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The Effect of Lane Changing on Long-Span Highway Bridge Traffic Loading

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ABSTRACT: Maximum loading on long-span bridges typically occurs in congested traffic conditions. As traffic becomes congested car drivers may change lane, increasing the tendency for trucks to travel in platoons. For long-span bridges this phenomenon may increase the regularity and severity of bridge repair programs, with potential significant associated costs. This research investigates the effect of lane changing by car drivers on bridge loading. A Monte Carlo simulation model in which individual car drivers probabilistically decide, based on a lane-changing bias probability, whether or not to change lane has been developed. The sensitivity of bridge loading to this factor is investigated for different bridge lengths and traffic compositions. This research concludes that the lane-changing behavior of car drivers has an effect on bridge loading for long-span bridges, and the magnitude of this effect is quite sensitive to the percentage of trucks in the traffic.

1 INTRODUCTION

With increasing numbers of freight trucks on European highways (Eurostat 2009), the likelihood of trucks travelling together in platoons has become greater. Trucks generally travel at lower speeds than lighter vehicles, and truck platoons occur in free-flowing traffic as cars change to a faster-moving lane. As traffic becomes congested car drivers may be even more likely to change lane, increasing the tendency for trucks to travel in platoons, consequently increasing the traffic loading on bridges. Congested traffic is the critical traffic loading scenario on long-span bridges (Buckland et al. 1978, Buckland 1981), and an increase in the frequency of occurrence of truck platoons may therefore cause an increase in the regularity and severity of bridge repair programs. The cost associated with bridge repair is twofold – the actual repair cost and the economic cost due to traffic delays during repairs. Reduced rehabilitation needs could therefore have significant associated costs savings.

To model congested traffic for bridge loading, researchers have used different vehicle arrival processes. Ivey et al. (1953) maintained actual vehicle patterns but reduced the gaps between vehicles from recorded free-flowing traffic. Harman et al. (1984) generated truck weights and dimensions randomly based on weigh-in-motion (WIM) recordings, allowing for 2.0 kN/m between trucks to replicate cars. The arrival of successive vehicles was assumed by

Flint & Jacob (1996) to be random, based on the percentage of each vehicle type in the recorded traffic. Nowak et al. (2010) kept trucks in the same order as recorded on-site and cars were ignored, with the gap between successive trucks reduced to 7.6 m to replicate congested traffic. Vrouwenvelder & Waarts (1993) modeled trucks statistically from traffic recordings and ignored cars. A Markovian vehicle arrival process based on a transition matrix computed from real traffic was used by Crespo-Minguillon & Casas (1997). In all the above congested traffic load models, the cause and effect of truck platoons has not been explicitly addressed.

This paper investigates how the lane-changing behavior of cars affects the formation of truck platoons, and the effect these platoons have on the traffic loading on long-span bridges. Traffic in two lanes in the same direction on the A4 (E40) motorway at Wroclaw, Poland, was recorded using WIM equipment over a six month period in 2008. This traffic is analyzed to determine the extent and nature of the platooning behavior of trucks and cars. The traffic at the measured site is free-flowing, and in this study it is passed through a traffic micro-simulation model to induce congestion, and the platooning behavior in this congested traffic is also studied. Further, a simple lane-changing model is developed that seeks to replicate the platooning behavior found in the congested traffic. The effect of the truck platoons on the traffic loading on long-span bridges is examined.

2 TRUCK PLATOONS

2.1 Truck Platoon Definition

A truck platoon is defined as a group of trucks travelling together with no cars in between, and with a maximum gap between successive trucks. In this work cars are defined to be any vehicle with two axles and a gross vehicle weight (GVW) of less than 3.5 t, with all other vehicles considered to be trucks. In order to analyze driver behavior, a conditional probability approach is adopted. In the measured stream of traffic in the slow lane at Wroclaw, the probabilities of the next vehicle being a truck are calculated *given* that there are successively no trucks, one truck, two trucks etc. in a platoon in front of the given vehicle. If vehicle occurrence is purely random (a sequence of vehicles that are mutually independent and identically distributed) then the probability of each successive vehicle being a truck should be constant, and equal to the proportion of trucks in the overall traffic stream, $P(T)$:

$$P(T | T_i) = \frac{P(T)^{i+1}}{P(T)^i} = P(T) \quad (1)$$

where T_i is a truck platoon of i trucks in length.

In free-flowing traffic, inter-vehicle gaps can be quite large, and it is reasonable to refine the definition of a truck platoon by specifying a maximum gap, above which a platoon is considered to have ended. Figure 1 shows the conditional probabilities of the next vehicle being a truck, given the platoon length in front, from measured free-flowing night-time (22:00-06:00) traffic in the slow lane at Wroclaw. The average measured flow is 109 vehicles/hr, giving an average gap between vehicles of 33 s. Four different platoon-defining maximum gaps, and a fifth case, with no maximum gap, are shown. There is a general trend for the probability of the next vehicle being a truck to increase as the platoon length increases. However, as this work focuses on long-span bridges for which the critical loading case is congestion, no maximum gap is applied in defining a truck platoon in the remainder of this paper. This is because the traffic will be travelling much more slowly, with very small distances between successive vehicles.

2.2 Truck platoons in congested traffic

As traffic breaks down from free flow to congested flow, lane-changing of cars might be expected to increase the length of truck platoons. To study truck platooning in congested traffic, the free-flowing traffic WIM traffic data from Wroclaw is passed through a traffic micro-simulation program (Caprani & O'Brien 2008, Caprani 2010a) based on the Intelligent Driver Model (IDM) of Treiber et al. (2001a, 2001b). The traffic micro-simulation program has the ability to reproduce complex traffic phenomena

based on individual driver's behavior. Congestion is induced in the micro-simulation by means of a speed limit and an increase in safe time headway for vehicles on the virtual road, similar to that of Treiber et al. (2001b). Figure 2 shows the frequency plot of truck and car platoons for congested night time traffic from the micro-simulation. (Car platoons are defined in a similar way to truck platoons).

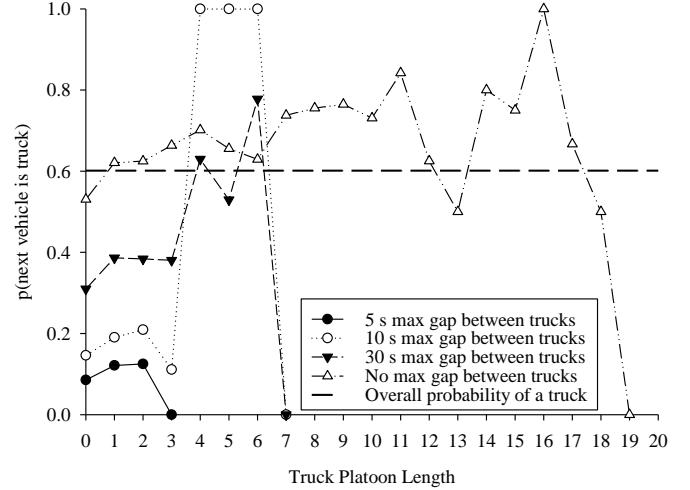


Figure 1. Platoon dependency plot for different maximum time gaps between vehicles in measured Wroclaw night-time traffic (22:00-06:00).

In Figure 2 purely random traffic is also shown. This is based on a randomly-generated traffic stream of the same time length and composition as the traffic from the micro-simulation. It can be seen that there are significantly longer platoons in the congested traffic than purely random arrivals suggests.

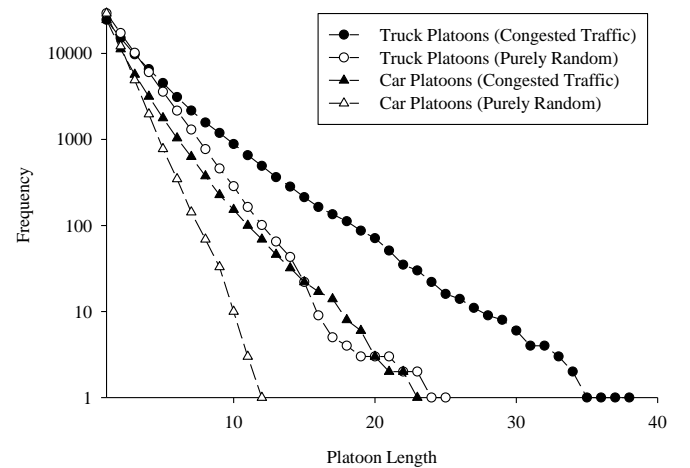


Figure 2. Platoon length frequency plot for congested traffic from micro-simulation.

Figure 3 shows the measured dependencies for different platoons obtained from Figure 2 as follows:

$$P(T | T_i) = \frac{T_{i+1}}{T_i} \quad (1)$$

Since all $i+1$ platoons contain i vehicle platoons. For example, from Figure 2:

$$P(T|T_8) = \frac{T_9}{T_8} = \frac{1188}{1573} = 0.76 \quad (1)$$

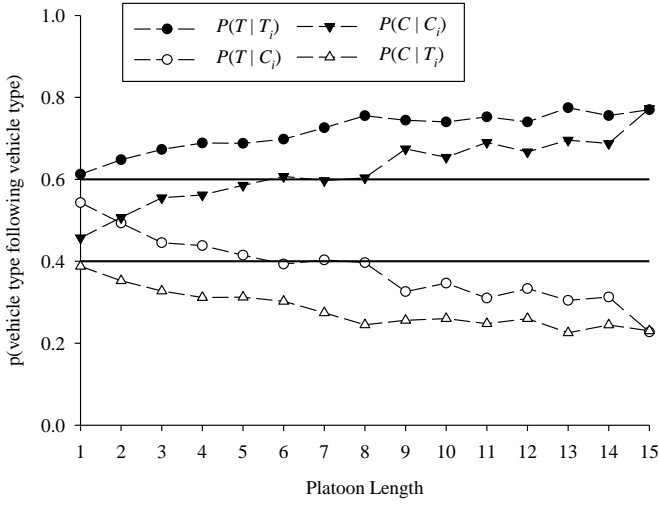


Figure 3. Platoon dependency plot for congested traffic from micro-simulation, where $P(T)$ is the probability that the next vehicle is a truck and $P(C)$ is the probability of the next vehicle being a car, T_i is a platoon of i trucks in length and C_i is a platoon of i cars in length.

Figure 3 also demonstrates the same phenomenon of platoon-length dependency for car platoons. Therefore the arrival of successive vehicles in congested traffic is not independent, with trucks and cars showing a tendency to group together. From Figure 2, this platooning tendency can be demonstrated as follows:

$$\frac{T_9^R}{T_8^R} = \frac{227}{376} = 0.60$$

where T_i^R is a truck platoon of i trucks in length from the purely random traffic. The difference between the purely random and measured platoons is:

$$m_8 = \frac{T_9}{T_8} - \frac{T_9^R}{T_8^R} = 0.16$$

Generalizing this concept gives:

$$m_i = \left(\frac{T_{i+1}}{T_i} - \frac{T_{i+1}^R}{T_i^R} \right) \quad (1)$$

From Figure 3, this difference is approximately linear for up to about 15-vehicle platoon lengths:

$$m_i \approx i \times 0.02, \quad 1 \leq i \leq 15 \quad (1)$$

It should be noted that this slope of the dependency plot is likely site- and traffic-specific, but in general, where a linear increase exists, we can write:

$$P(T|T_i) = \frac{T_{i+1}^R}{T_i^R} + m_i \quad (1)$$

Which, using Equations (1) and (1) gives a simple linear model for the dependency for this traffic:

$$P(T|T_i) = P(T) + 0.02(i) \quad (1)$$

The linear relationship used in this simple model becomes more unpredictable for platoon lengths greater than 15 vehicles as the platoon data gets sparse. The impact of this phenomenon on bridge traffic load effect is investigated next.

3 TRUCK PLATOON FORMATION

3.1 Model 1 – Lane-Changing Probability (LCP)

To investigate the observed formation of truck platoons, a single-lane traffic stream is generated randomly, assuming independence between successive vehicle types. The vehicle stream is then processed for random lane changing of cars, based on a simple lane-changing probability (LCP). Vehicles considered to have ‘changed lane’ are simply extracted from the single-lane vehicle stream, as illustrated in Figure 4(a). The platooning dependency of the resulting traffic stream is examined. This procedure is carried out for different percentage of trucks in the stream of traffic and also for a range of LCPs, and the results are given in Figure 5.

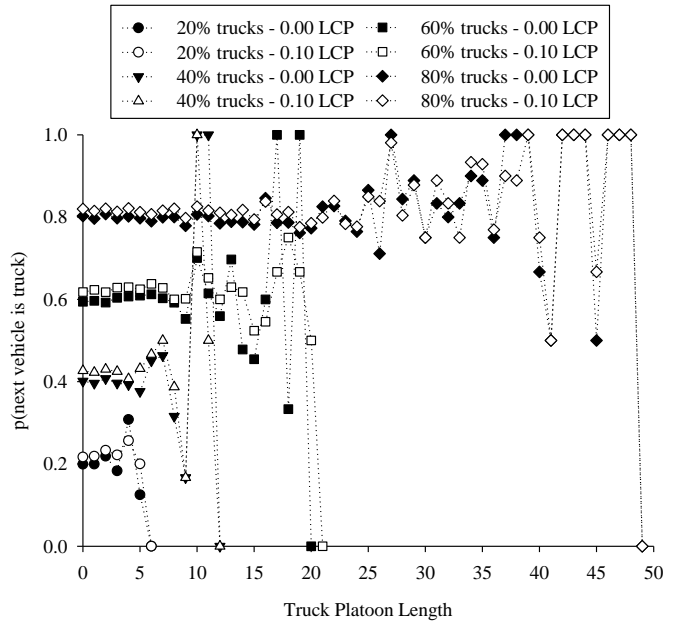


Figure 5. Platoon dependency plot for purely random traffic (LCP = 0.00) and traffic subsequent to lane changing simulation (LCP = 0.10) both with different percentage trucks.

From Figure 5, it is apparent that there is no dependency evident, suggesting that lane-changing alone does not explain the behavior found in Figure 3. As a result, further development of the simple model is examined.

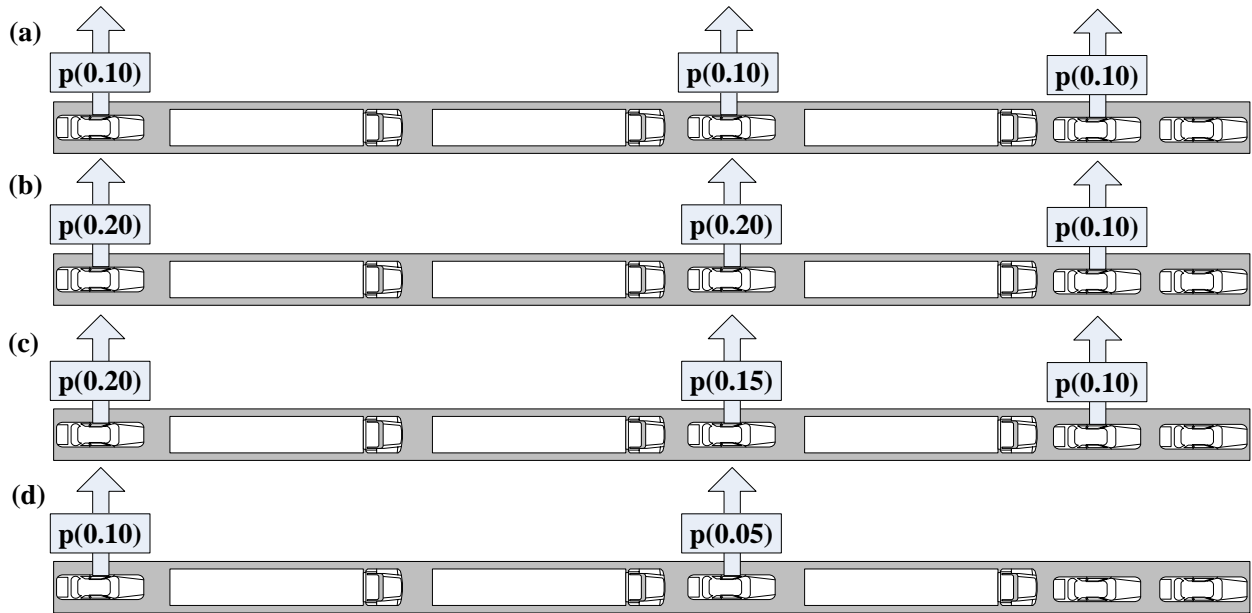


Figure 4. Overview for (a) Model 1: simple lane-changing model, (b) Model 2: lane-changing probability for cars following cars and lane-changing bias for cars following behind trucks, (c) Model 3: lane-changing probability for cars following cars and varying lane-changing bias for cars following behind trucks, (d) Model 4: varying lane-changing bias for cars following behind trucks.

3.2 Model 2 – Lane-Changing Probability (LCP) with Lane-Changing Bias (LCB) if car is behind truck/trucks

As trucks generally travel slower than cars on highways, there may be an increased tendency for cars to change lane if there is a truck travelling in front. To investigate the effect of such an increased probability of lane changing, cars following trucks are given a lane-changing bias (LCB), so that the traffic stream is processed as shown in Figure 4(b).

Figure 6 presents the platoon dependency plots for this model. It shows that with increasing LCB, the probability of the next vehicle being a truck also increases, as may be expected. However there is no obvious increase or decrease in platoon dependency under this model, contrary to the patterns evident in free-flowing and congested traffic and so further development to this model is necessary.

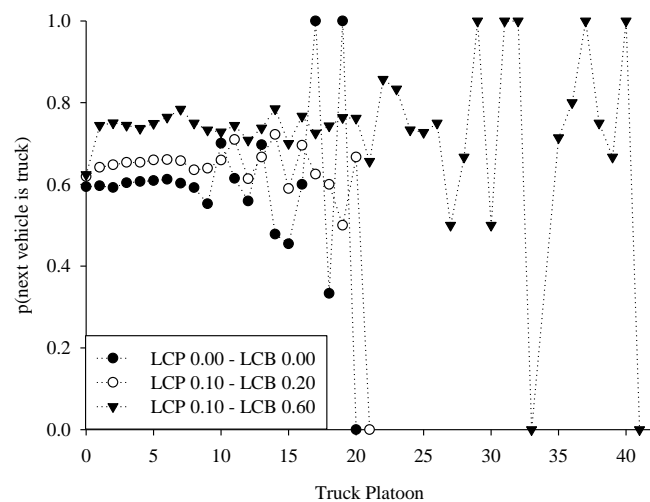


Figure 6. Platoon dependency plot with 60 % trucks, and a range of LCP and LCB factors.

3.3 Model 3 – Lane-Changing Probability (LCP) with varying Lane-Changing Bias (vLCB) if car is behind a truck or trucks

Since the probability of a truck being the next vehicle increases with the number of trucks travelling in front (Figure 3), the probability of a car being next decreases. Thus there is dependence between the occurrence of a car and the number of trucks in front. To examine this, a linearly increasing probability of a lane change, depending on the number of trucks in front is used. Each truck in front adds a probability of 0.05 that the following car will change lanes, as illustrated in Figure 4(c). The simulation is carried out for vLCBs of 0.05 and 0.10 with a LCP of 0.10 and the results are given in Figure 7, together with results for purely random traffic (with 60 % trucks) and for congested traffic from the micro-simulation.

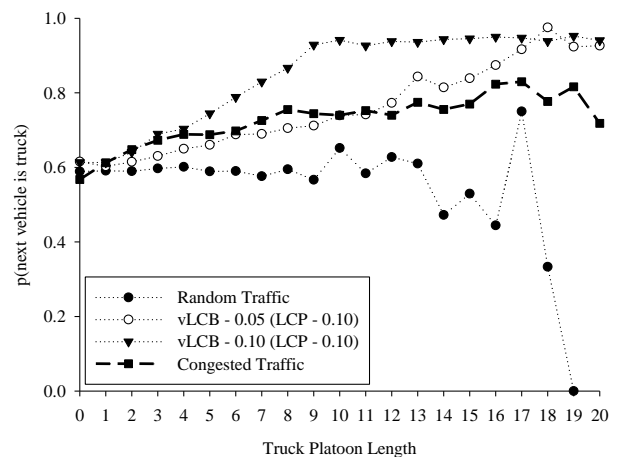


Figure 7. Platoon dependency plot for congested traffic from micro-simulation and random traffic with vLCB of 0.05 and 0.10, and a LCP of 0.10 for a truck composition of 60 %.

Figure 7 shows that this model gives a similar relationship to that of the ‘measured’ congested traffic of Figure 3. Interestingly, no one vLCB matches the congested traffic well in Figure 7, with a vLCB of 0.10 matching well up to a 4 truck platoon and a vLCB of 0.05 thereafter.

3.4 Model 4—varying Lane-Changing Bias (vLCB) if car is behind truck/trucks.

In this model, only the probability of a car changing lane whilst behind trucks, vLCB, see Figure 4(d), is considered. The platoon dependency results for this simulation are shown in Figure 8, also shown are the results for purely random traffic (with 60 % trucks) and for congested traffic from the micro-simulation. It can be seen that the dependency in truck occurrence on platoon length matches that of the congested traffic from the micro-simulation well for a vLCB of 0.05. Figure 9 shows the 95 % confidence intervals on the platoon dependency plot, for a vLCB of 0.05. The width of the confidence interval increases with increasing truck platoon length due to the decreasing number of truck platoons with increasing platoon size (Figure 2).

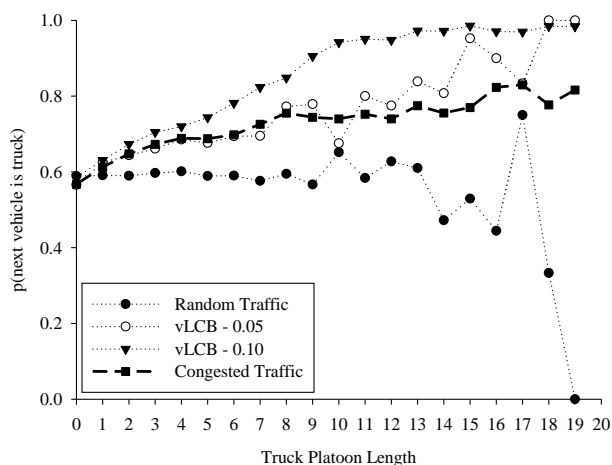


Figure 8. Platoon dependency plot for congested traffic from micro-simulation and random traffic with vLCB for a truck composition of 60 %.

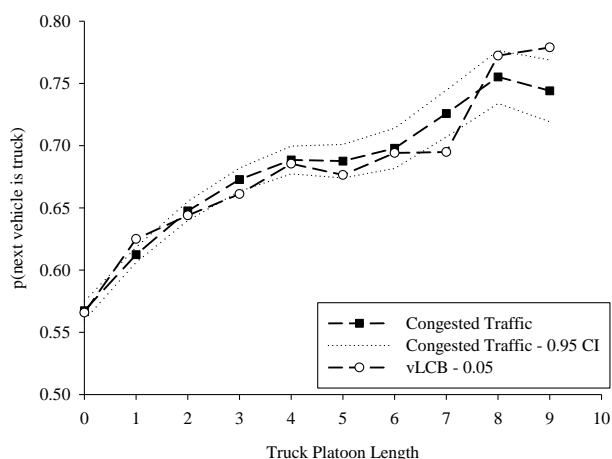


Figure 9. Platoon dependency plot for congested traffic from micro-simulation and random traffic with a vLCB factor of 0.05 for a truck composition of 60 % showing confidence intervals.

3.5 Summary of simple model findings

The simple lane-changing models presented here failed to match the ‘measured’ truck platoon dependency until an increased probability of lane-change, dependent on the number of trucks in front, was allowed for. Interestingly, the most accurate model (Figure 4(d)), does not require an overall lane-change probability, but one considering only the number of trucks in front. It is this model that will net be used to estimate the significant of lane-changing on bridge traffic load effect.

4 EFFECT OF CARS CHANGING LANE ON TRAFFIC LOADING ON BRIDGES

4.1 Simple traffic loading model

The platoon formation process described above will cause an increase in the number of trucks present on a bridge, particularly for longer bridges, thereby increasing the traffic loading on the bridge. To investigate the impact of this, a simple traffic stream was constructed with only two types of vehicles present: a typical heavy goods vehicle (HGV) and a typical car. Both vehicles configurations were based on average values found in the WIM traffic data and are given in Tables 1 and 2. Typical front and rear overhangs are used for each vehicle, based on a survey of manufacturer’s data (Caprani et al. 2010b). The gap between each vehicle is kept constant, as listed in Table 3, and these gaps are the average values obtained for congested traffic from micro-simulation.

Table 1. Axle weights for a typical car and truck.

Vehicle Type	Axle Weight kN				
	Axle 1	Axle 2	Axle 3	Axle 4	Axle 5
Truck	77.7	101.0	61.8	63.8	63.8
Car	9.8	9.8			

Table 2. Vehicle overhangs and axle spacing for a typical car and truck.

Vehicle Type	Axle Configuration m					
	Front Overhang	Axle Spacing			Rear Overhang	
		1-2	2-3	3-4		4-5
Truck	1.4	3.8	5.4	1.5	1.3	3.0
Car	0.9	3.0				0.8

Table 3. Bumper to bumper gaps for different vehicle combinations.

Vehicle Combination	Bumper-Bumper Gap m
Car-Car	5.72
Car-Truck	7.42
Truck-Truck	8.07
Truck-Car	6.51

Two types of platooning dependency are compared: purely random and the representative model developed above, Model 4 (Figure 4(d)). A range of percentage trucks in the traffic stream (20-80 %) is considered for the equivalent of 10 years of congested traffic (taking there to be 2 hours of congested traffic per day and 250 economic days per year). For each truck percentage a range of vLCBs (0.00, 0.025, 0.05, 0.075, and 0.10) are simulated. The total load caused by each traffic stream on bridges of different lengths is then obtained by crossing the traffic streams over a virtual bridge and calculating the load effect of interest at 0.10 s intervals. For this work, only the total load on the bridge length is considered, as this is typically modelled using an equivalent uniformly distributed load for long-span bridge codes of practice.

4.2 Sample load effect results

Figure 10 shows results for single-lane traffic with 40 % trucks on a 2 km bridge for randomly generated traffic (cars and trucks evenly distributed) and traffic subsequent to lane-changing with a vLCB of 0.05, plotted on Gumbel probability paper. It is evident that the platoon dependency model influences the total load on the bridge significantly. The maximum load effect (point B) from the traffic after platoon dependency is considered (using the Simple Model) is 22 % higher than the maximum value for purely random traffic (point A). Point B of Figure 10 is caused by an 81-truck platoon with only a single car present on the bridge, whereas point A is caused by a total of 65 trucks and 42 cars with the largest platoon being only 9 trucks long.

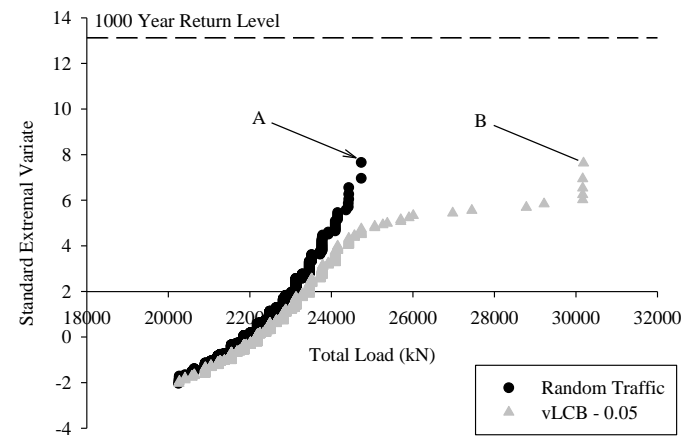


Figure 10. Standard extremal variate $[-\ln(-\ln(P))]$ versus hourly maximum total load on a 2 km bridge with traffic containing 40 % trucks.

Figure 11 shows the frequencies of truck platoon lengths for the two types of traffic streams. It is very clear that consideration of lane-changing through the platoon dependency model causes significantly longer platoons to form. The longest platoon in the purely random traffic contains 15 trucks; whereas

lane-changing produces platoons containing up to 5 platoons of 83 trucks or greater (this size platoon fills a 2 km bridge with only trucks).

The characteristic traffic load in the Eurocode (EC1.2 (2003)) is taken to have a value equivalent to a return period of 1000 years. To estimate this, the simplified traffic streams outlined above are used to simulate 1000 years of congested traffic. The characteristic total loading event for purely random traffic with 40 % trucks is found to be 26170 kN (consisting of 69 trucks and 31 cars with the largest truck platoon containing 9 trucks). As would be expected, this characteristic load is greater than the 10-year maximum load for the same traffic (247345 kN), but is still 15.3 % smaller the corresponding value for the platoon dependency modeled traffic. This suggests that the truck platooning caused by cars changing lane has a significant effect on traffic loading on long-span bridges.

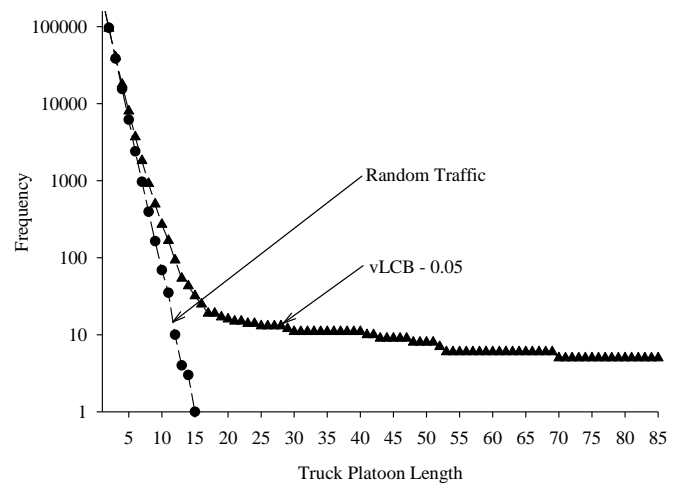


Figure 14: Frequency plot of truck platoon lengths, traffic streams with 40 % trucks.

4.3 Load effect results for simple model

Figure 15 shows the full set of load effect results, expressed as a ratio of the purely random traffic and the traffic considering platoon dependency. It can be seen from Figure 15 that the greatest sensitivity to different lane-changing probabilities occurs when the traffic contains 20 % trucks, for which a load effect ratio of up to 25 % is found. Also, it can be seen that for a vLCB of 0.10, the load effect ratio increases for bridge lengths up to 500 m and then begins to decrease.

For bridge lengths of 50 m and 100 m (not shown in Figure 15), lane-changing has no noticeable effect on total load because, for these bridge lengths, relatively small platoons with 2 or 3 truck determine the maximum total load, and platoons of this size occur regularly even in purely random traffic. As the percentage of trucks increases towards 80 %, the effect of lane-changing becomes negligible because suffi-

ciently large truck platoons form normally due to the very high percentage of trucks in the traffic.

For the higher truck percentages, different lane-changing biases have relatively little effect. For truck percentages of 40 % and 60 %, the effect of platoon dependency tends to increase with bridge lengths.

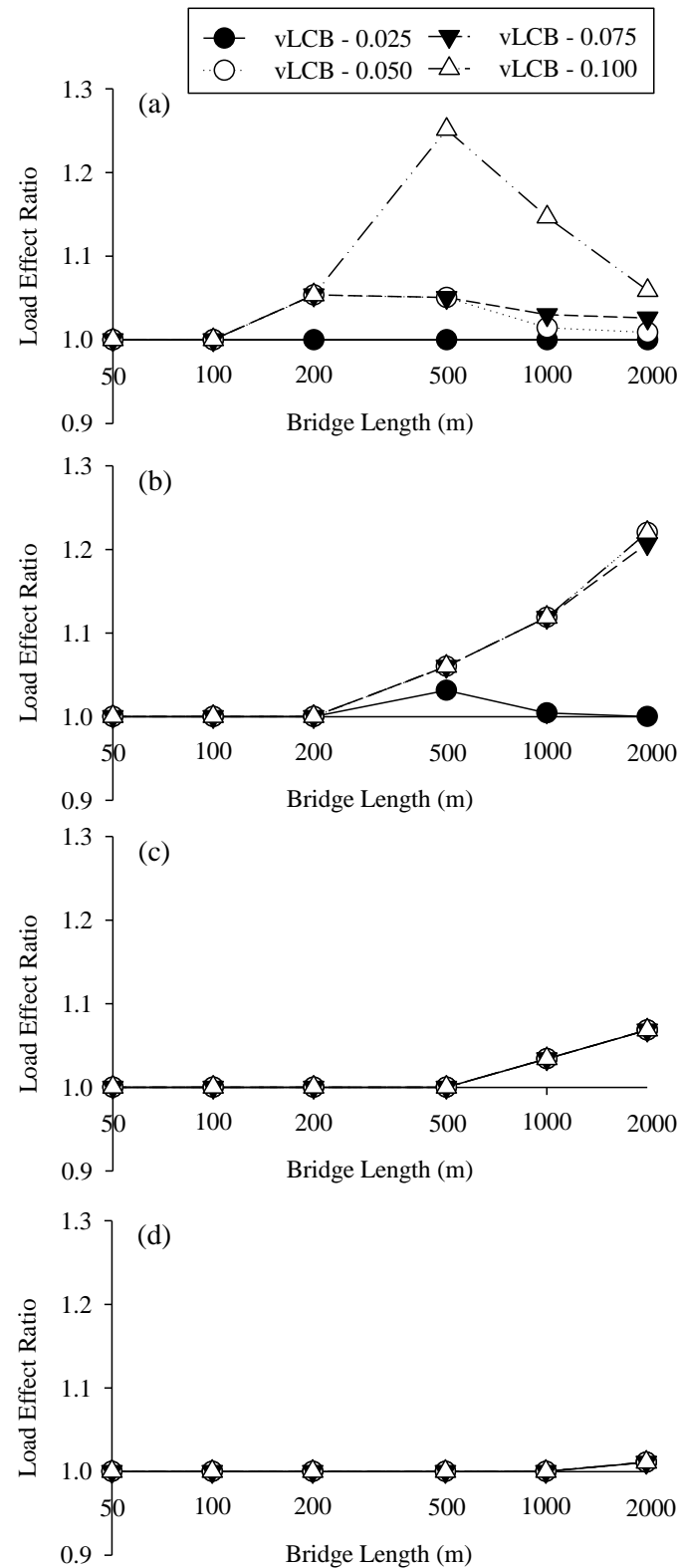


Figure 15: Load Effect Ratio (Total load for Model 4 with standard vehicles divided by total load for random traffic) versus percentage trucks for the 10 year maximum congested traffic load for different bridge lengths - for (a) 20 % trucks, (b) 40 % trucks, (c) 60 % trucks, (d) 80 % trucks, in traffic.

5 CONCLUSIONS & DISCUSSION

5.1 Conclusions

This research investigated the effect that the lane-changing behavior of cars may have on traffic loading on long-span bridges. It has been found that cars changing lane causes longer truck platoons to form. These truck platoons govern traffic loading on long-span bridges. The arrival of successive vehicles – cars and trucks – is found to be not independent in congested traffic produced by micro-simulation, and the probability of the next vehicle being a car or a truck increases with the number of similar vehicles travelling in front. This platooning dependency is also found in free-flowing traffic.

It is shown that the platooning behavior of trucks and cars may be replicated by assuming that the probability of a car changing lane increases as the number of trucks it is following increases. To model this, a varying lane-changing bias for cars following trucks (vLCB) is introduced and for the particular traffic examined it is found that a linear probability increase of 0.05 for each truck travelling immediately in front of a car produces a good match for the observed results.

Lane changing is found to affect the total load on long-span bridges. The effect of lane changing on the load effect is sensitive to bridge length and to the percentage of trucks in the traffic. Potential increases of up to 25 % in total load are shown to be possible due to lane changing.

5.2 Discussion

The vLCB found here to replicate measured platoon dependency is most likely site-specific, and is expected to change with the percentage of trucks and with the road topography (which will determine how many trucks that a car driver is actually able to see). A further factor may be the duration for which the same stream of traffic stays on a length of road, since truck platoons will tend to become longer as the traffic sorts itself into groups of traffic with similar desired velocities. This phenomenon would, in turn, affect the vLCB that best matches the traffic, and, of course, the resulting traffic load effect. Finally, the approximately linear relationship of dependency found here for up to 15 truck platoons could be generalized to allow for different dependency modeling, and the consequent importance to bridge traffic loading assessed.

6 ACKNOWLEDGEMENT

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7 REFERENCES

- Buckland, P.G., McBryde, J.P. and Francis, P.D., 1978. Traffic loading of long span bridges. *In: Conference on bridge engineering proceedings*, Transportation Record Research, 146-154.
- Buckland, P.G., 1981. Recommended design loads for bridges (Committee on loads and forces on bridges of the committee on bridges of the structural division), *Journal of the Structural Division, ASCE*, 1161 – 213.
- Caprani, C.C. & O'Brien, E.J. 2008. The governing form of traffic for highway bridge loading. *Proceedings of 4th Symposium on Bridge and Infrastructure Research in Ireland*, eds. E. Cannon, R. West & P. Fanning, National University of Ireland, Galway, 53-60.
- Caprani, C.C. 2010a. Using Microsimulation to Estimate Highway Bridge Traffic Load. *Proc. 5th Intl. Conf. on Bridge Maintenance, Safety and Management*, IABMAS, Philadelphia, July.
- Caprani, C.C., Carey, C. & Enright, B. 2010b. A new congested traffic load model for highway bridges. *Proceedings of 5th Symposium on Bridge and Infrastructure Research in Ireland*, Cork.
- Crespo-Minguillón, C. & Casas, J.R. 1997. A Comprehensive traffic load model for bridge safety checking, *Structural Safety*, 19, 339-359.
- EC1.2, 2003, Eurocode 1: Actions on Structures, Part 2: Traffic loads on bridges, European Standard EN 1991-2: European Committee for Standardisation, Brussels.
- European Commission Directorate-General for Energy, Transport and Environment Indicators, 2009. European Union Energy and Transport in Figures 2009.
- Flint, A.R. & Jacob, B.A. 1996. Extreme traffic loads on road bridges and target values for their effects for code calibration. *In Procs. of IABSE Colloquium Basis of Design and Actions on Structures*, IABSE, Delft, 469-477.
- Harman, D.J., Davenport, A.G. & Wong, W.S.S. 1984. Traffic loads on medium and long-span bridges. *Canadian journal of civil engineering*, v 11, n 3, p 556-573.
- Ivy, R. J. et al. 1953. Live loading for long-span highway bridges. *Trans.*, Paper No. 3708, ASCE.
- Kesting, A., Treiber, M. & Helbing D. 2007. General lane-changing model MOBIL for car-following models. *Transportation Research Record: Journal of the Transportation Research Board*, Volume 1999/2007, pp. 86-94. DOI 10.3141/1999-10.
- Nowak, A.S., Lutomirska, M. & Sheikh Ibrahim, F.I. 2010. The development of live load for long-span bridges. *Bridge Structures - Assessment, Design & Construction*. Volume 6, Number 1-2. 73-79.
- Treiber, M., Hennecke, A., & Helbing, D. 2000a. Microscopic Simulation of Congested Traffic. in: *Traffic and Granular Flow '99*, Eds. D. Helbing, H.J., Herrmann, M. Schreckenberg, & D.E. Wolf, Springer, Berlin, 365-376.
- Treiber, M., Hennecke, A. & Helbing, D. 2000b. Congested Traffic States in Empirical Observations and Microscopic Simulations. *Physical Review E*, 62(2), 1805-1824, [arXiv:cond-mat/0002177v2](https://arxiv.org/abs/cond-mat/0002177v2).
- Vrouwenvelder, A.C.W.M. & Waarts, P.H. 1993. Traffic loads on bridges. *Structural Engineering International*, 3/93, 169-177.