Stochastic Hybrid Embodied CO2-eq Analysis: an Application to the Irish Apartment Building Sector

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Stochastic hybrid embodied CO₂-eq analysis: An application to the Irish apartment building sector

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Abstract

The development of embodied CO₂-eq analysis has progressed significantly in recent years and has become a mainstream practice in many industries, as evidenced by the development of the ISO 14040 and 14044 life cycle assessment (LCA) standards. However, it is recognized that due to weaknesses in gathering data on product-related emissions, embodied CO₂-eq values are probabilistic. This paper therefore presents a stochastic analysis of hybrid embodied CO₂-eq in buildings to account for this weakness in traditional methods and, by way of example, applies it to an Irish construction-sector case study. Using seven apartment buildings, 70,000 results are simulated with Monte Carlo analysis and used to derive probabilistic and cumulative embodied CO₂-eq intensity distributions for apartment buildings in Ireland. A Wakeby distribution with known statistical parameters and uncertainty was derived for the average embodied CO₂-eq intensity of apartment buildings in Ireland. The mean hybrid embodied CO₂-eq intensity was estimated to be 1636 gCO₂-eq/m² with an uncertainty of 73 gCO₂-eq/m². The stochastic analysis helps to account for variability in input variables into LCA and embodied energy and CO₂-eq analysis. The application of the stochastic embodied CO₂-eq analysis as demonstrated in this study can be extended to other building sectors and countries and can form the basis for the development of evidence-based policy formulation since it provides greater information of embodied CO₂-eq intensities of buildings than deterministic approaches.

Keywords: Hybrid embodied CO₂-eq; Stochastic analysis; Construction sub-sector; Apartment buildings; Probabilistic and cumulative distributions; Monte Carlo analysis

1. Introduction

Greenhouse gas (GHG) mitigation is now a central policy of almost all developed economies. Because buildings account for approximately 40–50% of total emissions in these countries [1,2], such policies focus on emissions’ reductions from the built environment through measures such as promoting energy efficiency and the deployment of renewable energy supply (RES) technologies. These measures, however, fail to address the increasingly important role that embodied energy (the energy required to produce a building) plays in building-related emissions, which can represent as much as 40% of life cycle emissions for residential buildings [3].

Scheckels [4] define the embodied energy of a building as the energy consumed by all the process associated with its production. The embodied CO₂-eq of a building can therefore be defined as the equivalent carbon dioxide (CO₂-eq) gas emitted into the atmosphere as a result of all the associated energy used in the production of that building. Equivalent carbon dioxide (CO₂-eq) represents the most important anthropogenic energy-related greenhouse gases (GHGs) with the highest environmental impacts. These are: carbon dioxide – CO₂; nitrous oxide – N₂O; and methane – CH₄ [5]. These emissions are associated with the initial phase of a building’s life cycle, preceding emissions resulting from operational energy use and energy used in demolition and recycling.

It is recognized that operational energy analysis has dominated building energy research for many years when compared to embodied energy analysis. It has been shown, however, that the energy embodied in buildings is significant when compared to its operational energy use. For example, Yohanis et al. [6] showed that energy initially embodied in a single-storey office building could be as much as 67% of its operating energy over a 25-year period. Moreover, research carried out by the Commonwealth Scientific and Industrial Research Organisation [7] also shows that embodied energy of a building is a significant multiple of the annual operating energy consumed, ranging from around 10 times for typical dwellings to over 30 times for office buildings. It is also a well-established fact that as buildings become more operationally energy efficient, the embodied energy to operational energy ratio increases. Embodied energy and emissions are therefore likely to account for an increasingly large proportion of building-related life cycle CO₂-eq emissions in the future.

The importance of embodied energy and embodied CO₂-eq analysis should therefore not be underestimated when assessing life cycle energy requirements, resource depletion and related environmental impacts.

The development of embodied CO₂-eq analysis and life cycle assessment (LCA) has progressed significantly in recent years, and LCA has become a mainstream practice in many industries as evidenced by the development of the ISO 14040 and 14044 Life Cycle Assessment Environmental Standards. However, it is recognized that due to weaknesses in gathering data on product-related energy use and emissions, embodied energy values are probabilistic [8,9]. For example: designers and contractors are currently unable to obtain embodied emissions in the products they employ (apart from in exceptional circumstances); and the use of sectoral emissions intensities (derived using input–output (I–O) techniques) to estimate emissions for a particular product of process is normal practice, although the intensity relates to a wide range of products and processes aggregated into one sector. Despite these uncertainties regarding the applicability of data to the product being analysed, it is noted by commentators (inter alia [9,10]) that even with the recent methodological improvements, the general approach to estimating embodied emissions and energy remains deterministic, thus obscuring both the uncertainty and true variability in embodied energy and life cycle assessment results.

Best practice in embodied emissions analysis involves a hybrid approach incorporating both process and input–output analysis (inter alia [11–13]). These two approaches rely respectively on process-related data and national sectoral economic data combined with environmental accounts to give emissions per unit monetary output from a sector. For process data, uncertainties arise due to variations in manufacturing processes and supply chains, measurement error and the use of out-of-date data. In the case of input–output data, a significant source of error is due to its highly aggregated nature: for example, construction sector emissions intensity is equally applied to house building and motorway construction. Pacca and Horvath [14] identify that uncertainties can also arise from economic boundary and methodological constraints.

A number of studies have deterministically calculated embodied energy and LCA values for a variety of building types in different countries. For example, Fay et al. [15] have estimated the energy intensity of an Australian residential building to be 1803 GJ while Thormark [16] calculated an embodied energy of 2.9 GJ/m² for a Swedish apartment. Treloar et al. [17] also estimated the embodied energy of a three storey office building to be 10.7 GJ/m². Dixit et al. [18] compiled a list showing different deterministic embodied energy values in residential and commercial buildings. Due to the constraints mentioned above, these data may be representative of a very small sample of buildings which do not provide sufficient information for decision makers to identify methods for reducing energy consumption in the building and construction supply chain. If however, the distributions of embodied CO₂-eq can be estimated, then decision makers can design targeted policies to reduce the overall emissions in an industry sector or market segment. Understanding the distribution of embodied emissions in the construction sector or segment (for example in the apartment building sector) can therefore be useful in the formulation of effective policies. Furthermore, building designers and contractors will be better placed in terms of having more detailed information on their buildings which will enable them to take informed environmental decisions on their designs as well as in their choice of building products and processes.

The main aim of this work is to develop and implement methodologies which measure the nature and extent of uncertainty when estimating embodied CO₂-eq emissions in buildings. Specific objectives include:

- the presentation of a stochastic embodied CO₂-eq assessment methodology incorporating both process and input–output analysis;
- using industry data to estimate the probability distributions of embodied CO₂-eq intensities for a particular building sector; and
- an evaluation of the embodied CO₂-eq intensities and the uncertainty across a particular building type in Ireland.

The stochastic embodied emissions methodology employed in this study is applicable to any type of structure, sectors other than construction as well as to other countries.

2. Methodology

The methodology adopted involves:

- the use of hybrid analysis to develop relationships between input parameters such as product emissions intensities, input–output (I–O) sectoral emissions intensities, disaggregated construction emissions intensities, construction materials employed and construction expenditure;
- an analysis of industry and economic (input–output) data to estimate probability distributions for certain input parameters;
- an application of the model to 7 Irish apartment buildings;
- the use of Monte Carlo simulation to derive probability and cumulative distributions for emissions intensities for the seven apartment buildings; and
- analysis and interpretation of results.

Hybrid embodied CO₂-eq intensities are calculated where process analysis is used to determine the embodied CO₂-eq in the main building materials, sub-sector direct embodied CO₂-eq intensities to derive the direct embodied CO₂-eq emitted on site in constructing the buildings, and input–output analysis to estimate the indirect embodied CO₂-eq emitted in the construction of the building.

Seven apartment buildings in Dublin are investigated. For each apartment, the stochastic hybrid embodied CO₂-eq intensities are evaluated using Monte Carlo analysis by deriving input distributions for the stochastic input variables. A distribution representative of the hybrid embodied CO₂-eq distribution of apartment buildings in Ireland is derived by combining the distributions of the seven apartment buildings. This is based on the assumption that the samples are representative of the population of apartment buildings in Ireland. An analysis is then carried out on these distributions including deriving statistical parameters and the level of uncertainty in the results.

2.1. Stochastic hybrid embodied CO₂-eq (ECO₂-eq) intensity

Construction sector CO₂-eq emissions can be characterised as direct or indirect. The former are released as a result of activities directly related to construction processes on site (for example: excavation, fit-out, plant operation). The latter are associated with the use of energy in construction-related activities necessary for, but preceding site activities. These activities are ‘upstream’ of site work in the construction procurement supply chain (for example: energy used to manufacture building materials, excavation of raw aggregate, design team activities). The hybrid embodied CO₂-eq (ECO₂-eq) intensity can be broken down into three parts and expressed in terms of total grams of embodied CO₂-eq per Euro [gCO₂-eq/euro] of total expenditure. Building materials embodied CO₂-eq intensities are calculated by process analysis, direct embodied CO₂-eq emissions on the construction site are evaluated from disaggregated economic data of Irish construction firms and indirect embodied CO₂-eq emissions are evaluated by input–output anal-
Mathematically, the Hybrid ECO\textsubscript{2}-eq intensity is expressed as:

\[
\text{Hybrid ECO}_2\text{-eq intensity} = \frac{\sum_{x=1}^{n} Mx e_x + \left[ \sum_{j=1}^{5} S_j + \sum_{j=1}^{n} I_j q_j S_j \right]}{\sum_{j=1}^{n} S_j + C_p}
\]

where \(M_x\) is the mass of building material \(x\) (tonnes/\(t\)); \(n\) the number of building materials for which process emissions intensities and quantities exist; \(e_x\) the process embodied CO\textsubscript{2}-eq intensity of building material \(x\) (\(\text{gCO}_2\text{-eq}/\text{t}\)); \(i\) the input–output indirect embodied CO\textsubscript{2}-eq intensity of construction (\(\text{gCO}_2\text{-eq}/\text{e}\)); \(q_j\) the direct embodied CO\textsubscript{2}-eq intensity of each construction sub-sector \(j\) (\(\text{gCO}_2\text{-eq}/\text{e}\)); \(j\) the number of construction sub-sectors; \(S_j\) the expenditure classified by construction sub-sector, \(j\) on activities associated with the construction of the building \([\text{e}]\); and \(C_p\) the total cost of building materials analysed using process CO\textsubscript{2}-eq intensity inventory \([\text{e}]\).

The following steps were undertaken in the calculation to avoid double counting of input–output inputs into the model for which process data (associated with building materials) has already been collected:

- the total cost of building materials, \(C_p\), to which process data were applied was subtracted from the total expenditure extracted from the bill of quantities, therefore the remaining expenditure represented by \(\sum_{j=1}^{n} S_j\) is only attributed to I–O inputs;

- this sum is multiplied by the I–O construction sector indirect emissions to estimate total indirect emissions \(\left[ \sum_{j=1}^{5} S_j \right]\); and

- individual sub-sectoral expenditures, \(S_j\), are multiplied by the corresponding direct emissions coefficients, \(I_j\) and then summed to estimate total direct emissions \(\sum_{j=1}^{n} I_j q_j S_j\).

### 2.1.1. Material process embodied CO\textsubscript{2}-eq intensities

According to Goggins et al. [19] the sustainability credentials of construction materials are gaining increasing importance as the environmental impact of the construction industry becomes apparent. Data on the ECO\textsubscript{2}-eq intensities of building materials however are uncertain. Industry (process) data was therefore used to estimate probability distributions for all process embodied CO\textsubscript{2}-eq intensities of building materials. Available but limited data on building materials process CO\textsubscript{2}-eq intensities obtained from the Inventory of Carbon and Energy database, ICE v1.6a [20] are fitted into a probability density function using EasyFit Statistical Application and the distributions ranked according to Kolmogorov–Smirnov goodness of fit from a set of 57 different distributions. Kolmogorov–Smirnov test (K–S test) was preferred to other goodness of best fit tests such as Anderson–Darling goodness of best fit because it is sensitive to differences in both the location and shape of different distributions [21]. It is also an exact test, that is, the chi-square goodness-of-fit test depends on an adequate sample size for the approximations to be valid. Moreover, Anderson–Darling test is only available for a few specific distributions. Using the statistical parameters of the number one ranked fitted distribution, a set of 10,000 random embodied CO\textsubscript{2}-eq intensities are then generated for each of the building materials and used as input variables for the stochastic modeling. As an example, the embodied CO\textsubscript{2}-eq intensity probability distribution of insulation is shown in Fig. 1.

### 2.1.2. Direct sub-sectoral embodied CO\textsubscript{2}-eq intensities

Probability distributions are also derived for the direct embodied CO\textsubscript{2}-eq intensities, \(I_j\) of each of the five construction sub-sectors using disaggregated micro energy data collected by the Irish Central Statistics Office in their Census of Building and Construction [22–25] from 2003 to 2006. The sample data from the construction firms was chosen to be representative of the Irish construction sector and methodological notes are available from the Irish Central Statistics Office [26]. 682 firms were sampled in 2003, 628 in 2004, 728 in 2005 and 1291 in 2006. Table 2 shows a summary of the stochastic direct embodied CO\textsubscript{2}-eq intensity distributions and the statistical parameters of the construction sub-sector which was weighted from 2003 to 2006. It is assumed that all fuel used was diesel since the vast majority of plant and construction machinery in Ireland operates on diesel fuel [24].

Energy expenditure is divided among five construction sub-sectors defined by The General Industrial Classification of Economic Activities within the European Communities (NACE rev. 1). Construction sub-sectors 1–5 are hereafter referred to as ‘Ground Works’, ‘Structural Work’, ‘Services’, ‘Finishes’ and ‘Plant Operation’ respectively. The sub-sectors are defined in detailed below:

Table 1 shows some common building materials used in apartment buildings and the number one ranked distribution the process embodied CO\textsubscript{2}-eq intensities fits onto. In column 3 of Table 1, the statistical parameters used with the distribution type to generate the random embodied CO\textsubscript{2}-eq intensities are shown.
The equivalent primary energy [GJ] used in each construction sub-sector was calculated by multiplying energy expenditure [e] [22–25], average energy tariffs [GJ/e] derived from the energy balance for Ireland [27] and primary energy factors [G/G] [28]. The energy intensity for each construction sub-sector is then derived in terms of the energy in GJ per Euro output of each sub-sector. Irish emission factors [gCO2-eq/G] [29] and global warming potentials (GWP) of the energy related GHG emissions are then multiplied by the energy intensities to obtain the direct sub-sector embodied CO2-eq intensities $i_{Gj}$ [gCO2-eq/e]. The GWP of the energy-related GHGs regulated under the Kyoto Protocol over a 100-year time-frame which are relevant to this study are: CO2-1; N2O-298; and CH4-25. To normalise all data used to the 2005 baseline year in the analysis, energy and construction price indices published by the Central Statistics Office [30] are applied to the average energy tariffs and construction sub-sector output respectively. 2005 was taken as the baseline year because it is the most recent year in which the national supply and use and input–output table has been published for Ireland.

Direct sub-sector CO2-eq intensities $i_{Gj}$ of construction activities are treated as stochastic variables. The distributions for the direct sub-sector CO2-eq intensities of the Irish construction sector are derived for each of the five sub-sectors. The distribution and statistical parameters are then used to generate input parameters in the Monte Carlo modeling. As an example, Fig. 2 shows the direct embodied CO2-eq intensity distributions of Sub-Sector 1–Ground Works. The probability density functions of the distributions obtained for the sub-sectors are presented in Table 3.

### Table 3: Construction sub-sector distributions and the probability density functions

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Probability density function</th>
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<tbody>
<tr>
<td>4-parameter generalized gamma</td>
<td>$f(x) = \frac{1}{\text{beta}(\alpha,\beta)} \exp\left(-\frac{x}{\theta}\right)^{\alpha}$</td>
</tr>
<tr>
<td>Log-logistic</td>
<td>$f(x) = \frac{x^{-\alpha}}{\alpha \theta^{-\alpha} \Gamma(\alpha)} \exp\left(\frac{\alpha - 1}{\theta} - \alpha x\right)$</td>
</tr>
<tr>
<td>Frechet</td>
<td>$f(x) = \frac{\alpha}{\theta^\alpha \Gamma(\alpha)} \exp\left(-\left(\frac{x}{\theta}\right)\right)^{\alpha - 1}$</td>
</tr>
<tr>
<td>Dagum</td>
<td>$f(x) = \frac{\lambda^{-\lambda \alpha \beta}}{\Gamma(\alpha \beta)} \exp(-\theta x - \lambda x^{\alpha})$</td>
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</table>

2.2. Input–output indirect embodied CO2-eq intensity

Input–output (I–O) indirect embodied CO2-eq intensity of the construction sector as well as costs associated with each construction sub-sector ($S_1, S_2, S_3, S_4, S_5$) in the bill of quantities and costs associated with building materials $C_P$ are treated as deterministic input variables. Some uncertainties present in I–O analysis are outlined by eiocla.net [31] and some of these are addressed in Section 2.2.1. The level of uncertainty in input–output data is however difficult to estimate because the national input–output tables are compiled from a wide range of sources such as national systems of accounts, national economic sources, industry sector reports and statistical data. Lenzen and Dey [32] for instance stated that errors in I–O data depends largely on the error in the respective source data and estimated it to be in the region of 20%. The cost associated with energy use in the construction sub-sector and process analysis associated costs (building material costs) are used as deterministic input variables because they are assumed to be constant and are derived from standard construction industry approved costs of buildings material and construction activities.

Indirect I–O emissions are emissions arising from energy use not directly related to on-site construction but upstream of on-site construction and are calculated using I–O analysis. These were estimated using data from the Irish national I–O tables [33] which are compiled using data from national accounts as well as other national economic sources to show economic transactions between all product sectors of the national economy. The input coefficients of the economy-wide I–O tables are used to derive indirect I–O emissions intensities in the construction sector. This methodology is widely used and described in literature (see inter alia Bullard et al. [34], Lenzen and Dey [32] and Strømman and Solli [35]). In summary, the approach involves using the 2005 Irish I–O tables [33], average energy tariffs [27] and primary energy factors [28] to determine total I–O and direct I–O energy intensities per unit monetary value of construction sector output. The indirect I–O energy intensity is calculated as the difference between the total I–O and direct I–O energy intensities and is then converted to indirect I–O emissions intensity using the Irish emissions factors [29]. The direct requirement coefficient matrix of the Irish I–O tables was used to evaluate the direct I–O energy intensity and the Leontief inverse matrix used to calculate the total domestic energy intensity [13,36].

2.2.1. Limitations and the treatment of errors in I–O analysis

I–O analysis is known to suffer from well-documented limitations such as assumptions of homogeneity and proportionality [37]. For example, the proportionality assumption presumes that the inputs to each sector are proportional to their outputs so that if the output of a sector increases or decreases, then the consumption of intermediaries and primary inputs of that sector will also increase or decrease proportionally. In reality, there is not always a direct proportionality between activity and energy use. However, economies of scale should act to reduce marginal energy consumption. Homogeneity assumption proposes that each sector produces a single output using identical inputs and processes; however, this is obviously not the case with each sector containing many different products and services. Heterogeneity therefore occurs within the sector because of the aggregation of different production processes and products. Furthermore, I–O analysis assumes the uniform conversion of economic data into physical quantities (energy and emissions in this case) within a sector. For example, economic data are converted to energy consumed using national average energy tariffs, although such tariffs will vary across different industries.

This paper attempts to address some of these limitations. Disaggregation coefficients are used to disaggregate the energy supply sectors thus mitigating errors associated with the assumptions of
homogeneity and uniform conversion [38]. Another limitation of I–O analysis is the aggregation of many different products into one sector in the national I–O tables [39]. This reduces applicability of I–O derived embodied CO2-eq intensities to a specific product or product sector. In Ireland, the national I–O table contains three aggregated energy supply sectors, namely:

i. Coal, Peat, Crude Oil and Metal Ore Extraction;
ii. Petroleum and Other Manufacturing Products; and
iii. Electricity and Gas

Some energy supply sectors are aggregated together either with non-energy supply sectors or other energy supply sectors. For example, the ‘Petroleum and Other Manufacturing’ sector is an aggregation of an energy supply sector ‘Petroleum’ and non-energy supply sector ‘Other Manufacturing’. Therefore, to address the aggregation problem, disaggregation coefficients are introduced to separate the energy supply sectors into individual energy sources to which emissions factors can be applied. An analysis of the disaggregation of the energy supply sectors in Ireland and its application to embodied emissions was carried out by Acquaye and Duffy [38].

The use of the disaggregation constants has a two-fold advantage. Firstly, non-energy supply sectors are eliminated from the analysis. Secondly, it enables individual primary energy factors and specific energy tariffs to be used instead of average values for two or more aggregated energy supply sectors (for example, for the aggregated electricity and gas sector).

A further development of I–O analysis and its application to the embodied CO2-eq analysis and I–O tables relates to the system boundary in the I–O analysis. Direct Requirement and Leontief Inverse I–O coefficients for Ireland were derived for domestic product flows only, omitting energy inputs into imported products and services. For example, EuroStat [40] states that in order to account for the whole life energy use of a product using I–O analysis, the energy used to produce imported inputs should also be included in such an analysis. As such a methodology set out in the EuroStat European System of Accounts I–O Manual [40] is applied to re-derive the Irish direct and Leontief coefficients which are used to calculate the I–O indirect CO2-eq emissions. The estimation of the addition of energy inputs into imported construction sector goods and services is important in an open economy such as Ireland’s [41] and provides greater information for decision making by designers and policy makers by considering total global impacts. Furthermore, given that approximately 56% of Irish imports are from the EU [42], an understanding of the sources of emissions are important from an EU policy perspective.

2.3. Case studies

The seven apartment buildings are all located in Ireland. They comprise concrete strip foundations and ground floor slabs with walk-work rising elements, timber floors and roof structures. External finishes included brickwork and render, double-glazed timber-framed windows and concrete roof tiles. Internal finishes included timber stud partitions, plasterwork and painting. Further details are presented in Table 4.

3. Results

3.1. Hybrid embodied CO2-eq intensity distributions

For each apartment building, 10,000 Monte Carlo simulations were undertaken and the results (hybrid CO2-eq intensities) were plotted on a scatter diagram. The scatter diagram in Fig. 3 illustrates the variations that occur in the stochastically derived hybrid CO2-eq intensities of Apartment 1. Scatter diagrams for Apartments 2–7 are presented in the Appendix B Appendix.

Fig. 4 is an illustration the ECO2-eq intensity probability distributions of each of the apartment buildings analysed in the study. Each distribution shows the ECO2-eq intensity probability variations relating to the scatter plots in Fig. 3 and the Appendix B Appendix.

Fig. 5 shows the scatter plot of the combined seven apartment buildings representing 70,000 Monte Carlo simulated results and assumed to be the average for the apartment building sector in Ireland. It shows the dispersion in embodied emissions intensities of the apartment buildings due to the variability in input parameters such as emission intensities of building materials.

The average hybrid ECO2-eq intensity distribution for apartment buildings in Ireland shown in Fig. 6 was obtained by combining the individual distributions of the seven apartment buildings and also represents the combined ECO2-eq intensity scatter plots in Fig. 5. The distribution can be characterised as a Wakeby distribution with five parameters: $\beta = 1.4 \times 10^5$, $\gamma = 1.3 \times 10^5$, $\delta = 0.77$ are shape parameters while $\xi = 0$ and $\alpha = 1.5 \times 10^5$ are location parameters.

A general quantile function for a Wakeby Distribution is given by Eq. (2) below:

$$x(F) = \xi + \frac{\alpha}{\beta}(1 - (1 - F)^{\beta}) - \frac{\gamma}{\delta}(1 - (1 - F)^{-\delta})$$

Table 4 Description of apartment buildings used in the case studies

<table>
<thead>
<tr>
<th>Apartments</th>
<th>Description of apartment buildings</th>
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<tbody>
<tr>
<td>Apartment 1</td>
<td>Concrete piled foundation, reinforced concrete frame with infill 215 mm block-work; 320 mm thick reinforced concrete slab with 400 mm x 600 mm reinforced concrete columns on 9 m x 9 m grids. External finishes included brickwork and render, double-glazed timber-framed windows, thermofluid insulation and concrete roof tiles. Internal finishes included timber stud partitions, plasterwork and painting.</td>
</tr>
<tr>
<td>Apartment 2</td>
<td>Reinforced concrete frame with minor structural steel to roof; 300 mm thick reinforced concrete slab with 400 mm x 400 mm reinforced concrete columns on 8 m x 8 m grids. Thermofoil insulation and external finishes include plaster work with gloss paint to wood work. Roof work consists of mastic asphalt roofing with rigid sheet covering and deck. Extensive mechanical installations made up of waste, water, gas, heating, HVAC, and lift installations.</td>
</tr>
<tr>
<td>Apartment 3</td>
<td>Reinforced concrete substructure, block work external walls 440 x 215 x 100, concrete work in concrete frame structure, woodwork and precast prestressed concrete work for stairs, structural steel work fabricated members, internal walls partitioned with softwood and thermofluid insulation.</td>
</tr>
<tr>
<td>Apartment 4</td>
<td>Reinforced concrete substructure with reinforced concrete in situ concrete frame, fabricated members steel work, concrete work stairs 1.2 m wide, block work internal walls 100 x 215 x 440, in-situ concrete floors and slabs exceeding 150 mm thick and reinforced 200 mm thick with span &gt;5.00 m and &lt;7.00 m. Thermofluid insulation and structural steel work roof 254 x 146 x 37 kg/m Universal Beam.</td>
</tr>
<tr>
<td>Apartment 5</td>
<td>Reinforced concrete substructure, brickwork and concrete work size 440 x 215 x 100, floor insulation laid to underside of floor, in situ concrete floor exceeding 150 mm thick. Concrete walls consist of reinforced in situ concrete with thickness not exceeding 0.20 m². Concrete screed floor 75 mm thick with fabric reinforcement.</td>
</tr>
<tr>
<td>Apartment 6</td>
<td>Reinforced concrete substructure, brickwork and concrete work size 440 x 215 x 100; brick and block work external walls, coping to parapet 560 x 150. Precast concrete lintels, 100 x 65 mm, Insulation board 100 mm thick, structural steel work 50 x 90 x 10 kg/m stainless steel. Carcassing roof with insulation.</td>
</tr>
<tr>
<td>Apartment 7</td>
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</table>
Hence, the quantile function describing the derived average distribution for apartment buildings in Ireland is given by Eq. (3) below:

\[ x(F) = 1071\left(1 - (1 - F)^{1.4 \times 10^2}\right) - 168\left(1 - (1 - F)^{-0.77}\right) \]  

(3)

The \( \text{CO}_2\)-eq intensity distribution in Fig. 6 was derived using 100 class intervals with a bin or class size of 570 \( \text{gCO}_2\)-eq/\( \text{€} \). The mean \( \text{CO}_2\)-eq intensity was found to be 1636 \( \text{gCO}_2\)-eq/\( \text{€} \) while the median was 1127 \( \text{gCO}_2\)-eq/\( \text{€} \). This can be interpreted to imply that an ‘average’ design of Irish apartment buildings built in 2005 will result in the emissions of 1636 \( \text{gCO}_2\)-eq/\( \text{€} \). This is based on the assumption that the building samples analysed are representative of the population of apartment buildings in Ireland. Based on a class size of 570 \( \text{gCO}_2\)-eq/\( \text{€} \) (representing 100 class intervals) used in the distribution, the likeliest embodied \( \text{CO}_2\)-eq intensity of an apartment building is 1325 \( \text{gCO}_2\)-eq/\( \text{€} \) with a probability of 69%.

Using the principle that the uncertainty of a measured result can be taken to represent the estimated standard deviation [43],
the uncertainty associated with the stochastic ECO2-eq intensity distribution can be evaluated. It is therefore estimated that the mean of the stochastic distribution of ECO2-eq intensity across the apartment building sector is 1636gCO2-eq/€ with an uncertainty of 73gCO2-eq/€. An embodied CO2-eq intensity of 73gCO2-eq/€ was estimated as the standard deviation of the Wakeby derived average distribution of apartment buildings in Ireland after 70,000 stochastic simulations. It can therefore be assumed that an embodied CO2-eq intensity calculated for an apartment building in Ireland would have an uncertainty of 73gCO2-eq/€. It is however recognized that the addition of stochastic analysis for I–O indirect emissions will change the level of uncertainty in the overall results.

This is because Lenzen and Dey (2000) reported that the estimated inherent error and variability in I–O data is in the region of 20%. The cumulative hybrid ECO2-eq intensity probability distribution of apartment buildings in Ireland is presented in Fig. 7. The median (50th percentile) and the 90th percentile are respectively 1127gCO2-eq/€ and 1723gCO2-eq/€. This can be interpreted to mean that apartment buildings with embodied CO2-eq intensity greater or equal to 1723gCO2-eq/€ are in the top 10% of apartment buildings with the highest embodied emissions impacts in Ireland.

4. Discussion

In Fig. 4, the hybrid ECO2-eq intensity distributions of the individual apartment buildings can be observed. The differences in the distribution of Apartment 2 relative to the others can be attributed to two factors: firstly, Apartment 2 contained much greater quantities of mechanical and electrical services, the process probability density function for which resulted in negative skewing of the distribution for the overall building; secondly, indirect I–O data displaced a greater proportion of non-services-related process data, thus excluding more positively skewed distributions from the result. The importance of using stochastic techniques in ECO2-eq intensity analysis is seen in the ability of the model to capture the variability in the embodied emissions in each building. For the combined hybrid ECO2-eq intensity distribution, the uncertainty measured as the standard deviation of the distribution is estimated to be 73gCO2-eq/€. The uncertainty measured across the apartment building sector can therefore be factored into any calculation to account for any variability.

Obtaining the combined probability distribution represents an important step forward if embodied emissions policy measures are to be formulated. This helps both policymakers to formulate a basis for providing embodied CO2-eq intensity information in different sectors, and building designers to make informed decisions on material selection based on their embodied CO2-eq intensities (see Venkatarama Reddy and Jagadish [44]).

The combined ECO2-eq intensity probability distributions yielded a Wakeby distribution. While this was derived from analysing seven apartment buildings because of limited data, the shape should remain the same because of representative variability (that is, similarity in construction methods, design, materials used, etc.) when it is updated with new information and data. To assess the basis of this assumption, a sensitivity analysis is carried out using the derived ECO2-eq intensity cumulative distribution in Fig. 7. The sensitivity analysis is undertaken based on the premise that despite using only seven buildings as case studies, statistical parameters would not significantly change if large numbers of buildings were sampled. Hence a comparison is made between the cumulative distributions derived from the seven buildings and those derived from a much more limited number of buildings (5 and 6 apartment buildings represented by Apartments 1–5 and Apartments 1–6 respectively). As can be observed in Fig. 8, there are marginal differences between the cumulative distribution for the 5 apartment buildings (median: 1.06%; 90th percentile: 0.08%; and
mean: 5.04%) and that for 6 apartment buildings (median: 0.45%; 90th percentile: 0.08%; and mean: 2.5%) when compared to the average of the apartment building sector (7 buildings used in this study). While the reasoning behind the sensitivity analysis of CO$_2$-eq intensity distribution based on the number of cases analysed is valid, for it to be statistically rigorous, a much larger sample size is required, especially if the distributions are to be used to inform policy making.

5. Conclusions

This paper proposes a stochastic approach to estimating embodied emissions and, by way of example, applies it to an Irish case study of seven apartment buildings. Greater methodological and informational benefits are derived from the stochastic hybrid CO$_2$-eq intensity analysis of buildings compared to deterministic analysis. The stochastic CO$_2$-eq intensity employed integrates the accuracy of process analysis and the system boundary completeness of I-O analysis while providing a solution to the variability that exists in the CO$_2$-eq intensity data sets. Stochastic analysis also helps to establish the relationship between the CO$_2$-eq intensity of apartment buildings and the likelihood of obtaining a particular CO$_2$-eq intensity value. This can provide useful information if embodied CO$_2$-eq standards and regulatory measures are to be formulated. A Weibull distribution with known parameters and uncertainty was derived for the embodied CO$_2$-eq intensities of apartment buildings in Ireland. Such a stochastic distribution with known parameters provides more useful information to building designers and policy makers. The stochastic embodied emissions methodology employed in this study is applicable to any type of structure, sectors other than construction as well as to other countries.

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Appendix A. Supplementary data


References

[27] Sustainable Energy Authority of Ireland, Energy Balance by Energy Supply and Consumption, Year and Fuel Type, Dublin, Ireland, 2009.


