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A New Modified Smith Predictor Design

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Abstract – A new modified Smith predictor structure is presented with its associated tuning rules.

Keywords – Smith predictor, Dead-time compensator

I. INTRODUCTION

In the 1950’s, O.J. Smith [1] developed the Smith predictor structure to compensate systems with time delay, which are a feature of many industrial processes. The Smith predictor structure utilises a mathematical model of the process in a minor feedback loop. One of its advantages is that the Smith predictor approach for compensating a Single Input Single output (SISO) process may be directly extended to the compensation of a Multiple Input Multiple Output (MIMO) process with the same delay in each path.

Since the Smith Predictor structure was proposed, many modifications have been proposed to improve the servo response, the regulator response or both. Modifications were accomplished to adapt the structure to stable, integrative or unstable systems. Sourdille and O’Dwyer [2] present an extensive review of the literature concerning modifications to the Smith predictor; this review was used to develop a generalised form of the predictor. This paper discusses the generalised form of the Smith predictor and a new modified Smith predictor structure with its associated tuning rules.

II. GENERALISED FORM OF SMITH PREDICTOR STRUCTURE

The general form of the Smith predictor is obtained by combining several structures ([1], [3-16]), which have common points, in one general structure. A number of structures were considered but not included into the general structure due to their complexity ([17-24]). Figure 1 shows the generalised form of the Smith predictor structure developed and equations (1) and (2) represent the servo and regulator responses, respectively.

\[
\frac{y_p}{r} = \frac{[1+(G_3+G_a)G_p e^{s\tau_a}G_1(G_3G_6G_p e^{s\tau_p})]}{1+G_mG_1G_a e^{s\tau_a}+G_5G_p e^{s\tau_p}+(G_3+G_a)G_1G_6G_p e^{s\tau_p}+G_1G_6G_2(G_p e^{s\tau_p} - G_m e^{s\tau_a})} \tag{1}
\]

\[
\frac{y_p}{L} = \frac{(1+G_1G_m+G_4G_n e^{s\tau_n} - G_4G_2G_m e^{s\tau_a})G_3G_4 e^{s\tau_p}}{1+G_mG_1G_n e^{s\tau_a}+G_3G_p e^{s\tau_p}+(G_3+G_4)G_1G_3G_p e^{s\tau_p}+G_1G_3G_2(G_p e^{s\tau_p} - G_m e^{s\tau_a})} \tag{2}
\]
The requirements specified to design $G_{c1}$ to $G_{c6}$ are the obtaining of perfect servo and regulator responses (i.e. $\frac{\gamma_r}{r} = 1$ and $\frac{\gamma_L}{L} = 0$), and that the controller transfer functions are only expressed in terms of the model parameters. It turns out that three primary controllers need to be specified: one to optimise the servo response, one to optimise the regulator response and one to eliminate the mismatch between the process and the model. In the different cases, $G_{c1}$, $G_{c5}$ and $G_{c6}$ are equal to 1, and $G_{c2}$, $G_{c4}$ and $G_{c3}$ are equal to 0 when they are not used. After calculating each possible triplet of primary controllers, only fifteen cases are realisable, as some possibilities do not achieve the requirements specified above. From these realisable cases, only three cases are considered, as their controller transfer functions are of the simplest form to limit any necessary approximations. In the next section, one of these cases is explained in detail.

III. MODIFIED SMITH PREDICTOR STRUCTURE - CASE STUDIED

The servo and regulator responses are expressed by equations (3) and (4), respectively. As can be noticed from equations (3) and (4), $G_{c3}$ will optimise the servo response, $G_{c4}$ will optimise the regulator response and $G_{c2}$ will eliminate the mismatch between the process and the model.

$$\frac{y_p}{r} = \frac{[1+(G_{c3}+G_{c4})G_m e^{-\tau_{r}}]G_p e^{-\tau_{p}}}{1+G_m+G_{c3}G_p e^{-\tau_{r}} + G_{c4}G_p e^{-\tau_{r}} + (G_{c3}+G_{c4})G_m G_p e^{-\tau_{r}} + G_{c2}(G_p e^{-\tau_{r}} - G_m e^{-\tau_{r}})}$$

(3)
The following expressions may be calculated for $G_{c2}$, $G_{c3}$ and $G_{c4}$ by designing for perfect servo and regulator responses.

$$G_{c2} = 0 \quad (5), \quad G_{c3} = \frac{1}{e^{-s\tau_n} m} \quad (6), \quad G_{c4} = -\frac{1 + G_m}{G_m e^{-s\tau_n}} \quad (7)$$

Using the form of equation (8) for the non-delayed model, equation (7) becomes equation (9):

$$G_m = \frac{K_m}{T_m s + 1} \quad (8)$$

$$G_{c4} = -\frac{T_m s + 1 + K_m}{K_m e^{-s\tau_n}} \quad (9)$$

The controllers $G_{c3}$ and $G_{c4}$ are impossible to implement with real components, so approximations are needed. The inverse of the delay (equations (6) and (9) cannot be implemented), so the approximation detailed by Sourdille and O’Dwyer [2] is used:

$$\frac{1}{e^{-s\tau_n}} = \frac{1 + B(s)}{1 + B(s)e^{-s\tau_n}} \quad (10)$$

with $B(s) = \frac{T_m s + 1}{T_m s + p} \quad (11)$

Expressions (12) and (13) are the realisable controller forms of equations (6) and (9):

$$G_{c3} = \frac{B(s) + 1}{1 + B(s)e^{-s\tau_n}} \quad (12) \quad \text{and} \quad G_{c4} = -\frac{T_m s + 1 + K_m}{K_m (T_m s + K)} \quad (13)$$

IV. SIMULATION

To use the Smith predictor structure, a primary controller must be designed to achieve perfect responses (i.e. $\frac{y_s}{r} = 1$ and $\frac{y_L}{L} = 0$). This gives a primary controller of the following form (equation (14)) and its implementable approximation is given by equation (15).

$$G_p = -\frac{T_m s + 1}{K_m (1 - e^{-s\tau_n})} \quad (14) \quad \text{and} \quad G_p = -\frac{T_m s + 1}{K_m (s + 1)(1 - e^{-s\tau_n})} \quad (15)$$

For the modified Smith predictor, equations (12) and (13) are used. Using these equations, an offset is observed for the servo response and the regulator response. The value of this offset may be evaluated by calculating the steady state of the servo and regulator responses. These calculations give a common offset for the responses, which is given by equation (16):

$$\text{Offset} = \frac{K_p}{K_p + 1} \quad (16), \quad K_p = \text{process gain}$$

To solve this problem, a Proportional controller, $K_c$, is introduced at the command signal to eliminate the offset for the servo response and a filter is included in series with the controller $G_{c3}$ to eliminate the offset for the regulator response. As it is undesirable to express $K_c$ in term of the process gain, $K_p$ (as the process gain is, in
The approximation $K_p = K_m$ is used to determine $K_c$, with $K_c$ being the inverse of the offset. The expressions for the controllers in the modified Smith predictor (equations (12) and (13)) become equations (17) and (18), respectively.

$$G_3 = \frac{T_m s + 1}{T_m s + \frac{1 + B(s)}{1 + B(s)e^{-\tau_m \alpha}}}, \quad (17)$$

$$G_4 = \frac{T_m s + 1 + K_m}{K_m(T_m s + K_p)} \frac{1 + B(s)}{1 + B(s)e^{-\tau_m \alpha}}, \quad (18)$$

Table 1 presents tuning rules developed for each controller and the range of possible values for $\alpha$ and $p$ depending on the index $\frac{\tau_m}{T_m}$.

<table>
<thead>
<tr>
<th>$\frac{\tau_m}{T_m}$</th>
<th>$K_c$</th>
<th>$G_3$</th>
<th>$G_4$</th>
<th>$\alpha$</th>
<th>$p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 &lt; \frac{\tau_m}{T_m} \leq 0.5$</td>
<td>$K_m + 1$</td>
<td>$T_2 = \frac{T_m}{K_m}$</td>
<td>$T_1 = 50T_m, K_1 = 100$</td>
<td>$0.6 \leq \alpha \leq 1$</td>
<td>$3 \leq p \leq 5$</td>
</tr>
<tr>
<td>$0.5 &lt; \frac{\tau_m}{T_m} &lt; 1$</td>
<td>$K_m + 1$</td>
<td>$T_2 = T_m$</td>
<td>$T_1 = 50T_m, K_1 = 100$</td>
<td>$1 \leq \alpha \leq 1.5$</td>
<td>$2 \leq p \leq 3$</td>
</tr>
<tr>
<td>$1 &lt; \frac{\tau_m}{T_m} \leq 2$</td>
<td>$K_m + 1$</td>
<td>$T_2 = \frac{5T_m}{K_m}$</td>
<td>$T_1 = 50T_m, K_1 = 100$</td>
<td>$0.6 \leq \alpha \leq 1.3$</td>
<td>$2 \leq p \leq 3$</td>
</tr>
</tbody>
</table>

From these tuning parameter values, it can be concluded that $\alpha$ may be taken between 0.5 and 1.5 and $p$ may be chosen between 2 and 5.

The following table (Table 2) shows the number of simulations, in which improvement in response was detected, when the modified Smith predictor was used instead of the Smith predictor, with the responses evaluated using the four indices (Integral Absolute Error-IAE, Integral Squared Error-ISE, Integral Time multiplied by Squared Error-ITSE and Integral of Squared Time multiplied by Squared Error-ISTSE). Three simulations are conducted on each of seven benchmark processes and models, giving 21 simulation results altogether.

<table>
<thead>
<tr>
<th></th>
<th>IAE</th>
<th>ISE</th>
<th>ITSE</th>
<th>ISTSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Servo responses</td>
<td>18</td>
<td>14</td>
<td>17</td>
<td>15</td>
</tr>
<tr>
<td>Regulator responses</td>
<td>21</td>
<td>21</td>
<td>19</td>
<td>18</td>
</tr>
<tr>
<td>Corresponding</td>
<td>93%</td>
<td>83%</td>
<td>85%</td>
<td>79%</td>
</tr>
</tbody>
</table>

Table 2: Improvement in responses noted when the modified Smith predictor is used

From this table, it can be noticed that the modified Smith predictor structure gives improved performance compared to the use of the Smith predictor structure, especially for the regulator response. This is significant, as it is recognised that the Smith predictor structure facilitates relatively poor regulator response. One representative simulation result is shown in Figure 2. For this simulation,
$$G_n e^{-\tau_n} = \frac{2}{1 + 0.7s} e^{-1.4s}, \quad G_p e^{-\tau_p} = \frac{2}{1 + 0.7s} e^{-1.4s}, \quad \alpha = 1.1 \text{ and } p = 3.$$ In this simulation, the mismatch term is zero.

Figure 2: Responses for the modified Smith predictor and the Smith predictor

It is clear that the modified Smith predictor gives better servo and regulator responses.

V. CONCLUSION

A generalised Smith predictor structure is developed from several modified Smith predictor structures. A new modified Smith predictor structure with its associated tuning rules is subsequently presented. Our full panorama of simulation results show that better servo and regulator responses are not guaranteed with the modified Smith predictor compared with the responses achieved with a Smith predictor. However, the modified Smith predictor facilitates better overall responses, especially in regulator mode, for a significant majority of cases explored.

References


