A Linear Predictive Coding Filtering Method for Time-resolved Morphology of EEG Activity

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A Linear Predictive Coding Filtering Method for the Time-resolved Morphology of EEG Activity

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***Abstract*—This paper introduces a new time-resolved spectral analysis method based on the Linear Prediction Coding (LPC) method that is particularly suited to the study of the dynamics of EEG (Electroencephalography) activity. The spectral dynamics of EEG signals can be challenging to analyse as they contain multiple frequency components and are corrupted by noise. The LPC Filtering (LPCF) method described here processes the LPC poles to generate a series of reduced-order filter transform functions which can accurately estimate the dominant frequencies. LPCF is a parameterized time-frequency method that is suitable for identifying the dominant frequencies of multiple-component signals (i.e. EEG signal). We define bias and the frequency resolution metrics to measure the ability of LPCF to estimate the frequencies. The experimental results show that LPCF can reduce the bias of the estimates of LPC in the low and high frequency bands and LPCF provides finer frequency resolution than LPC. Furthermore, the LPCF method is less sensitive to the filter order and has a higher tolerance of noise than the LPC method. Finally, we apply LPCF to a real EEG signal where it can identify the dominant frequency in each frequency band and significantly reduce the redundant estimates of LPC.**

***Index Terms*—EEG analysis, modified linear predictive coding, time-frequency method.**

I. Introduction

EEG is an important bioelectric signal for researchers to explore the diagnosis and treatment of mental [1] and brain neuron-degenerative diseases [2] and abnormalities [3]. Dynamically exploring the key spectral characteristic information in EEG signals via time-frequency analysis can help researchers better understand human brain activity. Many of the traditional time-frequency methods are waveform methods such as the short-time Fourier transform [4] and the continuous wavelet transform [6]. They are excellent at demonstrating whether a certain frequency component exits or not by showing how the energy of the signal is distributed across the time-frequency domain. In this paper, we proposed a LPCF method which is a parameterized time-frequency method and it can robustly and accurately identify the dominant frequencies of noisy signals in the different frequency bands. An EEG signal is a multiple components signal and it has different frequency bands $(\delta, \theta, \alpha, \beta, \gamma)$ to analyse the different brain functions. The typical EEG signal is a high noise time-varying signal which requires the time-frequency method to be robust to noise. The LPCF method is particularly suitable for studying the dynamics of the dominant frequency at different EEG bands.

The LPC method can give us a numerical estimation frequency result. It has been extensively applied in speech signal processing for formant frequency identification [5]. However, standard LPC suffers from a sensitivity to noise and its performance is dependent on the filter order [8], [10]. In this paper, we propose a LPC Filtering (LPCF) method to further process the LPC poles into different frequency bands to generate a series of reduced-order filter transform functions to estimate the dominant frequency. The LPCF method is a further modification of our previous work [8], [9]. The LPCF method can overcome the shortcomings of the LPC method: sensitivity to noise and LPC order. We use the Monte Carlo simulation method to obtain the Probability Density Function (PDF) and use the mean and standard deviation of the PDF to define the bias and frequency resolution of the LPC-based method. These results show that (1) LPCF significantly reduces the bias of the estimates of LPC in the low and high frequency bands; (2) LPCF can provide finer frequency resolution than LPC; (3) The LPCF method has less sensitivity to the filter order and has a higher tolerance of noise than the LPC method. Furthermore, the LPCF method accurately identifies the dominant frequencies of different frequency bands of EEG.

This paper is organised as follows. In Section II, we first present details of the LPCF method. In Section III, we introduce the EEG frequency bands and the experimental metrics. Experimental results are presented in Section IV. Finally, the conclusions of the paper are presented in Section V.

II. Method Introduction

The LPC algorithm provides a method for estimating the parameters that characterize the linear time-varying system [10]. It is based on the assumption that the current signal sample $s(n)$ can be closely approximated as a linear combination of past samples

$$\hat{s}(n) = \sum_{i=1}^{P} a_i s(n-i), \tag{1}$$

where the factor a_i is the predictor coefficient, which is determined by minimizing the mean-squared error between the actual samples $s(n)$ and the predicted values $\hat{s}(n)$.

A. LPC Method

The LPC analysis operates on frames containing data samples. In the z-transform domain, a P^{th} order linear predictor is a system of the form

$$L(z) = \sum_{i=1}^{P} a_i z^{-i} = \frac{\hat{S}(z)}{S(z)} \tag{2}$$

where $\hat{S}(z)$ is the output of the filter. The prediction error $e(n)$ is of the form

$$e(n) = s(n) - \hat{s}(n) = s(n) - \sum_{i=1}^{P} a_i s(n-i) \tag{3}$$

where $\hat{s}(n)$ is the linear prediction and the z-transform for the prediction error can be written as

$$E(z) = S(z) - \sum_{i=1}^{P} a_i S(z) z^{-i}. \tag{4}$$

The prediction error is the output of a system with transfer function

$$A(z) = \frac{E(z)}{S(z)} = 1 - L(z) = 1 - \sum_{i=1}^{P} a_i z^{-i} \tag{5}$$

where $A(z)$ is an inverse filter for $H(z)$ given by

$$H(z) = \frac{1}{A(z)} = \frac{1}{1 - \sum_{i=1}^{P} a_i z^{-i}}. \tag{6}$$

The fundamental theorem of algebra tells us that $H(z)$ has P poles, which are the values of z for which $H(z) = \infty$. Therefore in finding the poles of $H(z)$ we obtain the set $\{z_1, z_2, z_3, \cdots, z_P\}$. As each pole z_i is complex, where each pole z_i can be expressed as

$$z_i = \gamma_i e^{j\omega_i}, \ (i = 1, 2, 3, \cdots, M) \tag{7}$$

in which $\omega_i = \tan^{-1}[\text{Im}(z_i)/\text{Re}(z_i)]$ is the angle corresponding to the pole. The magnitude of a pole is $m_i = |z_i|$ and the corresponding pole frequency is

$$p_i = \frac{\omega_i}{2\pi T_s}, \ (i = 1, 2, 3, \cdots, M) \tag{8}$$

where T_s is the sample period. The poles of $H(z)$ are often used to directly estimate the frequency content of signals [10] [16] [17]. The LPC method is the benchmark for our approach and the poles resulting from LPC are used as the frequency estimates for the analysed signals. Given that the poles occur in the filter as complex conjugate pole pairs, we only consider those poles with non-negative imaginary parts $\text{Im}(z_i) \geq 0$ and the number of LPC estimates is denoted by M.

B. LPCF Method

The proposed LPCF method further processes the LPC poles to generate a series of reduced-order transform functions to estimate the dominant frequencies in the different frequency bands. The details of further processing of LPC poles are as follows:

1) Obtain the set of poles of LPC filter $H(z)$, i.e. $\{z_1, z_2, z_3, \cdots, z_M\}$ and partition the poles into different frequency bands.
2) Organise the poles of each frequency band into the dominant pole and local poles. The LPC pole with the largest magnitude is classified as the dominant pole $\tilde{z}_i$ and the number of dominant poles is N, other poles are the non-dominant poles. The non-dominant poles around the dominant poles are called local poles $\hat{z}$, which can affect the final location of the spectral peak. A distance threshold λ is defined to identify the local poles. When the distance (frequency separation) Δf between the dominant poles $\tilde{z}_i$ and non-dominant poles is less than λ, we consider these non-dominant poles to be the local poles $\{\hat{z}_{i1}, \hat{z}_{i2}, \cdots, \hat{z}_{iL}\}$, where the L is the number of local poles for i^{th} dominant pole.
3) The dominant pole and its local non-dominant pole(s) form a new reduced order transform function which is denoted

$$\tilde{H}_i(z) = \frac{1}{(1 - \tilde{z}_i^{-1})} \times \prod_{j=1}^{L} \frac{1}{(1 - \hat{z}_{ij}^{-1})}. \tag{9}$$

As the new filter transfer function $\tilde{H}_i(z)$ has a lower order, it has fewer local maxima, which makes it easier to find the peaks.

4) Estimate the dominant frequencies. The maximum peak $\tilde{p}_i$ of the $\tilde{H}_i(z)$ is the dominant frequency component of the i^{th} frequency band. So the estimates of LPCF are $\{\hat{p}_1, \hat{p}_2, \cdots, \hat{p}_N\}$.

III. Performance Evaluation Metrics

A. EEG Frequency Bands

Many EEG research works have divided the spectra of EEG waveforms into several fixed frequency bands, they are named based on their frequency range using Greek letters $(\delta, \theta, \alpha, \beta, \gamma)$. Different researchers have defined different frequencies for these bands with little consensus between them [11]–[15]. In this paper, we use the EEG frequency band standard from [12], as shown in Table I.

TABLE I
EEG FREQUENCY BANDS.

Name of EEG Waves	δ	θ	α	β	γ
Frequency Range (Hz)	0-4	4-8	8-12	13-30	over 30

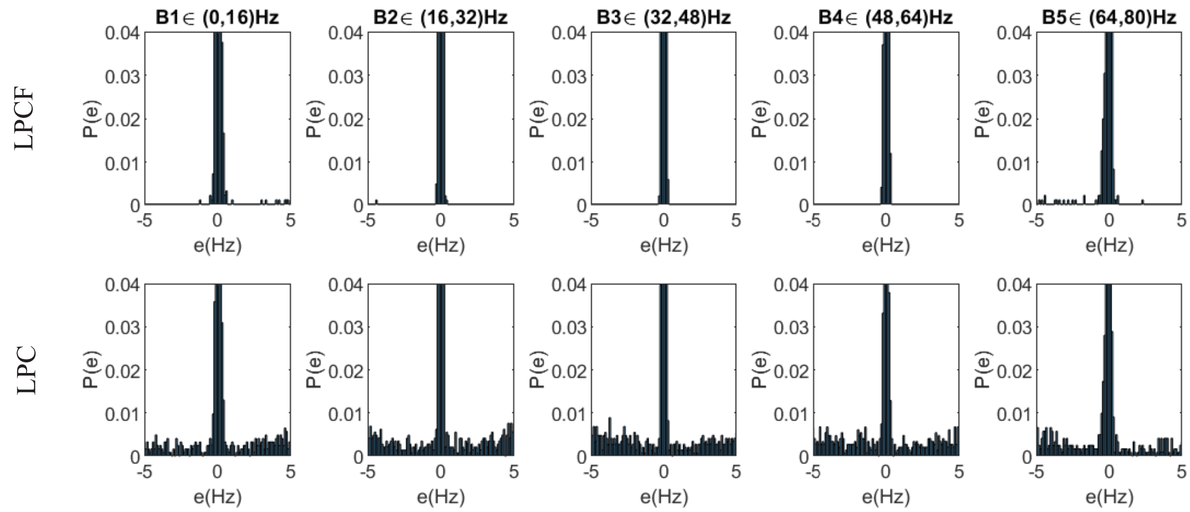


Fig. 1. The EPDF of the LPCF method and LPC method under the different frequency bands. The y-axis of each EPDF is fixed from 0 to 0.04 to zoom the EPDF so that it easier to observe the low $P(e)$ in EDPF.

B. Metrics

As LPCF is a parameterized method, Monte Carlo simulation is used to generate the Error Probability Density Function (EPDF) to measure the bias of the estimates and to estimate the frequency resolution of the LPCF method. The trials of Monte Carlo in all experiments of this paper are repeated $1,000$ times and the simulation signals are sinusoidal signals whose frequencies are uniform distribution. To ensure that we do not unfairly penalize the LPC method, we only considered the frequency estimates whose error was less than 5 Hz from the true frequency in the experimental evaluation conducted in this paper. In this paper, we use the mean value of each EPDF as the bias of LPC-based methods. A histogram of the frequency errors where the error e range is from -5 Hz to +5 Hz and the bin size is 0.1 Hz. For each histogram bar, we start by multiplying the central e-value to the corresponding bar height and the height of each histogram bar is expressed as a probability $P(e)$ (i.e. $\sum P(e) = 1$). The bias is defined as

$$\mu = \sum eP(e), \tag{10}$$

where the μ is the bias of the all estimates. The standard deviation σ of the EPDF is used to measure the frequency resolution of the LPC-based method. The frequency resolution Δf is defined as

$$\Delta f = \sigma = \sqrt{\Sigma(e-\mu)^2 P(e)} \tag{11}$$

The Heisenberg-Gabor uncertainty principle tells us what can be achieved with regard to time-frequency localization for the short-time Fourier transform [7], by referring to the dimensions of the tiles ($\Delta t \times \Delta f$) in the time-frequency plane. Therefore, the Time-Bandwidth Product (TBP) of the LPC-based method is

$$TBP = \Delta f \times \Delta t \tag{12}$$

where Δt represents the time resolution.

TABLE II
THE FREQUENCY RANGE OF DIFFERENT FREQUENCY BANDS.

Label	**B1**	**B2**	**B3**	**B4**	**B5**
Frequency Range (Hz)	0-16	16-32	32-48	48-64	64-80

IV. EXPERIMENTAL RESULTS

In this section, the first three experiments are Monte Carlo experiments and the experimental signals are sinusoidal signals with uniform frequency distribution. The last experiment applies LPCF to a real EEG signal and shows the parameterized time-frequency analysis results of LPCF.

A. Frequency Bands Analysis

In the first experiment, we demonstrate the EPDF of LPCF and LPC in the different frequency bands. The sampling frequency of simulation signal is $fs = 160$ Hz, the time resolution is Δt =1 s. These frequencies of experimental signals are uniform distribution for each frequency band. The details of the frequency bands are shown in Table II. The frequency domain is equally divided into 5 frequency bands (i.e. B1, B2, B3, B4 and B5). The bands B1 and B2 correspond to low frequencies, B3 corresponds to middle frequencies, B4 and B5 correspond to high frequencies. In order to simulate a high noise environment, Additive White Gaussian Noise (AWGN) is used to perturb the signal. The Signal-to-Noise Ratio (SNR) in dB is defined as the ratio of the power of the signal to the AWGN power. The SNR of this experiment is 3 dB, the order of filters is $P = 15$ and $\lambda = 5$ Hz.

The EPDF results are shown in Fig. 1. The spread of EPDF of the LPC method is bigger than that of LPCF method within each frequency band. For the bias analysis in Fig. 2, the bias μ of the LPC method is greater than 0 in the low frequency band (i.e. B1), and is less than 0 in the high frequency band (i.e. B5). This indicates that the LPC method overestimates the frequency at low frequency band and underestimates the frequency at high frequency band. The LPCF method can reduce this bias of the LPC method.

For the frequency resolution analysis in Fig. 3, the LPC method has a lower frequency resolution in the middle band. The reason is that the estimates of LPC in the low and high frequency EEG bands are biased to one side, while the EPDF in the middle frequency band is not biased, thus causing the Δf in the middle frequency band to be higher than in other frequency bands. This is also one of the reasons why the LPC method has a bigger bias in the high and low frequency bands than in the middle band. For the LPCF method. it can provide a finer frequency resolution than that of LPC in the different EEG frequency bands. The details of TBP are shown in Table III. The TBP value of the LPCF method is much lower than that of the LPC method. This result is consistent with the result of the bias analysis in Fig. 3 when the time resolution is fixed. In the following experiments (i.e. subsection IV-B and IV-C), we focused on selecting three representative frequency bands for detailed analysis, namely, B1 represents the high frequency band, B3 represents the middle frequency band, and B5 represents the high frequency band.

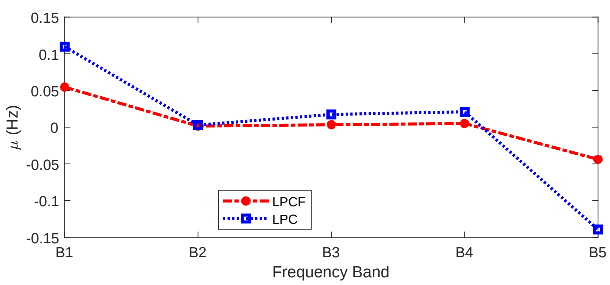


Fig. 2. The bias of the LPCF and LPC methods under the different frequency bands.

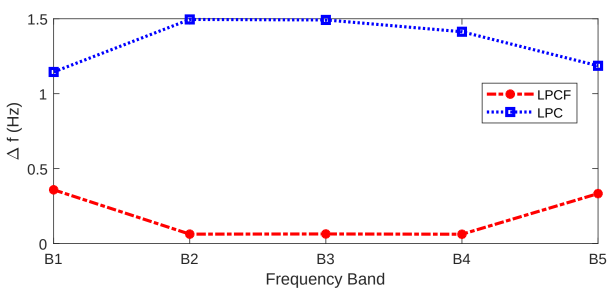


Fig. 3. The frequency resolution of the LPCF and LPC methods under the different frequency bands.

TABLE III
LPCF VS LPC: THE TBP UNDER DIFFERENT FREQUENCY BANDS.

Frequency Band	B1	B2	B3	B4	B5
TBP(LPCF)	0.3587	0.0628	0.0640	0.0622	0.3334
TBP(LPC)	1.1450	1.4955	1.4923	1.4134	1.1863

B. LPC Order Analysis

In this experiment, we analyse the influence of LPC order on the bias and frequency resolution of the LPC-based methods. The filter order P is changed from 5 to 25 and the step size is 5. The SNR of the signal is 3 dB; the other experimental parameters are the same as those used in the previous subsection. Fig. 4 and Fig. 5 show the bias analysis and the frequency resolution of LPCF and LPC for different filter orders. As we can see in Fig. 4, the bias μ of the LPC method in the low frequency band is greater than 0 and in the high frequency band is less than 0. This indicates the LPC method has a bigger bias in the low and high frequency bands than in the middle frequency band. The bias μ (Fig. 4) and the Δf (Fig. 5) of the LPC method first decreases and then increases with the increase of LPC order. The LPC method has the smallest bias value at $P = 15$ and it has the smallest Δf at $P = 10$. These results indicate the performance of the LPC method is dependent on the filter order. For the LPCF method, it can provide a smaller bias than the LPC method after P is greater than 10. The reason is that the number of filter order is too low to provide sufficient spectral information when $P = 5$. In Fig. 5, the LPCF method has very high frequency resolution under different filter orders and they are not affected by the filter order. So the performance of the LPCF method is much less sensitive to the filter order than that of the LPC method. Table IV shows the TBP results of this experiment in which the LPCF values are less than the LPC for all cases.

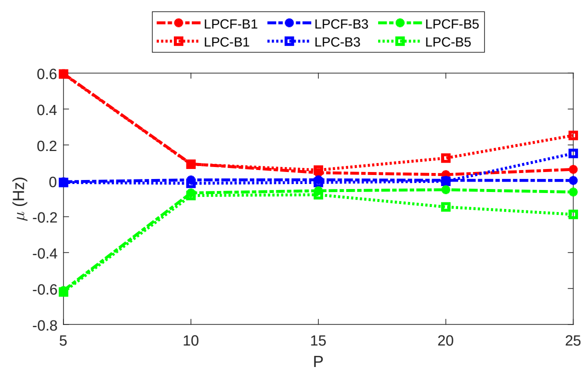


Fig. 4. The bias of the LPCF and the LPC methods under different filter orders.

TABLE IV
LPCF VS LPC: THE TBP UNDER DIFFERENT FILTER ORDER.

LPC order		5	10	15	20	25
B1(TBP)	LPCF	0.7398	0.2702	0.1945	0.1529	0.4639
	LPC	0.7399	0.2707	0.5647	1.1282	1.9428
B3(TBP)	LPCF	0.3036	0.1065	0.0756	0.0624	0.0588
	LPC	0.3049	0.2426	0.5803	1.3860	2.1785
B5(TBP)	LPCF	0.7558	0.2712	0.3370	0.3677	0.4467
	LPC	0.7559	0.3650	0.5968	1.0877	1.9075

C. Signal Noise Analysis

In this experiment, we analyse the effect of noise on the LPC-based methods. The LPC order P =15, and other

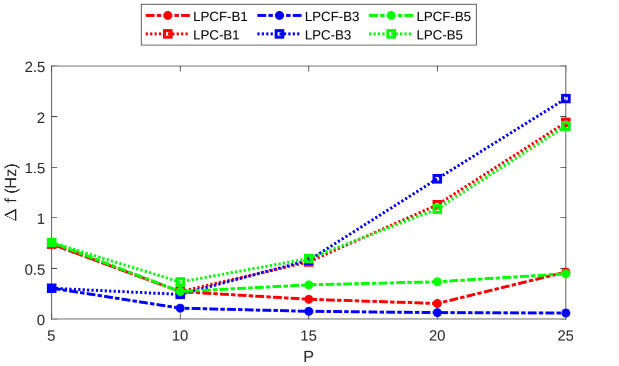


Fig. 5. The frequency resolution of the LPCF and LPC methods under different filter orders.

experimental parameters are the same as the experiment in the previous subsection. Fig. 6 and Fig. 7 demonstrate the bias μ and Δf of the LPCF method and LPC method under different SNR conditions. We can see that LPCF has a smaller μ than LPC under the same SNR level and LPCF can provide a higher frequency resolution than that of LPC at the same SNR level. In Fig. 7, the Δf of the LPC method becomes bigger as the SNR of noise is increased. The reason is that the range of EPDF only analyses frequency errors less than 5 Hz. But the error of estimates of the LPC method is over 5 Hz when the signal has a low SNR. So only the errors between the -5 and 5 Hz are counted, which is why the μ and Δf of the LPC method become bigger as the SNR increases. Fig. 8 and Fig. 9 show the results when the error range extends from -15 to 15 Hz. Fig. 8 shows that the bias of both methods is decreased as the SNR increases and Fig. 9 shows that the Δf of both methods is decreased as the SNR increases. The bias of the LPCF method still is much lower than that of LPC and the frequency resolution is much lower than that of LPC at B3. These results show that LPCF method has a higher tolerance to noise than LPC. Table V shows the TBP values of the LPC method and LPCF method and the error range of EPDF is from -5 to 5 Hz. In short, the TBP value of LPCF is lower than the LPC method for the different SNR levels.

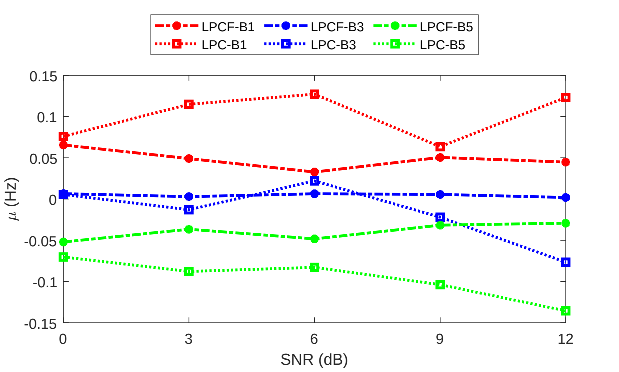


Fig. 6. The bias of the LPCF and LPC methods under the different SNR.

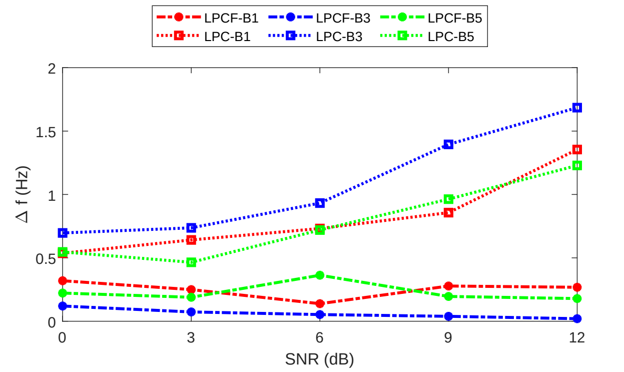


Fig. 7. The frequency resolution of the LPCF and LPC methods under the different SNR.

TABLE V
LPCF VS LPC: THE TBP UNDER DIFFERENT SNR LEVELS.

SNR(dB)		0	3	6	9	12
B1(TBP)	LPCF	0.3195	0.2497	0.1385	0.2779	0.2681
	LPC	0.5363	0.6414	0.7316	0.8568	1.3547
B3(TBP)	LPCF	0.1200	0.0734	0.0527	0.0388	0.0202
	LPC	0.6966	0.7367	0.9320	1.3950	1.6848
B5(TBP)	LPCF	0.2223	0.1892	0.3631	0.1957	0.1790
	LPC	0.5469	0.4649	0.7195	0.9631	1.2289

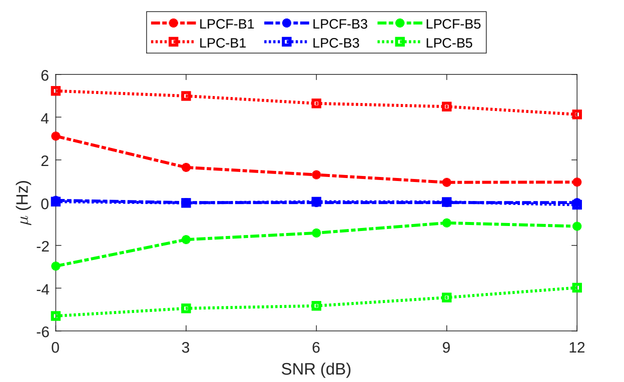


Fig. 8. The bias of the the LPCF and LPC methods under different SNR where the error range extends from -15 to 15 Hz.

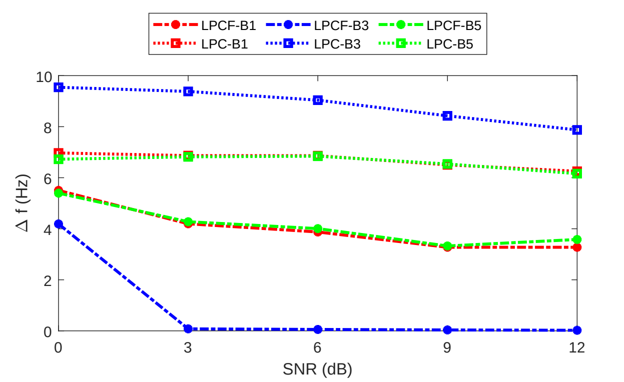
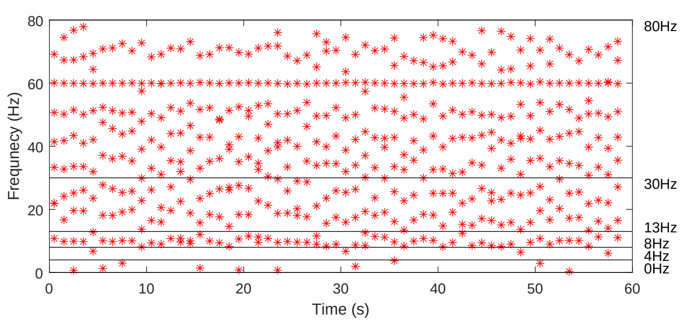


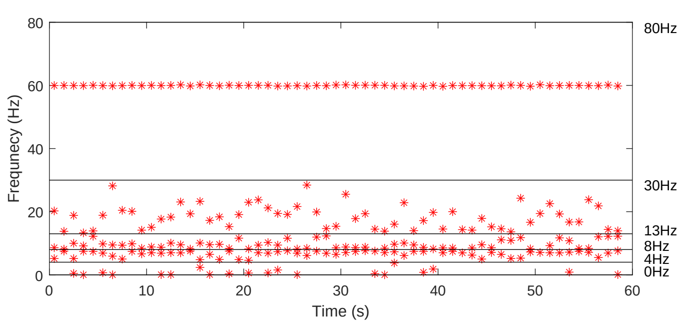
Fig. 9. The frequency resolution of the LPCF and LPC methods under different SNR where the error range extends from -15 to 15 Hz.

D. EEG Analysis

In this experiment, we demonstrate a real EEG signal application using LCP and LPCF to identify the dominant frequency components of different EEG waves $(\delta, \theta, \alpha, \beta, \gamma)$. The EEG signal used in our experiment comes from the public dataset BCI2000 [18]. The sampling frequency of the EEG signal is fs=160 Hz, the length of the EEG is 60 s. The order of LPC P=20, the time resolution $\Delta t = 1$ s. Other experimental parameters are the same as the experiments in the previous subsection. Fig. 10 compares the frequency estimations response between LPC and LPCF method. The black line is the reference line for different EEG frequency bands. It is particularly noticeable that both LPC and LPCF methods have identified the AC power supply frequency of 60 Hz. The LPC method directly generates many estimation frequencies as it does not distinguish between the dominant and non-dominant poles. These results show that LPCF has a greater ability to estimate the dominant frequency in different frequency bands than LPC. The LCPP method can reduce the bias of LPC in the same frequency band and it can provide higher frequency resolution at the same time resolution as the LPC method. The LPCF method allows us to estimate the dominant frequency component in each of the EEG bands and it can track the dominant frequency changes of the different EEG frequency bands.



(a) LPC Method



(b) LPCF Method

Fig. 10. Comparing time-resolved spectra of the LPC and LPCF methods in a EEG signal. The x-axis is the time. The left y-axis is the frequency. The right y-axis is the boundary value of EEG frequency band. The black line is the boundary line of different EEG frequency bands.

V. Conclusion

This paper introduces a parameterized time-frequency method LPCF which further processes the LPC poles to generate a series of reduced-order filter transform functions to estimate the dominant frequency at different frequency bands. The bias and the frequency resolution are defined to analyse the performance of the LPCF method. The experimental results show that the LPCF method can significantly reduce the bias of the LPC method in the low and high frequency bands. It can provide higher frequency resolution than LPC in different frequency bands and different orders. LPCF is a robust method, which has less sensitive to the filter order and has a higher tolerance of noise than LPC. Due to EEG is a noise multi-component EEG signal, LPCF is particularly suited to study the dynamic of EEG activity. It can estimate the dominant frequencies of different EEG frequency bands and effectively reduce redundant estimates compared to LPC. In further work, the LPCF method can support further processing of the EEG signal using machine learning techniques.

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