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Yue Zhang
Technological University Dublin

Aidan O'Dwyer
Technological University Dublin, aidan.odwyer@tudublin.ie

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Estimation of the parameters of a delayed process using an open loop time domain technique

Y.Zhang, A.O’Dwyer.

School of Control Systems and Electrical Engineering, Dublin Institute of Technology, Dublin8, Ireland.

Abstract

All process investigations should start by a determination of the process model. The modeling strategy used will influence the value of the model parameters, which will in turn affect the controller values determined from tuning rules. This paper will describe an analytical method to identify a model of the dynamic characteristics of a process with a delay by using a tangent and point method, as originally suggested by Murrill, 1967. The methodology has been applied to eight benchmark transfer functions suitable for testing PID controllers, as suggested by Åström and Hägglund, 2000. The main advantages of the modeling approach are that it is simple and it will give more accurate, repeatable results than one based on the more traditional graphical approach.

Introduction

The identification of a model of the dynamic characteristics of the system is a fundamental component of any model-based controller design. The modeling strategy used will influence the value of the model parameters, which will in turn affect the controller values determined from tuning rules. The alternative tangent and point method, as originally suggested by Murrill, 1967, is known to be a relatively straightforward method of process modeling. Model parameters are estimated from the open loop process step response. The intermediate values determined from the graph are the magnitude of the input change, the magnitude of the steady-state change in the output and the inflection point, which is at the maximum slope of the output-versus-time plot. A design approach, which performs graphical process reaction curve calculations, may be used to get the inflection point. It cannot give a satisfactory result if the initial design data are uncertain and inaccurate. Instead, an analytical method is proposed to identify a model of the dynamic characteristics of a process with a delay. The method is evaluated using MATLAB/SIMULINK. A MATLAB program has been written to identify the models of eight benchmark transfer functions, in both an ideal and noisy environment. These benchmarks were chosen through a process of experimentation over the course of a number of years that yielded a large collection of test examples, which have been used for research and for evaluation of commercial systems (Åström and Hägglund, 2000).

Modelling Method

Parametric identification methods are techniques to estimate parameters in given model structures. Basically, these techniques involve finding (by numerical search) those numerical values of the parameters that give the best agreement between the output of model (simulated or predicted) and the measured output. Model sets or model structure are usually expressed in a first order lag plus time delay (FOLPD) transfer
function form. The form of this model is as follows, with \( X(s) \) denoting the input and \( Y(s) \) denoting the output, both expressed in deviation variables.

\[
\frac{Y(s)}{X(s)} = \frac{K_m}{1 + T_m s} e^{-\tau_m s} \tag{1}
\]

where, \( K_m \) is the steady state gain; \( T_m \) is the time constant; \( \tau_m \) is the time delay.

![Process reaction curve of a step input response](image)

Figure 1: Finding the inflection point and showing the graphical calculations

Differentiating the output is the simplest and the most direct way to get the slope of the output for an ideal process when no noise is present on the process output signal.

Once the slope (\( m \)) of the line and one point \( (t_1, y_1) \) of the line are known, the equation of the line, which is named as ‘inflection line’, in that two-dimensional plane is found. For any other point \( (t, y) \) on the line, the slope \( m \) is, by definition, the ratio of \( (y - y_1) \) to \( (t - t_1) \). Thus for any point \( (t, y) \) on this line,

\[
m = \frac{y - y_1}{t - t_1} \tag{2}
\]

The point \( (\tau_m, 0) \) yields the value \( t = \tau_m \) when \( t = 0 \). This point lies on the inflection line and \( \tau_m \) is the \( t \) coordinate of the intersection of that line and the \( t \) axis.

According to the alternative tangent and point method approach, the time delay is the time value from the magnitude of the input change to the time at which the inflection line intercepts the \( t \) axis; the time constant is counted after the time delay to the time at which the output reaches 63 percent of its final value.

Using the graphical calculations shown in figure 1 and equation 1:

\[
m = \frac{y_1 - 0}{t_1 - \tau_m} \tag{3}
\]

From this equation, \( \tau_m \) is determined as:

\[
\tau_m = t_1 - \frac{y_1}{m} \tag{4}
\]

Then \( T_m \) is:

\[
T_m = t_{63\%} - \tau_m \tag{5}
\]

The Steady-State Gain \( K_m \) is the ratio of the steady state change in output to the steady state change in input.

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**The use of a filter in the method**

Consider the open-loop system shown in figure 2. The input signal and output measurement are corrupted by noise. Both these noise signals are of zero mean value. The signal-to-noise ratio is approximately 5.

![An open loop system corrupted by noise](image)

The main drawback of the identification algorithm is its sensitivity to inaccuracies in the process output, which is affected by noise and disturbances. Unfortunately, in the process industries, either before or after signal transmission, the transmitted
signal represents the result of many effects; the output data is usually corrupted by high-frequency noise. The output noise may be of rather large amplitude with respect to the signal level. One way of eliminating the high-frequency noise is to cascade a low-pass filter with the plant. The filter is implemented by the equation \( G_f(s) = \frac{1}{\tau_f s + 1} \) (7), where \( \tau_f < 0.05(T_m + \tau_m) \) (8).

The gain is unity because the filter should not alter the actual signal at low frequency, including the steady state. The filter time constant is selected to be small with respect to the dominant process dynamics so that the open-loop transfer function is not significantly changed. Also, the filter time constant should be large with respect to the noise period so that noise is attenuated (Marlin, 1995). These two requirements cannot usually be satisfied perfectly, because the signal has components of all frequencies and the cut-off frequency is not known. Thus, a fourth order low-pass filter (equation 9) is employed in the loop, in terms of maintaining the small time constant. The magnitude of the frequency response of this filter decreases by 80 dB for every increase in frequency of a factor of 10, resulting in a narrowing of the transition band.

\[
G_f(s) = \frac{1}{(\frac{\tau_f}{4} s + 1)^4} \quad (9)
\]

**Simulation and results**

To evaluate the efficiency of the method, this methodology has been applied to eight benchmark transfer functions in both an ideal or noisy simulation environment.

The 8 benchmark transfer functions (Åström and Hägglund [1]) are:

BM1: \( G(s) = \frac{1}{(s+1)^4} \)

BM2: \( G(s) = \frac{1}{(s+1)^{10.5}} \)

BM3: \( G(s) = \frac{1}{(s+1)(0.5s+1)(0.25s+1)(0.125s+1)} \)

BM4: \( G(s) = \frac{1}{(s+1)(0.2s+1)(0.04s+1)(0.008s+1)} \)

BM5: \( G(s) = \frac{1}{(s+1)(0.5s+1)(0.2s+1)(0.04s+1)(0.008s+1)} \)

BM6: \( G(s) = \frac{1}{(s+1)^2} \)

BM7: \( G(s) = \frac{1}{(0.5s+1)^2} \)

BM8: \( G(s) = \frac{1}{(2s+1)^2} \)

Figure 3 demonstrates the usefulness of the filter cascade after both the output and the derivative of the output for a representative example. Although the derivative of the output with noise is variable compared to the derivative of the output without noise, the time at which the maximum slope of the output-versus-time graph occurs, determined from the derivative of the output with noise, is quite close to that determined from the derivative of the output without noise.

Evaluation of this time is the only requirement from the derivative plot of the output for the modeling methods. The plot of the filtered output has a small “right shift” because of the effect of filtering. The model parameters can be evaluated by using the same calculation procedure as detailed previously. The comparison of the model parameters, for the non-noise and noise simulations, is displayed in figure 4.

Figure 3: The process reactor of BM5 ideal system & with the signal-to-noise ratio is 5

Figure 4: The comparison of the model parameter
Model validation

The goal is to develop a model that describes the input-output behaviour of the process adequately. The final check on the model is to look at both the time domain and frequency domain responses to show a validation of the process versus the model. The validation tests performed are:

- A simulation comparison of the step responses of these models, with the responses of the ideal process (i.e. processes without noise) and the processes with noise.
- A simulation comparison of the frequency responses of these models with the processes (with and without noise).

As an example, the results of the BM7 validation test are given in figure 5. Good agreement between the model and the process is obtained.

Conclusion

The philosophy of controller design is that controller parameters depends on the process model. Thus, how well the model faithfully represents the process is very important. This paper develops an automatic implementation of alternative tangent and point modelling method in either an ideal or noisy environment. The analytical method described, implemented using a MATLAB programme, is simple and will give more repeatable results than one based on the more traditional graphical approach. The method is simple when there is no noise on the process. However, in the process industries, it is not possible to find this condition. The filter used separates the signal from the noise, and reduces the noise amplitude. Selection of the filter time constant is a compromise between achieving good noise attenuation and preventing the open-loop transfer functions from changing significantly. Note that the model parameters obtained for the process without noise are similar to the results obtained when the process has added noise, but that slightly different values are determined for the dead time and time constant because it is impossible to separate the signal from the noise completely.

References