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Closed loop time domain gradient methods for parameter and time delay estimation

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Abstract

This paper discusses the estimation of process parameters and time delay, in a Smith predictor structure, using gradient algorithms in the time domain. A number of estimation algorithms are outlined and applied in simulation to the estimation of the parameters of an appropriate process model. An analytical exploration of the technique is also provided.

Keywords: Estimation, time delay, Smith predictor

1. Introduction

Gradient methods of parameter estimation are based on updating the process model parameter vector (which includes the time delay) by a vector that depends on information about the cost function to be minimised; the cost function is normally a function that depends on the square of the error between the process and model parameters. A number of such gradient algorithms have been defined (such as the Newton-Raphson and Gauss-Newton algorithms); Ljung [1] outlines these algorithms in detail. Applications of these algorithms to estimate the process model parameters in both open loop and closed loop environments have appeared in the literature. Open-loop applications, in both the time domain and frequency domain, have previously been dealt with by O'Dwyer and Ringwood [2], [3], amongst others; this paper concentrates on the application of the algorithms to estimate the model parameters in a Smith predictor structure. Marshall [4] and Bahill [5] reduced the mismatch between the process time delay and the model time delay, in such a structure, using a Gauss-Newton gradient algorithm; just one of the modifications of this approach subsequently proposed is that defined by Romagnoli *et al*. [6], who propose the use of a Newton-Raphson algorithm for the application. This paper takes a more unified approach to the problem by considering the estimation of the process parameters in a generalised model structure.

2. Development of the gradient algorithms

Marshall [4] and Bahill [5] have developed a parameter identification algorithm to estimate the corresponding model parameters and time delay of a first order lag plus time delay (FOLPD) process, in a Smith predictor structure. In the development of the algorithm, the authors assume that the plant output is linearly related to any changes in the plant parameters i.e.

$$y_p(t,\alpha+\Delta\alpha) = y_p(t,\alpha) + e(t,\alpha+\Delta\alpha) \tag{1}$$

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with $y_p(t,\alpha+\Delta\alpha) =$ process output after a change $\Delta\alpha$ in parameter α, $y_p(t,\alpha) =$ starting value of process output = model output $y_m(t)$. $e(t,\alpha+\Delta\alpha)$ may be estimated in a number of different ways; two such methods are as follows:

$$e(t,\alpha+\Delta\alpha) \approx \Delta\alpha \frac{\partial y_p}{\partial \alpha} \tag{2}$$

$$e(t,\alpha+\Delta\alpha) \approx 0.5\Delta\alpha\left(\frac{\partial y_p}{\partial \alpha} + \frac{\partial y_m}{\partial \alpha}\right) \tag{3}$$

Marshall [4] and Bahill [5] use equation (2) in their development of the identification method for updating the gain, time constant and time delay of a FOLPD process model. The authors develop a Gauss-Newton gradient method based on approximating the Hessian matrix as a function of $\partial y_p / \partial \alpha$. This development is provided in detail by O'Dwyer [7].

Alternative gradient algorithms may also be defined by using equation (3) in the development, for example (as it is a straightforward matter to analytically calculate $\partial y_m / \partial \alpha$). In addition, the Hessian matrix may be approximated as a function of appropriate second partial derivative as well as first partial derivative terms, giving a Newton-Raphson gradient algorithm. Five alternative algorithms are so defined by the authors. Following the example of Marshall [4], all six algorithms may be represented in block diagram form (O'Dwyer [7]).

3. Parameter estimation - simulation results

The algorithms defined have been simulated, for updating all of the parameters separately, using the SIMULINK package. It was decided to simulate the algorithms for seven process/model combinations, in a Smith predictor structure; the processes considered include high order, underdamped and non-minimum phase processes, which were modelled by equivalently ordered models or mismatched FOLPD and second order system plus time delay (SOSPD) models, as appropriate. The PI and PID primary controllers used (in the Smith predictor) are specified to be robust to the possible process/model parameter mismatches considered.

In each simulation, the excitation signal at the servo input is of band limited white noise form; such a signal was determined to be sufficiently exciting so that appropriate parameter updating is achieved. In all cases, the individual model parameters are updated at discrete intervals using a dedicated s-function in SIMULINK; the gradient algorithm implementations, which are in continuous time are also effectively set up in continuous time in the SIMULINK environment (by choosing a small step size for the simulations).

3.1 Time delay estimation

Representative simulation results are provided in Cases 1 to 5.

Case 1: Model $G_m e^{-s\tau_m} = 2e^{-1.4s}/(1+0.7s)$, primary controller $G_c = 1.75(1+1/0.7s)$. In the figures, process time delay τ_p = 1.2 seconds and non-delay process $G_p = G_m$. The Gauss-Newton (1) algorithm refers to the algorithm formulated by Marshall [4] and Bahill [5]; the other algorithms have been formulated by the authors.

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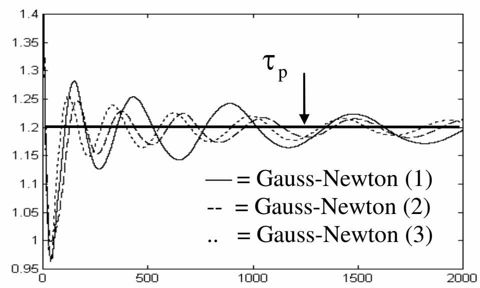
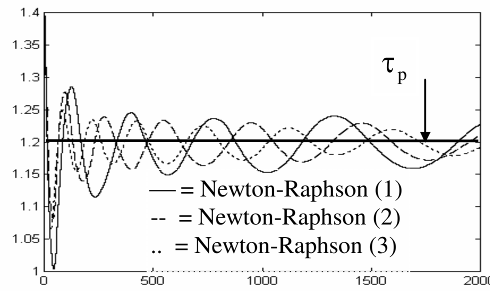
Figure 1: Model time delay updating



Time (seconds)

Figure 2: Model time delay updating

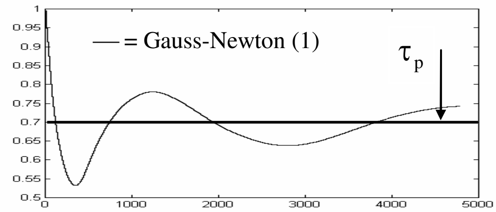


Time (seconds)

The figures show that the algorithms facilitate a reduction in mismatch between the process time delay and the model time delay.

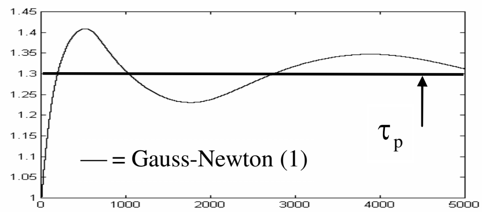
Case 2: $G_m e^{-s\tau_m} = 2e^{-s}/(1+4.5+4.5s^2), G_c = 1.17(1+1/4.07s+2.73s/(1+0.5s))$. $G_p = G_m$.

Figure 3: Model time delay updating



Time (seconds)

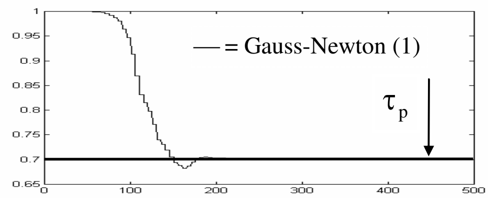
Figure 4: Model time delay updating



Time (seconds)

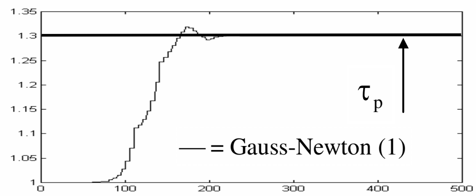
Case 3: $G_m e^{-s\tau_m} = 2e^{-s}/(1+18s+137s^2+567s^3+1403s^4+2103s^5+1846s^6+856s^7+158s^8)$, $G_c = 2.14(1+1/9.75s+3.31s/(1+0.61s))$. $G_p = G_m$.

Figure 5: Model time delay updating



Time (seconds)

Figure 6: Model time delay updating

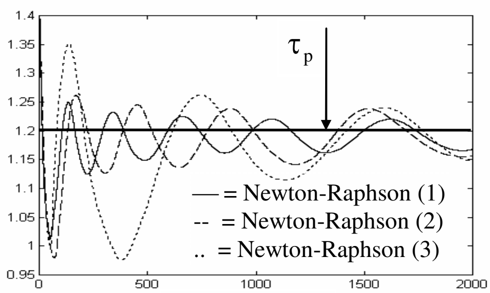


Time (seconds)

A similar comment to that made in Case 1 applies to the simulation results above.

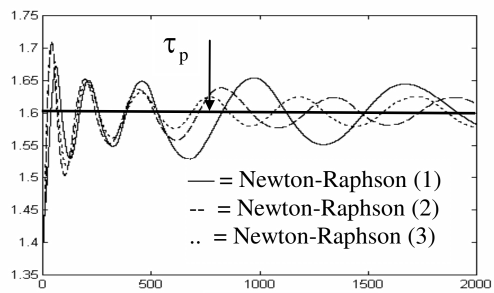
Case 4: $G_m e^{-s\tau_m} = 2e^{-1.4s}/(1+0.7s)$, $G_c = 1.75(1+1/0.7s)$. In Figure 7, τ_p = 1.2 seconds and $G_p = 1.6/(1+0.5s)$; in Figure 8, τ_p = 1.6 seconds and $G_p = 2.4/(1+0.9s)$.

Figure 7: Model time delay updating



Time (seconds)

Figure 8: Model time delay updating



Time (seconds)

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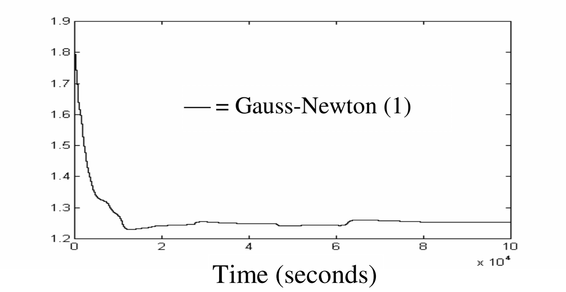
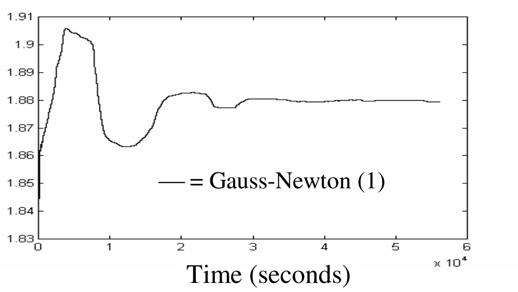
Figures 7 and 8 show that these algorithms facilitate a reduction in mismatch between the process and the model. These are significant results, as the non-delay process and model parameters are different.

<u>Case 5:</u> Model $G_m e^{-s\tau_m} = 1.96e^{-1.84s}/(1+6.7s)$. The process corresponding to this model is $(2+4.5s)e^{-s}/(1+8.5s+22.5s^2+18s^3)$; the model has been obtained from the frequency domain identification technique outlined by O'Dwyer and Ringwood [3]. $G_c = 6.84(1+1/6.7s)$. In Figure 9, $G_p = (1.2+3.1s)/(1+5.9s+15.7s^2+12.6s^3)$ and τ_p = 0.7 seconds; in Figure 10, $G_p = (2.8+6.1s)/(1+11s+29.3s^2+23.4s^3)$ and τ_p = 1.3 seconds.

<u>Figure 9</u>: Model time delay updating



<u>Figure 10</u>: Model time delay updating



Good fitting between the processes and their models is seen if the phase plots of the processes and models are obtained at higher frequencies. This implies that the model time delay estimates are appropriate, if it is desired to reduce the mismatch between the process and the model, as the time delay will be the dominant influence on the phase plot at higher frequencies. However, it is normally desirable when using a Smith predictor to reduce the mismatch between the process and model time delays; the matching of the process and the model at higher frequencies means that the difference between the process and the model, fed back in the Smith predictor, is small at these frequencies. This is not desirable, bearing in mind the large mismatch between the process and model time delays. Thus, the gradient algorithms may not be suited for updating the time delay in a Smith predictor structure, if the process and model orders are different.

Overall, the full panorama of simulation results (O'Dwyer [8]) show that when the order of the process equals that of the model, the mismatch between the model time delay and the process time delay is significantly reduced, irrespective of the match between the process and model parameters. When the order of the model differs from that of the process, then the model delay is updated to a final value. The performance of the six algorithms is more difficult to compare, though it is obvious that there is little to be gained (comparing Figures 1 and 2) in using a Newton-Raphson algorithm instead of a Gauss-Newton algorithm. On balance, taking the full panorama of simulation results obtained (O'Dwyer [8]), the Gauss-Newton (1) time delay updating algorithm is the most appropriate algorithm to use, with the Gauss-Newton (2) algorithm being the least appropriate one to use. It is interesting that it takes a long time for the model time delay to converge to the process time delay in most cases, even when the order of the process and model are the same. The oscillatory convergence pattern is a factor in this disappointingly slow convergence rate; an alteration in the learning rate of the gradient algorithms would improve this situation.

3.2 Estimation of the non-delay parameters

The six algorithms for separately updating the gain and the time constant have been simulated individually, for the process-model combination in which both the process and model are in FOLPD form. Representative simulation results that show the updating of the model gain and model time constant are shown in Figures 11 and 12; these results, and other supplementary

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simulation results provided by O'Dwyer [8], show that convergence of the model gain to the process gain occurs, for all of the gradient algorithms, when the non-gain model parameters are equal to the corresponding process parameters. However, if the non-gain model parameters differ from the corresponding process parameters, the model gain does not converge to the process gain (unlike the behaviour of the model time delay in corresponding circumstances). Similarly, convergence of the model time constant to the process time constant occurs when the non-time constant model parameters are equal to the corresponding process parameters for all of the algorithms; however, if the non-time constant model parameters differ from the corresponding process parameters, the model time constant does not converge to the process time constant. The simulation conditions for gain updating are $G_m e^{-s\tau_m} = 2e^{-1.4s}/(1+0.7s)$, $G_c = 1.75(1+1/0.7s)$, $\tau_p = \tau_m$, process time constant T_p = model time constant T_m and process gain K_p = 1.6; the simulation conditions for time constant updating are as above with $\tau_p = \tau_m$, K_p = model gain K_m and $T_p = 0.5$ seconds.

Figure 11: Model gain updating

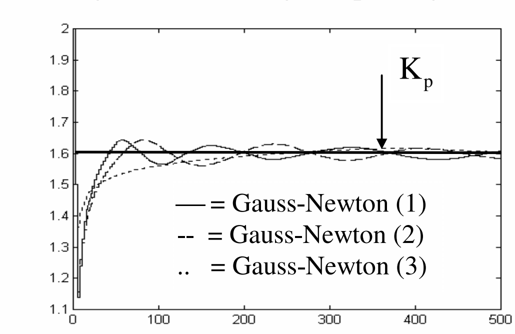
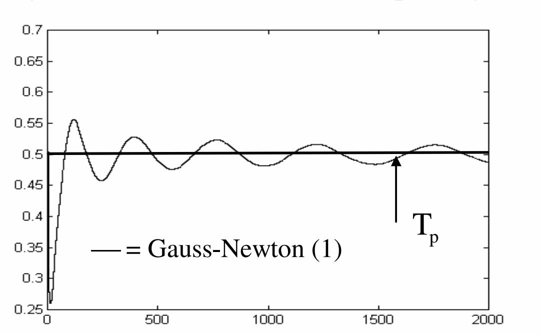


Figure 12: Model time constant updating



4. Analytical exploration of the algorithms used

An analytical exploration of the gradient techniques was performed in discrete time, for a number of process and model structures. These calculations are done in discrete time as integer values of the process time delay appear as appropriate power terms on the numerator transfer function of the process and that a standard procedure has been defined to calculate the mean squared error (MSE) surface, by Widrow and Stearns [9], in the domain. The closed loop gradient algorithms are, of course, defined in continuous time; the application of the analysis performed in the discrete time domain will need to take this into account.

It is required to prove that the MSE between the process and the model output is unimodal with respect to the relevant process parameters, and is minimised when the appropriate model parameter equals the equivalent process parameter.

4.1 Non-delay model parameter estimation

Theorem 1: For a first order discrete stable system, the MSE performance surface is minimised when the model gain equals the process gain and the model time constant equals the process time constant if (a) the model time delay index equals the process time delay index (b) measurement noise is assumed uncorrelated with the process input and output and (c) the input to the process and the model is assumed to be a white noise input. The model time delay index is the model time delay divided by the sample time.

Proof: The process difference equation is

$$y(n) = e^{-T_s/T_p} y(n-1) + K_p(1-e^{-T_s/T_p})u(n-g_p-1) + w(n) \tag{4}$$

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with $\tau_p = g_p T_s$, T_s = sample period, $g_p =$ process time delay index and w(n) = measurement noise. The model difference equation is (assuming the previous process output is used in its calculation)

$$y_m(n) = e^{-T_s/T_m} y(n-1) + K_m(1-e^{-T_s/T_m})u(n-g_m-1) \tag{5}$$

with g_m = model time delay index. The procedure defined by Widrow and Stearns [9] may be used to calculate the MSE performance surface as follows:

$$E[e^2(n)] = r_{yy}(0) + r_{ww}(0) + \frac{1}{2\pi j}\oint [G_m(z^{-1})\Phi_{uu}(z)G_m(z) - 2\Phi_{yu}(z)G_m(z)]\frac{dz}{z} \tag{6}$$

with $e(n) = y(n) - y_m(n)$, $\Phi_{uu}(z) = \sum_{n=-\infty}^{\infty} r_{uu}(n)z^{-n}$, $\Phi_{yu}(z) = \sum_{n=-\infty}^{\infty} r_{yu}(n)z^{-n}$, $r_{yy}(n)$, $r_{uu}(n)$ and $r_{ww}(n)$ are the autocorrelation functions of $y(n)$, $u(n)$ and $w(n)$ respectively; $r_{yu}(n)$ is the cross-correlation of $y(n)$ and u(n). The model $G_m(z)$ corresponds to the output $y_m(n)$.

Using the residue theorem to calculate the closed curve integral, the MSE function is calculated (from equation (6)) to be (O'Dwyer [7]):

$$E[e^2(n)] = \frac{{K_p}^2(1-e^{-T_s/T_p})^2}{(1-e^{-2T_s/T_p})} + \frac{{K_m}^2(1-e^{-T_s/T_m})^2}{(1-e^{-2T_s/T_m})} - \frac{2K_pK_m(1-e^{-T_s/T_p})(1-e^{-T_s/T_m})}{(1-e^{-T_s/T_p}e^{-T_s/T_m})}e^{-T_s(g_p-g_m)/T_m} + r_{ww}(0) \tag{7}$$

The MSE function is minimised when $\partial E[e^2(n)]/\partial K_m$ and $\partial E[e^2(n)]/\partial(1/T_m)$ equal zero simultaneously. The required calculations, determined by partially differentiating equation (7) (O'Dwyer [7]), show that $T_m = T_p$ and $K_m = K_p$ (assuming $g_p = g_m$). □

A corollary to this theorem is that if the process time delay index, g_p, is not equal to the model time delay index, g_m, then the MSE function is not minimised when $K_m = K_p$ and $T_m = T_p$. A further corollary to this theorem is that the MSE function is not minimised when $K_m = K_p$ unless $g_m = g_p$ and $T_m = T_p$, and the MSE function is not minimised when $T_m = T_p$ unless $g_m = g_p$ and $K_m = K_p$. In a closed loop environment, the excitation signal to the process is not of white noise form; nevertheless, it is interesting that the simulation results in Section 3.2 show that the conclusions indicated do apply to the closed loop identification case, provided the process input is sufficiently exciting. This is a less conservative criterion than that given in the theorem.

4.2 First order model time delay index estimation - non-delay parameters known

Elnagger *et al.* [10] prove that for a first order discrete stable system of known gain and time constant, the MSE performance surface versus time delay is minimised when the model time delay index equals the process time delay index, provided the measurement noise is uncorrelated with the open loop process input. The resolution on the process time delay is assumed to be equal to one sample period. The authors also show that the MSE surface is unimodal with respect to the time delay, and that this unimodality exists for any change in the process input (such a signal is consistent with the types of signals present at the process input in closed loop applications). These conclusions conform with the simulation results in Figures 1 and 2.

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4.3 First order model time delay index estimation - non-delay parameters unknown

Elnagger *et al*. [11] show that for a first order discrete stable system of unknown gain and time constant, the MSE performance surface versus time delay is minimised when the model time delay index equals the process time delay index. The input signal to the process is assumed to be white, though the authors state that this is a sufficient condition, rather than a necessary condition. However, the authors do not explicitly show that the MSE performance surface is unimodal with respect to the time delay, which is a requirement for the use of a gradient algorithm for time delay estimation. It is therefore appropriate to prove the following theorem.

Theorem 2: For a first order discrete stable system of unknown parameters, the unimodal MSE performance surface versus time delay is minimised when the model time delay index equals the process time delay index if (a) the measurement noise is uncorrelated with the process input (b) the input to the process is assumed to be a white noise signal and (c) the conditions provided in the theorem are observed on the model parameters.
Proof: The MSE performance surface, $E[e^2(n)]$, may be calculated to be (with e(n) = process output minus the model output)

$$\left(e^{-T_s/T_p} - e^{-T_s/T_m}\right)^2 r_{yy}(0) + \left(K_p{}^2(1-e^{-T_s/T_p})^2 + K_m{}^2(1-e^{-T_s/T_m})^2\right) r_{uu}(0) + r_{ww}(0) + 2\left(e^{-T_s/T_p} - e^{-T_s/T_m}\right) r_{uy}(g_p)$$
$$+2\left(e^{-T_s/T_p} - e^{-T_s/T_m}\right) r_{uy}(1) - 2K_m(1-e^{-T_s/T_m})\left[K_p(1-e^{-T_s/T_p}) r_{uu}(g_m - g_p) + (e^{-T_s/T_p} - e^{-T_s/T_m}) r_{uy}(g_m)\right] \tag{8}$$

assuming that the measurement noise is uncorrelated with the process input. The proof that the MSE function is unimodal with respect to the model delay, for $g_p > g_m$ and $g_p < g_m$, may be done by induction (O'Dwyer [7]); the full proof, together with the conditions required on the model parameter values, are omitted due to pressure of space. □

This theorem provides an analytical structure that helps to explain the simulation results given in Figures 7 and 8 (Section 3.1); it is interesting that these simulation results show that convergence of the model time delay to the process time delay is possible, when $K_m \neq K_p$ and $T_m \neq T_p$, even when the excitation signal to the process is not in white noise form, or when the conditions on the parameter values in the theorem are violated. This shows the conservative nature of the conclusions of the theorem.

4.4 Time delay index estimation for a general model

An analytical framework on the convergence of the model time delay index, in a general model structure, may also be put in place for the case where the non-delay process and model parameters are identical. The conditions for convergence were first calculated for a process and model in SOSPD form, as a prelude to calculating the convergence conditions for a process and model of arbitrary order; the conditions for convergence are wider when the process and model are in SOSPD form, compared to when the process and model are of arbitrary order. The relevant theorems are quoted below and are proven by O'Dwyer [7] in a manner similar to the proof of Theorem 2 above.
Theorem 3: For a second order discrete stable system of known non-delay parameters, the unimodal MSE performance surface versus time delay is minimised when the model time delay index equals the process time delay index if the measurement noise is uncorrelated with the process input.
Theorem 4: For a general order discrete stable system of known non-delay parameters, the unimodal MSE performance surface versus time delay is minimised when the model time delay

index equals the process time delay index if (a) the measurement noise is uncorrelated with the process input and (b) the conditions provided in the theorem are observed on the model parameters.

The conclusions reached in Theorem 3 conform with the simulation results given in Figures 3 and 4 and the conclusions reached in Theorem 4 conform with the simulation results given in Figures 5 and 6.

Overall, the conclusions of the theorems conform with the appropriate simulation results quoted in Section 3. Indeed, the results of the theorems are more conservative than many of the results achieved in simulation.

5. Conclusions

The work involves the estimation of model parameters (including time delay) using gradient methods, in a Smith predictor structure. A reduction in the mismatch between the (unknown) process and the model (particularly the time delay mismatch) facilitates an improvement in the performance of the Smith predictor. The original contributions of this work are as follows: (a) the development of five gradient algorithms for the estimation of the model parameters in a closed loop environment, following on from the work done in this area by Marshall [4] and Bahill [5], amongst others (b) the estimation of the appropriate parameters of a model in the Smith predictor structure, for a variety of processes using the gradient algorithms and (c) an analytical exploration of the technique, which demonstrates the appropriateness of using gradient algorithms for the identification problem, in certain applications.

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