(-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)\right)\right] \qquad \text{Equation 6}$$

where, v_0 is $\sigma'/2\pi\sigma$.

Cremona (2001) suggests the Kolmogorov test (DeGroot, 1986) to select the optimal number of bins in the outcrossing rate histogram and the threshold. Getachew (2003) adopts Cremona's approach for the analysis of traffic LE's on bridges induced by measured and Monte Carlo simulated vehicle data. O'Connor & OBrien (2005) compare the predicted extremes of simply supported moment for a range of span lengths by the Rice formula, Gumbel and Weibull Extreme Value distributions: they find about 10% difference between Rice and the others. Finally, Jacob (1991) uses Rice's formula to predict characteristic LE's for the cases of free and congested traffic in background studies for the development of the Eurocode.

2.6 Fitting Distributions to Extreme Data & Bayesian Inference

The concept of Bayesian Updating stems from Bayes' Theorem and is a major pillar of modern statistics. Bayesian Updating involves the adoption of an initial (prior) probability distribution, perhaps based on past experience, and updating it on the basis of measured data to give a posterior distribution (Basu, 1964, Bhattacharya, 1967, Holla, 1966).

Sinha & Sloan (1988) use Bayesian Inference to find the full 3-parameter Weibull distribution from measured data. They propose the use of Bayes Linear Estimate to approximate the posterior expectations and formulate the corresponding calculations for the Weibull parameters. Smith & Naylor (1987) work with the regular Weibull distribution with three parameters, comparing Maximum Likelihood with Bayesian estimators, using specially adapted versions of numerical quadrature to perform the posterior calculations. Although the priors they work with are arbitrary, they are chosen to reflect a range of potential scientific hypotheses. They report that the Bayesian inferential framework as a whole proves more satisfactory for their data analysis than the corresponding likelihood-based analysis. The issue of prior elicitation is pursued by Singpurewalla & Song (1988), who restrict attention to the '2-parameter' Weibull model, i.e., a Weibull model with a constant shape parameter. The predictive density function (Aitchison & Dunsmore, 1980) is defined as:

$$f(y|x) = \int f(y|\theta)f(\theta|x)d\theta \qquad \text{Equation 7}$$

where x represents historical data, y a future observation, θ the vector of parameters describing the distribution, $f(y|\theta)$ the likelihood and $f(\theta|x)$ the posterior distribution of θ given x. Thus, the predictive distribution averages the distribution across the uncertainty in θ as measured by the posterior distribution. Lingappaiah (1984) develops bounds for the predictive probabilities of extreme order statistics under a sequential sampling scheme, when sampling is carried out from either an exponential or Pareto population. From a practical viewpoint, the most important issues arising from the Bayesian literature are the elicitation and formulation of genuine prior information in extreme value problems, and the consequent impact such a specification has on subsequent inferences. Coles & Tawn (1996) consider a case study in which expert knowledge is sought and formulated into prior information as the basis for Bayesian analysis of extreme rainfall.

2.7 Predictive Likelihood

The relatively new theory of frequentist Predictive Likelihood can be used to estimate the variability of the predicted value, or predictand. Fisher (1959) is the first clear reference to the use of likelihood as a basis for prediction in a frequentist setting. A value of the predictand (z) is postulated and the maximized joint likelihood of the observed data (y) and the predictand is determined, based on a probability distribution with given parameters. The graph of the likelihoods thus obtained for a range of values of the predictand, yields a predictive distribution. Such a predictive likelihood is known as the profile predictive likelihood. It is found by maximising the likelihood of the data, L_y, and the predictand, L_z, jointly:

$$L_P(z|y) = \sup_{\theta} L_y(\theta;y)L_z(\theta;z) \qquad \text{Equation 8}$$

This formulation states that the likelihood of the predictand, z, given the data, y, is proportional to the likelihood of both the data (L_y) and the predictand (L_z) for a maximized parameter vector, θ (Caprani & OBrien, 2010).

Mathiasen (1979) appears to be the first to study Fisher's Predictive Likelihood and notes some of its problems. Foremost in this work is the problem that it does not take into account the parameter variability for each of the maximizations of the joint likelihood function required (Bjornstad, 1990, Lindsey, 1996). Lejeune & Faulkenberry (1982) propose a similar predictive likelihood, but include a normalizing function.

Predictive Likelihood is a general concept and in the literature many versions have been proposed. Cooley and Parke have a number of papers dealing with the prediction issue (Cooley & Parke, 1987, Cooley & Parke, 1990, Cooley et al., 1989). However, their method relies on the assumption that the parameters are normally distributed. Leonard (1982) suggests a similar approach while Davison & Hinkley (1997) use a different form of Predictive Likelihood.

Caprani & OBrien (2010) use the Predictive Likelihood method proposed by Butler (1986), based on that of Fisher (1959) and Mathiasen (1979) and also considered by Bjornstad (1990). Lindsey (1996) describes the reasoning behind its development. This Predictive Likelihood is the Fisherian approach, modified so that the variability of the parameter vector resulting from each maximisation is taken into account.

3. Simple Extreme Value Problem

To assess the safety of a bridge, a limited quantity of data is generally used to infer a probability of failure, a characteristic maximum or a statistical distribution of maximum load effects. Probability of failure is clearly the most definitive measure of bridge safety. However, it is strongly influenced by resistance which varies greatly from one example to the next. In order to retain the focus on load effect, the resistance distribution is here assumed to be a mirrored version of the exact LE distribution, shifted sufficiently to the right to give an annual probability of failure of 10^{-6}.

A simple example is used here to compare the alternative methods of extrapolation. A Normally distributed random variable (such as load effect in kNm) is first considered:

$$Z \sim \mathrm{N}(40, 5) \qquad \text{Equation 9}$$

Three thousand values of Z are considered in a given block, say per day, with maximum:

$$X = \max\left(Z_i\right) \quad i = 1, 2, \ldots, 3000 \qquad \text{Equation 10}$$

Typically, a finite number of days of data is available and extreme value distributions are inferred from a dataset of daily maximum values. Hence, a finite number of daily maxima (X values) may be used to infer, for example, annual maximum distributions. In all cases, the days are considered to be working days and a year is taken to consist of 250 such days.

The exact theoretical solution to this problem is readily calculated. The annual maximum can be expressed as:

$$F_Y(z) = [F_X(z)]^{250} = [F_Z(z)]^n \qquad \text{Equation 11}$$

where n is the number of values in a year, equal to (250×3000 =) 750 000.

In another study performed by the authors the generalized extreme value distribution is used as the parent as an alternative to Normal. It is found to not make a significant difference to the results except for GEV, Box-Cox and Predictive Likelihood, which is not surprising. The normal distribution is chosen here for simplicity and generality.

3.1 Methods of Inference

Three alternative quantities of daily maximum data are considered: 200, 500 and 1000 working days. A wide range of statistical extrapolation methods are tested in each case to estimate the distribution for annual maximum LE:

- Peaks Over Threshold (POT) data, fitted to the Generalized Pareto distribution;
- Generalized Extreme Value (GEV) fit to tail of daily maximum data;
- Box-Cox fit to tail of daily maximum data;
- Normal distribution fit to tail of daily maximum data;
- Fit of Upcrossing frequency data tail to Rice formula;
- Bayesian fit to all daily maximum data;
- Predictive Likelihood (PL) fit to all daily maximum data.

In each case, the probability distribution of LE is inferred and the theorem of total probability is used with the exact resistance distribution to determine the probability of failure (defined as LE exceeding resistance).

Figure 1 uses Gumbel probability paper to illustrate the first four methods of tail fitting to the CDF's: POT, GEV, Box-Cox and Normal. For all four cases, a least squares fit is found for the top 30% of values from 1000 daily maximum LE's. The exact distribution is shown for comparison. All distributions give good fits, with the Normal being more 'bounded' than the others in this example, i.e., tending towards a vertical asymptote at extremely low probabilities (Weibull-type behaviour).

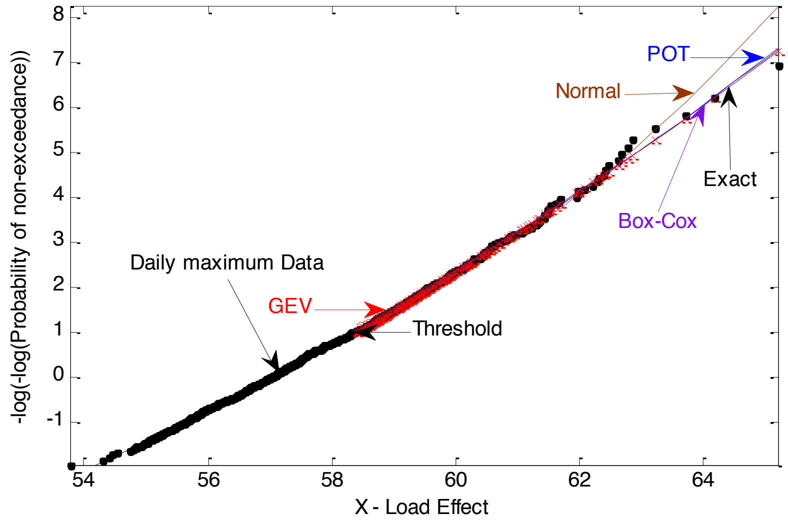


Figure 1 – Best fit distributions using four tail-fitting methods inferred from 1000 days of data

The Rice formula fit is illustrated in Figure 2 which gives the histogram of upcrossings above each threshold, for the same 1000 daily maxima. While Cremona (2001) has considered a variable quantity of data, the top 30% is used here to provide a direct comparison with the other tail fitting methods. Hence, the optimised parameters are found using a best fit to the normalised upcrossing histogram for the top 30% daily maximum data.

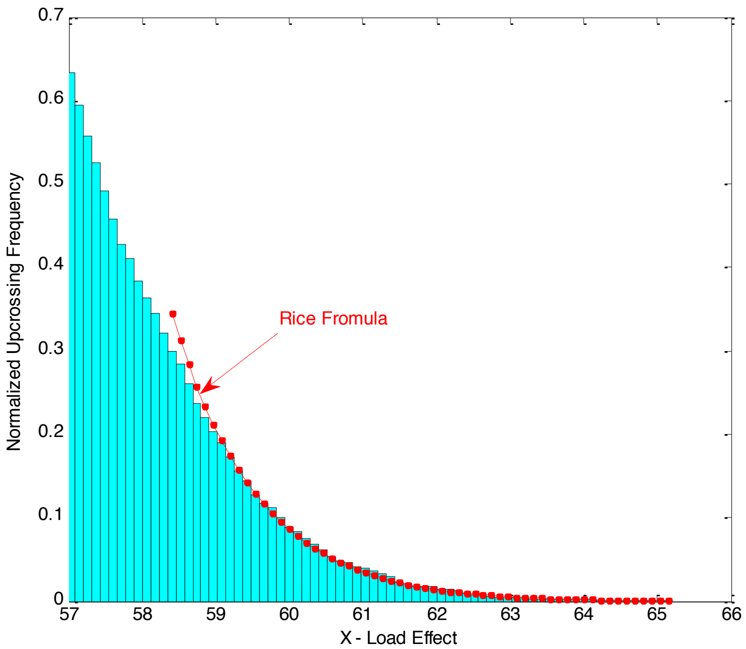


Figure 2 – Rice Formula Fit to Tail for 1000 days of data

Bayesian Updating is the sixth method considered. In this case, unlike the tail fitting methods, all 1000 daily maximum LE's are used. The method is therefore a Bayesian approach applied to block maximum data. The data is assumed to be GEV except that, in this case, a family of GEV distributions is considered. The GEV parameter values are initially assumed to be equally probable within specified ranges (uniform prior distributions). The daily maximum data is then used to update their probabilities.

The final method applied to this problem, Predictive Likelihood, is also based on the entire dataset of 1000 block maximum values and an assumed GEV distribution. The method is based on the concept of calculating the joint likelihood of a range of possible values at a given level of probability (predictands), given the value of that predictand and the available daily maxima. Figure 3 illustrates a schematic of a statistical extrapolation on probability paper. Two postulated predictands from the same set of observed data, are seen to yield different distributions. The figure shows the joint fit to Predictand Point A, given the daily maximum data and the joint fit to Predictand Point B, given that same set of daily maxima. The likelihood of actually observing Point A is less than that of Point B, given the measurements available. In this way, the joint likelihoods of a wide range of possible predictands are calculated and used to infer a probability distribution for a given time period, such as a year.

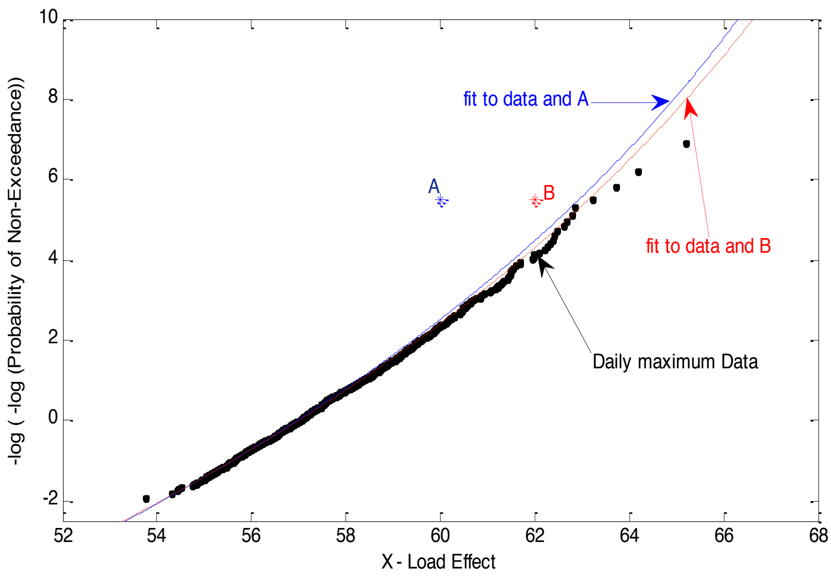


Figure 3 – Predictive Likelihood

3.2 Inference of Annual Maximum Results from Daily Maximum Data

For the first four tail fitting methods – POT, GEV, Box-Cox and Normal – the parameters of the daily maximum distributions are inferred from the best fits to the top 30% of the daily maximum data, i.e., the block size is one day. Allowing for public holidays and weekends, 250 days are assumed per year. The annual maximum distribution can then be found by raising the CDF for daily maximum to the power of 250.

The Rice formula approach is also a tail fitting method but, in this case, the CDF for annual maximum is found directly from Equation 6. Bayesian Updating and Predictive Likelihood both infer the annual maximum distribution directly as described above.

Table 1 gives a summary of the parameters and thresholds used in each method to infer the annual maximum distribution from daily maximum data.

Table 1 – Summary of parameters for the methods used

No.	Method	Parameters	Threshold	Converting daily maximum to annual

				maximum distribution
1	Peaks-over-Threshold	ξ, σ	Top 30% of data	Raised to the power of 250
2	Generalized Extreme Value	ξ, σ, μ	Top 30% of data	Raised to the power of 250
3	Box-Cox	$\xi, \sigma, \mu, \lambda$	Top 30% of data	Raised to the power of 250
4	Normal	σ, μ	Top 30% of data	Raised to the power of 250
5	Rice Formula	v_0, σ, m	Top 30% of data	Equation 6
6	Bayesian Inference	ξ, σ, μ	100% of data	Directly
7	Predictive Likelihood	ξ, σ, μ	100% of data	Directly

Figure 4 illustrates the annual maximum CDF's inferred from all 7 approaches, together with the corresponding exact distribution. For this example, most of the tail fitting methods and Predictive Likelihood are more bounded than or similar to the exact solution, while Bayesian Updating is less so. The horizontal line corresponds to a return period of 75 years and it can be seen that, for this particular example, all methods except Bayesian Updating, Box-Cox and GEV are slightly non-conservative.

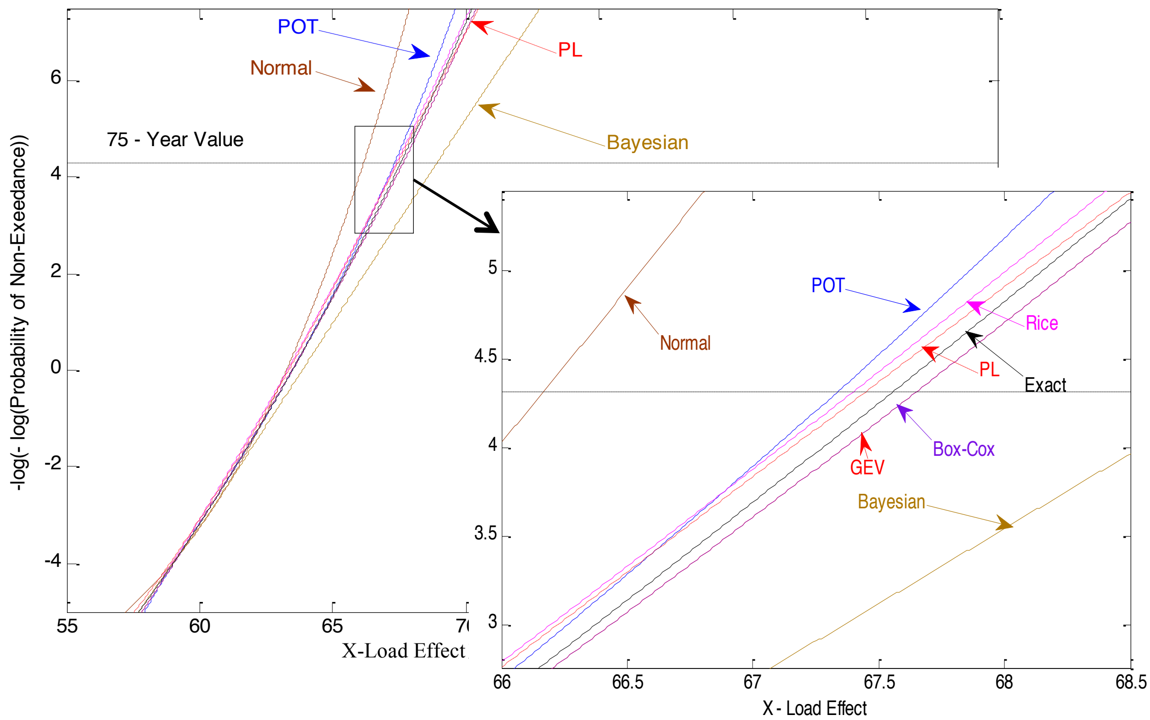


Figure 4 – Inferred Annual Maximum CDF's

Characteristic values are calculated for a 75-year return period. The process is repeated for three different quantities of daily maximum data: 200, 500 and 1000 days. For each of the three quantities, 20 samples are generated using the same distribution parameters (i.e., normal distribution) to investigate the variability associated with the results (i.e., each time, a new sample of normally distributed data is generated and the same calculation is performed to calculate the characteristic value). This process is repeated 20 times so that the variability in the results can be found. Figure 5 shows the mean of the 20 runs in each case, ± one standard deviation.

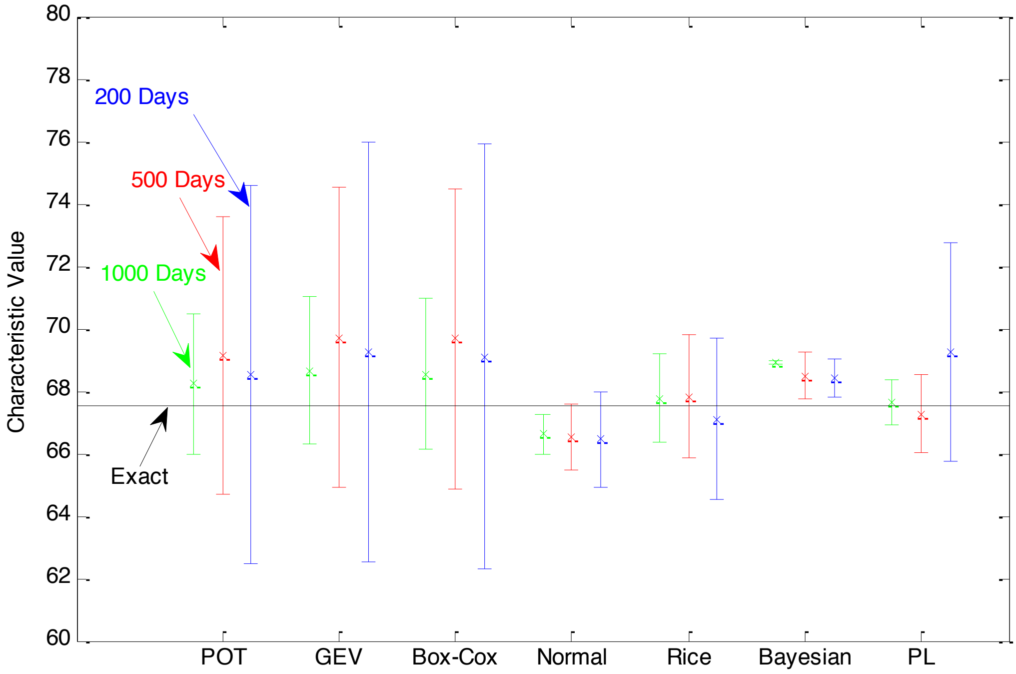


Figure 5 – Inferred 75-year Characteristic Values (mean ± one standard deviation)

Figure 5 illustrates that, not surprisingly, results are considerably less accurate when fewer days of data are available for inference. These results can be compared from different perspectives and points of view. For example, if the assessment criterion is the deviation of the mean characteristic value from the exact result, Predictive Likelihood and the Rice formula are the winners for 1000 days of data. The other methods are less accurate, at similar accuracy levels to each other. For small quantities of data, there is no clear winner but it is of note that Predictive Likelihood is less accurate than several of the other methods.

Root mean square can also be used for comparison of methods and leads to similar conclusions. However, comparing methods using the standard deviation gives different results. Bayesian Updating shows the smallest standard deviation (results are highly repeatable) and Normal is also small. The exact characteristic value (i.e., 67.5 kN) falls within the mean ± standard deviation range for the POT, GEV, Box-Cox, Rice and Predictive Likelihood for all quantities of data (i.e., 200, 500 and 1000 days). On the other hand, Bayesian and the Normal distribution, while giving good repeatability, do not include the exact value within the mean ± standard deviation range, i.e. they are less accurate than their low standard deviations might suggest.

In this example, the benchmark cumulative distribution for daily maximum load effects is expected to be linear on Gumbel probability paper. Therefore, a deviation from the linear behaviour can cause an error in estimated characteristic value. The 20 randomly generated samples exhibit different types of tail behaviour – Gumbel, Fréchet and Weibull – which resulted in a wide variation around the correct linear (Gumbel) behaviour. This explains the high standard deviation of predicted characteristic values for the first three methods.

The Normal distribution is more bounded than the Gumbel and Fréchet distributions, i.e., it tends to curve upwards in probability paper plots such as Figure 1. For data where the trend is to curve downwards (Fréchet-type), this results in an imperfect but consistent result. This is the reason for the small standard deviation in the results. Similarly, the Rice formula does not fit Fréchet-type data well, in comparison to the first three tail fitting methods (i.e., POT, Box-Cox and GEV).

Sensitivity studies of the Predictive Likelihood results show that there is a significant influence of the datasets that exhibit Fréchet-type behaviour. In PL, the distribution is found that jointly maximizes the likelihood of observing the data and the predictand. If the fit is limited so that Fréchet tails are not permitted (admitting only Weibull or Gumbel tails), as could be argued from the physical bounds of the traffic loading phenomenon, then the fits were found to improve.

In order to compare inferred probabilities of failure, the exact annual maximum probability density function is mirrored to give a resistance distribution that implies a failure probability of 10^{-6}. This resistance distribution is then used with each of the inferred distributions to determine the apparent probability of LE exceeding resistance. The calculated probabilities are illustrated in Figure 6.

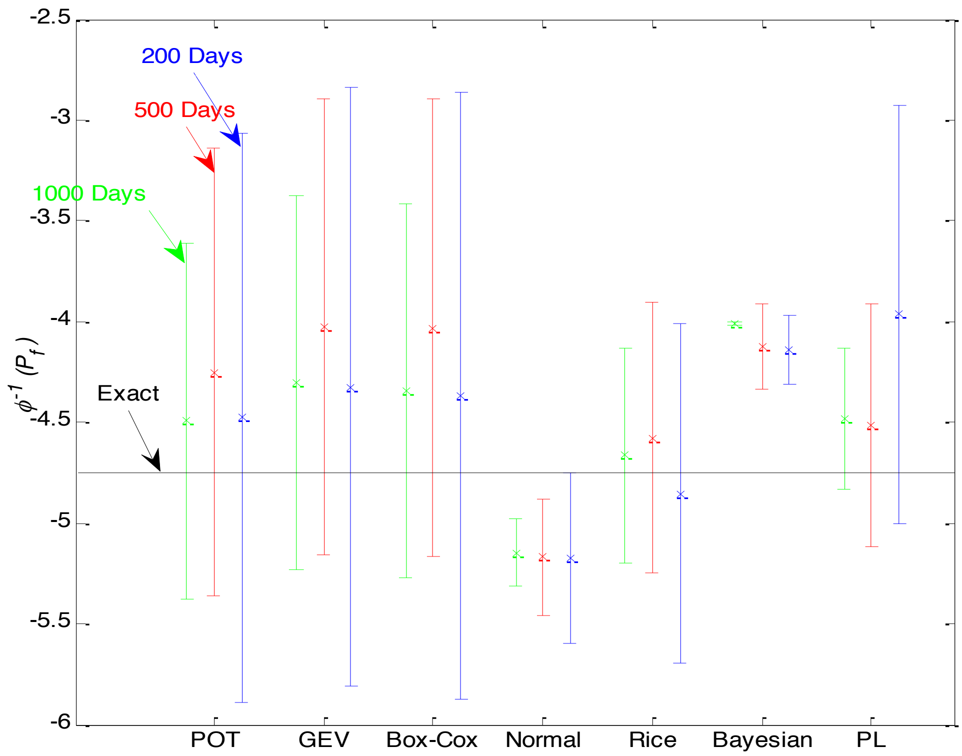


Figure 6 – Mean ± One Standard Deviation of Inferred Probabilities of Failure, P_f plotted to a Normal scale (ϕ = cumulative distribution function for Normal distribution)

This exercise is analogous to an extrapolation from 200 - 1000 days of data to 1 million years (i.e., annual probability of failure of 10^{-6}). The probability of failure calculation uses much more information from the tail area of the annual maximum distribution than the characteristic value calculation. As expected the results for probability of failure illustrated in Figure 6 exhibit quite a lot of variability. However, the relative performance of the seven methods is similar to that illustrated in Figure 5.

The Y-axis scale in Figure 6 is inverse normal (corresponding to the well known safety index, β). On this scale, the mean error from 1000 days of data is less than about 0.5 from the exact value. Errors in individual results are considerably worse, being as high as 2.1 in the case of one outlier for GEV. As before, it can be seen that for inference using POT, GEV, Box-Cox, Rice and Predictive Likelihood, the exact value falls within the error bars, for all three different quantities of daily maximum data.

4. Traffic Load Effect Problem

As part of the European 7th Framework *ARCHES* project [1], extensive WIM measurements were collected at five European sites: in the Netherlands, Slovakia, the Czech Republic, Slovenia and Poland. The *ARCHES* site in Slovakia is used as the basis for the simulation model presented here. Measurements were collected at this site for 750 000 trucks over 19 months in 2005 and 2006. The traffic is bidirectional, with average daily truck traffic (ADTT) of 1100 in each direction. A detailed description of the methodology adopted is given by Enright & OBrien (2012), and is summarised here. For Monte Carlo simulation, it is necessary to use a set of statistical distributions based on observed data for each of the random variables being modelled. For gross vehicle weight and vehicle class (defined here simply by the number of axles), a semi-parametric approach is used as described by OBrien et al. (2010). This involves using a bivariate empirical frequency distribution in the regions where there are sufficient data points. Above a certain GVW threshold value, the tail of a bivariate Normal distribution is fitted to the observed frequencies which allows vehicles to be simulated that may be heavier than, and have more axles than, any measured vehicle. Results for lifetime maximum loading vary to some degree based on decisions made about extrapolation of GVW, and about axle configurations for these extremely heavy vehicles, and these decisions are, of necessity, based on relatively sparse observed data.

Bridge load effects for the spans considered here (Table 2) are very sensitive to wheelbase and axle layout. Within each vehicle class, empirical distributions are used for the maximum axle spacing for each GVW range. Axle spacings other than the maximum are less critical and trimodal Normal distributions are used to select representative values. The proportion of the GVW carried by each individual axle is also simulated in this work using bimodal Normal distributions fitted to the observed data for each axle in each vehicle class. The correlation matrix is calculated for the proportions of the load carried by adjacent and non-adjacent axles for each vehicle class, and this matrix is used in the simulation using the technique described by Iman & Conover (1982).

Traffic flows measured at the site are reproduced in the simulation by fitting Weibull distributions to the daily truck traffic volumes in each direction, and by using hourly flow variations based on the average weekday traffic patterns in each direction. A year's traffic is assumed to consist of 250 weekdays, with the very much lighter weekend and holiday traffic being ignored. This is similar to the approach used by Caprani et al. (2008) and Cooper (1995).

For same-lane multi-truck bridge loading events, it is important to accurately model the gaps between trucks, and the method used here is based on that presented by OBrien & Caprani (2005). The observed gap distributions up to 4 seconds are modelled using quadratic curves for different flow rates, and a negative exponential distribution is used for larger gaps.

The modelled traffic is bidirectional, with one lane in each direction, and independent streams of traffic are generated for each direction. In simulation, many millions of loading events are analysed, and for efficiency of computation, it is necessary to use a reasonably simple model for transverse load distribution on two-lane bridges. For bending moment the maximum LE is assumed to occur at the centre of the bridge, with equal contribution laterally from each lane. In the case of shear force at the supports of a simply supported bridge, the maximum occurs when each truck is close to the support, and the lateral distribution is very much less than for mid-span bending moment. In this case a reduction factor of 0.45 is applied to the axle weights in the second lane. This factor is based on finite element analyses performed for different types of bridge (OBrien & Enright, 2012). The load effects and bridge lengths examined in the simulation runs are summarized in Table 2.

Table 2 – Load effects and bridge lengths

	Load Effect	**Bridge Lengths (m)**
LE1	Mid-span bending moment, simply supported bridge	15, 35
LE2	Shear force at start/end of a simply supported bridge	15, 35
LE3	Central support hogging moment, 2-span continuous bridge	35

Two series of simulation runs are performed – one to represent possible measurements over 1000 days, repeated 20 times, and another to represent the benchmark results, consisting of 5000 years of traffic. For the benchmark run, the outputs consist of annual maximum LE's, and, as this is an much less of an extrapolation (for the characteristic values, it is an interpolation from 5000 years to 75), these can be used to calculate the characteristic values and annual maximum distributions to a high degree of accuracy.

Sample results are plotted on Gumbel probability paper in Figure 7 for the 5000-year simulation run. Two load effects are shown – shear force (LE2) on a simply supported 15 m bridge, and hogging moment (LE3) over the central support of a two-span bridge of total length 35 m. Due to the randomness inherent in the process, there is some variability in the results, particularly in the upper tail region (top 1% of data approximately). Weibull fits to the upper 30% tail are used to smooth this variability (as shown in figure), and these are used to calculate the characteristic values. This long-run simulation process is considered to be highly accurate, subject to the assumptions inherent in the model and is used as the benchmark against which the accuracy of all other methods is measured.

It should be noted that the results in this example are strongly site-specific. Several studies have shown that truck traffic is highly dependent on the site characteristics, including a study of Wisconsin WIM data (Zhao & Tabatabai 2012), a study on WIM data from Idaho, Michigan and Ohio (Sivakumar et al. 2007) and in the research conducted by Pelphrey et al. (2008). Kim et al. (1997) also highlighted the site dependence of characteristic traffic load effects, even within a geographical region.

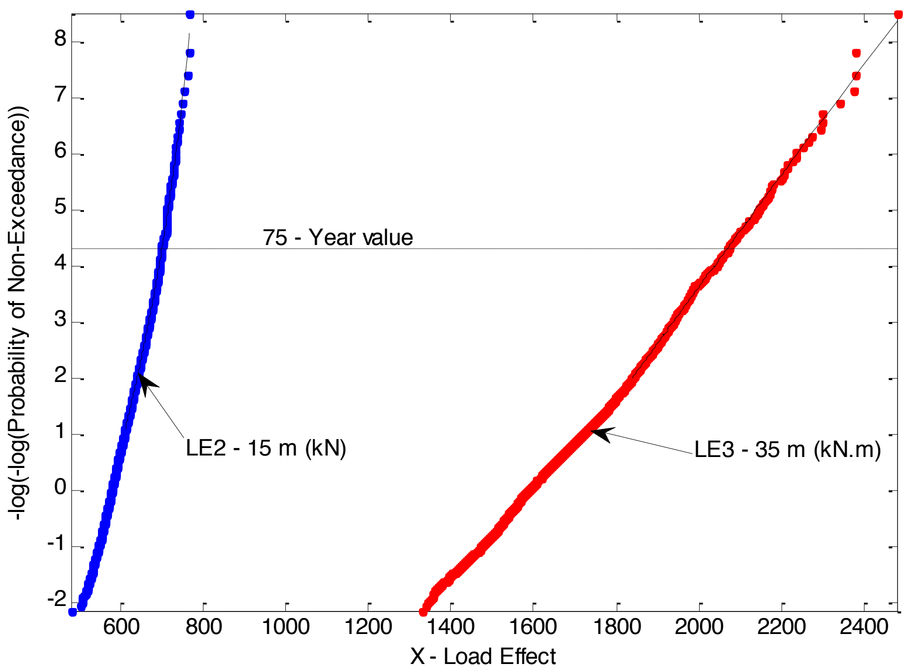


Figure 7 – Annual maximum LE's from 5000 years of simulation

4.1 Results of Inference Based on Long-run Simulation Data

The assumed measurements, consisting of 1000 simulated daily maxima, are used as the basis for extrapolation to estimate the benchmark results calculated from the long-run simulation. For the five tail fitting methods, the distributions are fitted to the top 30% of data. For some load effects and spans, the distribution of the data is multi-modal (see Figure 8), i.e., there is a change in slope – around 400 kN in this case – implying data from a different parent distribution. In the case illustrated, there is a change around this point from (i) daily maxima arising from heavily loaded regular trucks to (ii) maxima arising from extremely heavy (and rarer) low-loader vehicles and cranes.

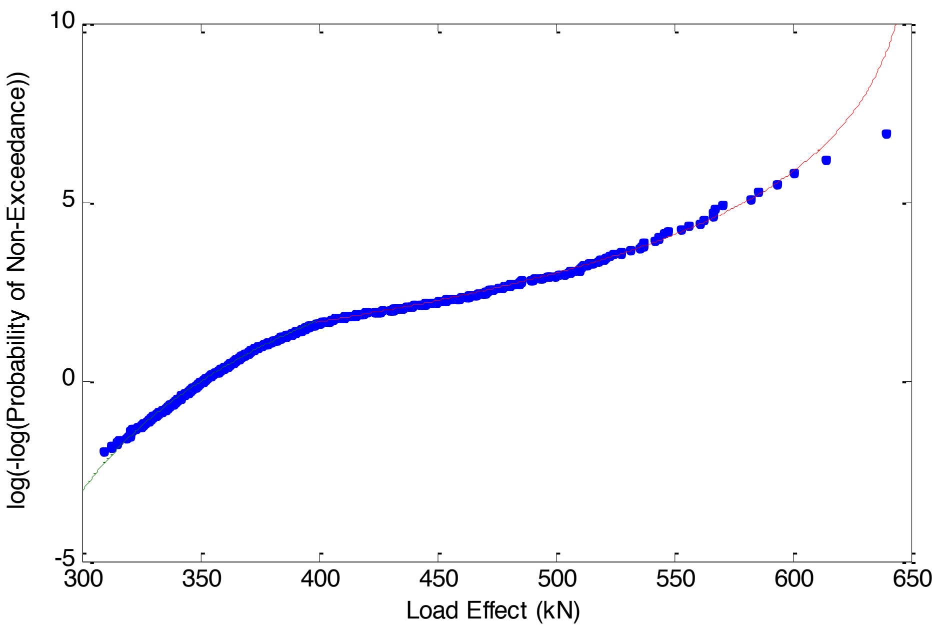


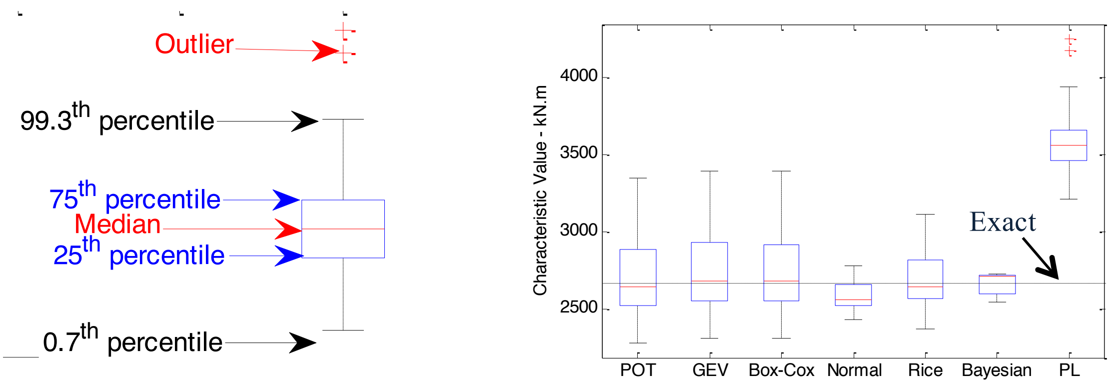
Figure 8 – Daily Maximum LE2 - 15 m span. Data and Inferred Distribution

The 75-year characteristic maximum LE's are inferred from the assumed measurements. This process is carried out for the 5 load effects and repeated 20 times to determine the variability in results. The results are illustrated in Figure 9 which shows, in each case (i) the median value, (ii) the 25% to 75% range (boxed), (iii) the 0.7% to 99.3% range (mean ± 2.7 standard deviations for normally distributed data) (dashed lines) and (iv) individual outliers beyond that range.

As for the first simple example, Figure 9 shows that the first three tail fitting methods are capable of covering the benchmark characteristic value within the boxed range, for all load effects except the hogging moment in a 2-span bridge (i.e., LE3). The influence line for this 3rd load effect has two peaks and is quite sensitive to the axle configuration of the vehicle. It is probably for this reason that the samples of 1000 daily maxima are highly variable, with different samples giving quite different results for the three tail fitting methods.

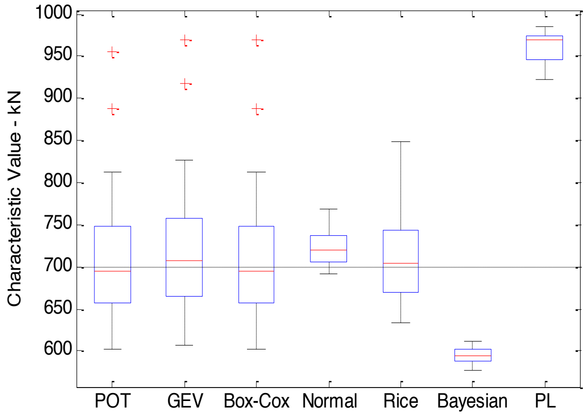
The Bayesian approach gives different ranges of accuracy for different load effects. Several variations were tested in attempts to find a Bayesian approach that is consistently good. The GEV distribution within the Bayesian approach was fitted to the top 30% of data, as an alternative to fitting it to all the data. Different numbers of parameters of the GEV distribution were updated: two (σ and ξ) and three (μ, σ and ξ). Different prior distributions were assumed for these parameters – Normal and Uniform. For the latter, different ranges were tested for the parameter values. None of these variations produced consistently better results for the five LE's and spans. The results shown are based on the use of all the data; updating just two parameters (σ and ξ) with a uniform prior distribution and a limit on the range of ξ to be non-positive.

Predictive likelihood clearly performs poorly in comparison to the simpler example. This can be explained by the fact that data resulting from long-run simulations comes from a mixture distribution rather than a single distribution. The tail region in the data is a good fit to a single distribution but this is not necessarily true for the entire data. Consequently a method such as predictive likelihood which takes account of all data and tends to fit a single distribution to the data generated from multiple distributions, fails to provide good results relative to the benchmark.

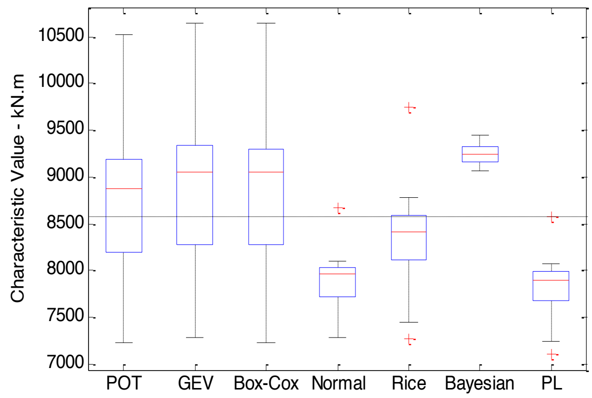


(a) Legend

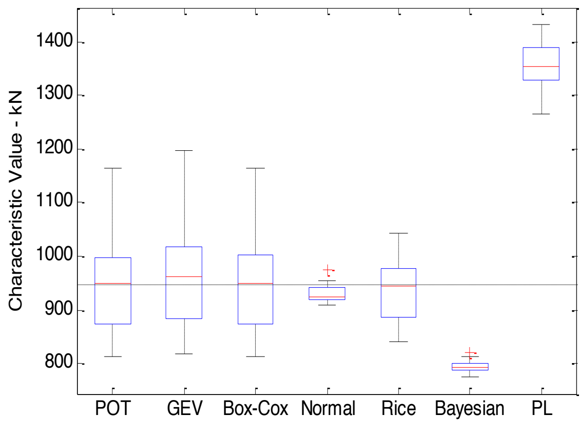
(b) LE1 – Mid-span Moment, 15 m Span



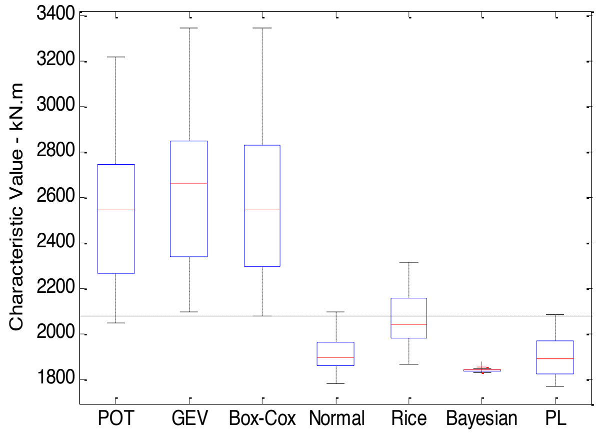
(c) LE2 – Support Shear, 15 m Span



(d) LE1 – Mid-span Moment, 35 m Span



(e) LE2 – Support Shear, 35 m Span

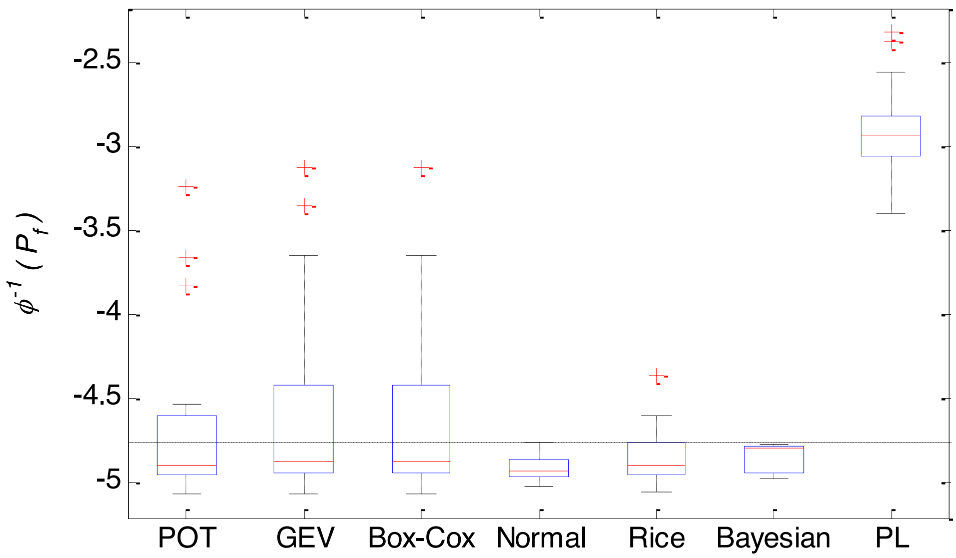


(f) LE3 – Support Moment, 2×17.5 m Spans

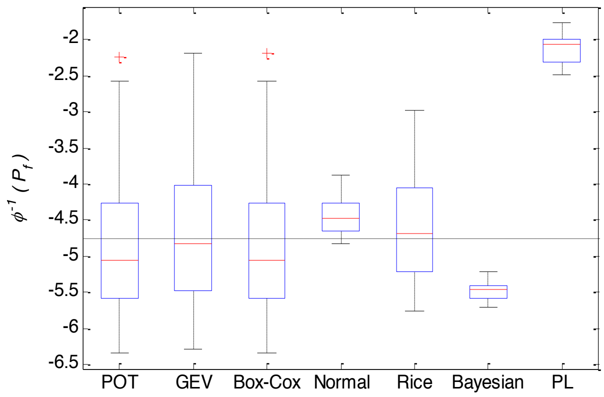
Figure 9 – Range of Inferred 75-year Characteristic Values from 1000 Days of Data

Annual probabilities of failure are also inferred for the five combinations of load effect and span. As before, the probability of failure for the benchmark example is set at 10^{-6} in each case and the resistance distribution is taken to be a mirrored version of the benchmark LE distribution. It has to be noted that using the mirror image of the load effect distribution as the resistance distribution amplifies the influence of any errors in the fitted load distribution. The resulting error in probability of failure may be quite different that would be the case with an accurate resistance distribution.

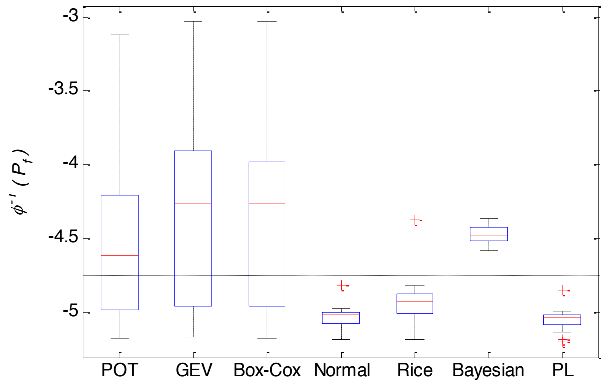
The results are illustrated in Figure 10. As for the simple example, the errors in the probabilities, even when plotted on a Normal scale, are much higher than for characteristic values. As before, when fitting to a Normal distribution, the benchmark result is sometimes outside the 25%-75% range, but not by a great deal.



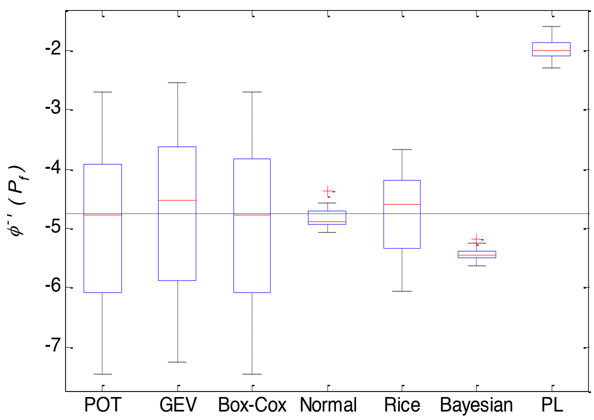
(a) LE1- Mid-span Moment, 15 m Span



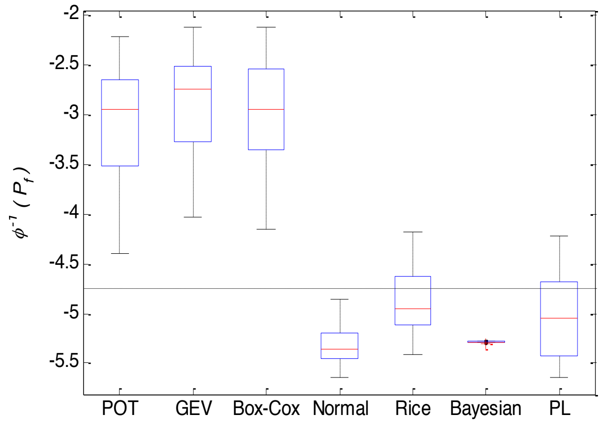
(b) LE2 – Support Shear, 15 m Span



(c) LE1 – Mid-span Moment, 35 m Span



(d) LE2 – Support Shear, 35 m Span



(e) LE3 – Support Moment, 2×17.5 m Spans

Figure 10 – Probabilities of Failure Inferred from Sets of 1000 Daily Maximum LE's

5. Conclusions

This study provides a rational evaluation of the performance of several statistical approaches to the assessment of load effects, with various quantities of data. Seven methods of statistical inference are critically reviewed. Each method is also tested using two examples. The first example is derived from a Normal distribution and the exact solution is known. A total of 3000 normally distributed values (e.g., vehicle weights) are considered per day and the daily maxima are used to infer the characteristic maximum and the probability of failure in a year. In the second example, a sophisticated algorithm is used to generate a train of vehicles with weights and axle configurations consistent with measured Weigh-in-Motion data. Five different combinations of load effect and span are considered and, in each case, characteristic values and probabilities of failure are again calculated. In these cases, the exact solutions are not known but the simulation is run for 5000 years to obtain accurate benchmark references against which inferences based on 1000 days of data can be compared.

Of the seven methods considered, five are tail-fitting approaches, i.e., a distribution is fitted to the tail of the data. Peaks-Over-Threshold (POT) is popular in some sectors but is not time-referenced and selecting the threshold is a subjective process. Fitting the tail of block-maximum data to a Generalized Extreme Value (GEV) is perhaps the most popular used for bridge traffic loading, with a typical block size of a day. Box-Cox could be considered to be a hybrid between POT and GEV. These three methods are generally good for inferring the characteristic values, both for the simple and the more complex examples. There is no theoretical justification for fitting block maximum data to the tail of a Normal distribution but it is sometimes done. It is found here to also give quite accurate results, with a small standard deviation. Finally, the Rice formula is an indirect approach as it is the upcrossing frequencies that are fitted to the formula, rather than the data itself. Nevertheless, it performs well in these tests, generally better than POT and GEV.

Bayesian Updating is used here to fit the block maximum data to a family of GEV distributions. The parameters of the GEV are allowed to vary, their associated probabilities being updated as the data is considered. Finally, Predictive Likelihood is considered, a method where the likelihood of each inferred characteristic value is considered, given the available data. Neither of these methods give good results on a consistent basis.

All seven methods are used to infer the annual probabilities of failure as well as the characteristic values. To avoid the need for any assumption on the distributions for resistance, the benchmark load effect distribution is mirrored and this mirrored version is used in the calculation of probability of failure. The inferred failure probabilities are considerably less accurate than the inferred characteristic values, perhaps not surprising given that such a small failure probability was being considered (10^{-6} in a year). As for characteristic values, the tail fitting methods are better than the others but none of the methods gives an accurate inference with 1000 days of data.

As a general conclusion it may be stated that the predicted characteristic value and probability of failure are more influenced by the database sample being used than the method adopted. Increasing the number of data in the sample can result in higher accuracy of approximations but it is not able to completely eliminate the uncertainty associated with the extrapolation. This highlights the importance of long-run simulations as a means of reducing the errors associated with the extrapolation process.

As traffic load effects result from a number of statistically dissimilar phenomena (different types of vehicle), the benchmark traffic problem exhibits mixture distribution behaviour. Bayesian and Predictive Likelihood, use all the data, not just that in the tail, in their predictions. Hence, these methods do not work well for the traffic problem, giving quite poor predictions for some load effects/spans. POT, GEV and Box-Cox are generally reasonably good and are consistent for most spans and load effects. Surprisingly, Normal and the Rice formula are quite good, despite the lack of a scientific basis for the former in particular. This would appear to be because they constrain the solutions to Weibull-type (bounded) behaviour. Allowing Fréchet-type behaviour, as happens in POT, GEV and Box-Cox, is more consistent with some data samples but gives a greater number of inaccurate results. In conclusion, it would seem sensible to constrain distributions to prevent Fréchet-type behaviour. This suggests that fitting to a Weibull distribution, a subset of GEV, should give good results and is scientifically more justifiable than Normal.

LE3, representing internal support moment in 2-span beams, causes problems for all the methods. Unlike simpler load effects such as mid-span moment in a single span, large LE3 values result from two clusters of heavy axles, as can happen in a low loader vehicle. The load effect is highly sensitive to the spacing between the groups of axles and data tends to be quite variable, with no clear trend on probability paper. While all of the methods are less effective with this data, the same general conclusions can be made, i.e., Normal and Rice are quite good and Weibull would be expected to be good.

Acknowledgment

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