A Virtual-system Concept for Exergy Analysis of Flow Network Plant; Part II: Exergetic and Exergonomic Analysis Illustration

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A Virtual-System Concept for Exergy Analysis of Flow Network Plant
Part II: Exergetic and Exergonomic Analysis Illustration

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University of Dublin
Trinity College, Ireland

ABSTRACT

This paper is a continuation of Part I — Principles, to which it refers extensively. The virtual system concept and the methodology introduced in Part I are illustrated by means of a numerical example. This is an exergetic and exergoeconomic analysis of the CHP steam plant that was described in Part I without operating values and parameters. Matrix methods developed by Valero et al. (1986) are adapted to the new concept. It is shown that these methods are made simpler and less subjective. Bond-graph-type diagrams for exergetic and economic costs are presented.

NOMENCLATURE

Abbreviations
CHP  Combined heating and power
FCS  Flow constraint system
LFCS  Linked flow constraint system
RN  Reversible node

Roman Symbols
A  Costs matrix
b  Specific flow exergy function \( b = h - T_o s \)
\( \dot{D} \)  Exergy destruction rate vector
\( \dot{F} \)  Fuel exergy rate vector
h  Specific enthalpy
I  Incidence matrix
\( I_f \)  Fuel incidence matrix
\( I_p \)  Product incidence matrix
\( \dot{m}_s \)  Mass flow rate identified by subscript
\( \dot{P} \)  Product exergy rate vector
\( p_s \)  Pressure at position identified by subscript
s  Specific entropy
\( T_o \)  Temperature of the environment
\( T_s \)  Temperature at position identified by subscript
\( x_o \)  Dryness at position identified by subscript
\( \dot{Y}^* \)  Input exergetic cost rate vector
A Virtual-System Concept—Part II

\[ \dot{Y}_z^* \] Vector of zeros, one for each subsystem
\[ \dot{Z} \] Input economic cost rate vector
\[ \dot{Z}_E \] Input non-fuel economic cost rate to subsystem identified by subscript
\[ \dot{Z}_f \] Input fuel economic cost rate

**Greek Symbols**
\[ \eta \] Rational efficiency vector
\[ \dot{\Xi} \] Exergy interaction rate vector
\[ \dot{\Xi}_* \] Exergetic cost rate vector
\[ \dot{\Xi}_{1*} \] Exergetic cost rate identified by subscript
\[ \dot{\Xi}_{(4-5-6)} \] Exergetic cost rate interaction that is the algebraic sum of the exergetic cost rate interactions identified by the signed subscripts \( (\dot{\Xi}_{(4-5-6)} = \dot{\Xi}_4 - \dot{\Xi}_5 - \dot{\Xi}_6) \)
\[ \dot{\Xi}_4 \] Exergy interaction rate identified by subscript
\[ \Pi \] Economic cost rate vector
\[ \Pi_3 \] Economic cost rate interaction identified by subscript
\[ \Pi_{(4-5-6)} \] Economic cost rate interaction that is the algebraic sum of the economic cost rate interactions identified by the signed subscripts \( (\Pi_{(4-5-6)} = \Pi_4 - \Pi_5 - \Pi_6) \)

**INTRODUCTION**

In presenting a numerical illustration of the virtual system concept reference is made to the description of the CHP steam plant and to Figs. 4, 5, and 6 in Part I.

The procedures for evaluating the exergy flows due to material streams and electric power, for locating the irreversibilities, and for presenting the analysis graphically as a Grassmann diagram are well established in the literature and are not reproduced here. The required data are given in Tables 1 and 2.

It is important, whatever analysis techniques are used, that the analyst has complete freedom to define the overall analysis boundary, the boundaries of any subsystems, and the external systems. If this freedom exists, there is an obligation to define these precisely and to define the environment: the boundaries of the systems and subsystems, including LFCS1, FCS2, FCS3, and the environment L have been described in Part I. In Table 1 in this Part values are assigned to parameters and properties to complete the definitions.

It has been recognised by other authors too that an understanding of the structure of a plant in exergy terms is vital to exergetic and exergoeconomic analysis; e.g., Valero and Torres (1988) have presented a methodology, suitable for use with symbolic mathematical computer packages, whereby the structure of a plant could be represented by algebraic expressions. In this paper it will be
shown how the new concepts presented in Part I contribute to the understanding of exergetic structure.

**EVALUATION OF NET EXERGY INTERACTIONS**

These are identified in Fig. 5, Part I, and can be evaluated from the exergy flows or by using the specific flow exergy function \( b \), as illustrated by the following examples:
Net exergy interaction rate from fuel source to FCS2.
$$\dot{\xi}_1 = \dot{m}_1b_{10} + \dot{m}_9b_9 - \dot{m}_{11}b_{11} = \dot{m}_1b_{10} = 67.7432 \text{ MW}$$

Net exergy interaction rate from LFCS1 to turbogenerator.
$$\dot{\xi}_4 = \dot{m}_4b_5 - \dot{m}_6b_6 - \dot{m}_8b_8 = 12.1067 \text{ MW}$$

There are fifteen such exergy interactions in Fig. 5 and these constitute the exergy interaction rate vector, $\dot{\Xi}$ (Table 3).

**THE INCIDENCE MATRIX**

The incidence matrix contains the information about which exergy interactions occur between which systems, and the directions of the interactions. Its columns correspond to the fifteen exergy interactions shown in Fig. 5 and its rows correspond to the eleven systems within the overall analysis boundary in the same figure. A plus one indicates that the exergy interaction represented by the column is *into* the system represented by the row. A minus one indicates that the exergy interaction is *out of* the system. The incidence matrix is thus written by inspection of Fig. 5.

$$I = \begin{bmatrix} 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

**EXERGY DESTRUCTION**

Having set up the incidence matrix and the exergy interaction vector the exergy destruction rate in each system within the overall analysis boundary can be calculated by matrix multiplication of these. The exergy destruction rates comprise the exergy destruction rate vector, $\dot{D}$, which is given by $\dot{D} = I\dot{\Xi}$ (Table 4).

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1In this evaluation only, $b$ is the absolute flow exergy including chemical exergy; with respect to the environment.

McGovern & O’Toole 1992
RATIONAL EFFICIENCIES OF SUBSYSTEMS

By replacing all negative numbers (-1’s) in the incidence matrix by zeros the fuel incidence matrix \( \mathbf{I}_f \) is formed. This identifies all exergy interactions that are into subsystems. Similarly by replacing all positive numbers (1’s) in the incidence matrix by zeros and multiplying the resulting matrix by minus one the product incidence

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<tr>
<td>FCS3</td>
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matrix $I_p$ is formed: this identifies all exergy interactions that are out of subsystems within the overall analysis boundary.

The fuel exergy rate vector $\dot{F}$ and the product exergy rate vector $\dot{P}$ contain the sum of the exergy input rates and the sum of the exergy output rates respectively for each of the fifteen subsystems within the overall analysis boundary shown in Fig. 5: these are calculated as follows:

$$\dot{F} = I_f \Xi \dot{P} = I_p \dot{\Xi}.$$

The rational efficiencies of all subsystems within the analysis boundary are obtained by dividing each row of the product exergy vector by the corresponding row of the fuel exergy vector:

$$\eta = \frac{\dot{P}}{\dot{F}}.$$

**EXERGETIC COST**

**Definition**

Exergetic cost is defined as the exergy input from sources outside the overall analysis boundary that can be associated with a particular net exergy interaction between systems or subsystems. Valero et al. (1986a) proposed a similar definition: the amount of exergy required per unit time to produce a particular exergy flow. The distinction between a net exergy interaction and an exergy flow is an important one, the very basis of Part I.

It is not possible, on the basis of the exergy flows alone, objectively to assign exergetic costs to material flow streams, such as the streams of steam that leave the turbine in Fig. 4, since to attempt to do so would not take into account the structural constraints on net exergy interactions due to material transport. The FCS concept automatically incorporates these constraints: the exergy flows with the steam that enters the turbine at 5 and with the steam that leaves at 6 and 8 are not independent. Therefore, in applying the FCS concept the definition of exergetic costs is less subjective than in previous methodologies.

**Cost Balances and Propositions**

For the overall analysis boundary an exergetic cost balance applies; i.e., the net exergetic cost input across the boundary is zero, or the sum of the exergetic cost inputs equals the sum of the exergetic cost outputs. A similar exergetic cost balance must be satisfied for each subsystem within the overall analysis boundary. Each exergy interaction has a corresponding exergetic cost. Thus, there are fifteen exergetic cost interactions corresponding to the fifteen exergy interactions shown in Fig. 5. Fifteen equations are required to solve for these: cost balances for the eleven subsystems provide eleven independent equations.

Valero et al. (1986a) put forward the hypothesis that “Nature has all the necessary information for calculating costs” and proposed
Fuel-Product-Residue propositions as a means of augmenting the cost balance equations to evaluate the costs. In their methodology, based on exergy flows rather than net exergy interactions, the definitions of fuels, products, and residues were subjective — some could be based on net exergy interactions, but this might not always be feasible, or desired.

In the FCS approach described here fuels and products are objectively and unambiguously defined: for the overall analysis boundary or a subsystem boundary the fuel is the sum of all exergy interactions into the system or subsystem; the product is the sum of all the exergy interactions out of it. A residue (i.e., an exergy output that is not useful) would be described as an output to an exergy destruction sink (which could be inside or outside the overall analysis boundary).

Cost propositions are also required as part of the FCS methodology. These are as follows:

**Fuel proposition.** The exergetic cost of an exergy interaction into the system enclosed by the overall analysis boundary is equal to the exergy input.

**Exergy destruction-sink proposition.** The exergetic cost of an exergy interaction to an exergy destruction sink is zero. It should be noted that for exergy destruction sinks within the overall analysis boundary the exergy destruction-sink proposition is already incorporated in the cost balance equations. This proposition will yield one additional equation for each exergy interaction to an external exergy destruction sink.

**Recycle proposition.** At a node or subsystem let any pair of exergy interactions, one in and one out, that can form part of the same recycle be called a recycle pair. There may be a number of recycle pairs at a node or subsystem. Also, a given interaction at a node may be a member of more than one recycle pair: interactions that are structurally linked in this way will be said to belong to the same recycle group. In exergetic costing, at any node where a recycle group represents a net exergy output all the interactions of the group are structurally constrained to behave as one net exergy interaction. Due to the structural constraint the interactions must have the same exergetic cost per unit of exergy. The proposition yields an additional independent equation for each recycle of the group.

In order to apply this proposition, recycles must be identified. This can be done by inspection of the exergy interaction diagram or by application of a suitable algorithm to the incidence matrix.

**Fork proposition.** Where a subsystem has more than one output exergetic cost interaction, the output exergetic costs are distributed in proportion to the exergy values; that is, the exergetic costs per unit of exergy are the same for all output exergetic cost interactions. At a subsystem where multiple output exergetic cost interactions occur, the fork proposition yields a number of equations, which is one less
than the number of output exergetic cost interactions. This proposition is subject to two preconditions, as follows:

Precondition 1. Any output exergy interaction that has a zero exergetic cost due to the exergy destruction-sink proposition and exergetic cost balances for subsystems (which are always “downstream” in this case) is not involved in the fork proposition since it does not have an associated exergetic cost. Therefore the cost balances and the exergy destruction-sink proposition must be applied before the fork proposition.

Precondition 2. All interactions of a recycle group are treated as a single net exergy interaction in applying the fork proposition. Hence a recycle group is included in the fork proposition if it represents a net exergy output. This can conveniently be done by including just one of the individual exergy output interactions of the recycle group in the fork proposition (rather than the net exergy interaction of the group). If the recycle group is a net exergy input none of its interactions feature in the fork proposition.

THE COSTS MATRIX

A costs matrix $A$ and an input exergetic cost rate vector $\dot{Y}^*$ contain the cost balances and constraints of the cost propositions for the interactions within the analysis boundary.

The exergetic costs (as yet unknown) are contained in a vector $\dot{\Xi}$ and the cost balances can be written in matrix form as: $I\dot{\Xi} + \dot{Y}^* = 0$, where $\dot{Y}^*$ is a vector of zeros, one for each subsystem.

The $A$ matrix and the $\dot{Y}^*$ vector are formed by augmenting the rows of the $I$ matrix and those of the $\dot{Y}^*$ vector so as to incorporate the cost propositions.

The fork proposition for LFCS1 in Fig. 5 can be written as

$$\frac{\dot{\Xi}_4}{\dot{\Xi}_4} - \frac{\dot{\Xi}_8}{\dot{\Xi}_8} = 0.$$

The recycle proposition for LFCS1 in Fig. 5 can be written as

$$\frac{\dot{\Xi}_4}{\dot{\Xi}_4} - \frac{\dot{\Xi}_5}{\dot{\Xi}_5} = 0 \quad \text{and} \quad \frac{\dot{\Xi}_4}{\dot{\Xi}_4} - \frac{\dot{\Xi}_6}{\dot{\Xi}_6} = 0.$$

The fuel proposition for subsystem FCS2 is

$$\dot{\Xi}_1 = \dot{\Xi}_1.$$

The $I$ matrix is thus augmented to give the $A$ matrix
and the $\mathbf{Y}^*_Z$ vector to give the $\mathbf{Y}^*$ vector as in Table 3.

The cost balance equations and cost proposition equations are thus represented by the following matrix expression:

$$\mathbf{A}\dot{\mathbf{X}}^* + \dot{\mathbf{Y}}^* = 0.$$ 

Hence, the exergetic cost vector $\mathbf{X}^*$ (Table 3) can be evaluated from:

$$\mathbf{X}^* = \mathbf{A}^{-1}(\dot{\mathbf{Y}}^*).$$

The exergetic cost interactions involve recycles similar to the exergy interaction recycles shown in Fig. 5. However, these can be eliminated using exactly the same type of transformation as was described in Part I for exergy interactions that form recycles. Fig. 7 is the resulting diagram, which illustrates the real structure of the exergetic cost interactions. Considerable simplification and clarification is achieved.

**EXERGOECONOMIC ANALYSIS**

In exergoeconomic analysis there are two types of economic input costs:

1. Fuel economic costs: the economic costs of the net exergy interactions from external systems into the system surrounded by the overall analysis boundary; and

2. Non-fuel economic costs: the capital amortisation costs, operating costs, and maintenance costs of the subsystems within the analysis boundary.

In addition to the economic costs of fuel exergy (there is only one for the overall analysis boundary shown in Fig. 5) there will be the non-fuel economic cost inputs to various subsystems within the analysis boundary: these are all known (Table 5). An economic cost balance applies for the system enclosed within the analysis boundary and for each of the subsystems. The sum of the economic cost outputs equals the sum of the economic cost inputs: since the non-fuel economic cost inputs are all known, the number of equations required is the same as the number of equations required to find the exergetic costs. The cost propositions apply to economic costs in the same way as they apply to exergetic costs.

The same costs matrix $\mathbf{A}$ that was used in the exergetic cost analysis can be used with an economic cost input rate vector $\mathbf{Z}$ to represent the economic cost balance and cost proposition equations.
Hence the economic costs of the exergy interactions are given by the following matrix expression:

\[ \mathbf{\dot{H}} = \mathbf{A}^{-1} (-\mathbf{Z}). \]

The economic cost input rate vector \( \mathbf{Z} \) contains the non-fuel economic cost inputs in the rows corresponding to the cost balance equations (one for each subsystem) and contains the fuel economic cost inputs in the rows corresponding to the fuel proposition. For the overall analysis and subsystem boundaries shown in Fig. 5, \( \mathbf{Z} \) is given in Table 3. Hence the economic cost rate interaction vector \( \mathbf{\dot{H}} \) is evaluated (Table 3).

The cost rate interactions \( \dot{H}_{12} \), \( \dot{H}_{13} \), and \( \dot{H}_{14} \) are negative. This indicates they have the opposite directions to the corresponding exergy interaction rates shown in Fig. 5.

As for the exergy interactions and exergetic cost interactions, the economic cost interactions can be represented on a bond graph type of diagram. Fig. 8 is such a diagram based on \( \dot{P} \) in Table 3. The economic cost interactions have been transformed in the same way as the exergy interactions in Fig. 6 (a diagram without RN1, analogous to Fig. 5, could also be drawn).

**On the Aggregation Level of Analysis Subsystems**

Valero et al. made it clear that their exergetic cost depended on the aggregation level of the subsystems; for example, the exergetic...
cost of an electric power output would depend on the selection and number of subsystems within the overall analysis boundary. They also pointed out that there were basic subsystems that could not be reduced to others: a fact that lessened the problem of lack of robustness of exergetic costs, which depended on the aggregation level of analysis boundaries.

The flow constraint system concept and the transformation of recycle interactions to equivalent direct interactions yields a new and simpler structure of the exergetic and economic costs, as shown in Figs. 7 and 8. The fork proposition is the basis for the division of costs at any node. The definitive output exergetic costs at the overall analysis boundary are those when all forks occur at reversible nodes; e.g., an LFCS or any system within which there is negligible exergy destruction. If this is the case, further partitioning into subsystems does not affect the output exergetic costs of the plant.
CONCLUSIONS

A numerical illustration of the application of the flow constraint system concept to the analysis of a CHP steam plant has been presented. The matrix techniques for exergetic and exergoeconomic analysis of Valero et al. (1986) have been adapted as necessary to the FCS concept and a less subjective methodology has been shown to result. Bond-graph-type diagrams have been used to illustrate exergetic cost and economic cost interactions. These provide for a clear visualisation of the structure of the plant in exergetic and economic cost terms.

The FCS concept is well suited to use with matrix analysis techniques, and thus to the analysis of plant of virtually unlimited complexity. However, matrix techniques are not essential and the FCS concept results in such a simplification of structure that solution of the cost balance and cost proposition equations is easy enough for pencil and paper calculations for a plant such as the CHP steam plant as described.

ACKNOWLEDGEMENT

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REFERENCE