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Probabilistic Study of Lifetime Load Effect Distribution of Bridges

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PROBABILISTIC STUDY OF LIFETIME LOAD EFFECT DISTRIBUTION OF BRIDGES

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ABSTRACT

Assessment of highway bridge safety requires a prediction of the probability of occurrence of extreme load effects during the remaining life of the structure. While the assessment of the strength of an existing bridge is relatively well understood, the traffic loading it is subject to, has received less attention in the literature. The recorded traffic data are often limited to a number of days or weeks due to the cost of data collection. Studies in the literature have used many different methods to predict the lifetime maximum bridge load effect using a small amount of data, including fitting block maximum results to a Weibull distribution and raising maximum daily or maximum weekly distributions to an appropriate power.
Two examples are used in this study to show the importance of the quantity of data in predicting the lifetime maximum distribution. In the first, a simple example is used for which the exact theoretical probabilities are available. Hence, the errors in estimations can be assessed directly. In the second, 'long-run' simulations are used to generate a very large database of load effects from which very accurate estimates can be deduced of lifetime maximum effects. Results are presented for bidirectional traffic, with one lane in each direction, based on Weigh-in-Motion data from the Netherlands.

NOMENCLATURE

- GVW Gross vehicle Weight
- WIM Weigh in Motion
- GEV Generalised Extreme Value
- GPD Generalized Pareto Distribution
- G Generalised Extreme Value Probability Density Function

1. INTRODUCTION

Highway bridges deteriorate over time and a programme of inspection, maintenance and repair represents a substantial portion of the total lifetime cost of the structure. The costs of maintenance, including disruption to traffic and the cost of resulting delays have received much attention in the literature over the past number of years. Brady [1] estimates the EU expenditure on the repair, rehabilitation and maintenance of bridge structures to be €4-6 bn annually. Therefore the assessment of existing highway infrastructure is known as an area with high potential for savings.

All key design/assessment parameters such as accurate traffic load and resistance models must be incorporated in an analysis to allow a full investigation of the consequence of different design specifications or maintenance strategies. Despite a considerable quantity of work being done on methods of evaluating the capacity of structures, predictions of safety are still not accurate. This issue is particularly important when it comes to traffic loading estimation which is more variable than bridge load carrying capacity.
In traditional bridge assessment, the extreme load effect is calculated using deterministic loading models provided by standard/code specifications [2]. Improved statistical analysis capabilities for highway bridge traffic loading may result in more accurate prediction of the maximum loading to which a bridge may be subject in its lifetime. As part of the background studies for the Eurocode for bridge loading based on measured traffic,

Bruls et al. [3] and Flint and Jacob [4] consider several methods of extrapolation of the histogram of load effect. These include half normal curve fitted to the end of the histogram, a Gumbel distribution fitted to the tail of the histogram, Monte Carlo simulation of artificial traffic and Gumbel extrapolation and Rice formula for a stationary Gaussian process. The Eurocode for traffic loading on bridges requires the bridge to be designed for the characteristic load effect with a 5 % probability of exceedance in a 50-year bridge lifetime. Based upon a fitted distribution, the extrapolation can be made to this return period, resulting in a single value. However, the inherent variable nature of traffic loading means that this process can yield a different characteristic value for different samples. There is a need to acknowledge both this variability and the variability from the modelling itself. While codes traditionally have used characteristic value, lifetime maximum distributions are of increasing interest as they can be used in reliability analysis.

Various methods exist in the statistical literature to estimate such a distribution, the delta method and boot strapping being two. Caprani and OBrien [5] introduce predictive likelihood as a further method to find the lifetime extreme load effect distribution. European and North American codes are based on relatively small amounts of collected data. The U.S. and Canadian codes are based on data collected in Ontario in 1975 for 9250 trucks [6, 7]. The Eurocode [8] was initially based on a number of weeks of data from Auxerre, France, in the 1980s [9], and was confirmed using data from number of French sites in 1997 [10]. Even with the relatively large amount of data gathered in recent years, the extrapolation to return periods of 75 or 1000 years is still considerable. Using small amounts of measured traffic to calculate a distribution of load effect and then extrapolating from this to lifetime maxima which implicitly incorporate the patterns in the traffic, involves considerable uncertainty due to extrapolation process. Gindy and Nassif [11] report up to 33% variation in results from extrapolation and up to 20% for the estimation of characteristic load for the Eurocode.

Bailey and Bez [12], O'Connor and OBrien [2] and OBrien and Enright [13] use an alternative approach which generates synthetic traffic data based upon the measured traffic characteristics such as vehicle weights and inter-vehicle gaps; however, even with this form of simulation, lifetime maximum load effects usually require some form of statistical extrapolation. To avoid the problem of extrapolating from short simulation runs, it is necessary to run the simulation for a sufficiently long time that an interpolation is possible. These long-run simulations provide samples of the types and combinations of vehicle expected to feature in extreme bridge loading events [14].

This study presents the results from a Monte Carlo simulation which has been analysed to calculate lifetime maximum distributions using the well known Generalised Extreme Value distribution (GEV). This study focuses on short to medium span bridges, up to 45 m long. In longer spans, static loading produced by congested traffic has generally been considered to be more critical whereas in short spans the combined static and dynamic load effects produced by free-flowing traffic are taken to govern [4]. (In longer spans, modelling is complicated by a lack of information on the minimum gaps between vehicles in congested conditions).

Two extreme value examples are considered here as benchmark tests. The first example is a simple GEV distributed variable, for which the exact theoretical solution for maximum lifetime effect is known. The second example is based on a carefully calibrated traffic load simulation model. The simulation is run for 5000 years so that, while the exact solution is unknown, there is a high degree of confidence in the lifetime maximum results.

2. WIM DATA AND MONTE CARLO SIMULATION

Site specific traffic data can be generated using Weigh-in-Motion (WIM) data to calibrate the model. WIM is the process of weighing trucks travelling at full highway speed . The WIM traffic records used in this study were collected at one site - at Woerden in the Netherlands, as detailed in Table 1. The data were filtered in order to remove unreliable values and photographic evidence from the site was used in this regard. Vehicle records with speed less than 40 km/h or greater than 120 km/h were rejected. Other filters included zero or one recorded number of axles, different wheelbase from the sum of axle spacing, different GVW from sum of axle weights, axle weight less than 35 t, axle spacing less than 1 m or greater than 15 m, wheelbase less than 1 m, Maximum axle load greater than 15 t or more than 85% of GVW and inconsistent number of axles, axle spacings and axle loads were the other criteria used to support this data cleaning.

Table 1. WIM data

Country	Netherlands (NL)
site	Woerden
Road Number	A12 (E25/E30)
Number of measured lanes	2
Number of directions	1
Total trucks (cleaned)	646 548
Time period	Feb to Jun 2005
Number of weekdays with full traffic record	77
Average Daily Truck Traffic (ADTT) in one direction	7 102
Time stamp precision (s)	0.01
Maximum number of axles	13
Average GVW (t)	21.8
Number over 60 t	1 716
Number over 70 t	892
Number over 80 t	609
Number over 100t	238
Maximum GVW (t)	165.6

These data are measured using piezo electric sensors embedded in the pavement of the lane so no inaccuracies were introduced by side by side combinations of vehicle. These measurements are assumed to be typical highway traffic in the region. It should be noted that the legal GVW limit for standard trucks is 50 t in the Netherlands.

The parameters such as GVW, vehicle class, axle spacings, etc. for each individual truck and for the arrangement of trucks at each lane, are generated using different statistical distributions depending on parameters derived from the traffic measured at the site. For instance, for GVW and vehicle class, a semi-parametric approach is adopted. The maximum axle spacing for each individual truck is generated using an empirical distribution (bootstrapping directly form histogram). Individual axle weight is simulated using a bimodal Normal distribution fitted to the observed data for each vehicle class. A fitted Weibull distribution to the daily truck traffic volume in each lane at the site is used to reproduce traffic flow. The distribution of inter vehicle gaps within each lane is modelled using the quadratic curves for different flow rates for gaps up to 4 seconds and a negative exponential distribution is used for larger gaps [15].

It should be noted that no allowance has been made for growth in the volumes of freight during the lifetime of the bridge. This means that the traffic is assumed to be statistically stationary. Furthermore a year's traffic is assumed to consist of 250 weekdays, ignoring the much lighter volumes of weekend and holiday traffic.

The approach used for vehicle modelling is described in more detail by Enright [16]. The optimised simulation process is achieved through a program written in C++, utilising parallel processing and considering only significant loading events (Importance Sampling). The simulation process has been optimised by ignoring individual trucks less than some chosen span-dependent threshold (for example 40 t on a 15 m bridge) which greatly improves the computational efficiency. The very long runs reduce the variability of results and largely avoids issues about the selection of suitable statistical methods for the extrapolation process.

3. METHODOLOGY

Characterizing the extremal behaviour of the past history of a process in order to design against extreme excursions of future values of the process is the most practical application of an extreme value analysis. The methods of statistical inference used in literature to predict extreme traffic load effects are quite diverse. According to traffic load effects calculated directly from WIM data, it is found that the majority of the peaks are relatively light and are due to cars. Excluding consideration of these sorts of data results in a significant improvement in computational efficiency. The concept of considering only values above an appropriate threshold level or considering block maximum have emerged to address this concern. The block maximum approach has the advantage of time referencing the data which is necessary to calculate lifetime maximum probabilities of exceedance.

Fisher and Tippett [17] recognized that the maximum of *N* sets of observations of *n* values of *X*, must also be the maximum of *n* values of *X*. The limiting form of the distribution of the maximum from a parent distribution is:

$$[F_x(y)]^n = F_x(a_n + b_n y) \qquad \text{Equation 1}$$

Fisher and Tippett gave three solutions to this equation, based on the values for a_n and b_n. Gnedenko [18] established the strict mathematical conditions under which the Type I, II and III distributions form the limiting distributions for various forms of parent distribution. The three forms of limiting distribution (Types I, II and III), are the Gumbel, Frechet and Weibull distributions [19]. Jenkinson [20] and von Mises [21] independently solved Equation 1 for a single form: the Generalized Extreme Value distribution (GEV), given by

$$G(y) = exp\left\{-\left[1+\xi\left(\frac{y-\mu}{\sigma}\right)\right]_+^{-\frac{1}{\xi}}\right\} \qquad \text{Equation 2}$$

where $[x]_+ = max(x,0)$ $[\mathrm{x}]_+ = \max(\mathrm{x},0)$ and where the parameters satisfy $-\infty < \mu < \infty, \sigma > 0$ and $-\infty < \xi < \infty$. The GEV provides a model for the distribution of block maxima. Its application consists of separating the data into blocks of equal length/time, and fitting the GEV to the set of block maxima.

According to the stability postulate, a distribution of maximum values from a parent distribution that is of GEV form is itself a GEV distribution, given by:

$$F_Y(x) = [F_X(x)]^n \qquad \text{Equation 3}$$

The stability postulate result is a linear transform of the variable x:

$$G^n(x;\theta) = G(a_n + b_n x; \theta^*) \qquad \text{Equation 4}$$

where a_n and b_n are reliant on *n* and θ and θ^* are distribution parameter vectors. Once θ^* is found, the form of the lifetime maximum distribution is known. In linear transformation the shape parameter does not alter. Location and scale parameters are as follows:

$$\sigma_n = \frac{\sigma}{n^{-\xi}} \qquad \text{Equation 5}$$

$$\mu_n = \mu + \frac{\sigma}{\xi}\left(\frac{1}{\mathrm{n}^{-\xi}} - 1\right) \qquad \text{Equation 6}$$

4. BENCHMARK TESTS

4.1 THEORETICAL PROBLEM

A GEV-distributed random variable is first considered as a load effect event.

$$z \sim GEV\ (40{,}5{,}0.05) \qquad \text{Equation 7}$$

2500 days (i.e., 10 years) are taken, with an assumed 3000 events per day, to infer a range of distributions, including daily maximum, annual maximum and 50-year lifetime maximum.
Using the stability postulate, the parameter vector for daily maxima, annual maxima and 50-year lifetime maxima are calculated. Using the parameters for annual maxima, 5000 annual maximum events are generated to use as another source of information in order to predict the 50-year lifetime maximum distribution. .

It should be noted that, in all cases, the days are considered to be working days and a year is taken to consist of 250 such days. The exact solutions to these problems are readily calculated. For example, the daily maximum is the maximum of 3000 events per day:

$$CDF(x) = F^{3000}(x, 40{,}5{,}0.05) \qquad \text{Equation 8}$$

where $F(x, 40{,}5{,}0.05)$ is cumulative density function for a GEV distribution with shape of 0.05, scale of 5 and location of 40. In general, Equation

8 can be written for the block maximum of n values:

$$CDF(x) = F^n(x, 40, 5, 0.05) \qquad \text{Equation 9}$$

where n is 3000 for daily maxima, (3000×250=750 000) for annual maxima and (3000×250×50=37 500 000) for 50-year lifetime maxima.

It should be noted that the distribution of Equation 9 is the same as the distribution based on postulated stability parameters.
Two sets of data are considered:

- 2500 daily maximum values (10 years) of 3000 events per day and
- 5000 generated annual maxima.

The best fit GEV distribution is found in each case from the simulated data sets. Then daily maximum, predicted annual maximum and predicted 50-year lifetime maximum distributions are found for the first set of data. Using the second set of data, annual maximum and predicted 50-year lifetime maximum distributions are found. Figure 1 illustrates the generated daily maximum data, fitted GEV to daily maxima, predicted annual maximum distribution and fitted GEV to annual maximum data. All cumulative distribution functions are plotted to a Gumble scale.

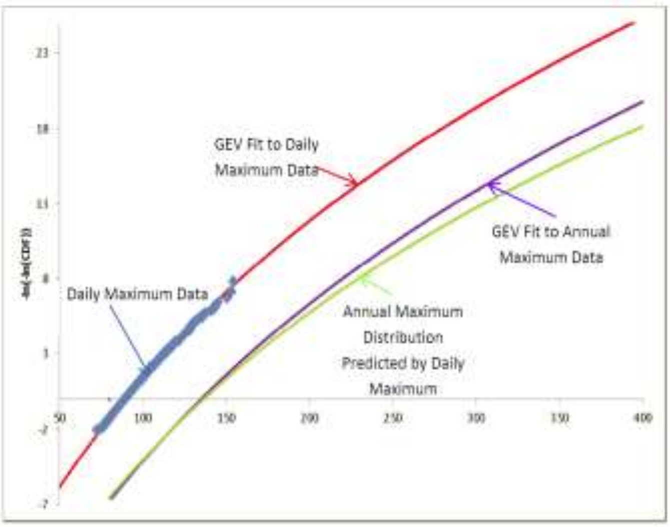

Figure 1. Theoretical example: Daily and annual maximum distributions

Even though the data generating source is similar for both daily and annual maxima, the predicted annual maximum distribution is different from the fitted annual maximum distribution.

This difference becomes even greater in predictions of the 50-year lifetime maximum distribution (Figure 2).

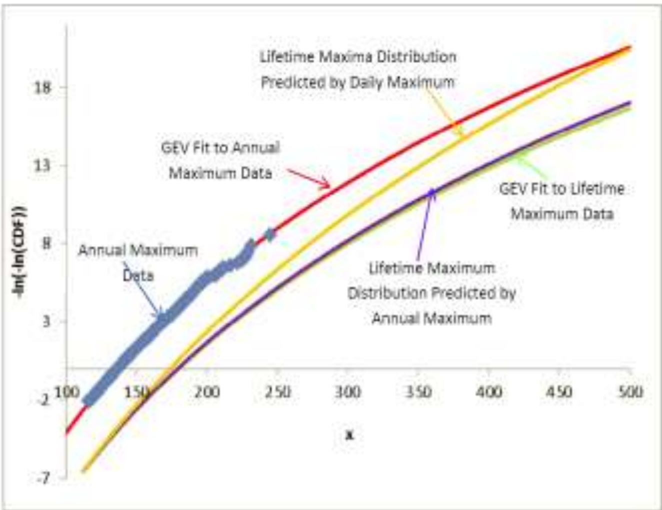

Figure 2. Theoretical example: Annual and 50-year lifetime maximum distributions

As could be expected, the significant amount of additional data for annual maxima results in a more accurate distribution than daily maxima. This difference can also be viewed as being due to the different power used in the predictions for these sets of data: 250×50 = 12 500 using daily maxima to predict the lifetime maximum distribution and 50 using annual maxima for this prediction. Parameters for fitted GEV and the power used for prediction are summarised in Table 2 and Table 3 respectively.

In order to make an appropriate comparison between the different predicted 50-year lifetime maximum distributions the probability of failure is calculated. Rather than having results that are dependent on an arbitrarily assumed distribution of load carrying capacity/resistance, a mirror image of the fitted 50-year lifetime maximum load effect is used as a resistance distribution (Figure 3). This resistance distribution is chosen to give probabilities of failure in the region of $1{\times}10^{-6}$. The results are summarised in Table 4.

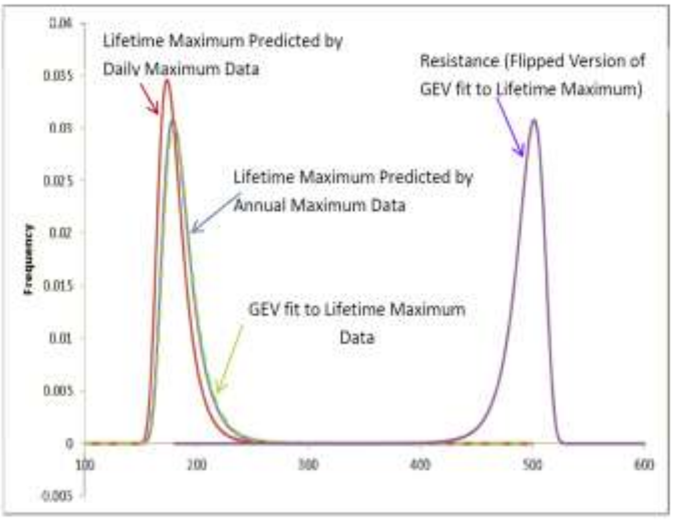

Figure 3. Theoretical example: 50-year lifetime maximum load effect and resistance distributions

Table 2. GEV parameters

Data	ξ	σ	μ
Daily Event	0.05	5	40
Daily Maxima	0.05	7.461	89.230
Annual Maxima	0.05	9.834	136.677
50-year Lifetime Maxima	0.05	11.958	179.167

Table 3. '*n*' values for Equation 9

Data	Daily Maxima	Annual Maxima	50-year Lifetime Maxima
Daily Event	3000	750 000	37.5×10^{6}
Daily Maxima	---------	250	12 500
Annual Maxima	---------	---------	50
50-year Lifetime Maxima	---------	---------	---------

Table 4. Probability of Failure and Values with 95% Probability (50-year Lifetime Maxima)

Theoretical Example	95%	Probability of Failure
GEV fit to 50-year Lifetime Maxima	217.22	0.693×10^{-6}
Predicted by Annual Maxima	219.07	0.857×10^{-6}
Predicted by Daily Maxima	207.22	0.184×10^{-6}

4.2 LONG-RUN SIMULATION PROBLEM

As a second benchmark test, 2500 daily maximum and 5000 annual maximum load effects are generated using a simulation of vehicles crossing bridges developed by Enright [16], calibrated using traffic data from the Netherlands. This set of data, in contrast to the previous example does not have a known distribution. However, the long-run simulations are considered to be highly accurate so the best fit distributions to these simulations are used as the benchmark against which other estimated distributions are measured. A range of statistical distributions with different goodness of fit tests such as Kolmogorov-Smirnov, Anderson Darling and Chi-Squared have been used in this regard. In general GEV fits were found to be better in comparison to other distributions such as lognormal, log-gamma, log-logistic etc. Kernel Density Estimation has also been used as an estimation of the lifetime maximum probability function from sample data.

A histogram gives an estimate of probability density at discrete points. However the choice of the bin size and origin influence the results. For instance, if the band width is too small, not enough variability is introduced to the empirical data, whereas too large a band width over-smooths the data.

As can be seen in Figure 4 there is a significant difference between the 50-year lifetime maximum distribution predicted by daily maximum data and either GEV fitted density to 50-year lifetime maxima or predicted 50-year lifetime maxima using annual maxima.

In fact, the way that the 50-year lifetime maximum distribution is obtained, has also amplified the difference. This set is based on 5000 annual maxima, i.e., 100 50-year lifetime maximum values. It should be noted that GEV itself is an approximation in this example and raising it to a certain power has a significant effect on results.

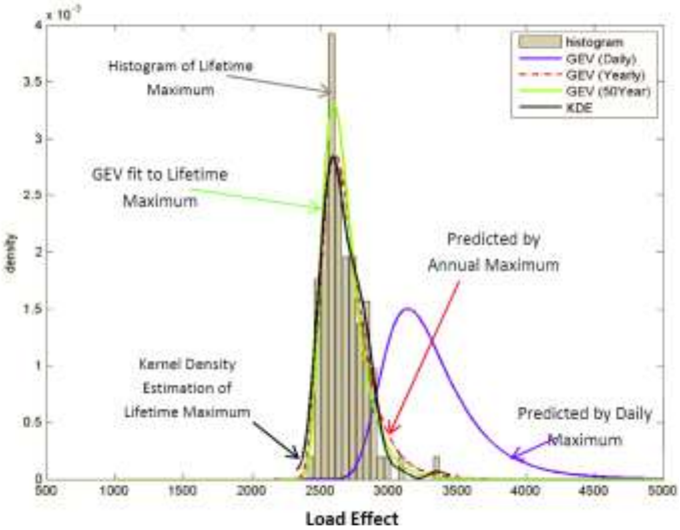

Figure 4. Long-run problem: Estimated 50-year lifetime maximum distributions

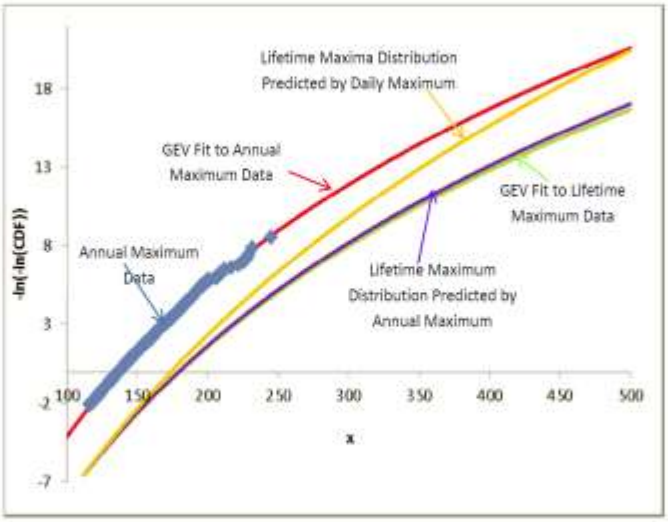

Figure 6. Long-run problem: Annual and 50-year lifetime maximum distributions

Figure 5 shows considerable difference even among annual maximum distribution predictions.

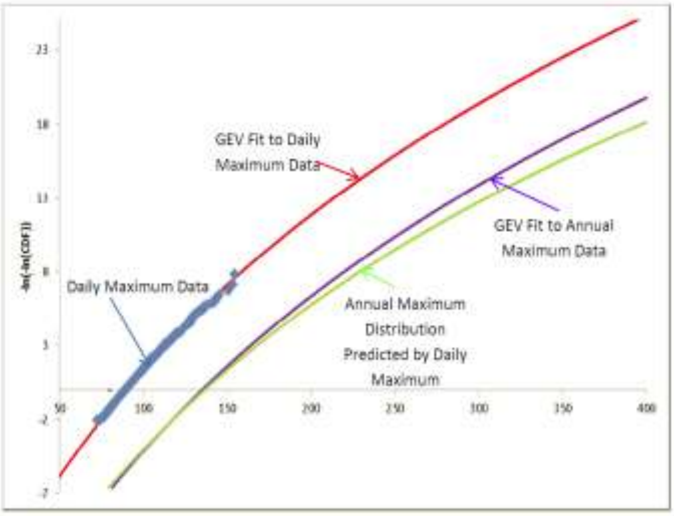

Figure 5. Long-run problem: Daily and annual maximum distributions

Figure 6 illustrates annual and 50-year lifetime maximum distributions predicted using daily maximum data, annual maximum data and the 50-year lifetime maximum data itself. It can be seen that GEV fits better to annual maximum data in comparison with daily maximum data, which means GEV is a better approximation in the annual maximum case. This results in better prediction of 50-year lifetime maximum distributions.

With the same mirror image concept for the resistance distribution for the 50-year lifetime maximum GEV fit, the probability of failure is calculated for predicted distributions for lifetime maxima. The results are presented in Table 5.

Table 5. Probability of failure and values with 95% probability (50-year lifetime maxima)

Long-run Simulation results	95%	Probability of Failure
GEV fit to 50Year Lifetime Maxima	2949	0.632×10^{-6}
Predicted by Annual Maxima	3017	4.32×10^{-6}
Predicted by Daily Maxima	3941	0.00353

5. CONCLUSIONS

It can be concluded that using more data results in a more accurate lifetime maximum distribution. Despite the fact that the distributions for daily and annual maxima are known, the amount of data is able to have a considerable effect on accuracy. Collecting such amounts of annual maxima is not practical but optimised long-run simulation makes accurate predictions possible.

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