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Development of Neuro-Fuzzy Models to account for temporal and spatial variations in a lumped rainfall-runoff model

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Abstract

For many good and practical reasons, lumped rainfall-runoff models are widely used to represent a catchment's response to rainfall. However, they have some acknowledged limitation, some of which are addressed here using a neuro-fuzzy model to combine, in an optimal way, a number of lumped-sub-models. For instance, to address temporal variation, one of the sub-models in the combination may perform well under flood conditions and another under drier conditions and the neuro- fuzzy system would combine their outputs for each time-step in a manner depending on the prevailing conditions. Similarly to address spatial variation, one of the sub-models may perform well for the upland parts of the catchment and another for the lowland parts and again the neuro-fuzzy system is expected to combine the different outputs appropriately. The proposed combination method can use any lumped catchment model, but has been tested here with the Simple Linear model (SLM) and the Soil Moisture and Accounting Routing (SMAR) models. Eleven catchments with different hydrological and meteorological conditions have been used to assess the models with respect to temporal variations in response while one catchment is used to address the effect of spatial variation. The neuro-fuzzy combined-sub-models of SLM and SMAR modelled the temporal and spatial variation in catchment response better than the

lumped version of each model. Also the SMAR model significantly outperformed the SLM either as a lumped model or as a sub-model in any of the combinations.

Keywords: Neuro-fuzzy; lumped model; combined-sub-models; Simple Linear model; Soil Moisture and Accounting Routing model; rainfall-runoff modelling

1. Introduction

Mathematical models are widely used in water resources applications despite considerable difficulties arising from catchment heterogeneity, strong non-linearity in its response to precipitation and uncertainties in parameter estimation. In many practical cases, simple lumped models of either the black-box or conceptual type often perform adequately, and compare well with the more complex distributed-models, particularly for flood modelling in small catchments and for behaviour within the range of the data used to calibrate its parameters (Beven, 2000). Larger catchments can be modelled by associating different sub-lumped-models with different spatial units within the catchment (e.g. Chen and Adams, 2006; Marechal and Holman, 2005; Ajami et al., 2004). Similarly different sub-models could be used to represent the various temporal patterns in the system's response (e.g. Shamseldin and O'Connor, 1996; Ahsan and O'Connor, 1994; Kachroo and Natale, 1992). The success of this approach is primarily because of its ability to capture some of the non-linearity in the catchment behaviour resulting from its spatial heterogeneity and time-varying character. The choice of a suitable lumped model for use in each of the sub-catchments is critical to its success and practicality. It should have a small number of parameters to reduce the total number to be estimated for the combined model thereby reducing the computational requirements. This also is likely to reduce potential problems caused by model over-parameterisation, such as ill-conditioning (Bruen and Dooge, 1992), or equifinality (Beven, 1993) in which a number of different combinations of parameter values give similar model fits and so a single optimal parameter set is difficult to determine.

One obvious symptom of non-linearity is the very different responses of the catchment to different flow regimes. The direct way of dealing with this is to build the complicated non-linear physical relationships into the model. An alternative is to have a different, but simple, sub-model for each different flow regime. For instance, Chen and Adams (2006) used a number of sub-models to simulate spatial variation in the rainfall-runoff relationship. The estimated runoffs from all sub-models were combined together using an artificial neural network to estimate the total runoff. Moreover, they investigated the suitability of using sub-models of three different conceptual models including the Xinanjiang Model (Zhao and Liu, 1995), the Soil Moisture Accounting and Routing (SMAR) Model (O'Connell et al., 1970) and the Tank Model (Sugawara, 1995). A significant improvement was obtained when using different sub-models compared to a single lumped model. Kachroo and Natale (1992) also used three sub-models using the same Simple Linear Model (SLM) (Nash and Foley, 1982) structure with different parameter sets to represent the response during low, medium and high flow regimes. Although the total number of parameters is tripled, all of the sub-lumped-model parameters could be calibrated using the least-squares criterion. The choice of which of the sub-model to use at each time step is guided by a type of wetness index taken as the current observed discharge in this case. When no observed discharge is available at the current time step, (e.g. when either (a) simulating or (b) forecasting beyond a single time step) the discharge simulated by the lumped model is used for this index. The combined-sub-lumped models have shown significant improvement over the lumped one.

Building on these efforts to improve the performance of combined-sub-lumped-models, this paper reports the investigation of a fuzzy method proposed to combine sub-lumped-models of two types, black box model and conceptual model. The former is the Simple Linear Model (SLM) (Nash and Foley, 1982) and the latter is the Soil Moisture Accounting and Routing model (SMAR) (O'Connell et al., 1970). Each of the two models has been included into a framework of a special type of Neuro-Fuzzy Model (NFM), called an Adaptive Neuro-Fuzzy Inference System. The first objective is to produce a combined-lumped-model better able to represent the spatial and temporal variability of the catchment's response to rainfall. The resulting NFM addresses the temporal variations in response by using a number of sub-models for the SLM and the SMAR models for different regimes (e.g. separate sub-models for floods and low flow situations). Each of the sub-models describes a particular feature in the temporal pattern of the catchment's response. The NFM is assessed by applying it to eleven different catchments from around the world. In the second part of this study an NFM (for the SLM and for the SMAR model) is developed that is able to identify homogenous spatial units within a catchment on which the sub-models can be based. In this, the NFM structure of the first part is further coupled to a subtractive fuzzy clustering algorithm (Vernieuwe et al., 2005) to determine the homogeneous spatial units using a number of spatial variables specified on a catchment grid. Finally, using one of the catchments which has the required spatial database, namely, the Brosna, the NFM developed in the second part of the study is tested and its results compared with those of the corresponding model developed in the first part of the study.

The proposed method is described in section 2 and the NFM is reviewed in section 3. The two rainfall-runoff models, SLM and SMAR, are briefly described in sections 4 and 5 respectively. In section 6, a detailed description is given of the two NFMs applied in this study. In the final sections, 7 and 8, the results of the NFM applications are presented and conclusions are drawn. Suggestions for further work are added in section 9.

2. Interpretation of the proposed sub-models combination method

The method of sub-model combination used in this study is different from the flood forecast model combination methods proposed in earlier work (e.g. Shamseldin et al., 1997; See and Openshaw, 2000; See and Openshaw, 1999; Xiong et al., 2001; Abrahart and See, 2002; Coulibaly et al., 2005; Fenicia et al. 2007). In those methods, a number of models each with different internal structures were individually applied to the entire study catchment and their simulated outputs were combined. Each model was attempting the same task, to simulate the entire catchment. In contrast, in our approach each model is truly a sub-model, assigned to simulate a particular part of the catchment or a specific range of responses, e.g. for a particular flow regime.

Following the multi-linear model approach pioneered by Bruen (1985), Becker and Kundzewicz (1987), Kachroo and Natal (1992), and Todini and Wallis (1997), our proposed sub-model approach was previously used to build different rainfall-runoff models. For instance, Bruen (1985) constructed a quasi-linear model from a combination of linear sub-models. An illustration of the structure of this quasi-linear model, with a single threshold, is given in Fig. 1. Note: (i) The input series (I) is effectively divided into a number of separate series (e.g. I_1, I_2, etc.), each of the same length as the original. The division procedure is preformed in two steps. First, the range of values in the input series is divided into a number of parts by threshold levels (partitions) of fixed values. Then the magnitude of each input value determines the band or division in which it lies, and the entire input in that band is then assigned to the corresponding time series. (ii) The output from each of the separated input series (e.g. O_1, O_2) is obtained from a number of separate models (e.g. $model_1$, $model_2$). (iii) The total output (e.g. O_f) is the sum of the outputs from each of the different models applied to the corresponding separated inputs. This allows the overall model to respond differently to low rainfall compared to high rainfall.

In essence a number of sub-models are constructed to describe the relationship between the input and the output for different ranges of their values representing different hydrologic conditions. This requires that each input value should be assigned to a specific sub-set (e.g. low values, medium values, high values). Such an approach assumes the inputs can be assigned to the sub-sets with certainty but there are times where uncertainty might occur, such as when the magnitude of an input value is close to a partition threshold value. The method proposed addresses this uncertainty using fuzzy logic theory whereby different levels of memberships of input to all sub-sets are estimated. These degrees of memberships can be taken as the weights given to the outputs from the models corresponding to each of the input sub-sets.

To illustrate our proposed method Fig.1 has been extended in Fig. 2 which shows, still for the case of a single threshold, how the concept of the membership of fuzzy sub-sets is used to define weights given to the sub-models. Unlike in Bruen's method (Bruen, 1985), the input series (e.g. I) is not separated here but alternatively it is assumed that for certain hydrologic conditions there is a sub-model (e.g. $model_1$, $model_2$) and a membership function (e.g. mf_1, mf_2) associated with it. The former produces the output (e.g. O_1, O_2) from the sub-model while the latter calculates membership values used to estimate the weight given to that output (e.g. w_1, w_2). The final output value (e.g. O_f) from the combination is the weighted average of the outputs from the models used for each sub-set. It is worth mentioning that the method described above is valid for the case of a lumped catchment. However, if the catchment is divided into sub-catchments, then the method can be applied separately to each sub-catchment and the final output can be estimated as the area-weighted average of the outputs of each of the sub-catchments (where routing to the catchment outlet is considered part of the sub-model).

3. Neuro-Fuzzy Model (NFM)

The Neuro-Fuzzy Model (NFM) used in this study implements the Takagi-Sugeno fuzzy approach (Takagi and Sugeno, 1985) to obtain a direct crisp value for the output variable(s) from fuzzy input variable(s). Jacquin and Shamseldin (2006) explored the application of Takagi-Sugeno fuzzy inference systems to rainfall-runoff modelling. They developed two different fuzzy models to account for the non-linearity in the catchment response due to both antecedent catchment wetness and seasonality. Vernieuwe et al. (2005) also investigated fuzzy rule-based models of the Takagi-Sugeno type relating rainfall to catchment discharge. Their models differed in the methods used to partition the spaces of the input and output variables and hence the identification of the number of membership functions and their locations for each variable. Earlier, the Takagi-Sugeno model was used by Xiong et al (2001) in the multi-model output combination context. All these studies produced models by combining different sub-models and this also has been followed in the present study.

A fuzzy number consists of a number of sub-sets each of which has an interval of possible values between specified minimum and maximum limits. For every point in the interval a corresponding membership function is defined that represents, within the interval, the degree of confidence one might have for a particular value of the fuzzy number (Ganoulis, 1994).

Generally the NFM consists of five layers configured analogously to any multi-layer feed-forward neural network. Chen et al. (2006) named these five layers according to their operative function, as ‘input nodes’, ‘rule nodes’, ‘average nodes’, ‘consequent nodes’, and ‘output nodes’ respectively. Fig. 3 illustrates an NFM with two input variables, x and y, each of which has two fuzzy sub-sets, A1 and A2 for x and B1 and B2 for y. The first layer in the Figure thus has four nodes, one for each of the two fuzzy sets of each of the two input variables. Each node in the first layer receives a crisp value of one of the input variables (e.g. x) and, for each fuzzy sub-set of this input variable, it uses a membership function (e.g. A1) to generate a membership grade (e.g. u_{A1}). Different shapes for the membership function, such as Gaussian, Generalised bell shaped, trapezoidal shaped, and triangular, can be used.

Although the second, third and fourth layers have different functions each has the same number of nodes. Each node in these three layers is assigned to a certain IF-THEN rule, called “the antecedent part” of the NFM. The total number of IF-THEN rules is determined by the number of possible combinations of the fuzzy sub-sets of the input variables. This procedure gives the neuro-fuzzy model an advantage over the ordinary fuzzy logic model because the former does not require the modeller to specify in advance the number of rules.

The function of each node in the second layer is to multiply the membership grades of all fuzzy sub-sets involved in a specific IF-THEN rule (e.g. u_{A1} and u_{B1}) to obtain the weight for this rule (e.g. w_1) which is normalised in the corresponding node in the third layer. The normalised weight (indicated by a bar $\bar{w}_1$) is obtained by dividing the weight assigned to that particular IF-THEN rule by the sum of the weights for all rules (e.g. $\bar{w}_1 = w_1 /(w_1 + w_2 + w_3 + w_4)$). The nodes in the fourth layer compute the fractional contribution to the final model output(s) of each IF-THEN rule and this layer represents "the consequent part" of the NFM. This fraction is the product of the normalised weight of the associated IF-THEN rule (e.g. $\bar{w}_1$) by a value calculated from a function associated with this rule to transform the crisp values of the inputs into a scalar output (e.g. $f_1(x,y)$). In the original NFM formulation, a first order polynomial model, such as a Linear Transfer Function (Box and Jenkins, 1976), was used for this purpose. However, in this study the black-box SLM and conceptual SMAR catchment models are used instead.

In the fifth layer, each output variable is represented by a neuron. The final output produced by each neuron in the fifth layer is the aggregation of the outputs contributed by all the associated IF-THEN rules. Fig. 3 illustrates the case of a single output Z, from the single neuron in the fifth layer, having the form:

$$Z = \bar{w}_1 * f_1(x,y) + \bar{w}_2 * f_2(x,y) + \bar{w}_3 * f_3(x,y) + \bar{w}_4 * f_4(x,y) \tag{1}$$

4. Simple Linear Model

The Simple Linear Model (SLM) was introduced by Nash and Foley (1982) as a naïve, benchmark, model against which the performance of more substantive and sophisticated rainfall-runoff models could be compared. The SLM assumes a linear time invariant relationship between rainfall and discharge, expressed by a convolution summation relation. Here, an additional term has been added in order to include, albeit crudely, losses due to evaporation in the modelling, giving the equation:

$$q_i = G\sum_{j=1}^{m} r_{i-j+1} h_j + \alpha . e_i + \varepsilon_i \qquad (2)$$

where q_i, r_i, and e_i are the discharge, rainfall and evaporation respectively at the i^{th} time step, h_j is the j^{th} ordinate of the discrete pulse response function, m is the memory length of the system, G is the gain factor, α is the coefficient of the evaporation term (this can be set to zero if evaporation is to be ignored) and ε_i is the error term. Usually, the sum of the h_j terms is unity.

This is a multiple linear regression of the observed discharge on the m previous observed rainfall values and the current evaporation value. For the pulse response terms, h_j, either a parametric or non-parametric form can be used, and the two-parameters Nash cascade model (Nash, 1957) is used here. The discrete h_j terms are calculated from its impulse response function $h(t)$ which has the following form:

$$h(t) = \left(1/k\Gamma(n)\right)\left(t/k\right)^{n-1} \exp^{-t/k} \qquad (3)$$

where $\Gamma(n)$ is the gamma function.

Thus the SLM, with the pulse response function in parametric form, has four parameters, G, n, k, and α.

5. Soil Moisture Accounting and Routing (SMAR) model

O'Connell et al. (1970) developed a quasi-physical rainfall-runoff model known as the layers model but later on renamed the Soil Moisture Accounting and Routing (SMAR) model. This model consists of two complementary components. The first implements a water balance (the soil moisture accounting procedure) between rainfall, evaporation, runoff, and simulated soil storage for each time step. The second routes the calculated runoff to the catchment outlet, taking account of attenuation and wave diffusive effects. A number of modifications to the original structure of the model have been introduced (Khan, 1986; Liang, 1992) and the latest version by Tan and O'Connor (1996) is used here. It has eight parameters in the water balance component and three parameters in the routing component. In addition, the initial condition of the groundwater storage is considered as a parameter bringing the total number of parameters to twelve.

6. Description of the proposed NFM

Ozelkan and Duckstein (2001) described any catchment model as a system composed of sub-modules to represent the sub-elements of this modelled system coupled together in order to produce a synergic effect reflected at the output of the system. The representation of the catchment model in this modal structure is equivalent to the branching structure in an algorithm flow diagram resulting from 'IF-THEN' fuzzy rules (Gupta and Sorooshian, 1983). In the present work, the aim is not to utilise the 'IF-THEN' fuzzy rules as the model core but rather to improve the performance of deterministic catchment models by using a number of 'IF-THEN' fuzzy rules to create specific localised versions of these models which are better able to respond to local variations in the pattern of temporal and spatial data. The approach is similar to that of Jaquin and Shamseldin (2006) who investigated the combination of different empirical sub-models, using a fuzzy logic model, to account separately for variation in catchment wetness and for catchment seasonality.

In this study, temporal variations are accounted for in a separate modelling scenario, called NFM_T, and the spatial variations in another one, called NFM_S. The NFM structure for both scenarios is similar to the one illustrated in Fig. 1. All NFMs used in this study employ the Gaussian function to represent the membership function of all temporal input variables to the models. This function has the following analytical expression:

$$u(x) = \exp^{-(x-c)^2/2\sigma^2} \tag{4}$$

where $u(x)$ is membership value of a variable x to certain fuzzy sub-set, and parameters c and σ specify the location and spread of the function and require calibration.

As mentioned earlier, the two models, SLM and SMAR, are used in the consequent part of the NFM in both modelling scenarios. It is worthwhile stressing at this point that the resulting consequent part of the NFM for each scenario can be visualised as a collection of either SLM or SMAR sub- or local-models determined according to the IF-THEN rules acting in parallel. Indeed it is the generation of such a configuration, as an alternative method of involving the temporal and spatial pattern variations of the variables in modelling the rainfall-runoff relationship, that is sought in this study.

6.1. NFM_T modelling scenario

In the NFM_T scenario there are two inputs, rainfall and evaporation, and the output, discharge, is calculated using one or other of the catchment models. To distinguish between the NFM_T variant which uses SLM and the other which uses SMAR in the consequent part they are called NFM_T_SLM and NFM_T_SMAR respectively. For each model a total of ten possible rainfall and evaporation fuzzy sub-set combinations are formulated as indicated in Table 1. The performances of all ten cases are evaluated separately for eleven catchments from different parts in the world. Details of these eleven catchments are given in Table 2.

The total number of parameters (*npar*) requiring calibration is determined from

(i) number of fuzzy subsets for the rainfall (nr_{fsub}) and the evaporation (ne_{fsub});

(ii) number of the IF-THEN rules (this is equal to $nr_{fsub} * ne_{fsub}$); and

(iii) number of the model parameters (P) (4 for SLM and 12 for SMAR).

The relation used to calculate *npar* is as follow:

$$npar = 2 * \left(nr_{fsub} + ne_{fsub}\right) + \left(nr_{fsub} * ne_{fsub}\right) * P \qquad (5)$$

The first term in the above equation gives the total number of the Gaussian function parameters for all fuzzy sub-sets while the second term gives the total number of the SLM or SMAR model parameters. Thus there are two sets of parameters that need to be determined by the calibration process. The first set is the parameters of the Gaussian membership functions of the rainfall and evaporation. The second set is the parameters of the models (SLM and SMAR) which are used to relate the rainfall and evaporation (input variables) with the discharge (output variable). The overall optimisation problem is non-linear and it has been found that if the two sets of parameters are determined simultaneously the calibration is often poor. Hence the calibration is performed in a sequential iterative procedure as follows; (i) Initial values are given to the parameters of the SLM and SMAR models, (ii) Holding the SLM and SMAR model parameters constant, the parameters of the Gaussian function sub-sets of the rainfall and evaporation are determined by using the Genetic algorithm (Holland, 1975). (iii) The Gaussian function parameters are then held constant and the parameters of SLM and SMAR models are recalibrated in a second optimisation step. The least squares method is used for the linear optimisation problem required by the NFM_T_SLM whereas the Genetic algorithm is used for the non-linear one in the NFM_T_SMAR. (iv) If the resulting objective function is less than a specified tolerance the calibration stops otherwise step (ii) to (iii) are repeated. Note that the initial values of the parameters of SLM and SMAR models in this case are the ones obtained from the calibration in step (iii).

A split sampling approach was used for model testing, in which the available data for each catchment was split into two parts. The first part (67% of the data) was used in the model calibration while the second (33% of the data) was used in verifying the calibrated models. Two criteria are used in calibration and validation, (i) the Nash-Sutcliffe index (R^2) (Nash and Sutcliffe, 1970) and (ii) the average relative errors (*ARE*) of the estimated discharge peaks over a threshold, conservatively set here as the mean discharge. In addition to these numerical criteria, the observed and the simulated hydrographs for some catchments, for each calendar year, have been plotted to illustrate the fit of the hydrograph shapes.

6.2. NFM_S modelling scenario

Here the performance of the NFM_S model with the SLM and the SMAR sub-models is assessed. The first case is called NFM_S_SLM while the latter is called NFM_S_SMAR. However, unlike the NFM_T scenario the modelled catchment in the NFM_S scenario is divided spatially into a number of Homogenous Hydrologic

Characteristics Units (HHCUs). Although, analogous to Hydrologic Response Units (HRUs) (e.g. Quiroga et al., 1996), HHCUs are defined and determined in a somewhat different way. The inputs to each HHCU are the catchment averages of rainfall and evaporation.

If the rainfall and evaporation for each HHCU are used as fuzzy variables then their fuzzy sub-sets can be used to determine the number of IF-THEN rules in the consequent part of each sub-NFM model for each HHCU. However, as only one fuzzy sub-set is used for rainfall and likewise only one for evaporation the resulting combined sub-NFM models is essentially a model describing different homogenous spatial units, i.e. each IF-THEN rule represents a sub-model describing the rainfall-runoff relationship for a given HHCU and the final estimated runoff value is the weighted sum of the contribution from all the HHCUs. This is a type of semi-distributed modelling that can be easily implemented either within or in conjunction with a GIS by overlaying three map layers, the catchment boundary, land use map, and soil map. The number of the HHCUs obtained with this GIS procedure is based only on elevation, land use and soil type and here they are determined with an innovative approach based on the subtractive clustering algorithm (Vernieuwe et al., 2005).

6.2.1. Determination of the HHCUs for the Brosna catchment

Each HHCU is expected to have a unique rainfall-runoff relation used to estimate its contribution to the catchment outflow. A large number of spatially-related parameters such as elevation, soil permeability, soil roughness, bedrock transmissivity, etc. could influence the rainfall-runoff response and could be used to characterise the HHCU.

However, for this study, the number of such variables is limited to elevation, land use, soil type and these were used to test the NFM_S for the Brosna catchment only. From these three basic maps, four spatial variables are calculated by the GIS (i) elevation, (ii) slope, (iv) land use, and (iv) soil type. Although the original land use map had nineteen different categories, here land use has been aggregated into four main types, agriculture, urban, forest, and wetland. Similarly the slopes obtained directly from the DEM have been assigned to one of three groups: (i) for slopes between 0 % and 8 % a slope index is taken as 4 %; (ii) for slopes between 8 % and 15 % a slope index is taken as 12 %; and (iv) for slope greater than 15 % a slope index is taken as 20 %. The original categories of soil types and elevation bands are used without any changes since they are primary governing parameters in characterising the response to the rainfall.

Various combination alternatives, summarised in Table 3, of the four input spatial variables are passed on to the subtractive clustering algorithm in order to obtain different number of HHCUs. The resolution of the resulting clusters in each combination alternative can be adjusted by changing the parameters in the subtractive clustering algorithm. In this study, the reject ratio (RR) (c.f. Vernieuwe et al., 2005) had the most influence on the cluster resolution. The RR is used by the subtractive clustering algorithm as a stopping criterion to halt any further attempts to determine new clusters. For each combination alternative the RR was varied from 0.1 to 0.5 in increments of 0.1 and from 0.5 to 1 in increments of 0.05. The calculated numbers of clusters are plotted against reject ratio in Fig. 4. It is clear that for all combination alternatives changing the RR value between 0.1 and 0.65 did not change the number of the resulting clusters. Then there is a gradual drop in the number of clusters corresponding to an increase in RR up to 0.8 which is followed again by a constant number of clusters until RR reaches the value of 0.95. The RR value of 1 corresponds to one cluster and this is consistent with a lumped catchment. Note that for the combination alternatives 3A and 4 the number of clusters corresponding to RR values less than and equal to 0.75 is significantly higher than the corresponding values for the other cases.

6.2.2. NFM_S_SLM and NFM_S_SMAR modelling cases

For each combination of spatial variables an upper limit of 40 clusters (shown by section ¢-¢ in Fig. 4) is applied to select cases to be considered in the NFMs tested here. The choice of 40 is aimed to avoid an excessive number of parameters in the NFMs. As the number of clusters remains constant for a range of RR values, the number of cases tested for the NFM_S_SLM and NFM_S_SMAR models in the Brosna catchment, varies from one combination alternative to another (Table 4).

Generally when multiple fuzzy sub-sets are used for banding the rainfall and evaporation then the number of parameters to be calibrated for each case in the NFM_S scenario is obtained by multiplying the number of parameters for the NFM_T scenario, given by Eqn. 5, by the number of clusters or HHCUs involved. However, as one fuzzy sub-set is used for both the rainfall and evaporation in the NFM_S scenario only the parameters of the models (SLM and SMAR) must be calibrated. Therefore there is no need for the sequential iterative procedure used in the NFM_T scenario and instead only the least squares method is used for the linear optimisation problem in the NFM_T_SLM whereas the Genetic algorithm is used for the non-linear one in the NFM_T_SMAR.

7. Results

The key issue is to determine whether the introduction of combined sub-models to account for temporal or spatial pattern variations improves the simulation compared to that of a single lumped catchment model. First, the results corresponding to the lumped case (case 1 in Table 1 for NFM_T, and cases 1 of all combination alternatives in Table 4 for NFM_S) are calculated. These provide a baseline to be used in assessing the second set of results corresponding to the best combined case. In each scenario, the best combined case can be described as the one with the highest R^2 during the calibration period compared to the others in the same group. The best combined case is an improvement over the lumped case if it scores a higher value for the R^2 criterion and a smaller value of the *ARE* criterion. In addition to these two numerical criteria, a graphical comparison of the simulated and the observed hydrographs allowed a visual assessment of model fit.

In addition, the suitability of using a linear model, such as SLM, or a non-linear model, such as SMAR, in the fuzzy model is also addressed in the discussion.

7.1. Results of the NFM_T scenario

7.1.1. Lumped case vs. the best combined case

For the NFM_T_SLM and NFM_T_SMAR models, the R^2 and *ARE* values for the calibration and validation periods are summarised for the eleven test catchments in Table 5. There is an improvement in the R^2 values during calibration for the best combined case over the lumped case. However, the best combined case improved the R^2 values for validation in nine catchments, the exceptions being Halda and Sg.

Bernam, for the NFM_T_SLM and in seven catchments, the exceptions being Halda, Kelantan, Sg. Bernam, and Shiquan-3, for the NFM_T_SMAR model. Only in one of these catchments, Sg. Bernam, the R^2 values during validation of the best combined case were markedly lower than the corresponding values for the lumped case in both the NFM_T_SLM and NFM_T_SMAR models, the differences being insignificant in the rest of the catchments.

For the *ARE* criterion during calibration, the best combined model case was better than the lumped case for the NFM_T_SLM in all but three catchments (Bird Creek, Kelantan, and Sg. Bernam). During validation the combined models of the NFM_T_SLM gave better *ARE* values than the lumped case in five catchments but was worse in six catchments (Blue Nile, Halda, Kelantan, Nan, Sg. Bernam, and Wolombi Brook). The values of *ARE* for calibration of the NFM_T_SMAR model exhibited a consistent improvement of the best combined case over the lumped case whereas the values of the corresponding validation were worse in six catchments (Bird Creek, Halda, Nan, Sg. Bernam, Sunkosi-1 and Wolombi Brook).

The best combined case was not consistent for the NFM_T_SLM and NFM_T_SMAR models. For the former model each of cases 8 and 9 was the best in four catchments while case 10 was the best in three catchments. Different trends was obtained in the latter model as each of cases 4, 5, and 10 was the best in three catchments and case 7 was the best in two catchments.

7.1.2. NFM_T_SLM vs. NFM_T_SMAR

The values of R^2 and *ARE* criteria shown in Table 5 for the eleven catchments and for both the lumped case and the best case did not show which of NFM_T_SLM or NFM_T_SMAR is the overall best model. For the lumped case, the R^2 values for the calibration of NFM_T_SMAR were higher than the values of NFM_T_SLM in all catchments. The same occurred in validation except in two catchments, Sg. Bernam and Wolombi Brook. For the best case, only in Shiquan-3 catchment was the value of R^2 for calibration of NFM_T_SMAR lower than for NFT_T_SLM and in validation the same was true for three catchments, Sg. Bernam, Shiquan-3, and Sunkosi-1.

The *ARE* values showed even more mixed results as NFM_T_SMAR did not outperform SFM_T_SLM in terms of *ARE* for the lumped case at two catchments (Chu and Shiquan-3) for calibration and at five catchments (Blue Nile, Chu, Sg. Bernam, Shiquan-3, and Wolombi Brook) for validation. Similar results hold for the best combined case in calibration. It holds also in validation but with the addition of two more catchments (Halda and Sunkosi-1).

7.1.3. Hydrographs matching in the NFM_T scenario

The observed and simulated hydrographs of the best combined cases of NFM_T_SLM and NFM_T_SMAR for four catchments, Blue Nile, Brosna, Chu, and Wolombi Brook, are plotted in Figs. 5 to 8. Each of the four catchments exhibits different hydrological behaviour and this is reflected in the shape of its hydrograph. In addition, the period of each hydrograph is chosen to be within the validation period for two reasons: (i) to verify the model parameters; and (ii) to ensure minimal influence of the initial conditions on the models comparison.

The four graphs demonstrate the ability of the NFM_T_SMAR to capture most of the hydrograph features. This model showed an outstanding performance in reproducing the observed hydrograph in the Chu catchment, (Fig. 7), and to some extent the one in the Blue Nile catchment, (Fig. 5). However, in the Brosna and Wolombi Brook (Figs. 6 and 8 respectively), features such as rising limb, recession, and base flow were better generated by this model than the individual peak values.

The NFM_T_SLM was able to match the non-linearity in the two flashy catchments, Chu and Wolombi Brook, Figs. 7 and 8 respectively. In contrast, this model, with its linear component, was particularly bad for the Brosna, which has a large base flow component, and for the Blue Nile, which has a strong seasonal pattern.

7.2. Results of the NFM_S scenario in the Brosna catchment

7.2.1. Lumped case vs. the best combined case

Table 6 shows the values of the R^2 and *ARE* model efficiency criteria for the NFM_S_SLM and the NFM_S_SMAR model for both calibration and validation and for the lumped case and the best case for all spatial combination alternatives in the Brosna catchment. The results for the lumped case (treating the catchment as a single unit) of each of the NFM_S_SLM and the NFM_S_SMAR models were identical for all combination alternatives since this involved a single HHCU and only one fuzzy sub-set for rainfall and evaporation.

The R^2 results for calibration and validation for the NFM_S_SLM do not differ significantly from each other. In contrast, for the NFM_S_SMAR the R^2 values for calibration for the best cases were significantly higher than for the lumped case. In validation, a significant improvement in R^2 was obtained by the best case of the combination alternatives 2A, 2B, and 4 for which the *ARE* values were amongst the lowest. There were no significant differences among the *ARE* values for the NFM_S_SLM and likewise among those of the NFM_S_SMAR models. However, the *ARE* values of the NFM_S_SLM were all much greater than those of the NFM_S_SMAR. The results in Table 6 suggest that, while the 2A combination alternative performs significantly better than the lumped case, the improvement is not as impressive as that obtained for the NFM_T scenario.

7.2.2. NFM_S_SLM vs. NFM_S_SMAR

The superiority of SMAR over SLM can be easily seen from the R^2 and the *ARE* values. The introduction of non-linearity in the SLM through the combination of its sub-models did not produce any significant improvement. This is not surprising because the use of HHCUs in this context has no effect on the SLM itself but it rather assigns weights to similar sub-models with the same characteristics as the lumped model. In contrast, in the SMAR model each sub-model adds to the non-linearity of the combined model and this in turn provides the greater flexibility required in modelling the rainfall-runoff relationship.

For both NFM_S_SLM and NFM_S_SMAR models, using large number of HHCUs, i.e. sub-models, did not improve the results significantly and this means there is an upper limit for the number of HHCUs above which no significant improvement can be expected. Thus using an excessive number of HHCUs might result in including some redundant HHCUs which add little to the model's performance. Again this behaviour is not surprising because the spatial parameters of the HHCUs have no influence on the models. Different responses would be expected if some inputs to the sub-models depended on the characteristics of HHCUs.

7.2.3. NFM_T vs. NFM_S

The important question arising out of the results for the two combination scenarios is which combination NFM scenario performs best. To answer this requires a comparison between the best models of the two scenarios. For illustration only, we do this here for the Brosna catchment as the NFM_S was applied for that catchment only. From Table 5 and 6 it is possible to identify the NFM_T_SMAR_4 (case 4) and NFM_S_SMAR_4_14 (combination alternative 4 and HHCUs = 14) as the best models for the two scenarios respectively in the Brosna catchment. The R^2 and *ARE* results for these two models are not substantially different from each other. The fit between the observed hydrograph and the simulated hydrographs for each model are shown in Fig. 9 and they represent the same period used in Fig. 4. The visual comparison between the observed hydrograph and the two models does not show any major differences between models to the extent that one can be declared consistently superior to the other. Thus the use of the NFM_S scenario, which requires more data than the NFM_T scenario, is not justified if the intention of the modelling is to produce outputs only for the outlet of a catchment.

8. Conclusions

In this study, the NFM has been proposed to account for spatial and temporal variations in modeling the rainfall-runoff relationship. The proposed procedure was implemented with two simple lumped models, SLM and SMAR. For each model two scenarios (NFM_T and NFM_S) were used to construct sub-models to address the temporal and spatial pattern variations respectively. In the NFM_T scenario, the two models NFM_T_SLM and NFM_T_SMAR, were applied to eleven catchments from around the world. A split sample technique was used and in most cases the neuro-fuzzy combined sub-models were better than the lumped model. The NFM_T_SMAR model was, in general, better than the NFM_T_SLM.

To address spatial variation in response, a subtractive clustering algorithm was used in the NFM_S scenario to derive a number of HHCUs which exhibit homogenous hydrologic responses. Three spatial layers representing DEM, land use and soil maps of the Brosna catchment (Ireland) have been processed by a GIS software to prepare data of four variables (elevation, slope index, generalised land use types, and soil types) used in the clustering algorithm. For all possible combination alternatives between the four variables the relation between the reject ratio parameter (RR) of the subtractive clustering algorithm and the resulting number of HHCUs was investigated. A remarkable improvement was achieved by the best case of the sub-models of NFM_S_SMAR compared to the lumped model. The NFM_S_SMAR model significantly outperformed the NFM_S_SLM and this is probably due to its inclusion of non-linearity. Only a small number of HHCUs were required to obtain improved results and using a larger number of HHCUs did not improve the results of the NFM_S_SMAR model.

9. Further work

This work has shown that combinations of relative simple models can extend their ability to model a range of catchment behaviour without requiring fully distributed time-varying, physically-based models. While the combination approach has proved useful in our Brosna catchment, it should be applied to other catchments with a wider range of climatic variation and conditions to test its generality. In addition, it should be possible to extend the approach to modelling other types of data, particularly water quality time series, where the output at a single point is all that is required. In such cases, the effort to generate and calibrate a physically-based distributed model may not be justified and a calibrated combination of simple models may suffice. The method can be used in all investigations that compare time-series or model the relationship between two time-series, such as investigating tele-connections between climate variables at different locations.

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Figure Captions

Figure 1. Structure of quasi-linear model proposed by Bruen (1985)

Figure 2. Sub-models combination using fuzzy logic principle of membership function

Figure 3. NFM architecture

Figure 4. Number of clusters vs. reject ratio (RR) for all combination alternatives used in the subtractive clustering algorithm

Figure 5. Simulated and observed hydrographs of the Blue Nile catchment

Figure 6. Simulated and observed hydrographs of the Brosna catchment

Figure 7. Simulated and observed hydrographs of the Chu catchment

Figure 8. Simulated and observed hydrographs of the Wolombi Brook catchment

Figure 9. Comparison between NFM_T and NFM_S best models in the Brosna catchment

Tables

Table 1. Cases representing the rainfall and evaporation fuzzy sub-sets combination for the NFM_T_SLM and NFM_T_SMAR

Model	Case*	No of fuzzy sub-sets	
		Rainfall	evaporation
NFM_T_SLM_*, NFM_T_SMAR_*	1	1	1
	2	1	2
	3	2	1
	4	2	2
	5	3	1
	6	3	2
	7	3	3
	8	4	4
	9	5	5
	10	6	6

Table 2. Details of the test catchments

Catchment name	Country	Area (km^2)	Starting date of data	No. of data points	Memory length (day)
Bird Creek	USA	2344	1-Jan.-1955	2922	15
Blue Nile	Sudan	175125	1-Jan.-1992	1461	15
Brosna	Ireland	1207	1-Jan.-1969	3652	30
Chu	Vietnam	2370	1-Jan.-1965	3652	15
Halda	Bangladesh	779	1-Jan.-1980	2556	15
Kelantan	Malaysia	12867	1-Jan.-1975	2922	20
Nan	Thailand	4609	1-Jan.-1978	3287	20
Sg. Bernam	Malaysia	1090	1-Jan.-1977	2556	25
Shiquan-3	China	3092	1-Jan.-1973	2922	15
Sunkosi-1	Nepal	18000	1-Jan.-1975	2922	30
Wolombi Brook	Australia	1580	1-Jan.-1963	1826	15

Table 3. Combination alternatives of the four spatial variables used in the subtractive clustering algorithm

id.	No of variables	Variables
2A	2	Elevation + Land use
2B	2	Elevation + Soil
2C	2	Slope index + Land use
2D	2	Slope index + Soil
2E	2	Land use + Soil
3A	3	Elevation + Land use + Soil
3B	3	Slope index + Land use + Soil
4	4	Elevation + Slope index + Land use + Soil

Table 4. Description of the NFM_SPT_SLM and NFM_SPT_SMAR cases tested in the Brosna catchment for each combination alternative

Model	Case*	No of HHCUs
NFM_S_SLM_2A_*, NFM_S_SMAR_2A_*	1	1
	2	9
	3	12
	4	29
	5	36
	6	37
NFM_S_SLM_2B_*, NFM_S_SMAR_2B_*	1	1
	2	32
NFM_S_SLM_2C_*, NFM_S_SMAR_2C_*	1	1
	2	10
NFM_S_SLM_2D_*, NFM_S_SMAR_2D_*	1	1
	2	2
	3	3
	4	4
	5	15
NFM_S_SLM_2E_*, NFM_S_SMAR_2E_*	1	1
	2	4
	3	5
	4	7
	5	20
NFM_S_SLM_3A_*, NFM_S_SMAR_3A_*	1	1
	2	23
NFM_S_SLM_3B_*, NFM_S_SMAR_3B_*	1	1
	2	5
	3	6
	4	8
	5	37
NFM_S_SLM_4_*, NFM_S_SMAR_4_*	1	1
	2	14

Table 5. R^2 and *ARE* results for the lumped case and the best combined case of the NFM_T_SLM and NFM_T_SMAR models in the eleven catchments

Model	Catchment	Best case	R^2				ARE			
			Lumped case		Best combined case		Lumped case		Best combined case	
			Calib.	Valid.	Calib.	Valid.	Calib.	Valid.	Calib.	Valid.
NFM_T_ SLM	Bird Creek	9	15.86	23.34	24.37	42.54	71.99	81.57	72.99	79.31
	Blue Nile	10	71.69	71.38	87.44	77.82	28.94	22.45	25.43	24.08
	Brosna	8	49.36	32.36	60.24	40.53	29.92	34.37	28.35	33.27
	Chu	9	15.23	29.13	39.27	56.95	57.64	55.73	55.01	52.72
	Halda	8	53.43	69.84	67.63	67.78	38.95	49.57	38.60	52.75
	Kelantan	10	28.88	22.78	52.39	34.78	33.16	29.25	26.79	30.45
	Nan	10	65.29	68.94	69.57	69.54	39.05	33.69	39.98	44.46
	Sg. Bernam	8	60.35	52.14	62.73	47.90	24.88	26.77	27.00	31.47
	Shiquan-3	8	13.45	6.32	28.33	24.40	54.16	49.95	51.76	49.80
	Sunkosi-1	9	77.80	78.78	80.73	82.10	27.86	25.95	27.35	23.55
	Wolombi	9	10.27	-17.03	30.03	17.31	80.33	71.88	71.60	92.62
NFM_T_ SMAR	Bird Creek	7	85.85	66.58	89.72	75.27	67.70	60.50	67.45	63.73
	Blue Nile	4	93.26	83.00	94.57	86.53	17.37	29.65	16.22	24.28
	Brosna	4	87.93	83.86	89.81	86.18	15.66	19.17	15.37	18.01
	Chu	10	35.30	43.20	81.46	64.72	69.68	70.52	59.56	65.51
	Halda	10	62.42	69.56	83.76	68.82	34.45	43.35	33.26	54.25
	Kelantan	5	84.67	47.70	87.26	46.81	20.06	27.70	19.70	27.63
	Nan	7	76.36	80.48	83.88	80.70	34.71	26.29	33.29	27.30
	Sg. Bernam	4	73.51	21.49	76.40	5.93	23.38	43.05	23.22	46.19
	Shiquan-3	5	19.69	17.96	23.32	17.24	78.37	83.93	75.46	79.82
	Sunkosi-1	5	80.49	79.90	82.78	80.28	26.57	24.93	25.64	25.56
	Wolombi	10	34.74	-33.82	89.15	58.39	70.41	108.54	59.11	112.46

Table 6. *R^2* and *ARE* results for the lumped case and the best combined case for all the combination alternatives of the NFM_S_SLM and NFM_S_SMAR models in the Brosna catchment

Model	id.	case	No of HHCUs	R^2		ARE	
				Calib.	Valid.	Calib.	Valid.
NFM_S_SLM	2A,2B,…,4	1	1 (lumped model)	49.36	32.36	29.92	34.37
	2A	2	9	50.18	32.68	29.47	33.98
	2B	2	32	48.87	31.04	30.26	34.71
	2C	2	10	50.07	32.65	29.54	33.99
	2D	3	3	50.44	32.93	29.15	33.76
	2E	3	5	50.42	32.92	29.29	33.90
	3A	2	23	49.26	31.75	30.29	34.75
	3B	2	5	50.22	32.95	29.34	33.90
	4	2	14	49.94	32.35	29.81	34.32
NFM_S_SMAR	2A,2B,…,4	1	1 (lumped model)	87.96	84.18	15.44	18.60
	2A	6	37	91.17	87.91	13.68	16.38
	2B	2	32	90.25	86.50	14.08	16.67
	2C	2	10	90.31	82.90	14.19	19.64
	2D	4	4	91.28	85.82	13.63	17.86
	2E	4	7	91.16	84.53	13.86	19.23
	3A	2	23	90.67	85.68	14.44	18.03
	3B	3	6	91.23	85.59	13.99	17.98
	4	2	14	91.42	86.00	13.47	17.57