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Impact of a national professional development programme for out-of-field teachers of mathematics in Ireland

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Out-of-field teaching refers to the practice of assigning teachers to teach subjects that do not match their training or education. This paper reports on a study evaluating the impact of a national professional development programme for out-of-field teachers of post-primary school mathematics in Ireland – the Professional Diploma in Mathematics for Teaching. Evidence of impact was collected from three surveys. Two surveys evaluated changes in the prevalence of out-of-field teaching before and six years after the introduction of the programme. The third survey investigated programme graduates' beliefs about mathematics, mathematics teaching and mathematics learning, and reported changes in teaching practices. Outcomes of the programme included a reduction in out-of-field teaching of mathematics and increased opportunities for graduates to teach higher level mathematics in the senior post-primary years. These teachers also endorsed child-centred beliefs and reported teaching practices consistent with the problem-solving orientation of the new mathematics curriculum. The findings go some way towards testing a theory of teacher change in order to enhance our understanding of how professional development works to upskill out-of-field teachers.

Keywords: Out-of-field teaching; mathematics teachers; professional development; beliefs; classroom practices

Out-of-field teaching is an international phenomenon that involves teachers being assigned to teach subjects that do not match their training or education (Ingersoll 2002). This practice can arise when there is mismatch between supply and demand of appropriately qualified, subject-specialist teachers, in particular, where there are insufficient specialist teachers of mathematics. These teachers are referred to as out-of-field teachers of mathematics: generally, such teachers possess a teaching qualification but have limited advanced studies of mathematical content and little or no specific training in mathematics education.

Research highlighting the complexities involved in understanding and addressing out-of-field teaching is beginning to emerge in many countries (Hobbs and Törner 2019). There is also growing recognition of the need for professional development programmes that meet the specific needs of out-of-field teachers (Du Plessis,

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Gillies, and Carroll 2015). To date, however, there has been little research on the effectiveness of such programmes (Faulkner et al. 2019). Thus, the aim of this paper is to contribute to the emerging literature on effective professional development for out-of-field teachers of mathematics. We report on aspects of a study that is evaluating the impact of a long-term, large-scale, government-funded, university-accredited programme offered nationally – the Professional Diploma in Mathematics for Teaching (PDMT).

Background to the study

A national system of primary and post-primary school education operates in the Irish context. At post-primary level, students (aged 12–18 years) undertake six years of schooling comprising three years in Junior Cycle and three years in Senior Cycle.[1] A combination of two formal in-school assessments and a final state examination results in the award of a Junior Cycle Profile of Achievement at the end of Year 3. At the end of Senior Cycle, students sit a further state examination leading to the award of a Leaving Certificate that is recognised for further study and progression both nationally and internationally. Although mathematics is not a compulsory subject at post-primary level, virtually all students study mathematics at some level. In Junior Cycle, mathematics is offered at Ordinary or Higher Level, while in Senior Cycle three levels of mathematics are offered – Foundation, Ordinary, or Higher Level.

There are two entry routes into post-primary teacher education in Ireland, with both routes usually qualifying graduates to teach as a specialist in two subject areas. In the concurrent route (four-year undergraduate degree) pre-service teachers study both the required subject content and teacher education components. In the consecutive route, pre-service teachers complete undergraduate degree level studies in their chosen subject area(s) and then apply for entry to a two-year Master's degree (since 2012) in which the focus is on educational and pedagogical studies.

In Ireland, concerns about underperformance in post-primary school mathematics at the beginning of the twenty-first century led to the introduction in 2010 of a new curriculum that shifted emphasis away from memorisation and procedures towards understanding and problem-solving (National Council for Curriculum and Assessment 2005). Concurrently, the Teaching Council of Ireland (2013) introduced new accreditation requirements for initial teacher education programmes. In mathematics, fully qualified teachers must have a degree-level qualification with at least one-third of the degree comprising of the specific study of mathematics. There are also minimum credit requirements in analysis, algebra, geometry, and probability and statistics with additional credits to be obtained in a variety of optional topics. Despite these strict requirements, school principals in Ireland have autonomy in recruiting staff and assigning teachers to subjects and classes, thus leaving open the possibility of placing teachers in out-of-field positions.

Ní Ríordáin and Hannigan (2009) speculated that the phenomenon of out-of-field teaching of mathematics could be a possible obstacle to achieving the goals of the new mathematics curriculum. In the international context, measuring the extent of out-of-field teaching is not as straightforward as it may appear, and such measures can vary according to how a 'qualified' teacher is defined; how measures define the boundary between teaching fields; whether they focus on the number of teachers in out-of-field positions, or the number of classes or students taught by out-of-field teachers; and which school grades or year levels are included in the analysis (Ingersoll 2019). Ní Ríordáin and Hannigan (2009) conducted a national survey of teachers of mathematics in Irish post-primary schools, collecting data on respondents' teaching assignments, degree qualifications and the subjects they were qualified to teach according to the requirements specified by the Irish Teaching Council. This survey established that 48% of respondents were teaching mathematics without the necessary subject-specific qualifications, thus providing a measure of the number of teachers in out-of-field positions. In response to this finding, the Department of Education and Skills (DES) funded the PDMT to develop the content and pedagogical content knowledge of out-of-field teachers of mathematics to the level required by the Teaching Council. The programme also aims to support participants in developing beliefs and pedagogical practices aligned with the goals of the new mathematics curriculum. Six cohorts comprising almost 1100 teachers have participated in the PDMT since it began in 2012. This represents around 20% of the estimated number of teachers in post-primary schools who are assigned to teach mathematics classes. Although there are a number of emerging international models of professional development for out-of-field teachers (e.g. Vale, McAndrew, and Krishnan 2011; Crisan and Rodd 2017; Faulkner et al. 2019), to our knowledge Ireland offers the only example of a nationally consistent, government-funded, university-accredited programme that is available to teachers throughout the country. The scale and longevity of the PDMT provide a unique opportunity to investigate its impact on the teachers who have graduated from the programme.

In conceptualising impact, we draw on Desimone's (2009, 184) path model for studying the effects of professional development on teachers and students. This model proposes the following steps linking inputs with intermediate and final outcomes:

1. Teachers experience effective professional development (characterised by content focus, active learning, coherence, sustained duration, and collective participation).
2. The professional development increases teachers' knowledge and skills and/or changes their attitudes and beliefs.
3. Teachers use their new knowledge and skills, attitudes, and beliefs to improve the content of their instruction or their approach to pedagogy, or both.
4. The instructional changes foster increased student learning.

Desimone (2009) notes that it is rare for a single study to investigate all four elements of this model; in particular, there are significant methodological difficulties in designing evaluations that measure the specific effects of professional development on student achievement. Our own research has concentrated on analysing the critical features of the PDMT programme (Step 1 in Desimone's model; see Goos et al. 2020) and its effect on the teachers who participated in the programme (Steps 2 and 3; see Lane and Ní Ríordáin 2020; Ní Ríordáin, Paolucci, and O'Dwyer 2017; O'Meara and Faulkner 2021). In this paper, we further examine the impact of the PDMT on teachers' *beliefs* and *pedagogical approaches* as key elements in Desimone's (2009) model of teacher change. Given the well-known challenges of empirically measuring out-of-field teaching (Ingersoll 2019), we argue that it is also important to track changes in the *prevalence of out-of-field teaching of mathematics* in Irish post-primary schools since the introduction of the PDMT. Such information is of vital interest to the education policy makers who funded the programme with the goal of reducing the incidence of out-of-field teaching of mathematics.

The research questions guiding this investigation are:

(1) What changes were observed in the prevalence of out-of-field teaching of mathematics in Ireland during the time in which the PDMT was delivered?
(2) What epistemological and pedagogical beliefs are held by formerly out-of-field teachers of mathematics who have completed the PDMT?
(3) What approaches to teaching mathematics are reported by PDMT graduates, and how have their perceptions of their classroom practices changed since they completed the programme?

The next section provides a brief outline of the PDMT's content, structure, and delivery modes. We then discuss our theoretical rationale for studying mathematics teacher beliefs and their relationship to classroom practices in the context of out-of-field teaching, before describing our methodological approach, findings, and implications of the study.

The Professional Diploma in Mathematics for Teaching

The PDMT is a two-year, part-time, blended learning programme worth 75 ECTS credits.[2] Delivery of the programme is led by the University of Limerick in conjunction with a national consortium of higher education institutions. PDMT participants teach full-time in their schools while they undertake the programme in the evening, on weekends and during school vacations via a blended learning approach. They are not given time off from their teaching duties to meet study commitments, because release time for study is not generally available to teachers participating in professional development programmes in Ireland. Teachers' tuition fees are fully funded by the DES and participation in the programme is voluntary, but strongly motivated by the prospect of gaining continuing, full-time employment as fully qualified teachers of mathematics.

The content comprises ten mathematics content modules delivered online with additional face-to-face and online support and two mathematics pedagogy modules delivered face-to-face. The mathematics modules, each worth 6 ECTS credits, are presented in 30-hour blocks in six-week sessions (24 lectures, 6 tutorials) and cover topic areas such as calculus, algebra, probability, statistics, geometry, problem solving and modelling, and history of mathematics. The two mathematics pedagogy modules, worth 6 and 9 ECTS credits respectively, are spread over a full academic year and summer. For these latter modules, compulsory attendance is required at five 3-hour workshops and a week-long summer school. The mathematics pedagogy modules develop teachers' pedagogical content knowledge and emphasise classroom practices that support problem-solving and promote conceptual understanding (Stigler and Hiebert 2004). Each mathematics pedagogy module is explicitly linked to the corresponding mathematics content module. One of the pedagogy modules also requires participants to complete a supervised action research project on their practice in the mathematics classroom. (See Goos et al.

2020 for an analysis of the design principles underpinning development and delivery of the PDMT).

Conceptualising teacher beliefs and their relationship with practices

Teachers' beliefs have received considerable attention from researchers in recent decades (Zhang and Morselli 2016), particularly in light of the relationship between teachers' beliefs and their classroom practices (Beswick 2012) and resultant impact on students' experience and learning (Kunter et al. 2013). While no single definition of 'belief' exists, various researchers view beliefs as a cognitive construct, related to knowledge (e.g. Furinghetti and Pehkonen 2002). Beliefs differ from knowledge, however, in that they incorporate affective and evaluative components, captured by Pajares (1992, 316) as 'an individual's judgement of the truth or falsity of a proposition, a judgement that can only be inferred from a collective understanding of what human beings say, intend, and do'. Teachers' pedagogical beliefs 'develop during the many years teachers spend at school, first as students, then as student teachers and teachers, and over time and use, these beliefs then become robust' (De Vries, Van De Grift, and Jansen 2014, 339).

Beliefs about teaching and learning alone, however, do not fully capture the relationship between teacher beliefs and practices in the mathematics classroom. Beswick (2012) has highlighted the importance of teachers' epistemological beliefs about the nature of mathematics in considering the impact on their classroom praxis. Teachers' beliefs about mathematics have been categorised by Ernest (1989) as the Instrumentalist view (mathematics as an unrelated collection of rules and procedures to be followed), the Platonist view (mathematics as an established and unified body of knowledge), and the Problem-solving view (mathematics as a dynamic and creative process, socially and culturally constructed). In his study of out-of-field teachers of mathematics in Germany, Bosse (2014) found a prevalence of the Instrumentalist view of mathematics, with little or no evidence of the Problem-solving view.

In an Australian study, Perry, Howard, and Tracey (1999) developed and validated a model for teachers' beliefs which incorporates beliefs about mathematics (epistemological beliefs), mathematics teaching and mathematics learning (pedagogical beliefs). From their factor analysis of survey responses made by head teachers of mathematics and classroom mathematics teachers, they identified two categories of beliefs they labelled as transmission and child-centredness. Transmission beliefs are consistent with a view of mathematics teaching and learning as the transfer of knowledge from teacher to learner and tends to adhere to Ernest's (1989) Instrumentalist and/or Platonist views of mathematics. Child-centred beliefs, on the other hand, reflect a view of learners as actively constructing their own knowledge of mathematics, facilitated by teachers and encompassing a more Problem-solving view of mathematics as described by Ernest. These two categories of beliefs appear in various other models of teachers' beliefs and are not unique to the study by Perry, Howard, and Tracey (1999); however, the authors make it clear that, unlike in some other perspectives, transmission and child-centredness are conceptualised as independent factors and not as opposite ends of a single belief spectrum. We chose to use their survey because their model allows for meaningful analysis of teachers' beliefs while taking into account the sometimes contradictory nature of those beliefs (Sosniak, Ethington, and Varelas 1991).

Teachers' beliefs are critical in shaping classroom practice (Speer 2005). This is not to deny that inconsistencies also occur, which might stem from the fact that beliefs are self-reported while practices can be observed (Zhang and Morselli 2016). Others have highlighted the importance of the teachers' contexts in determining which beliefs they enact in their practices (Beswick 2004), including the context of teaching mathematics out-of-field. In particular, Lane and Ní Ríordáin (2020) analysed the action research reports produced by PDMT participants and found evidence of teacher development in the form of a majority shift towards constructivist beliefs and practices. However, they also highlighted the complex interplay between the teachers' reflective self-study and their interpretation of constraints within their professional environment. Mathematics teachers' self-efficacy beliefs are also related to their classroom practice. O'Meara and Faulkner (2021) investigated changes in PDMT participants' self-efficacy beliefs and self-reported teaching practices after engaging with the programme's mathematics pedagogy modules. This study, which focused on only one cohort of PDMT participants, found substantial improvements in self-efficacy beliefs that were accompanied by changes in mathematics teaching approaches, moving away from teacher-led, procedural methods and towards student-centred methods that emphasise conceptual understanding.

An earlier study undertaken by Ní Ríordáin, Paolucci, and O'Dwyer (2017) examined the mathematical knowledge of out-of-field teachers on commencement of the PDMT, finding that participating teachers exhibited low levels of knowledge in relation to curriculum-aligned mathematical content and high levels of conceptual errors. The analysis also revealed that, despite demonstrating difficulties with the content of the curriculum, out-of-field teachers reported feeling somewhat to very confident in teaching mathematics upon enrolling in the PDMT. This suggests a culture in Irish schools that supports the belief that mathematics is a subject that can be taught well, even without advanced studies of mathematical content or mathematics-specific pedagogy. Each of these studies highlights the importance of attending to context when investigating teacher beliefs and the relationship between beliefs and classroom practices. In the present study, we examine teachers' beliefs and practices in the context of professional development that was designed to challenge preconceived notions about what teachers need to know in order to teach mathematics well.

Research design and methods

The findings reported in this paper come from three surveys (Survey 1, Survey 1R, and Survey 2) of post-primary mathematics teachers in Ireland, which form part of our larger research programme investigating the impact of the PDMT. Survey 1 was the baseline study of out-of-field teaching of mathematics in Ireland, conducted by Ní Ríordáin and Hannigan (2009). Survey 1R re-evaluated the prevalence of out-of-field teaching of mathematics in 2018, several years after the PDMT was first offered in 2012. Survey 2, also conducted in 2018, was sent only to graduates of the PDMT and investigated their perceptions and experiences of teaching mathematics since completing the programme. Survey research on the impact of professional development is sometimes criticised for eliciting biased self-reports that over-estimate socially desirable beliefs and 'good' implementation of teaching practices. However, Desimone (2009) argues that common notions about the supposed strengths and weaknesses of surveys, compared to interviews and classroom observations, are unwarranted. She maintains that properly constructed surveys can provide valid and reliable data on teacher behaviour, especially when they include questions that elicit teacher beliefs so that responses can be interpreted in the context of those beliefs.

Survey responses were analysed using quantitative and qualitative methods. Descriptive statistics were calculated using SPSS (Statistical Package for the Social Sciences) to determine the proportions of teachers with various qualifications and teaching assignments, and the frequency of selected response options regarding beliefs and teaching practices. A qualitative content analysis was also conducted of open-ended responses to questions about teaching practices.

Surveys 1 and 1R

The aims of Surveys 1 (conducted in 2009) and 1R (conducted in 2018) were to determine the prevalence of out-of-field teaching of post-primary mathematics, not only in terms of teacher qualifications but also the deployment of out-of-field and fully qualified teachers of mathematics. Both surveys sought information on teachers' undergraduate and postgraduate qualifications, number of years of experience in teaching mathematics and other subject areas, and the year group(s) and level of mathematics (Higher, Ordinary, Foundation) being taught by the teacher. Full details of the design of Survey 1 (and hence Survey 1R, which is an identical instrument) are provided in the report by Ní Ríordáin and Hannigan (2009).

The sampling frame for Survey 1 was a list of all 731 post-primary schools operating in Ireland in 2008. The targeted sample size was 400 teachers of mathematics, giving a margin of error for the estimate of the percentage of unqualified teachers of mathematics of ±5%, with a 95% confidence level. A stratified random sample of 60 schools was selected, and copies of the survey were mailed to the school principals for distribution to teachers of mathematics. Stamped addressed envelopes were also enclosed to facilitate return of completed surveys. Teachers from 51 schools (85% of the targeted sample) responded to Survey 1, with 324 questionnaires returned from teachers teaching mathematics in these schools.

A similar sampling and distribution method was used for Survey 1R. The sampling frame was a list of all 711 post-primary schools in Ireland in 2018. The targeted sample size was 700 teachers of mathematics, giving a margin of error for the estimate of the percentage of unqualified teachers of mathematics of ±5%, with a 95% confidence level. A stratified random sample of 100 schools was selected. There were 114 teachers from 20 schools (20% of the targeted sample) who responded to Survey 1R.

The very high response rate recorded for Survey 1 was unusual for social sciences research (Denscombe 2017), where it is more common to achieve survey response rates in the range of 20-30% – as was the case for Survey 1R. The high response rate (and hence sample size) for the initial survey may reflect the significant levels of interest in mathematics curriculum reform at the time, and the fact that this was the first time out-of-field teachers of mathematics in Ireland had been recognised as a distinct group and invited to submit information about their backgrounds and pedagogical practices. We anticipated a lower response rate for Survey 1R, and hence the need for a larger targeted sample. Regardless of the difference in sample sizes between Surveys 1 and 1R, the same probability sampling methods were used and yielded samples of suitable size for social science research. There were thus no adverse effects on the credibility of the survey findings.

Survey 2

Survey 2 examined the perceptions and experiences of teachers of mathematics since graduating from the PDMT. As well as collecting information about graduates' personal and professional backgrounds, the survey explored perceptions of their preparedness for teaching mathematics, development of their identity as a teacher of mathematics, beliefs, classroom practices, and perceptions of the effectiveness of the PDMT programme.

The survey section investigating epistemological and pedagogical beliefs was taken from the study of Perry, Howard, and Tracey (1999) and consisted of 20 items examining teachers' beliefs about mathematics, mathematics learning, and mathematics teaching. Responses to each belief item were given on a six-point Likert scale: strongly disagree (SD), disagree (D), somewhat disagree (SWD), somewhat agree (SWA), agree (A) and strongly agree (SA). There were two reasons why this scale differed from that used by Perry et al., which had only three response options: disagree, undecided, agree. First, we wanted consistency with the response options offered for other items in our survey which also worked off a six-point scale, and second, a larger number of response options increases the reliability and validity of the scale (Lozano, García-Cueto, and Muñiz 2008). Each item was classified as representing either transmission (T) or child-centred (C) beliefs.

Two sections of Survey 2 investigated teachers' perceptions of their classroom practices. The first used items from Question 14 in the Trends in International Mathematics and Science Study (TIMSS) Grade 8 Teacher Questionnaire Mathematics (International Association for the Evaluation of Educational Achievement 2014), asking respondents to indicate how often they used the listed strategies while teaching their mathematics class: every/almost every lesson, about half the lessons, some lessons, or never. In addition, the survey included two open-ended questions inviting teachers to describe their approach to mathematics teaching before and since completing the PDMT pedagogy workshops.

In November 2018, an invitation email was sent to the four graduated cohorts of the PDMT from 2014, 2015, 2016 and 2017, with two follow-up email remainders. Included in the email was a URL for the online survey, which was developed using SurveyMonkey. The survey was sent to the last known email addresses of 822 PDMT graduates; 26 of these invitations were undeliverable due to changes in email addresses, which means that the survey was successfully delivered to 796 graduates. A total of 224 responses to the online survey were received. However, 6 were excluded from the analysis due to the respondents not completing any question items. Accordingly, the sample for Survey 2 was 218 respondents (27% response rate).

Research findings

Changes in the prevalence of out-of-field teaching and deployment of teachers

Figure 1 displays the profile of mathematics teaching qualifications of respondents to Surveys 1 (2009) and 1R (2018). Respondents typically held a Bachelor of Science

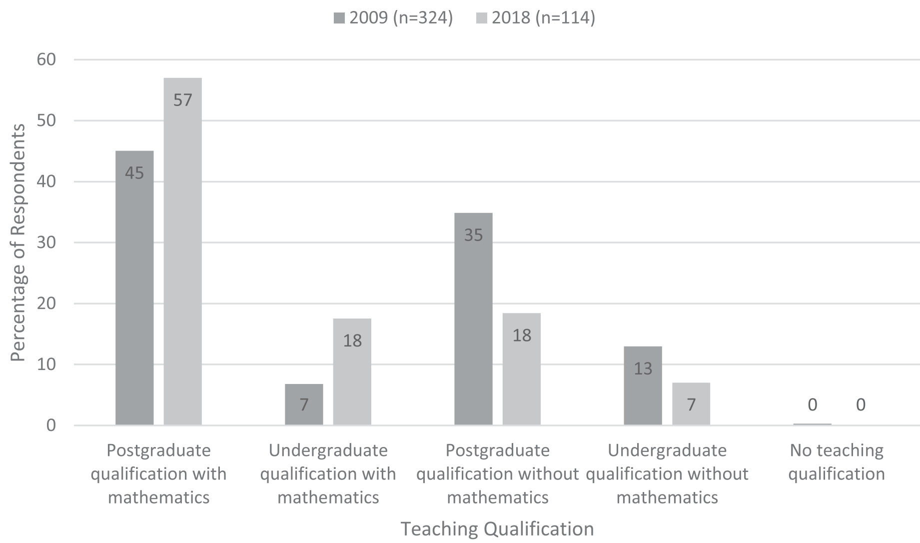

Figure 1. Teaching qualifications of respondents to Surveys 1 and 1R.

degree without a significant mathematics component together with a postgraduate teaching degree (35% of respondents in 2009; 54% in 2018). However, examples of other types of qualifications were also observed amongst out-of-field teachers of mathematics: such as a Bachelor of Commerce/Business degree and a postgraduate teaching degree (34% in 2009; 7% in 2018); a Bachelor of Arts degree and a postgraduate teaching degree (18% in 2018); and a concurrent Bachelor's degree in teaching subjects other than mathematics (27% in 2009). Overall, in 2009, 48% of respondents were teaching mathematics without adequate qualifications while by 2018 the proportion had fallen to 25% of respondents. As there were no known changes in the supply of fully qualified mathematics teacher graduates from initial teacher education degrees during this time, this finding is most likely explained by the upskilling effect of the PDMT on teachers who were formerly teaching mathematics out-of-field.

Tables 1–4 summarise information about the deployment of out-of-field and fully qualified teachers of mathematics from Surveys 1 and 1R. Shading is used to identify

Table 1. Distribution of Fully Qualified and Out-of-Field Teachers of Mathematics by Year Level Taught, Survey 1 (2009).

	Year Level Taught					
Mathematics teaching qualification?	First Year	Second Year	Third Year	Fourth Year	Fifth Year	Sixth Year
Yes (n = 168)	85 (51%)	100 (60%)	134 (80%)	95 (57%)	133 (79%)	131 (78%)
No (n=156)	81 (52%)	94 (60%)	79 (51%)	18 (12%)	45 (29%)	38 (24%)

Table 2. Distribution of Fully Qualified and Out-of-Field Teachers of Mathematics by Year Level Taught, Survey 1R (2018).

	Year Level Taught					
Mathematics teaching qualification?	First Year	Second Year	Third Year	Fourth Year	Fifth Year	Sixth Year
Yes (n = 85)	48 (57%)	47 (55%)	57 (67%)	36 (42%)	50 (59%)	55 (65%)
No (n=29)	20 (69%)	19 (66%)	16 (55%)	5 (17%)	18 (62%)	16 (55%)

Table 3. Distribution of Fully Qualified and Out-of-Field Teachers of Mathematics by Year Level Taught, Higher Level Mathematics, Survey 1 (2009).

	Year Level Taught (Higher Level Mathematics)			
Mathematics teaching qualification?	Second Year	Third Year	Fifth Year	Sixth Year
Yes (n = 168)	52 (31%)	71 (42%)	75 (45%)	68 (40%)
No (n=156)	7 (4.5%)	4 (3%)	0 (0%)	2 (1%)

majority responses. Note that, since it was possible for teachers to be assigned to classes across more than one year level, the sum total of frequencies in each table row is greater than the corresponding number shown in the first column. The percentages teaching each year level are calculated by dividing the number of teachers for that year level by the total number of teachers shown in the first column.

Responses to the 2009 survey revealed that out-of-field teachers were more likely than fully qualified teachers to be assigned to teach across year levels of the Junior and Senior Cycles in which there were no external examinations (first, second, and fifth years; see Table 1). This general pattern had not changed a great deal in 2018 (Table 2). In 2009, it was rare to find out-of-field teachers assigned to Higher Level mathematics classes (Table 3). However, by 2018 this situation had changed

Table 4. Distribution of Fully Qualified and Out-of-Field Teachers of Mathematics by Year Level Taught, Higher Level Mathematics, Survey 1R (2018).

	Year Level Taught (Higher Level Mathematics)			
Mathematics teaching qualification?	Second Year	Third Year	Fifth Year	Sixth Year
Yes (n = 85)	25 (29%)	36 (42%)	27 (32%)	25 (29%)
No (n=29)	9 (31%)	8 (28%)	6 (21%)	5 (17%)

somewhat (see Table 4), with slightly less than one-third of the out-of-field respondents reporting that they were teaching a Higher Level mathematics class in Junior Cycle.

A different perspective on the deployment of teachers comes from Survey 2, which asked PDMT graduates to indicate the year groups and level of mathematics that they taught before and after completing the programme. While there was very little change in individual teachers' class assignment from first to fourth year classes, a 32% decrease was observed in the proportion of teachers assigned to Ordinary Level mathematics in fifth year (Senior Cycle) and a corresponding large (35%) increase in teaching Higher Level mathematics for this year group.

Epistemological and pedagogical beliefs of PDMT graduates

Table 5 provides a summary of the PDMT graduates' responses to the 20 belief statements included in Survey 2. The response rate for each item ranged from 72-75%; that is, not every teacher answered each item in the online survey. Consistent with Perry, Howard, and Tracey (1999), the statements are grouped to identify beliefs about mathematics, beliefs about mathematics learning, and beliefs about mathematics teaching. Each item is labelled to indicate either a transmissive (T) or child-centred (C) belief (Perry, Howard, and Tracey 1999). Shading is used to identify majority responses favoured by at least half the respondents.

Beliefs about mathematics

Almost all PDMT graduates expressed at least some agreement that 'mathematics is a beautiful, creative, and useful human endeavour that is both a way of knowing and a way of thinking'. Similarly high levels of agreement were recorded with the statement that 'mathematics is the dynamic searching for order and pattern in the learner's environment', and there was clear disagreement that 'right answers are much more important in mathematics than the ways in which you get them'. Each of these response patterns aligns with the child-centred intent of the new post-primary mathematics curriculum and is similar to the findings reported in the study of Perry, Howard, and Tracey (1999). However, two-thirds of respondents agreed that 'mathematics is computation', suggesting a transmission orientation that was also observed to a somewhat lesser degree by these researchers.

Beliefs about mathematics learning

A very high proportion of respondents agreed with the following statements aligned with child-centred beliefs: 'mathematics learning is enhanced by activities which build upon and respect students' experiences', 'mathematics knowledge is the result of the learner interpreting and organising the information gained from experiences', and 'mathematics learning is enhanced by challenge within a supportive environment'. Respondents largely rejected the view that 'mathematics learning is being able to get the right answers quickly'. Yet there were equivocal views about the importance of memorisation, with 43% of teachers expressing some level of agreement – again mirroring the contradictory transmission response pattern found in the Perry, Howard, and Tracey (1999) study.

Table 5. Percentage Distribution of PDMT Graduates' Responses to Belief Statements.

Belief Statement	SD	D	SWD	SWA	A	SA
Mathematics						
1. Mathematics is computation (T)	4	14	15	45	20	2
2. Mathematics problems given to students should be quickly solvable in a few steps (T)	10	29	29	21	9	2
3. Mathematics is the dynamic searching for order and pattern in the learner's environment (C)	2	5	8	48	30	7
4. Mathematics is no more sequential a subject than any other (C)	11	31	27	19	11	1
5. Mathematics is a beautiful, creative and useful human endeavour that is both a way of knowing and a way of thinking (C)	0	2	2	26	49	21
6. Right answers are much more important in mathematics than the ways in which you get them (T)	50	35	10	3	2	0
Mathematics Learning						
7. Mathematics knowledge is the result of the learner interpreting and organising the information gained from experiences (C)	1	1	2	31	53	12
8. Students are rational decision makers capable of determining for themselves what is right and wrong (C)	1	11	22	38	24	4
9. Mathematics learning is being able to get the right answers quickly (T)	30	39	22	6	3	0
10. Periods of uncertainty, conflict, confusion, surprise are a significant part of the mathematics learning process (C)	1	2	6	28	47	16
11. Young students are capable of much higher levels of mathematical thought than has been suggested traditionally (C)	6	12	17	36	26	3
12. Being able to memorise facts is critical in mathematics learning (T)	4	19	20	35	18	4
13. Mathematics learning is enhanced by activities which build upon and respect students' experiences (C)	0	1	3	21	55	20
14. Mathematics learning is enhanced by challenge within a supportive environment (C)	2	2	2	20	50	24
Mathematics Teaching						
15. Teachers should provide instructional activities which result in problematic situations for learners (C)	1	3	6	29	47	14
16. Teachers or the textbook – not the student – are the authorities for what is right or wrong (T)	20	37	18	19	5	1
17. The role of the mathematics teacher is to transmit mathematical knowledge and to verify that learners have received this knowledge (T)	1	6	10	33	40	10
18. Teachers should recognise that what seem like errors and confusions from an adult point of view are students' expressions of their current understanding (C)	2	2	7	33	44	12
19. Teachers should negotiate social norms with the students in order to develop a cooperative learning environment in which students can construct their knowledge (C)	1	4	7	32	47	9
20. It is unnecessary, even damaging, for teachers to tell students if their answers are correct or incorrect (C)	32	36	15	11	5	1

Beliefs about mathematics teaching

A large majority of PDMT graduates agreed that teachers should 'provide instructional activities which result in problematic situations for learners', 'recognise that what seem like errors and confusions from an adult point of view are students' expressions of their current understanding', and 'negotiate social norms with the students in order to develop a cooperative learning environment in which students can construct their knowledge', and disagreed that the teacher or textbook are the authorities for what is right or wrong. On the other hand, there was equally strong agreement that the role of the teacher is to 'transmit mathematical knowledge and to verify that learners have received this knowledge': 83% of PDMT graduates expressed agreement with this transmission-oriented practice compared with 48% of mathematics head teachers and 61% of mathematics classroom teachers in the Perry, Howard, and Tracey (1999) study.

Teaching practices of PDMT graduates

Table 6 shows Survey 2 responses to an item taken from the TIMSS Grade 8 Teacher Questionnaire. Shading is again used to identify majority responses favoured by at least half the respondents. The response rate for this item was 57% (124 responses). Although both the PDMT and TIMSS surveys were designed to be administered to teachers, the latter results are reported at the student level, for example, by saying that '25% of *students* were taught by teachers who did X'. Also, in the TIMSS survey we do not know what proportion of respondents were teaching mathematics out of field. In contrast, for Survey 2 the responses are reported at the teacher level, and all participants were formerly out-of-field. Despite these differences, some comparisons may be drawn between the two samples. We do so by examining the proportions of teachers who regularly reported engaging in a particular classroom practice, that is, *in at least half of lessons* (as reported in the TIMSS Grade 8 Teacher Questionnaire).

Table 6 Percentage Distribution of PDMT Graduates Who Engage in Various Classroom Practices.

Classroom Practice	Every/ almost every lesson	About half of lessons	Some lessons	Never
Relate the lesson to students' daily lives	40	27	32	2
Ask students to explain their answers	58	25	16	<1
Ask students to complete challenging exercises that require them to go beyond the instruction	20	33	40	7
Encourage classroom discussion among students	36	32	30	3
Link new content to students' prior knowledge	80	13	7	0
Ask students to decide their own problem solving procedures	23	29	45	3
Encourage students to express their ideas in class	59	23	17	2

For PDMT graduates, linking new content to students' prior knowledge was the most common teaching practice, reportedly engaged in by 93% of teachers. Asking students to explain their answers and encouraging students to express their ideas in class were also common (reported respectively by 83% and 82% of teachers). These teachers (52%) were least likely to ask students to decide their own problem-solving procedures. All of these results were similar to the response patterns reported by Irish teachers of second year post-primary mathematics classes in TIMSS 2015 (Clerkin, Perkins, and Chubb 2018). However, there are two interesting differences between the responses of these two groups of teachers. Two-thirds (67%) of PDMT graduates reported regularly encouraging classroom discussion between students, whereas in the TIMSS sample only a little over half (55%) of Irish students were taught by teachers who did this. On the other hand, only 53% of PDMT graduates regularly asked their students to complete challenging exercises that went beyond their direct instruction, compared with the TIMSS sample in which 61% of Irish students indicated that their teachers did so. These findings might suggest that PDMT graduates found it easier to endorse and implement discussion-based teaching strategies than approaches that might have pushed students beyond the boundaries of the teachers' own content knowledge.

Table 7 provides a summary of PDMT graduates' responses to two open-ended questions asking them to describe their approaches to teaching mathematics before and after completing the PDMT pedagogy workshops, respectively. The response rate was 47% for the first question (102 responses) and 48% for the second question (104 responses). We classified responses as indicating practices aligned with either transmission or child-centred beliefs (aligned with the beliefs survey of Perry, Howard, and Tracey 1999), a mixture of these beliefs, no change, or other.

Typical *transmission* approaches reported by teachers referred to 'chalk and talk', 'board work', 'follow the book', and 'learning steps in a procedure', as some of the following sample responses illustrate:

> Less confidence in my teaching of maths led to more "chalk and talk" rather than more purposeful/engagement including practical activities *(before PDMT)*
>
> More traditional than now! I would have used the textbook much more and done less interactive, discovery learning tasks *(before PDMT)*
>
> Explain how to do the sum. Get students to do examples *(before PDMT)*
>
> Drill and repeat *(after PDMT)*

Table 7. Percentage Distribution of PDMT Graduates' Descriptions of Mathematics Teaching Approaches Before and After Completing Pedagogy Workshops.

Teaching Approach	Before PDMT	After PDMT
Transmission	56	5
Child-centred	23	51
Mixed	9	11
No change	9	26
Other	3	7

Responses categorised as *child-centred* referred to approaches that teachers described as 'problem-solving', 'teaching for understanding', 'group work', 'concrete examples', and 'investigation':

> Group work, activities involving movement around the room, problems with multiple solutions, pair work *(before PDMT)*

> I'm a lot more confident in my ability to stand aside and let the answer evolve from the group. Group learning has been successful for me *(after PDMT)*

> A lot more pupil centred. For example, I have begun to hand the problems over to the students a lot more and give them a chance to discuss/solve and also a lot more group work. I think this is mainly down to my confidence in the subject following the course *(after PDMT)*

Mixed responses reported some combination of these approaches, for example:

> A mix of teacher and pupil centred, leaning more towards teacher demonstration and instruction *(before PDMT)*

> Chalk and talk but with active learning strategies also implemented *(after PDMT)*

Some of the *no change – after PDMT* responses explicitly indicated that no new teaching approaches were learned in the PDMT pedagogy workshops:

> My approach was not overly influenced by the PDMT workshops I'm afraid *(after PDMT)*

> The PDMT helped with my maths base, it did not influence my pedagogy to any great degree *(after PDMT)*

Table 7 shows teachers who responded to these questions reporting a substantial decrease in transmission-oriented mathematics teaching practices (from 56% to 5% of respondents) after completing the PDMT pedagogy workshops, and a corresponding increase in child-centred approaches (from 23% to 51% of respondents). It was interesting that 9% of teachers responded 'no change' to the question asking about their approaches to mathematics teaching *before* completing the PDMT. This question does not invite teachers to think about how their practices have changed, and so we have no way of knowing what they meant by this response. On the other hand, a further 26% of teachers responded 'no change' to the question asking about their approaches to mathematics teaching *after* completing the PDMT. Here, a 'no change' response can be interpreted as meaning that these teachers were teaching in the same way before and after completing the PDMT. Yet we cannot unambiguously interpret the meaning of 'no change': for example, it could mean that some teachers had not changed their transmission-oriented approaches and others had not changed their child-centred approaches. Nevertheless, even if all who reported no change in their teaching approach after completing the PDMT were in fact maintaining transmissive practices, this would still indicate a clear shift towards the problem-solving orientation promoted by the new mathematics curriculum.

Discussion

Learning to teach is considered to be a lifelong journey that begins with initial teacher education and continues through the process of formal professional development (Teaching Council 2016). Out-of-field teaching presents substantial challenges for teacher professional development because out-of-field teachers need to learn new content and new ways of teaching. Ireland's PDMT is significant because it was a large-scale, university-accredited professional development programme aligned with a government policy objective of reducing the prevalence of out-of-field teaching of mathematics. Recent reviews of the international literature on out-of-field teaching have found no other cases of such a sustained, national professional development programme (Faulkner et al. 2019). The main purpose of this paper was to examine evidence of the impact of the programme on teachers' pedagogical and epistemological beliefs and their approaches to teaching mathematics. To provide a context for understanding the significance of the programme, we also measured changes in the prevalence of out-of-field teaching of mathematics over the time in which it was delivered.

Findings show a substantial decrease in the prevalence of out-of-field teaching of mathematics in Irish post-primary schools since the introduction of the PDMT. This is not a surprising result since this was a remit of the PDMT. However, there are several reasons why this result is significant. First, we have tackled the internationally documented challenges of measuring out-of-field teaching (Ingersoll 2019) by applying a consistent survey methodology and definition of 'out-of-field'. Second, the results provide trustworthy data to the government Department that funded the PDMT and no doubt seeks to justify its substantial investment in teacher upskilling. Third, the survey showed that a sizable proportion of the mathematics teaching workforce – estimated at 25% – still lacks adequate qualifications to teach mathematics, thus providing an evidence base to argue for continued funding. Our analysis also revealed little change over time in the pattern of deployment of teachers of mathematics, with out-of-field teachers most commonly assigned to teach Ordinary Level mathematics to students in non-examination years. While school principals might argue that their fully qualified teachers should be reserved for examination preparation of Higher Level mathematics classes, this practice gives insufficient priority to developing younger students' interest, procedural fluency and conceptual understanding of mathematics. Research on effective mathematics teaching highlights the complexities and importance of teachers' mathematical and pedagogical content knowledge in order to teach the content, modify it and adapt it to the needs of its learners (Darling-Hammond and Hudson 1990). This task is no less complex when teaching younger students or those taking lower level mathematics, and so to assign less qualified teachers to such groups undermines the mathematics teaching profession (Darling-Hammond and Hudson 1990). A positive outcome of the PDMT, however, is evidence of graduates being deployed to teach Higher Level mathematics in the Senior Cycle years, demonstrating evidence of value placed by principals on graduates of the programme.

Although the PDMT was designed with the primary goal of developing out-of-field teachers' knowledge of mathematics content and pedagogy, attention is also given to enhancing teachers' beliefs and classroom practices, emphasising problem-solving and conceptual understanding (Stigler and Hiebert 2004). Graduates of the PDMT report beliefs about mathematics, mathematics teaching, and mathematics learning that could be described as largely child-centred. It is not possible to make any claims about changes in their reported beliefs. However, we can make some international comparisons with reported teacher beliefs to provide a context for interpreting our findings. In Germany, for example, Bosse (2014) found little evidence of out-of-field teachers of mathematics holding problem-solving beliefs. On the other hand, our findings mirror the largely child-centred beliefs of Australian secondary school mathematics teachers in the study conducted by Perry, Howard, and Tracey (1999), even though some teachers may have responded to the set of beliefs statements in contradictory ways. Nevertheless, it is also important to remember some teachers came into the PDMT with misplaced feelings of confidence in their ability to teach most of what is in the curriculum (Ní Ríordáin, Paolucci, and O'Dwyer 2017), signaling the need for a broader cultural change around dispositions toward what it takes to be able to teach mathematics.

The second finding from the survey of PDMT graduates suggests that they perceived a substantial change in their mathematics teaching practices, shifting from transmission towards more child-centred approaches. They also reported a profile of teaching practices emphasising links to students' prior knowledge and encouraging student explanation and discussion, similar to the results for the Irish sample of post-primary mathematics teachers who participated in TIMSS 2015. While we cannot be sure about the composition of the TIMSS sample, these findings collectively suggest that PDMT graduates are aware of and endorse student-centred teaching approaches and recognise changes in their own classroom practice.

Although we cast our findings in a largely positive light, we acknowledge that caution is needed in interpreting evidence of teacher change resulting from their participation in a professional development programme. Desimone (2009) notes that survey research has been criticised for 'eliciting biased, socially desirable responses that overreport 'good' implementation and underreport 'bad' implementation' (188). However, she additionally points out that social desirability bias can occur in any form of data collection, whether this involves surveys, interviews, or observation of classroom practice.

There are three ways in which we might evaluate the validity of our findings, reported in this paper, regarding the impact of the PDMT. First, to partially counter the possibility that teachers may have overreported the effects of participation in the programme, we followed Desimone's (2009) recommendation to interpret teachers' self-reports of behavioural change in the context of the epistemological and pedagogical beliefs elicited via other survey items. Second, our claims are supported by previous studies, conducted by Lane and Ní Ríordáin (2020) and O'Meara and Faulkner (2021), that involved different cohorts of PDMT participants and different data sources and analysis methods. Both of these studies collected data on self-reported beliefs and classroom practices before and after teachers experienced specific components of the PDMT programme (action research for the former study; pedagogy workshops for the latter study); and both studies found evidence of a reported shift away from a traditional, teacher-led style to more student-centred, inquiry-based pedagogies that emphasise development of mathematical understanding. Finally, we are conducting comparative case study research involving structured classroom observations, interviews, and surveys of three groups of mathematics teachers: (i) those who have been upskilled via the PDMT; (ii) those who are still teaching mathematics out-of-field; and (iii) those who have always been fully qualified and hence in-field. Initial analysis of classroom observation data reveals that the upskilled teachers may be adopting pedagogical practices that are more like those of in-field teachers than those who are still teaching mathematics out-of-field, especially in relation to promoting higher order thinking, problem solving, and connectedness between mathematical concepts (Goos and Guerin 2021).

Conclusion

The results of this study provide some evidence of the impact of a national professional development programme designed to support out-of-field teachers of post-primary school mathematics. The study illuminates those elements of Desimone's (2009, 184) 'operational theory of how professional development works to influence teacher and student outcomes' relating to teacher beliefs and how these are implicated in changing teachers' approach to pedagogy. As the data were gathered through national surveys, the findings comprise teacher self-reports and reveal large-scale breadth and trends rather than depth and nuance. Currently, our research team and doctoral students are conducting classroom-based case studies that will yield richer and more detailed accounts of how PDMT graduates are using the new knowledge and skills gained through their participation in this programme. This work will allow us to make valid and reliable comparisons between the observed teaching practices of PDMT graduates, teachers of mathematics who are still out-of-field, and those who have always been fully-qualified, thus yielding a more rigorous evaluation of the impact of the PDMT on teachers' approaches to pedagogy. The most important challenge for future research will be to design longitudinal evaluations capable of linking teachers' participation in professional development with changes in their pedagogical approach and student achievement. This is a challenge not only for research on out-of-field teaching, but for teacher professional development in general.

Notes

1. Year 4 is an optional year known as Transition Year (TY). Not all schools offer this year and it is not compulsory for students to complete TY.
2. ECTS refers to the European Credit Transfer System representing the workload and defining learning outcomes of a given course or programme. 1 ECTS typically corresponds to between 20–25 h of student learning activity, including, for example, class contact time, reading and research, and assessment preparation and completion

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