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## Reliability-Based Bridge Assessment Using Enhanced Monte Carlo to Simulate Extreme Traffic Loading

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# Reliability-based bridge assessment using enhanced Monte Carlo to simulate extreme traffic loading

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**ABSTRACT:** A framework is presented for the assessment of the safety of a bridge deck under actual traffic loading using an enhanced Monte Carlo method which attempts to reduce computational cost while preserving the advantages of more conventional, computationally intensive, simulation. To generate the bridge loading scenarios, an extensive Weigh-in-Motion (WIM) database is used to calibrate a sophisticated simulation model of two-directional traffic. Traffic and vehicle characteristics are generated from statistical distributions derived from measured traffic data. Two examples are used in this study to assess the usefulness and accuracy of the enhanced method. In the first, a simple example is used for which the exact theoretical probability of failure is available. Hence, the error in estimation can be assessed directly. In the second, ‘long-run’ simulations are used to generate a very large database of load effects from which very accurate estimates can be deduced of lifetime maximum effects.

## 1 INTRODUCTION

The application of reliability theory to the assessment of the safety of bridge structures requires the accurate modeling of both the applied loading and the strength and stiffness of the bridge structure. In its simplest form, a bridge is safe when its capacity to resist load exceeds the load applied. More precisely, a bridge can be considered safe when there is an acceptably low probability that load exceeds resistance capacity. A great deal of work has been carried out on methods of evaluating the load-carrying capacity of bridges and the associated uncertainties. Load-carrying capacity can be reduced by different forms of deterioration, depending on factors such as the structural material, the quality of workmanship during construction, the age of the structure, the environment and the loading history.

The modeling of traffic loading is usually based on measurements of highway traffic taken at selected sites over a period of some months, typically using weigh-in-motion (WIM) technology. One approach is to fit a statistical distribution to the calculated load effects for the measured traffic and to use these distributions to estimate characteristic lifetime maximum effects (Miao & Chan 2002, Nowak 1993, Vrouwenvelder & Waarts 1993). An alternative approach adopted here and by many authors is to use Monte Carlo simulation to generate typical traffic and hence load effects (Bailey & Bez 1999, O’Connor & OBrien 2005, OBrien et al. 2006). For reliability analysis, Monte Carlo simula-

tion can be used to generate values from the statistical distributions of both the applied loading and the resistance of the structure, and thus calculate the probability of failure (and hence the reliability) of the structure. Long-run simulations reduce the variability of the estimated probability but are computationally demanding. An enhanced Monte Carlo method has been developed recently by Naess et al. (2009). This method attempts to reduce computational cost while preserving the advantages of more conventional, computationally intensive, simulation. In this paper, this enhanced Monte Carlo method is applied to two sample problems. In the first, a simple example based on the Normal distribution is used for which the exact theoretical probability of failure is available. Hence, the error in estimation can be assessed directly. In the second, ‘long-run’ simulations are used to generate a very large database of load effects from which very accurate estimates can be deduced for the probability of failure, based on a theoretical model of resistance. In both examples, sub-samples are drawn from the loading data, and three different methods are used to estimate the probability of failure from these samples. The first method uses the enhanced Monte Carlo technique to estimate the probability of failure. In the other two methods, the Generalized Extreme Value (GEV) and Normal distributions are fitted to the load effect data and used to calculate the probability of failure. The results of each method are then compared with the known ‘true’ probability. Five

sub-sample sizes are considered - 1000, 2500, 5000, 10 000 and 20 000 working days. In the case of the 1000-day sample, the exercise is repeated 20 times to investigate the variability of the results from each of the methods.

## 2 LITERATURE REVIEW

### 2.1 Structural reliability and enhanced Monte Carlo

The basic structural reliability problem is described by Melchers (1999). A structural element is considered to have failed if its resistance  $R$  is less than the load effect  $S$  acting on it. Both  $R$  and  $S$  are random variables, described by probability distributions  $f_R(\cdot)$  and  $f_S(\cdot)$  respectively. It is common for  $R$  and  $S$  to be independent and in this case, the probability of failure,  $P_f$ , is given by the convolution integral:

$$P_f = P(M = R - S \leq 0) = \int_{-\infty}^{\infty} F_R(x) f_S(x) dx \quad (1)$$

where  $F_R(\cdot)$  is the cumulative distribution function for the resistance  $R$ , and  $M$  (i.e.  $R-S$ ) is the safety margin. A widely-used approach to calculating the probability of failure is use Monte Carlo simulation to generate values of load and resistance from their respective assumed distributions and to count the number of times the load exceeds the resistance as a proportion of the total number of values generated. As the probability distribution of loading becomes more complex, as in the case of traffic loading on bridges, the simulation requires significant computational effort. Naess et al. (2009) have proposed an enhanced Monte Carlo technique that can yield a substantial reduction in the computation time. It exploits the regularity of tail probabilities to calculate the far tail failure probabilities based on estimates of the failure probabilities obtained by Monte Carlo simulation at more moderate (near tail) levels. It achieves this using a parameterized class of safety margins using a scaling parameter,  $\lambda$ , for the safety margin,  $M$ :

$$M(\lambda) = M - (1 - \lambda)\mu_M \quad (2)$$

where  $0 \leq \lambda \leq 1$ , and  $\mu_M$  is the mean safety margin, i.e.  $\mu_M = E[M]$ .

Using this parameter the following assumption is made about the behavior of the failure probability:

$$P_f(\lambda) \approx q(\lambda) \exp\{-a(\lambda - b)^c\} \quad (3)$$

where  $q(\lambda)$  is slowly varying compared with the exponential term. Using this expression, the target probability of failure ( $P_f = P_f(1)$ ) can be estimated from values of  $P_f(\lambda)$  for  $\lambda < 1$ . In other words this parametric form of the failure probability allows us

to estimate the target value by extrapolation. Values of  $P_f(\lambda)$  for  $\lambda < 1$  are generally easier to estimate accurately than the target value, since they are larger, and hence will require less simulation. The method requires the estimation by simulation of values of  $P_f(\lambda)$  for a range of values of  $\lambda$  between 0 and 1, and then using Equation 3 to extrapolate to  $P_f(1)$ . As a simplification, Naess et al. (2009) propose that  $q(\lambda)$  may be assumed to be constant. Levenberg-Marquardt least-squares optimization is used here to find values for the parameters  $a$ ,  $b$ ,  $c$  and  $q$  by optimizing the fit on a log probability scale.

### 2.2 Block Maximum – Extreme Value Distributions

Extreme value theory is based on the extreme value theorem (Gnedenko 1943), following initial work by Fisher & Tippett (1928) and Gumbel (1935), and the Generalized Extreme Value (GEV) distribution is:

$$F_{GEV}(x) = \begin{cases} \exp\left(-\left(1 + \xi\left(\frac{x-\mu}{\sigma}\right)\right)^{-\frac{1}{\xi}}\right), & \text{if } \xi \neq 0 \\ \exp\left(-\exp\left(-\frac{x-\mu}{\sigma}\right)\right), & \text{if } \xi = 0 \end{cases} \quad (4)$$

It is defined in terms of parameters  $\mu$ ,  $\sigma$  and  $\xi$ , where  $\mu \in R$  is the location parameter,  $\sigma > 0$  the scale parameter and  $\xi \in R$  the shape parameter.

Extreme value distributions have been applied by many authors to bridge load effects (such as bending moments and shear forces) that have been calculated from either measured or simulated traffic. The distributions are usually fitted to block maximum values – daily, monthly or yearly maxima. Many studies (Caprani & O'Brien 2006, Caprani et al. 2008, Kanda & Ellingwood 1991, O'Connor & O'Brien 2005) indicate that load effect data is either Weibull or Gumbel. Given that Gumbel is a special case of Weibull (with shape parameter,  $\xi = 0$ ), an assumption that load effect is always of the form of Equation 4, with  $\xi \leq 0$ , seems reasonable.

### 2.3 Block Maximum – Normal Distribution

Block maximum data is often fitted with extreme value distributions as each data point represents the maximum of a number of parent values. However, block maximum data is also sometimes fitted with a Normal distribution. Nowak (1993) uses 2.4 hours as the block size and fits a Normal distribution to the maximum-per-block data. This distribution is then raised to an appropriate power to obtain the 75-year maximum load effect distribution.

To calibrate the traffic load model for the AASHTO load and resistance factor design (LRFD)

approach, Nowak and others use Normal probability paper to extrapolate the maximum load effects for time periods from 1 day to 75 years, based on a set of 9250 heavy vehicles representing about two weeks of heavy traffic measured on a highway in Ontario (Kulicki et al. 2007, Moses 2001, Nowak 1994, Nowak 1995, Nowak 1999, Nowak & Hong 1991, Nowak et al. 1993, Sivakumar et al. 2011). The expected value of the lifetime maximum is found by fitting a straight line to the tails of the data on Normal probability paper.

Kulicki et al. (2007) identify the fact that block maximum load effects due to measured trucks are not Normal but fits the Normal distribution to tail data. In the background studies for Eurocode 1, Flint & Jacob (1996) fit half-normal curves to the ends of the histograms of load effects. They adopt a least-squares best fit method to estimate the distribution parameters. Multimodal (bimodal or trimodal) Gumbel and Normal distributions are also used.

## 2.4 Traffic simulation

As part of the European 7th Framework ARCHES project [1], extensive WIM measurements were collected at five European sites: in the Netherlands, Slovakia, the Czech Republic, Slovenia and Poland. The ARCHES site in Slovakia is used as the basis for the simulation model presented here. Measurements were collected at this site for 750 000 trucks over 19 months in 2005 and 2006. The traffic is bi-directional, with average daily truck traffic (ADTT) of 1100 in each direction. Very heavy trucks were recorded at all sites, with a maximum gross vehicle weight (GVW) of 117 t being recorded in Slovakia.

A detailed description of the methodology adopted is given by Enright & OBrien (2012), and is summarized here. For Monte Carlo simulation, it is necessary to use a set of statistical distributions based on observed data for each of the random variables being modeled. For gross vehicle weight and vehicle class (defined here simply by the number of axles), a semi-parametric approach is used as described by OBrien et al. (2010). This involves using a bivariate empirical frequency distribution in the regions where there are sufficient data points. Above a certain GVW threshold value, the tail of a bivariate Normal distribution is fitted to the observed frequencies which allows vehicles to be simulated that may be heavier than, and have more axles than, any measured vehicle. Results for lifetime maximum loading vary to some degree based on decisions made about extrapolation of GVW, and about axle configurations for these extremely heavy vehicles. These decisions are, of necessity, based on relatively sparse observed data.

Bridge load effects for short- to medium-span bridges are very sensitive to wheelbase and axle lay-

out. Within each vehicle class, empirical distributions are used for the maximum axle spacing for each GVW range. Axle spacings other than the maximum are less critical and trimodal Normal distributions are used to select representative values. The proportion of the GVW carried by each individual axle is also simulated in this work using bimodal Normal distributions fitted to the observed data for each axle in each vehicle class. The correlation matrix is calculated for the proportions of the load carried by adjacent and non-adjacent axles for each vehicle class, and this matrix is used in the simulation using the technique described by Iman & Conover (1982).

Traffic flows measured at the site are reproduced in the simulation by fitting Weibull distributions to the daily truck traffic volumes in each direction, and by using hourly flow variations based on the average weekday traffic patterns in each direction. A year's traffic is assumed to consist of 250 weekdays, with the very much lighter weekend and holiday traffic being ignored. This is similar to the approach used by Caprani et al. (2008) and Cooper (1995). For same-lane multi-truck bridge loading events, it is important to accurately model the gaps between trucks, and the method used here is based on that presented by OBrien & Caprani (2005). The observed gap distributions up to 4 seconds are modeled using quadratic curves for different flow rates, and a negative exponential distribution is used for larger gaps.

The modeled traffic is bi-directional, with one lane in each direction, and independent streams of traffic are generated for each direction. In simulation, many millions of loading events are analyzed, and for efficiency of computation, it is necessary to use a reasonably simple model for transverse load distribution on two-lane bridges. The load effect considered for this paper is the mid-span bending moment on a simply-supported 15 m bridge, and it is assumed that for this there is an equal contribution laterally from each lane, i.e., the girder considered is located where the two lanes meet.

## 3 ANALYSIS AND RESULTS

### 3.1 Normal example

To assess the safety of a bridge, a limited quantity of data is generally used to infer a probability of failure, a characteristic maximum or a statistical distribution of maximum load effects. Probability of failure is clearly the most definitive measure of bridge safety. However, it is strongly influenced by resistance which varies greatly from one example to the next. In order to retain the focus on load effect, the resistance distribution used here is taken to be a Normal distribution with parameters chosen so as to give a daily probability of failure of  $10^{-5}$ .

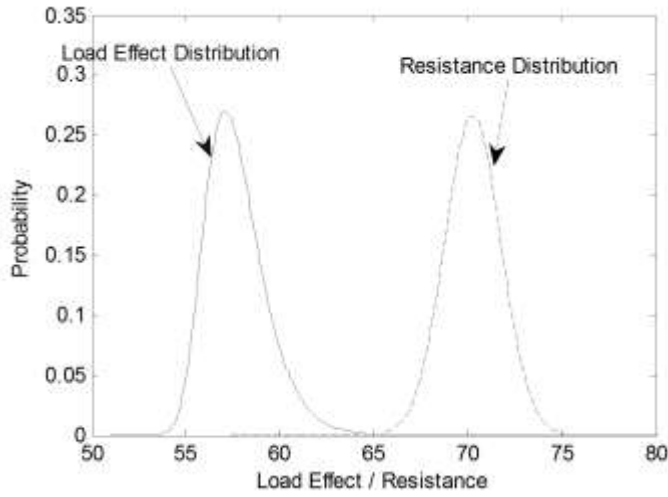


Figure 1. Load effect and resistance

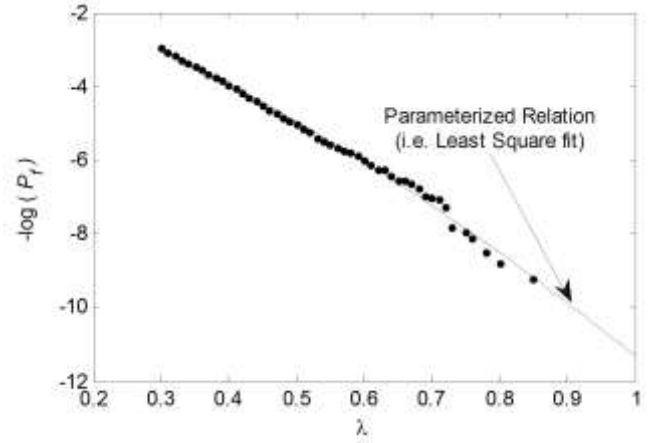


Figure 2. Curve fitting for enhanced Monte Carlo method

Three different extrapolation methods are tested to estimate the daily probability of failure:

- Enhanced Monte Carlo Simulation
- Generalized Extreme Value (GEV)
- Normal distribution

In this first sample problem, the loading is represented by a Normally distributed random variable (such as vehicle weight in tonnes), with a mean of 40 and a standard deviation of 5:

$$S \sim N(40, 5) \quad (5)$$

Values are generated from this distribution and three thousand values of  $S$  are considered as a block, say per day, with a maximum for day  $j$  of:

$$S_j = \max(S_{i,j}) \quad i = 1, 2, \dots, 3000 \quad (6)$$

The resistance is also assumed to be Normally distributed, with parameters chosen to give a benchmark probability of failure of  $10^{-5}$ :

$$R \sim N(70.2, 1.5) \quad (7)$$

The following simple safety margins are used for the enhanced Monte Carlo method:

$$M_j = R_j - S_j, \quad j = 1, \dots, m \quad (8)$$

where  $m$  is the number of days (blocks) of data. It should be noted that all random variables are assumed to be independent. The probability of failure is calculated for a range of values of  $\lambda$ , as defined in Equation 2. The mean safety margin is calculated from the simulated  $M_j$  values, and for each value of  $\lambda$ , the proportion of days on which failure occurs (i.e.,  $M(\lambda) \leq 0$ ) is calculated. An example of the curve fitting for Equation 3 is shown in Figure 2.

In the other two methods, GEV and Normal distributions are fitted to the daily maximum data, and the probability of failure is calculated by numerical evaluation of the convolution integral in Equation 1. The process is repeated for five different quantities of daily maximum data: 1000, 2500, 5000, 10 000 and 20 000 days. For the first case (i.e., 1000 days), the probability of failure is calculated 20 times so that a measure of the variability in the results can be found.

The results are illustrated in Figure 3 which shows how the estimation error generally reduces with increasing quantities of data, and in Figure 4 which shows, in each case (i) the median value (dashed line), (ii) the 25% to 75% range (boxed), (iii) the 0.7% to 99.3% range (median  $\pm$  2.7 standard deviation for normally distributed data, shown as error bars) and (iv) individual outliers beyond that range. Figure 3 shows that the Enhanced Monte Carlo and GEV fitting methods converge to the benchmark result as the quantity of data considered increases. Fitting to a Normal distribution gives a more ‘bounded’ distribution in this example, i.e., tending more towards an asymptote at extremely low probabilities. This results in a smaller probability of failure in comparison to the other two methods.

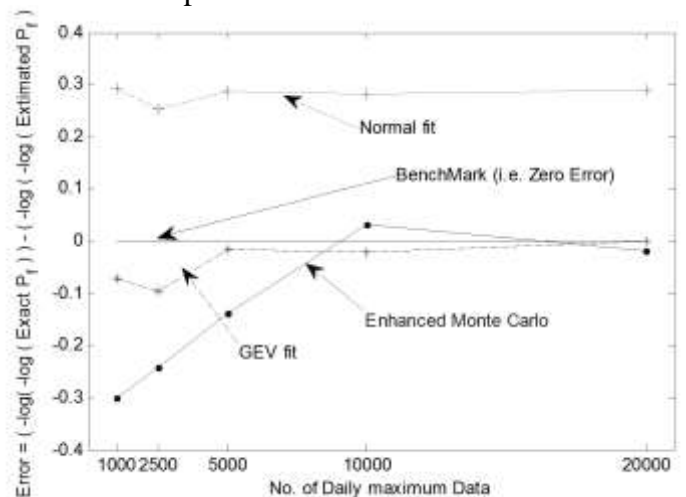


Figure 3. Effect of quantity of data on estimate of probability of failure – Normal example

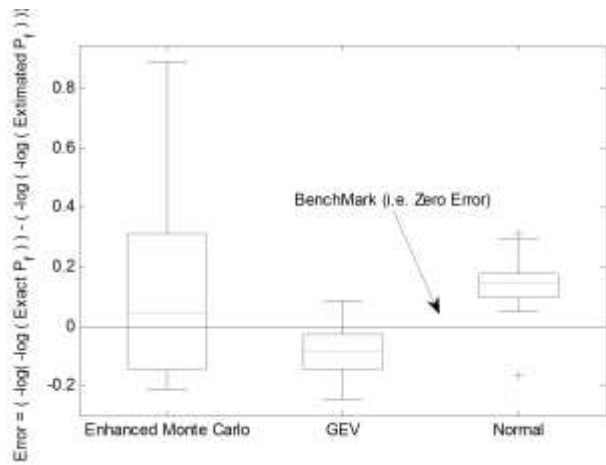


Figure 4. Variability of estimates of probability of failure – Normal example

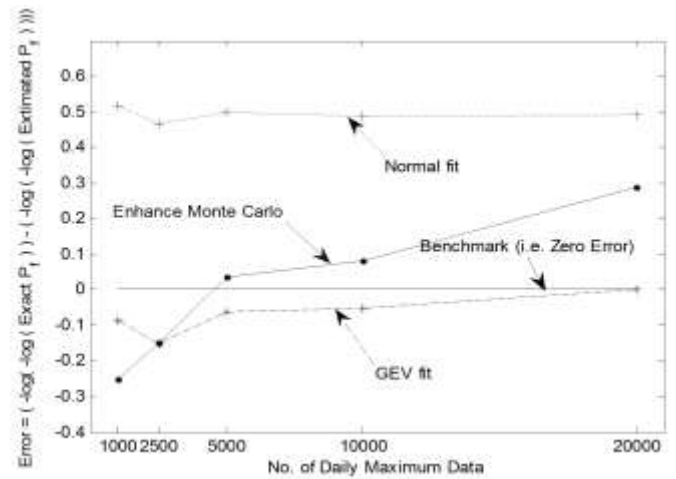


Figure 5. Effect of quantity of data on estimate of probability of failure – long run example

### 3.2 Long-run simulation example

A set of runs is performed consisting of 20 000 days of simulated traffic at the site in Slovakia, as described earlier. The benchmark probability of failure is calculated by fitting a GEV distribution to the daily maxima and numerically evaluating the convolution integral. In order to retain the focus on load effect, the resistance distribution chosen is a Generalized Extreme Value distribution (Equation 4) with parameter values  $\mu = 30.2$ ,  $\sigma = 1.5$  and  $\xi = 0.001$  which gives a probability of failure of  $10^{-5}$  for this simulated traffic. As in the Normal example, the process is repeated for five different quantities of daily maximum data: 1000, 2500, 5000, 10 000 and 20 000 days. For the first case (i.e., 1000 days), the probability of failure is calculated 20 times so that a measure of the variability in the results can be found.

The results are shown in Figures 5 and 6, which are quite similar to the corresponding results for the simpler Normal example, although in this case the enhanced Monte Carlo method does not converge to the same result as the GEV method. As before, the Normal method gives a smaller probability of failure (i.e. is non-conservative), and does not vary significantly as the quantity of data increases.

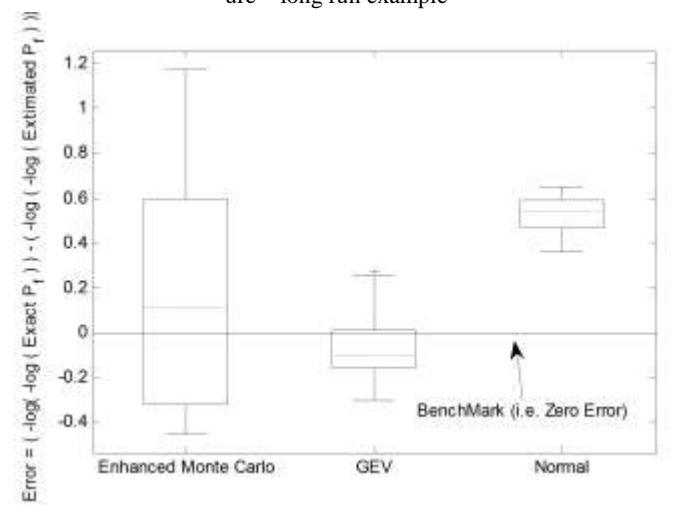


Figure 6. Estimates of probability of failure – long run example

## 4 CONCLUSION

The enhanced Monte Carlo method of estimating probabilities of failure is compared with two other methods (GEV and Normal), using two sample problems. In both problems, the loading distribution is defined by a sample of daily maximum values, and an analytical resistance distribution is used. In the enhanced Monte Carlo method, samples are drawn from the resistance distribution and the number of failures is counted. The GEV and Normal methods entail fitting a GEV and Normal distribution respectively to the sample loading data, and using the fitted distribution and the assumed resistance distribution to evaluate the convolution integral numerically. The first, simple, problem is based on a Normal distribution of both loading and resistance, and the second problem is based on a long-run simulation of traffic loading, and a GEV distribution of resistance. In both problems, the ‘true’ or ‘near-true’ probability of failure is known, allowing the estimation error to be calculated. The calculations are done for five different sample sizes for the loading data, and are repeated twenty times for the smallest sample (1000 daily maximum values) to establish the variability of the results. The results may be summa-

rized by describing the enhanced Monte Carlo method as relatively accurate, but not very precise (i.e., results are variable); the GEV method as being similarly accurate and significantly more precise, and the Normal method as having similar precision to the GEV method, but being less accurate.

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