1990

Exergy Analysis - A Different Perspective on Energy Part 2: Rational Efficiency and Some Examples of Energy

Jim McGovern
Technological University Dublin, jim.mcgovern@tudublin.ie

Recommended Citation

McGovern, J. (1990). Exergy Analysis - A Different Perspective on Energy Part 2: Rational Efficiency and Some Examples of Energy. *Proceedings of the Institution of Mechanical Engineers, Part A: Journal of Power and Energy*, vol. 204, no. 4, pp. 263-268. doi.org/10.1243/PIME_PROC_1990_204_035_02

Exergy analysis—a different perspective on energy Part 2: rational efficiency and some examples of exergy analysis

Eur Ing **J A McGovern,** PhD, CEng, MIMechE, FIEI, MASHRAE, MASME
Department of Mechanical and Manufacturing Engineering, Trinity College, University of Dublin, Ireland

The concept of exergy having been introduced in the first part of this paper, a universal rational efficiency is now defined and a number of examples of exergy analyses is given.

1 RATIONAL EFFICIENCY

There has been much confusion within the literature concerning the rational or second law efficiency. Such efficiencies have been defined in many different ways [see for example reference (**1**), Ch. 4]. It is hereby proposed that there is one universal rational efficiency which can be defined with respect to a specified reference environment for any system across whose boundary exergy interactions occur with any number of specified systems.

According to equation (46) in Part 1 a system in which energy crosses the boundary and energy transformations take place is perfectly good in the thermodynamic sense if there is no exergy destruction (irreversibility) within it. Such a system has a rational efficiency of unity. No system can be less efficient than a system in which all exergy that enters and any excess of the system's initial exergy over its final exergy is destroyed. Such a system has a rational efficiency of zero.

In general, a system may interact with any number of other systems including the specified reference environment at its boundary (Fig. 1). In defining the rational efficiency these external systems must be specified in order that a single net exergy interaction with each can be evaluated.

In Fig. 1 an exergy output to the specified reference environment is shown. This represents a loss of exergy from the system being analysed, rather than exergy destruction within the system. Any such exergy output can always be made zero by redrawing the analysis boundary at a position within the specified reference environment where its equilibrium is undisturbed. This is not unreasonable since exergy analysis imposes no restrictions on the definition of a system boundary.

The rational efficiency is therefore defined as follows. Let Ξ_{1-2} represent the decrease in the exergy of the system over the period for which the rational efficiency is defined. If $\Xi_1 > \Xi_2$ then the rational efficiency, ψ, is given by

$$\psi = \frac{\sum_k \Xi_{\text{out, k}}}{\sum_l \Xi_{\text{in, l}} + \Xi_{1-2}} \tag{1}$$

where the subscript k refers to an external system that serves as an exergy sink and subscript l refers to an external system that serves as an exergy source.

If $\Xi_1 < \Xi_2$ then

$$\psi = \frac{\sum_k \Xi_{\text{out, k}} + \Xi_{2-1}}{\sum_l \Xi_{\text{in, l}}} \tag{2}$$

If the system undergoes no change, as in the case of a steady flow system, or returns to its initial state, as where it undergoes a cycle, then $\Xi_{1-2} = \Xi_{2-1} = 0$ and

$$\psi = \frac{\sum_k \Xi_{\text{out, k}}}{\sum_l \Xi_{\text{in, l}}} \tag{3}$$

The definition of the universal rational efficiency can be summarized in a single expression as follows:

$$\psi = \frac{\begin{bmatrix}\text{any exergy increase}\\ \text{of the system}\end{bmatrix} + \sum \begin{bmatrix}\text{exergy outputs}\\ \text{to all systems}\end{bmatrix}}{\begin{bmatrix}\text{any exergy decrease}\\ \text{of the system}\end{bmatrix} + \sum \begin{bmatrix}\text{exergy inputs}\\ \text{from all systems}\end{bmatrix}} \tag{4}$$

For a given item of plant the exergy transfers and quantities of exergy that are transported depend on where the boundary is drawn. It is therefore of the utmost importance that the boundary that is used to quantify the overall performance of a plant is carefully selected and precisely defined. The identification and specification of the various external systems with which exergy interactions occur stem from the constraints of the physical situation being analysed (see Section 7.3 in Part 1 and the worked examples which follow in Section 2).

2 APPLICATIONS OF EXERGY ANALYSIS

2.1 Exergy analysis of a hot water boiler

Consider the case of a conventional oil-fired boiler which has an overall thermal efficiency based on the net calorific value of the fuel of 85 per cent. The water flow temperature is 95°C, with a return temperature of 60°C. Boiler electrical auxiliaries consume an amount of power equivalent to 0.5 per cent of the net calorific value of the fuel burned. The air flowrate is 110 per cent

The MS was received on 21 February 1990 and was accepted for publication on 9 June 1990.

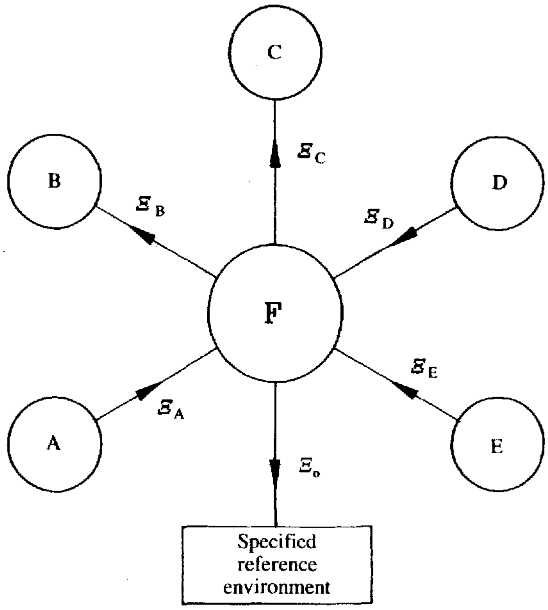

Fig. 1 A system, F, which undergoes exergy interactions with five external systems, A to E, in addition to the specified reference environment. Some of the exergy interactions are into system F while others, including the interaction with the reference environment, are out of system F

of the stoichiometric requirement and the temperature of the flue gases is 250°C. The reference environment, which corresponds with the state of the intake air, is specified as air at 25°C and 0.101 325 MPa. The pressure drop of the water in passing through the boiler is negligible.

Figure 2 shows an analysis boundary for the plant. According to a conventional energy analysis, for 100.5 units of energy that enter the system 85 units are transferred to the water and 15.5 units are lost to the environment. Although the thermal efficiency is normally stated without including the energy input to the auxiliaries a strict energy-based efficiency would be 85/100.5, or 84.6 per cent.

In carrying out an exergy analysis it is necessary to identify the external systems with which exergy interactions occur. These are

(a) the system that provides the electric power,
(b) the system that provides the fuel,
(c) the steady flow stream of water that receives heat transfer.

Note that the analysis boundary has been drawn in such a way that the flue gases have zero exergy as they have fully mixed with the specified reference environment. Also the air that enters the analysis boundary has zero exergy.

The exergy transfers corresponding to flow work are zero where the fuel, air and flue gases cross the analysis boundary since the pressure in each case is taken to be p_o.

2.1.1 The exergy input equivalent to the electrical input

The exergy input, Ξ_A, is equal to the electrical energy input, that is 0.5 exergy units per 100 units of calorific value that enter the analysis boundary.

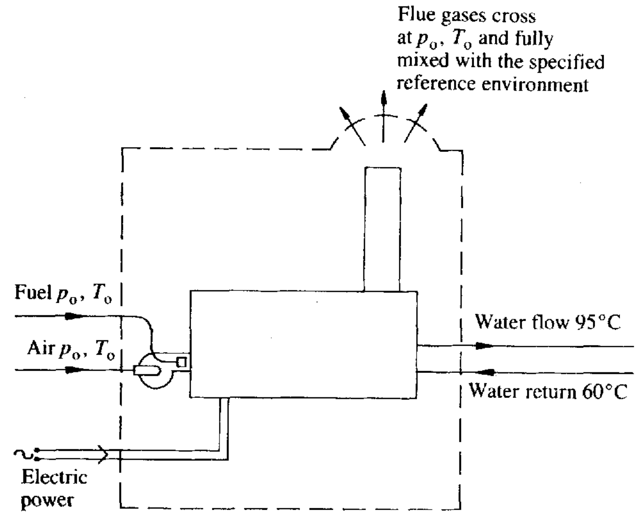

Fig. 2 Analysis boundary for a water boiler plant

2.1.2 The exergy input with the fuel

The exergy that crosses the analysis boundary with the fuel is Ξ_{ch} of the fuel. Methods have been developed to estimate the standard chemical exergy of multiple-component hydrocarbon fuels with respect to a standard reference environment [see references (**2**), Ch. 3, Sec. 5 and (**3**), App. C].

Gas oil with a net calorific value of 42 800 kJ/kg has an estimated chemical exergy of 45 726 kJ/kg, or 1.068 times the NCV, with respect to a standard reference environment at 25°C and 0.101 325 MPa.

Therefore, for 100 units of calorific value that enter with the fuel the exergy that enters the analysis boundary, Ξ_B, is 106.8 units.

2.1.3 The exergy output to the hot water

From equation (28) in Part 1, for 85 energy units of heat transfer to the water the exergy transfer is given by

$$\Xi_C = \left[1 - \frac{298.15 \ln\{(95 + 273.15)/(60 + 273.15)\}}{95 - 60}\right] 85$$
$$= 12.67$$

2.1.4 Rational efficiency of the boiler

$$\psi = \frac{\Xi_C}{\Xi_A + \Xi_B} = \frac{12.67}{106.8 + 0.5} = 11.8\%$$

This low figure indicates that there is very considerable scope for improvements in energy utilization in the case of conventional boilers such as this one. This fact is easily overlooked when conventional thermal efficiency values of the order of 85 per cent are quoted.

2.1.5 The source of inefficiency (rational)

Exergy analysis can be readily applied to identify the regions of a plant in which significant irreversibility (exergy destruction) occurs. Consider an analysis boundary around the isolated combustion and heat-transfer system of the boiler, as shown in Fig. 3. In this case exergy interactions occur with three external systems:

(a) the steady flow stream of reactants and products,

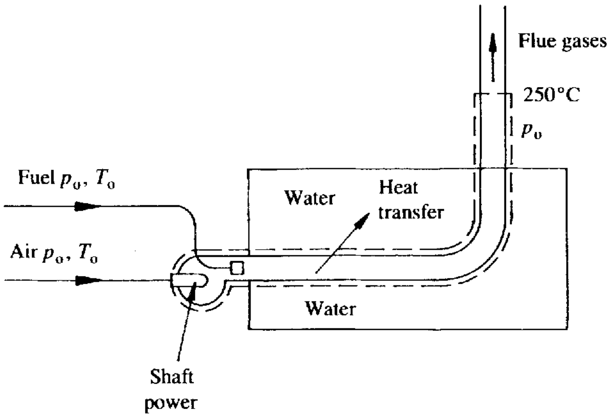

Fig. 3 Analysis boundary for the combustion and heat-transfer region of the water boiler represented in Fig. 2

(b) the shaft of the fan (the fuel pump power input is regarded as negligible for simplicity),
(c) the water surrounding the analysis boundary.

Assume the shaft power of the fan is determined to be 0.3 units per 100 units of fuel calorific value. Assume that the temperature on the water side of the heat exchange surface at the analysis boundary is roughly uniform at 98°C. The gas oil is taken to have an ultimate mass analysis of C, 86.7%; H, 13.3%.

As the composition of the reference environment (and the intake air) is required for the combustion and exergy calculations this is specified in Table 1.

Assuming combustion is complete, the combustion equation on a mass basis is as follows:

$$1 \text{ kg fuel} + 15.805 \text{ kg dry air} + 0.307 \text{ kg } H_2O_{vap} \rightarrow 3.179 \text{ kg } CO_2 + 1.504 \text{ kg } H_2O_{vap} + 12.091 \text{ kg } N_2 + 0.338 \text{ kg } O_2$$

Hence, the mole fractions of the products are: CO_2, 0.1208; H_2O, 0.1397; N_2, 0.7219, O_2, 0.0177.

The mean specific heat capacity of the flue gas is estimated to be 1.100 kJ/kg K, using tabulated specific heat data for the constituents. Assuming the gas is ideal the thermomechanical exergy [equation (10) in Part 1] can be expressed per unit mass as

$$\xi_{tm} = c_p\left(T - T_0 - T_0 \ln\frac{T}{T_0}\right) + RT_0 \ln\frac{p}{p_0}$$

$$= 1100\left(523.15 - 298.15 - 298.15 \ln\frac{523.15}{298.15}\right) + 0$$

$$= 63\,095 \text{ J/kg} = 63.1 \text{ kJ/kg}$$

Thermomechanical exergy per 100 units of

calorific value $= 100 \times 63.1 \times 17.112/42\,800 = 2.52$

The chemical exergy of the flue gas can be calculated from equation (13) in Part 1 once its composition has been determined by measurement or by estimation from the air–fuel ratio and the combustion equation, as in this case. Per 100 units of calorific value, this is found to be 3.05 units.

The total exergy loss with the flue gas is thus 2.52 + 3.05, or 5.57 units per 100 units. The comparable figure for the conventional energy loss is found to be 9.9 units.

Table 1 Composition of the specified reference environment

Substance	Mole fraction	Mass fraction
N_2	0.7567	0.7401
O_2	0.2035	0.2275
H_2O_{vap}	0.0303	0.0190
CO_2	0.0003	0.0005
Other	0.0092	0.0129

The net exergy input from the stream of reactants and products is thus

$$\Xi_A = 106.8 - 2.52 - 3.05 = 101.2$$

Therefore,

$$\Xi_{in} = 101.2 + 0.3 = 101.5$$

The heat transfer to the water across the analysis boundary is equal to the energy released in combustion minus the energy lost with the flue gases, that is 100 − 9.9 or 90.1 units. The exergy transfer across the analysis boundary to the water from the combustion and heat-transfer region is therefore given approximately by equation (23) in Part 1 as

$$\Xi_C = \frac{98 - 25}{273.15 + 98} 90.1 = 17.7$$

This is the only exergy output. Thus $\Xi_{out} = 17.7$.

Exergy destruction (irreversibility)

$$= \Xi_{in} - \Xi_{out}$$
$$= 101.5 - 17.7 = 83.8$$

Rational efficiency of the combustion and heat

$$\text{transfer region} = \frac{\Xi_{out}}{\Xi_{in}} = \frac{17.7}{101.5} = 17.4\%$$

Thus, most of the exergy destruction within the boiler occurs within the combustion and heat-transfer region. Exergy is destroyed because of lack of reversibility in the combustion process and the large temperature differences between the combustion products and the water that is heated. The exergy that is lost to the environment with the flue gases is relatively small at 5.57 units per 100 units of calorific value. Exergy analysis thus yields a very different perspective on energy use in the boiler to that given by a conventional energy-based analysis.

Two remedies which are available to reduce the exergy destruction that occurs in the combustion and heat-transfer region of a boiler are

(a) to extract the heat of combustion at the highest possible temperature, by interposing a power generating cycle if necessary;
(b) to oxidize the fuel by a method that involves less irreversibility, such as in a fuel cell.

2.2 Provision of heated air to a building

What ideal mechanical power would be required to supply a building with 0.4536 kg/s of air heated from an outside temperature of 10°C to a temperature of 26.67°C?

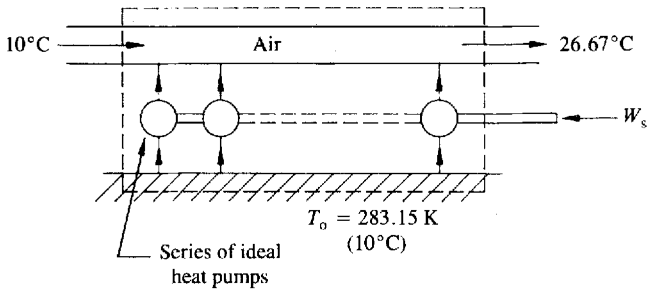

Fig. 4 Analysis boundary for an ideal device that provides heat transfer to a steady flow air stream

The required rate of heat transfer to the air is given by

$$\dot{Q} = \dot{m}c_p \Delta T = 0.4536\,(1005)(26.67 - 10)$$
$$= 7599\text{W} = 7.60\text{ kW}$$

Kelvin posed the question and answered it with the figure 0.2135 kW in December 1852 (**4**). This represented what would now be called an ideal heat pump coefficient of performance of 35.

Consider an ideal reversible system as represented in Fig. 4. The exergy output to the air stream per unit time is given by equation (28) in Part 1:

$$\dot{\Xi}_{out} = 7.60\left\{1 - \frac{283.15\ \ln(299.82/283.15)}{299.82 - 283.15}\right\}$$
$$= 0.2153\text{ kW}$$

Since the system shown in Fig. 4 is assumed to be ideal the exergy input rate as shaft power must equal the exergy output rate, that is the required mechanical power is 0.2153 kW. Thus, Kelvin's answer was correct to within 1 per cent. This was quite remarkable at a time when the second law of thermodynamics had not yet been postulated in mathematical terms and the absolute temperature scale had not been defined. Kelvin arrived at his result by considering reversible gas cycles, the known gas relationships and, in particular, Carnot's previous work (**5**).

Kelvin stated that it was not his objective to show how closely the ideal could be approached, but rather to show how 'the limit which [had] hitherto appeared absolute, [could] be surpassed'. By continuing to use techniques and particularly performance criteria based on the first law of thermodynamics, rather than on the first and second laws, engineers remain blinkered and continue to aim for and fall short of limits that can in fact be surpassed. This is the case in many areas, not only in relation to providing useful heating.

A very good conventional natural-gas-fired air heater with a thermal efficiency of 95 per cent would provide a heating effect equivalent to 95 per cent of the net calorific value of the fuel consumed. The chemical exergy of methane gas is about 1.03 times its net calorific value. An ideal reversible machine would thus provide a heating effect as described above of $1.03 \times 35 = 36.1$ times the NCV. The ideal machine would thus provide a heating output equivalent to $36.1/0.95 = 38$ times the output of the current technology 'high-performance' fuel-fired heater. Assuming that the temperature of the heated air need not be higher than 26.67°C the rational efficiency of the gas-fired heater can be expressed as the inverse of the latter figure, that is 2.6 per cent—a very different perspective on energy use.

2.3 Exergy analysis of an air-to-water heat pump

A small electrically driven air-to-water heat pump operating in the steady state takes heat from the outside ambient air which is at a temperature of −5.5°C and provides heat transfer to a steady stream of water flowing at the rate of 1 kg/min, raising its temperature from 14.5 to 31.5°C. The total electrical power input to the heat pump to drive the compressor and the evaporator fans is 550 W, of which 80 W goes to the fans.

According to a conventional energy analysis the rate of heat transfer to the water is given by

$$\dot{Q} = \dot{m}c_p \Delta T = (1/60)(4180)(31.5 - 14.5)$$
$$= 1184\text{ W}$$

The coefficient of performance (COP) is given by

$$\text{COP} = \frac{Q}{W} = \frac{1184}{550} = 2.15$$

With respect to an analysis boundary which intersects the inlet and outlet water pipes, the electric power cable and encompasses part of the specified reference environment, exergy interactions occur with two external systems:

(a) the supply of electrical energy and
(b) the steady stream of water that accepts heat transfer.

From equation (45) in Part 1 the exergy transfer rate to the water stream is given by

$$\dot{\Xi}_B = \dot{m}(\beta_{out} - \beta_{in}) = \dot{m}(h_{out} - h_{in}) - \dot{m}T_o(s_{out} - s_{in})$$

For an incompressible fluid and assuming the pressure drop between inlet and outlet is negligible, this can be written as

$$\dot{\Xi}_B = \dot{m}c_p\left\{(T_{out} - T_{in}) - T_o \ln\left(\frac{T_{out}}{T_{in}}\right)\right\}$$
$$= \frac{1}{60}\,4180\left\{(31.5 - 14.5) - (273.15 - 5.5)\ln\left(\frac{304.7}{287.7}\right)\right\}$$
$$= 113.9\text{ W}$$

The exergy input rate, $\dot{\Xi}_A$, is equal to the electric power input and hence the overall rational efficiency of the heat pump is given by

$$\psi = \frac{\Xi_B}{\Xi_A} = \frac{113.9}{550} = 20.7\%$$

This rational efficiency, which is based on the operating data of an actual plant, is low and indicates considerable potential for improvement. This fact illustrates the inadequacy of the energy-based 'coefficient of performance' as a figure of merit for heat pumps. A COP value which may be quoted for a heat pump gives no idea of how closely it approaches the thermodynamic

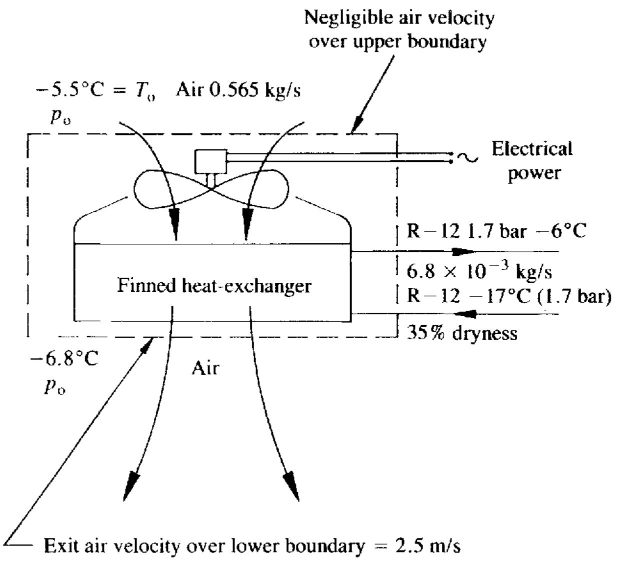

Fig. 5 Analysis boundary around the evaporator of a heat pump

ideal and especially to the non-engineer conveys the impression of 'getting something for nothing,' even when the true performance is very poor.

Exergy analysis can be used to identify the sources of irreversibility within the plant. Consider the evaporator of the same heat pump as shown schematically in Fig. 5. Exergy interactions occur with three external systems:

(a) the supply of electrical energy for the fans,
(b) the steady flow refrigerant stream,
(c) the steady flow air stream.

The net exergy that crosses the analysis boundary to the steady flow air stream is given by equation (45) in Part 1, modified to take account of the kinetic energy at outlet:

$$\dot{\Xi}_C = \dot{m}(h_{out} - h_{in}) - \dot{m}T_0(s_{out} - s_{in}) + \dot{m}\,\frac{C_{out}^2}{2}$$

Considering the air to be an ideal gas and since the inlet and outlet pressures are equal to p_0 this reduces to

$$\begin{aligned}\dot{\Xi}_C &= \dot{m}c_p\left\{(T_{out} - T_{in}) - T_0 \ln\left(\frac{T_{out}}{T_{in}}\right)\right\} + \dot{m}\,\frac{C_{out}^2}{2}\\ &= 0.565(1005)\\ &\quad\times\left\{(-6.8 + 5.5) - (273.15 - 5.5)\ln\left(\frac{266.35}{267.65}\right)\right\}\\ &\quad+ 0.565\,\frac{2.5^2}{2}\\ &= 1.799 + 1.765 = 3.56\ \text{W}\end{aligned}$$

The rate at which exergy crosses into the analysis boundary from the refrigerant (at the conditions shown in Fig. 5) is given by

$$\begin{aligned}\dot{\Xi}_B &= \dot{m}(h_{in} - h_{out}) - \dot{m}T_0(s_{in} - s_{out})\\ &= 6.80 \times 10^{-3}\{(240.5 - 351) \times 10^3\\ &\quad - 267.65(1.166 - 1.59) \times 10^3\}\\ &= 20.29\ \text{W}\end{aligned}$$

The electric power input to the fans, $\dot{\Xi}_A$, is 80 W. Hence, the exergy destruction rate $\dot{I}$ in the evaporator is given by

$$\begin{aligned}\dot{I} &= \dot{\Xi}_A + \dot{\Xi}_B - \dot{\Xi}_C\\ &= 80 + 20.29 - 3.56 = 96.7\ \text{W}\end{aligned}$$

The rational efficiency for the evaporator as enclosed by the analysis boundary shown in Fig. 5 is given by

$$\psi = \frac{\Xi_C}{\Xi_A + \Xi_B} = \frac{3.56}{80 + 20.29} = 3.5\%$$

The rate of heat transfer in the evaporator is given by

$$\dot{Q} = 6.8(351 - 240.5) = 751\ \text{W}$$

In energy terms, the evaporator fan power of 80 W does not seem large in comparison to the rate of heat transfer in the evaporator. This is seen in a different light when the exergy transfer corresponding to the electric power, 80 W, is compared with the net exergy provided to the evaporator by the refrigerant stream, which is just 20.3 W.

3 SOME LIMITATIONS OF EXERGY ANALYSIS AND AREAS FOR FURTHER WORK

1. There is a lack of consensus on terminology and methodology. This paper is intended as a further step towards consensus.
2. The procedures for applying exergy analysis can be tedious. Computer-based algorithms will minimize this problem.
3. The detailed application of exergy analysis places greater demands on experimental measurement techniques than conventional energy analysis. This presents an ongoing challenge.
4. In order to apply exergy analysis for conceptual design there is a need for a data base that would describe the rational efficiencies achievable in current technology processes and plant. Manufacturers of plant components such as turbines and expanders will need to provide performance data in a suitable form.
5. Data on chemical substances suitable for exergy analysis is not always readily available. Undoubtedly the gaps will be filled as the demand for such data increases.
6. Improved methods are required for estimating the exergy of complex substances and mixtures.
7. There is a need for further work to develop a methodology for optimizing plant and to relate the overall rational efficiency to the rational efficiencies of its components or sub-systems.

4 CONCLUSIONS

In this paper an account has been given of exergy analysis and this has included some significant new conceptual developments and interpretations. Part 1 describes a toolbox which engineers can use in applying exergy analysis to actual problems. Part 2 is concerned with describing the degree to which actual systems approach the thermodynamic ideal of reversibility and includes worked examples to illustrate the concepts and the use of the toolbox.

REFERENCES

1 **Moran, M. J.** *Availability analysis: a guide to efficient energy use*, corrected edition, 1989 (ASME Press, New York).

2 **Szargut, Jan, Morris, David R.** and **Steward, Frank R.** *Exergy analysis of thermal, chemical, and metallurgical processes*, 1988 (Springer Verlag).

3 **Kotas, T. J.** *The exergy method of thermal plant analysis*, 1985 (Butterworths).

4 **Thomson, William (Lord Kelvin)** On the economy of heating or cooling of buildings by means of currents of air. In *Mathematical and physical papers*, Vol. 1, 1911, Art. LX, pp. 515–520 (Cambridge University Press, London); also in *Proc. Phil. Soc. of Glasgow*, 1852.

5 **Thomson, William (Lord Kelvin)** An account of Carnot's theory of the motive power of heat; with numerical results deduced from Regnault's experiments on steam. In *Mathematical and physical papers*, Vol. 1, 1911, Art. XLI, pp. 113–155 (Cambridge University Press, London); also in *Trans. R. Soc. of Edinburgh*, 1849.