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An Approach to Unification using a Linear Systems Model for the Propagation of Broad-Band Signals

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***Abstract*— We review the inhomogeneous scalar Helmholtz equation in three-dimensions and the scattering of scalar wavefields from a scatterer of compact support. An asymptotic solution is then considered representing the effect of the frequency approaching zero when a 'wavefield' reduces to a 'field'. The characteristics of ultra-low frequency Helmholtz scattering are then considered and the physical significance discussed of a model that is based on the scattering of Helmholtz wavefields over a broad frequency spectrum. This is equivalent to using a linear systems approach for modelling the propagation, interaction and detection of broad-band signals and provides an approach to the classification of a field from a wavefield that is intrinsically causal and thus, consistent with the basic principle of information theory. The approach leads to the proposal that all fields are derived from wavefields interacting over a broad frequency spectrum and that there are two principal field types: (i) fields generated by low frequency scattering - a 'gravitational field'; (ii) fields generated by high frequency eigenfield tendency - an 'electric field'.**

***Index Terms*— Helmholtz equation, asymptotic solutions, scattering theory, gravitational fields, electric fields**

I. INTRODUCTION

THE ideas presented in this paper are a first attempt to develop a universal physical model in which 'fields' and 'particles' do not exist along with such concepts as 'charge'. All that is considered is a universe consisting of scalar wavefields whose governing equation is the (inhomogeneous) Helmholtz equation over a broad frequency spectrum with a bandwidth that is determined by the Planck length

$$\ell = \sqrt{\frac{\hbar G}{c_0^3}} \sim 1.16 \times 10^{-35} \text{metres}$$

where $\hbar$ is Dirac's constant (Planck's constant divided by 2π), G is the gravitational constant and c_0 is the speed of light. The frequency associated with the Planck length is $c_0/\ell \sim 2.59 \times 10^{43}$Hz.

The rationale for a Planck bandwidth is as follows: Consider the hypothetical case where the de Broglie wavelength λ associated with a non-relativistic particle with constant velocity $v << c_0$ is continually decreased. The rest mass m of the particle will then increase according to $m = 2\pi\hbar/(v\lambda)$. As the mass increases, its Newtonian gravitational field will increase as will the escape velocity $v_e = \sqrt{2Gm/r} = \sqrt{4\pi\hbar G/(v\lambda r)}$ where r is the distance required to escape the gravitational field. Suppose that the wavelength becomes so small that the escape velocity is equal to the speed of light (i.e. the particle becomes a micro black hole), then $\lambda r = 4\pi\hbar G/(v c_0^2)$. We define the Planck length for the limiting case when $r \to 4\pi\lambda$ and $v \to c_0$, i.e. the length associated with the case when the velocity of a particle approaches the speed of light and the distance required to escape the gravitational field approaches the de Broglie wavelength of the particle. The Planck frequency sets a upper limit on the band width of a universal spectrum since, beyond this frequency, any particle (and the de Broglie wavefield associated with it) will not be detectable. The breadth of the spectrum is taken to be a consequence of the 'big-bang' (i.e. a broad frequency spectrum is the product of a short impulse).

Although the approach considered in this paper has some philosophical similarities to string theory, which is increasingly being challenged by a number of authors (e.g. [1], [2]), it is different in its 'scale'. If string theory is concerned with the interpretation of physics through wavefields with a wavelength of the order of ℓ, then, in this paper, we consider wavefields interacting (scattering) at all scales greater than the Planck length (i.e. over all frequencies less than the Planck frequency). In a sense, we consider the universe itself to be a single 'string' composed of a broad spectrum of (scalar) wavefields. This is a 'waves within waves' approach and can thus be interpreted in terms of a universal fractal model [3], not in terms of the 'shape of the universe' but in terms of the wavefields from which it is taken to be composed. In this paper, we adopt a formal scattering theory approach for a scalar Helmholtz wavefield and derive both standard and some non-standard results which are considered in terms of two fundamental experimental observations, the Poisson spot and the Einstein ring.

II. FIELD EQUATIONS

The field equations for electromagnetic and gravitational fields (i.e. Maxwell's equations [4] and Einstein's equations [5], respectively) appear to have only one thing in common: they both predict wave behaviour (the wavefields being composed of very different 'fields' with different properties), namely, electromagnetic waves and gravity waves respectively where, in the latter case, no direct experimental observations have been made, to date. In quantum mechanics, the quantum fields that are modelled through equations such as the Schrödinger [6], Dirac [7], [8], [9], Klein-Gordon (e.g. [10], [11]) and Rarita-Schwinger [12] equations, are not fields in

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the sense of an electric (vector) field or a gravitational (tensor - a curved vector space) field but wavefields of different types, i.e. scalar (Klein-Gordon and Schrödinger equations for the relativistic and non-relativistic case, respectively), scalar-spinor (Dirac equations), vector (Proca equations [13], [14]) and vector-spinor (Rarita-Schwinger equations) fields. The theoretical origin of these wavefields is a direct result of the fundamental postulates of quantum mechanics, namely, that energy $E = \hbar\omega$ and momentum $\mathbf{p} = \hbar\mathbf{k}$ for a wavefield with (angular frequency) ω and wavenumber $\mid \mathbf{k} \mid = 2\pi/\lambda$. Relating energy and momentum (particulate concepts associated with Newtonian mechanics) to frequency and wavelength respectively immediately raises the issue of particle verses wave. It also brings into focus the question of whether a field or a wavefield is more fundamental as discussed in this paper.

Apart from the Schrödinger equation, all of the equations listed above describe relativistic quantum fields. They are all 'products' of the fact that, given the postulates of quantum mechanics, Einstein's special theory of relativity allows for the existence of scalar, scalar-spinor, vector, vector-spinor and tensor fields. In each case, the field, as characterised by a given operator, is taken to describe a 'particle' (a localised entity) that is classified in terms of a Boson or Fermion which have integer or half-integer spin (the intrinsic angular momentum) respectively. This is compounded in the following table (where m denotes the rest mass):

Equation name	**Field Type**	**Spin $s\hbar$**	**Example**
Klein-Gordon	Scalar	$s = 0$	Higgs boson
Dirac	Scalar Spinor	$s = 1/2$	leptons: electrons, muons
Proca-Maxwell	Vector	$s = 1$	$m = 0$: photons gluons; $m \neq 0$: mesons
Rarita-Schwinger	Vector Spinor	$s = 3/2$	None discovered
Gravitation	Tensor	$s = 2$	gravitons

Note that, like the graviton, the Higgs boson is a hypothetical particle that is taken to explain the origins of mass m which has, to date, not been verified experimentally. The terms 'Boson' and 'Fermion' relate to the fact that the statistical behaviour of integer spin particles can be classified in terms of Bose-Einstein statistics and half-integer spin particles, in terms of Fermi-Dirac statistics.

Vector bosons are considered to mediate three of the four fundamental interactions in 'particle' physics, i.e. electromagnetic, weak and strong interactions, and tensor bosons (gravitons) are assumed to mediate the gravitational force as summarised in the following table:

Force	**Range**	**Transmitted by Bosons**
Gravitational	Long	Graviton, $m = 0$, $s = 2$
Electromagnetic	Intermediate	Photon, $m = 0$, $s = 1$
Weak	Short	$W^{\pm}$, Z_0, $m \neq 0$, $s = 1$
Strong	Short	gluons, $m = 0$, $s = 1$

Of the four fundamental forces in nature, gravity was the first to be 'invented' but, to this day, remains the most elusive. With just criticism over his universal theory of gravity and, in particular, the principle of instantaneous action at a distance, upon which the theory is based, Isaac Newton rightly stated that '*... I have told you how it works, not why*'. Here, we consider a causal approach to explaining the 'why'.

III. Fields, Wavefields and the Proca Equations

In electromagnetism and general relativity, the field equations are considered to be fundamental, the wave properties of these fields being a consequence of decoupling (under certain conditions) the field equations. In other words, the wave properties of these fields are, in a sense, a by-product of writing a set of coupled equations in terms of a single or set of equations of the same (wave) type. What if a wave equation was to determine the form of the field equations and thus the characteristics of the field(s)? The first to consider such an approach was the Romanian born Alexandru Proca who derived the Proca or Proca-Maxwell equations.

For a three-dimensional space $\mathbf{r} = \hat{\mathbf{x}}x + \hat{\mathbf{y}}y + \hat{\mathbf{z}}z$, with time denoted by t and with the Laplacian operator defined as

$$\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2},$$

it is well known that Maxwell's equations (specifically, the microscopic equations for point 'charges') can be decoupled to produce the inhomogeneous wave equations (e.g. [9], [15], [16])

$$\left(\nabla^2 - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)\phi(\mathbf{r}, t) = -\frac{\rho}{\epsilon_0}$$

and

$$\left(\nabla^2 - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{A}(\mathbf{r}, t) = -\mu_0\mathbf{j}$$

for the magnetic vector potential $\mathbf{A}$ and the electric scalar potential ϕ where ρ is the charge density, $\mathbf{j}$ is the current density and ϵ_0 and μ_0 are the permittivity and permeability of free space, respectively. This requires use of the gauge transforms

$$\mathbf{A} \rightarrow \mathbf{A} + \nabla X \quad \text{and} \quad \phi \rightarrow \phi - \frac{\partial X}{\partial t}$$

where the gauge function X is taken to satisfy the homogeneous wave equation

$$\left(\nabla^2 - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)X = 0.$$

The solutions at $(\mathbf{r}_0, t_0)$ for the 'retarded potentials' ϕ and $\mathbf{A}$ are then given by

$$\phi(\mathbf{r}_0, t_0) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\mathbf{r}, \tau)}{\mid \mathbf{r} - \mathbf{r}_0 \mid} d^3\mathbf{r}, \quad \tau = t_0 - \mid \mathbf{r} - \mathbf{r}_0 \mid /c_0$$

and

$$\mathbf{A}(\mathbf{r}_0, t_0) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{j}(\mathbf{r},\tau)}{\mid \mathbf{r}-\mathbf{r}_0 \mid} d^3\mathbf{r}$$

which show that a change in ρ and $\mathbf{j}$ affects ϕ and $\mathbf{A}$ $\mid \mathbf{r} - \mathbf{r}_0 \mid /c_0$ seconds later. The change propagates away from the sources ρ and $\mathbf{j}$ at a velocity c_0 which is the theoretical basis for the propagation of electromagnetic waves.

In quantum mechanics, energy E and momentum $\mathbf{p}$ are replaced by the wave operators

$$-i\hbar\frac{\partial}{\partial t} \text{ and } i\hbar\nabla$$

respectively. Thus, the non-relativistic 'free energy' (no potential energy component) equation

$$E = \frac{p^2}{2m}$$

yields Schrödinger's equation [17]

$$i\hbar\frac{\partial}{\partial t}U = -\frac{\hbar^2}{2m}\nabla^2 U$$

for a unit amplitude plane wave of the form

$$U(\mathbf{r},t) = \exp[i(\mathbf{k}\cdot\mathbf{r} - \omega t)].$$

In the relativistic case when

$$E = \pm\sqrt{p^2c_0^2 + m^2c_0^4} \text{ or } E^2 = p^2c_0^2 + m^2c_0^4$$

we obtain the (homogeneous) Klein-Gordon equation [9]

$$\left(\nabla^2 - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)U - \kappa^2 U = 0$$

where $\kappa = mc_0/\hbar$. This equation is taken to describe massive scalar Bosons (spin 0 particles) such as the Higgs boson. In contrast, the classical wave equation is taken to describe massless scalar (for the electric field potential) or vector (for the magnetic vector potential) Bosons, i.e. the photon.

Given that Maxwell's equations can be decoupled to produce inhomogeneous wave equations for ϕ and $\mathbf{A}$, Proca's idea was to modify Maxwell's equations in order to produce inhomogeneous Klein-Gordon equations for ϕ and $\mathbf{A}$ given by

$$\left(\nabla^2 - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)\phi(\mathbf{r},t) - \kappa^2\phi = -\frac{\rho}{\epsilon_0}$$

and

$$\left(\nabla^2 - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{A}(\mathbf{r},t) - \kappa^2\mathbf{A} = -\mu_0\mathbf{j}$$

respectively. The modifications required to do this yield the Proca equations given by

$$\nabla\cdot\mathbf{E} = \frac{\rho}{\epsilon_0} - \kappa^2\phi, \quad \nabla\cdot\mathbf{B} = 0$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}, \quad \nabla\times\mathbf{B} = \mu_0\mathbf{j} + \epsilon_0\mu_0\frac{\partial\mathbf{E}}{\partial t} + \kappa^2\mathbf{A}$$

where

$$\mathbf{B} = \nabla\times\mathbf{A}, \quad \text{and} \quad \mathbf{E} = -\nabla\phi - \frac{\partial\mathbf{A}}{\partial t}.$$

Note that the Klein-Gordon equations for ϕ and $\mathbf{A}$ imply that ϕ and $\mathbf{A}$ and thus $\mathbf{E}$ and $\mathbf{B}$ are effected by mass.

The Proca equations are relativistic field equations that describe massive electromagnetic fields or massive photons (spin 1 vector bosons). They form the foundations for the electro-weak theory (the unification of electromagnetism with the 'weak' force) where it is assumed that the electromagnetic fields of the early universe had significantly greater (relativistic) energies than now, i.e. the electromagnetic and the weak force are manifestation of the same force at relativistic energies. Vector Bosons ($W^\pm$ and Z_0 bosons) are taken to be mediators of the weak interaction. However, the Proca equations, as a description for massive photons, have a number of other implications. These include variations in light speed, the possibility of charged black holes, the existence of magnetic monopoles and superluminal (faster than light) particles (Tachyons) with an imaginary mass that can be described by a Proca field with a negative square mass [18], [19] and [20].

The principle associated with deriving the Proca equations can be applied to other field equations such as the Einstein equations for a gravitational field. The Proca-Einstein equations have been used as a basis for modelling the interaction of gravitational fields with dark matter, for example [21]. In string theory, there is tentative evidence that non-Riemannian models such as the Einstein-Proca-Wyle equations may account for dark matter [22]. However, in the context of this paper, the Proca equations are an example of the modification and extension of a set of field equations in order that a given wave equation is satisfied. Thus, in the derivation of the Proca equations, the wavefield U is the governing function and not the fields $\mathbf{E}$ and $\mathbf{B}$. In other words, the Proca equations are based on 'tailoring' a field to 'fit' a wavefield. This leads us to consider an approach in which unification is attempted, not in terms of a unified field theory but in terms of a unified wavefield theory where a wavefield U is not just the governing function but the governing principle.

If a unified field theory (unifying gravity and electromagnetism, for example) were available, then, by induction, we might expect that the unifying field equations yield a unifying wave equation. Since a unified field theory is not currently available, our approach is to attempt to construct a unified wavefield theory in which a field is the product of certain characteristics of a wavefield. Thus, the basic idea is to develop a universal physical model that is based on a wavefield equation alone and attempt to explain the characteristics of a field from the wavefield. In this paper, we adopt the (inhomogeneous) Helmholtz equation and study some of its properties over a broad frequency band including the case when the wavelength approaches infinity. We show how this approach can, for example, be used to explain phenomena such as the 'diffraction' of light by a field that we interpret to be a gravitational field.

IV. THE INHOMOGENEOUS HELMHOLTZ EQUATION

The three-dimensional inhomogeneous scalar Helmholtz equation can be derived from the (inhomogeneous) time dependent wave equation

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)U(\mathbf{r},t) = 0$$

by letting

$$\frac{1}{c^2}=\frac{1}{c_0^2}(1+\gamma)$$

where $\gamma(\mathbf{r})$ is a dimensionless quantity (the scattering function) and U is a time-dependent scalar wavefield (which is also taken to be dimensionless). We make no demands on the physical nature of U or γ.

With

$$U(\mathbf{r},t)=u(\mathbf{r},\omega)\exp(i\omega t)$$

for constant ω (the angular frequency), or with

$$U(\mathbf{r},t)=\frac{1}{2\pi}\int\limits_{-\infty}^{\infty}u(\mathbf{r},\omega)\exp(i\omega t)d\omega$$

for variable ω, we obtain the inhomogeneous Helmholtz equation in the form

$$(\nabla^2+k^2)u(\mathbf{r},k)=-k^2\gamma(\mathbf{r})u(\mathbf{r},k)$$

where k $(=2\pi/\lambda)$ is given by

$$k=\frac{\omega}{c_0}.$$

We consider a scattering function γ which is of compact support, i.e.

$$\gamma(\mathbf{r})\quad\exists\quad\forall\quad\mathbf{r}\in V$$

where V is an arbitrary volume. In electromagnetism, for example, the Helmholtz equation can be derived by decoupling Maxwell's (macroscopic) equations where u describes the scalar electric field and the scattering function is given by $\gamma=\epsilon_r-1$ where $\epsilon_r\geq 1$ is the isotropic relative permittivity, the relative permeability being taken to be 1 and the conductivity being taken to be zero [23].

V. Green's Function Solution for an Incident Plane Wave

Using Green's theorem, the general solution to the inhomogeneous Helmholtz equation at a point $\mathbf{r}_0$ is given by [9], [23],

$$u(\mathbf{r}_0,k)=\oint\limits_S(g\nabla u-u\nabla g)\cdot\hat{\mathbf{n}}d^2\mathbf{r}+k^2\int\limits_V g\gamma ud^3\mathbf{r}$$

where g is the 'outgoing free space' Green's function given by [23], [24]

$$g(\mathbf{r}\mid\mathbf{r}_0,k)=\frac{\exp(ik\mid\mathbf{r}-\mathbf{r}_0\mid)}{4\pi\mid\mathbf{r}-\mathbf{r}_0\mid}$$

which is a solution to the equation

$$(\nabla^2+k^2)g(\mathbf{r}\mid\mathbf{r}_0,k)=-\delta^3(\mathbf{r}-\mathbf{r}_0)$$

where δ^3 denotes the three-dimensional delta function. Here, S denotes the (closed) surface of the scattering function γ with volume V and $\hat{\mathbf{n}}$ is a unit vector that is perpendicular to an element of the surface $d^2\mathbf{r}$. Note that

$$g(\mathbf{r}\mid\mathbf{r}_0,k)=\frac{1}{4\pi\mid\mathbf{r}-\mathbf{r}_0\mid},\quad k\to 0$$

and thus,

$$\nabla^2\left(\frac{1}{4\pi\mid\mathbf{r}-\mathbf{r}_0\mid}\right)=-\delta^3(\mathbf{r}-\mathbf{r}_0).$$

To compute the surface integral, a condition for the behaviour of u on the surface S of γ must be chosen. We consider the case where a simple plane wave of unit amplitude given by

$$u_i(\mathbf{r},k)=\exp(ik\hat{\mathbf{n}}_i\cdot\mathbf{r})$$

and satisfying the homogeneous Helmholtz equation

$$(\nabla^2+k^2)u_i(\mathbf{r},k)=0$$

is incident on the surface of the scatterer. In this case,

$$u(\mathbf{r},k)=u_i(\mathbf{r},k),\quad\forall\mathbf{r}\in S$$

and we therefore obtain

$$u(\mathbf{r}_0,k)=\oint\limits_S(g\nabla u_i-u_i\nabla g)\cdot\hat{\mathbf{n}}d^2\mathbf{r}k^2\int\limits_V g\gamma ud^3\mathbf{r}=u_i+u_s$$

where

$$u_s=k^2\int\limits_V g\gamma ud^3\mathbf{r}.$$

The function u_s is the scattered wavefield which we shall write in the form

$$u_s(\mathbf{r},k)=k^2g(r,k)\otimes_3\gamma(\mathbf{r})u(\mathbf{r},k),\quad r=\mid\mathbf{r}\mid$$

where $\otimes_3$ denotes the three-dimensional convolution integral.

VI. Evaluation of the Scattered Field

To evaluate the scattered field (i.e. to compute u_s), we must define u inside the volume integral. Unlike the surface integral, a boundary condition will not help here because it is not sufficient to specify the behaviour of u at a boundary. In this case, the behaviour of u throughout V needs to be known. This requires a model to be chosen for u inside V that is compatible with a particular physical problem. The simplest model for the internal field is based on assuming that $u\sim u_i\forall\mathbf{r}\in V$. The scattered field is then given by

$$u_s(\mathbf{r}_0,k)=k^2g(r,k)\otimes_3\gamma(\mathbf{r})u_i(\mathbf{r},k).$$

This assumption - known as the Born approximation - provides an approximate solution for the scattered field which is valid if

$$k^2\|g(r,k)\otimes_3\gamma(\mathbf{r})\|<<1.$$

This result can be considered to be a first approximation to the (Born) series solution given by

$$u_s(\mathbf{r},k)=u_i(\mathbf{r},k)+k^2g(r,k)\otimes_3\gamma(\mathbf{r})u_i(\mathbf{r},k)$$
$$+k^4g(r,k)\otimes_3\gamma(\mathbf{r})[g(r)\otimes_3\gamma(\mathbf{r})u_i(\mathbf{r},k)]+...$$

which is valid under the condition

$$k^2\|g(r,k)\otimes_3\gamma(\mathbf{r})\|<1.$$

Each term in this series expresses the effects due to single, double and triple etc. scattering events. Because this series scales as k^2, k^4, k^6, ..., for a fixed $k<<1$ (long wavelength wavefields), the Born approximation becomes an exact solution.

VII. LOW FREQUENCY HELMHOLTZ SCATTERING

If a Helmholtz wavefield oscillates at lower and lower frequencies, then we can consider an asymptotic solution of the form

$$u_s(\mathbf{r}_0,k)=\frac{k^2}{4\pi}\int\limits_V \frac{\gamma(\mathbf{r})}{\mid \mathbf{r}-\mathbf{r}_0\mid}u_i(\mathbf{r},k)d^3\mathbf{r},\quad k\to 0.$$

This is a consequence of the fact that the higher order terms in the Born series can be ignored leaving just the first term as $k\to 0$ and because

$$\frac{\exp(ik\mid \mathbf{r}-\mathbf{r}_0\mid)}{4\pi\mid \mathbf{r}-\mathbf{r}_0\mid}=\frac{1}{4\pi\mid \mathbf{r}-\mathbf{r}_0\mid},\quad k\to 0$$

giving an exact solution to the problem.

If the incident field is a unit plane wave, then

$$u(\mathbf{r}_0,k)=1+u_s(\mathbf{r}_0,k)$$

where

$$u_s(\mathbf{r}_0,k)=\frac{k^2}{4\pi}\int\limits_V \frac{\gamma(\mathbf{r})}{\mid \mathbf{r}-\mathbf{r}_0\mid}d^3\mathbf{r},\quad k\to 0$$

which we write in the form

$$u_s(\mathbf{r},k)=\frac{k^2}{4\pi r}\otimes_3\gamma(\mathbf{r}),\quad k\to 0.$$

Here, the wavelength of the incident plane wavefield is assumed to be significantly larger than the spatial extent V of the scatterer. For a given scattering function $\gamma(\mathbf{r})$ the wavefield is a 'weak field' because of the low values of k required to produce this (asymptotic) result. But this result is the general solution to Poisson's equation

$$\nabla^2 u_s(\mathbf{r},k)=-k^2\gamma(\mathbf{r})$$

since, using the result

$$\nabla^2\left(\frac{1}{4\pi r}\right)=-\delta^3$$

we have

$$\nabla^2 u=\nabla^2 u_s=k^2\nabla^2\left(\frac{1}{4\pi r}\otimes_3\gamma\right)$$

$$=k^2\gamma\otimes_3\nabla^2\left(\frac{1}{4\pi r}\right)=-k^2\gamma\otimes_3\delta^3=-k^2\gamma.$$

By considering u_s to be a potential, we can write

$$\nabla\cdot\mathbf{U}_s(\mathbf{r},k)=k^2\gamma(\mathbf{r}),\quad \mathbf{U}_s(\mathbf{r},k)=-\nabla u_s(\mathbf{r},k).$$

Integrating over the volume of the scatterer V, we obtain

$$\int\limits_V\nabla\cdot\mathbf{U}_s(\mathbf{r},k)d^3\mathbf{r}=k^2\int\limits_V\gamma(\mathbf{r})d^3\mathbf{r}$$

and using the divergence theorem we can write

$$\oint\limits_S\mathbf{U}_s(\mathbf{r},k)\cdot\hat{\mathbf{n}}d^2\mathbf{r}=k^2\Gamma,\quad \Gamma=\int\limits_V\gamma(\mathbf{r})d^3\mathbf{r}.$$

If we now consider a scatterer that is a sphere, then the field $\mathbf{U}$ will have radial symmetry, i.e. $\mathbf{U}_s=\hat{\mathbf{n}}U_s$. In this case, the surface integral becomes $4\pi r^2U_s$ and we obtain

$$U_s=\frac{k^2\Gamma}{4\pi r^2},\quad k\to 0.$$

Hence, in the limit as $k\to 0$, Helmholtz scattering provides an exact solution for a weak field whose gradient (for the radially symmetric case) is characterized by a $1/r^2$ scaling law.

VIII. DIFFRACTION

For $k\to 0$, $u_s(\mathbf{r},k)$, which we now denote by $u_s^0(\mathbf{r},k_0)$, is the solution to

$$\nabla^2u_s^0(\mathbf{r},k_0)=-k_0^2\gamma(\mathbf{r})$$

where k_0 denotes a value for $k,\quad k\to 0$. Consider a Born scattered Helmholtz wavefield $u_s(\mathbf{r},k)$ for $k>>1$ given by

$$u_s(\mathbf{r},k)=k^2g(r,k)\otimes_3\gamma(\mathbf{r})u_i(\mathbf{r},k).$$

We can then write

$$u_s(\mathbf{r},k)=-\frac{k^2}{k_0^2}g(r,k)\otimes_3 u_i(\mathbf{r},k)[\nabla^2u_s^0(\mathbf{r},k_0)]$$

from which we can derive an expression for the far field scattering amplitude generated by the field $\mathbf{U}_s^0$ given by

$$u_s(\mathbf{r},k)=-\frac{k^2}{k_0^2}g(r,k)\otimes_3 u_i(\mathbf{r},k)[\nabla\cdot\mathbf{U}_s^0(\mathbf{r},k_0)]$$

$$=\frac{\exp(ikr_0)}{4\pi r_0}A(\hat{\mathbf{n}}_0,\hat{\mathbf{n}}_i),\quad \frac{r}{r_0}<<1$$

where, with $u_i(\mathbf{r},k)=\exp(ik\hat{\mathbf{n}}_i\cdot\mathbf{r}),\quad \hat{\mathbf{n}}_0=\mathbf{r}_0/\mid\mathbf{r}_0\mid$ and

$$\mathbf{U}_s^0=\hat{\mathbf{n}}U_s^0=\hat{\mathbf{n}}\frac{k_0^2\Gamma}{4\pi r^2},$$

$$A(\hat{\mathbf{n}}_0,\hat{\mathbf{n}}_i)=-\frac{k^2\Gamma}{4\pi}\int\limits_V\exp[-ik(\hat{\mathbf{n}}_0-\hat{\mathbf{n}}_i)\cdot\mathbf{r}]\nabla\cdot\left(\frac{\hat{\mathbf{n}}}{r^2}\right)d^3\mathbf{r}.$$

Hence, the wavefield $u_s(\mathbf{r},k)$ (for $k>>1$) generated by a scatterer that is simultaneously generating a scattered wavefield $u_s^0(\mathbf{r},k_0)$ is, in the far field (under the Born approximation) determined by the Fourier transform of the scattering function (assuming radial symmetry) $f(r)=\nabla\cdot(\hat{\mathbf{n}}r^{-2})$. In other words, the weak field generated by very low frequency scattering will diffract a high frequency Helmholtz wavefield, the diffraction pattern (i.e. the far field scattering pattern) being determined by $f(r)$.

A. Diffraction by an Infinitely Thin Scatterer

Consider the case where an incident plane wavefield is travelling in the z-direction, i.e. $u_i=\exp(ikz)$ and is incident on an infinitely thin scatterer defined by the function $\gamma(\mathbf{r})=\gamma(x,y)\delta(z)$. The scattered wavefield is then given by

$$u_s(x,y,z,k)$$

$$=k^2\frac{\exp(ik\sqrt{x^2+y^2+z^2})}{4\pi\sqrt{x^2+y^2+z^2}}\otimes_3\gamma(x,y)\delta(z)\exp(ikz)$$

$$= k^2 \frac{\exp(ik\sqrt{x^2+y^2+z^2}}{4\pi\sqrt{x^2+y^2+z^2}} \otimes_2 \gamma(x,y), \quad \gamma \;\; \exists \;\; \forall(x,y) \in S$$

where $\otimes_2$ denotes the two-dimensional convolution integral over area S. Writing out this result in the form

$$u_s(x_0, y_0, z_0, k)$$

$$= k^2 \int\int \frac{\exp[ik\sqrt{(x-x_0)^2+(y-y_0)^2+z_0^2}]}{4\pi\sqrt{(x-x_0)^2+(y-y_0)^2+z_0^2}} \gamma(x,y)dxdy,$$

it is clear that if the scattered wavefield is now measured in the far field, i.e. for the case when $x/z_0 << 1$ and $y/z_0 << 1$, then

$$z_0 \left(1 + \frac{(x-x_0)^2}{z_0^2} + \frac{(y-y_0)^2}{z_0^2}\right)^{\frac{1}{2}}$$

$$\simeq z_0 - \frac{xx_0}{z_0} - \frac{yy_0}{z_0} + \frac{x_0^2}{2z_0} + \frac{y_0^2}{2z_0}$$

and thus,

$$u_s(x_0, y_0, z_0, k) = \frac{\exp(ikz_0)}{4\pi z_0} \exp\left(ik\frac{x_0^2+y_0^2}{2z_0}\right) A(u,v)$$

where

$$A(u,v) = k^2\widetilde{\gamma}(u,v) = k^2\mathcal{F}_2[\gamma(x,y)]$$

$$= k^2 \int\int \exp(-iux)\exp(-ivy)\gamma(x,y)dxdy$$

with spatial frequencies u and v being defined by

$$u = \frac{kx_0}{z_0} = \frac{2\pi x_0}{\lambda z_0}$$

and

$$v = \frac{ky_0}{z_0} = \frac{2\pi y_0}{\lambda z_0}.$$

Here, $\mathcal{F}_2$ denotes the two-dimensional Fourier transform, the result being the standard expression for a diffraction pattern in the far field or Fraunhofer zone [23].

B. Diffraction by an Infinitely Thin Field

In the previous section, we derived the far field diffraction pattern for an infinitely thin scatterer. However, suppose this scatterer also radiates a field generated by low frequency Helmholtz scattering from the same scattering function. What is the contribution of this field to the diffraction of the same incident plane wave within and beyond the extent of the scatterer[1]? In this case, the scattered wavefield is given by (under the Born approximation)

$$u_s = -\frac{k^2}{k_0^2} g \otimes_3 u_i \nabla^2 u_s^0, \quad u_s^0 = \frac{k_0^2}{4\pi r} \otimes_3 \gamma.$$

For an infinitely thin scatterer given by $\gamma(x,y)\delta(z)$,

$$u_s^0(x,y,z,k_0) = \frac{k_0^2}{4\pi\sqrt{x^2+y^2+z^2}} \otimes_2 \gamma(x,y)$$

so that in the (x,y) plane located at $z = 0$,

$$u_s^0(x,y,k_0) = \frac{k_0^2}{4\pi\sqrt{x^2+y^2}} \otimes_2 \gamma(x,y).$$

For an incident plane wave $u_i = \exp(ikz)$, the scattered wavefield u_s is thus, given by

$$u_s(x,y,z,k) = -k^2 g(r,k) \otimes_3 \exp(ikz)...$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left(\frac{1}{4\pi\sqrt{x^2+y^2}} \otimes_2 \gamma(x,y)\right).$$

Repeating the calculation given in the previous section (for $z \to 0$), the diffracted wavefield now becomes

$$u_s(x_0, y_0, z_0, k) = \frac{\exp(ikz_0)}{4\pi z_0} \exp\left(ik\frac{x_0^2+y_0^2}{2z_0}\right) A(u,v)$$

where

$$A(u,v) = -2zk^2\mathcal{F}_2\left[\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\frac{1}{4\pi\sqrt{x^2+y^2}} \otimes_2 \gamma(x,y)\right].$$

Note that although the scatterer is taken to be 'infinitely thin' because $\gamma(\mathbf{r}) = \gamma(x,y)\delta(z)$, we still consider the physical thickness of the scatterer to be finite[2], i.e. $z \neq 0$. Now, for an arbitrary function $f \Longleftrightarrow \widetilde{f}$, where $\Longleftrightarrow$ denotes the transform from real space to Fourier space [23],

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) f \Longleftrightarrow -(u^2+v^2)\widetilde{f},$$

$$\frac{1}{\sqrt{x^2+y^2}} \Longleftrightarrow \frac{2\pi}{\sqrt{u^2+v^2}},$$

and we obtain

$$A(u,v) = zk^2\sqrt{u^2+v^2}\widetilde{\gamma}(u,v).$$

Figure 1 shows numerical simulations of the diffraction patterns compounded in the (intensity) functions

$$|\; \widetilde{\gamma}(u,v) \;|^2 \quad \text{and} \quad u^2+v^2 \;|\; \widetilde{\gamma}(u,v) \;|^2$$

using a two-dimensional Fast Fourier Transform for the case when the scattering function is given by the rotationally symmetric functions (for $r = \sqrt{x^2+y^2}$)

$$\gamma(r) = \exp(-r^2/\sigma^2)$$

(a unit amplitude Gaussian function[3] with standard deviation σ) and (a unit amplitude disc function)

$$\gamma(r) = \begin{cases} 1, & r \leq a; \\ 0, & \text{otherwise.} \end{cases}$$

The analytical solutions, for the intensity

$$I_1 = |\; u_s \;|^2$$

generated by diffraction from the scatterer γ and

$$I_2 = |\; u_s \;|^2$$

[1]Note that the scattered wavefield u_s^0 is taken to exist within and beyond the finite spatial extent of the scatterer $\gamma(\mathbf{r}), \mathbf{r} \in V$, i.e. u_s^0 is not of compact support since it is given by the convolution of a function of compact support with r^{-1}.

[2]z should be taken to be a positive real 'infinitesimal' for all real k.

[3]Taken by default, to be of finite extent.

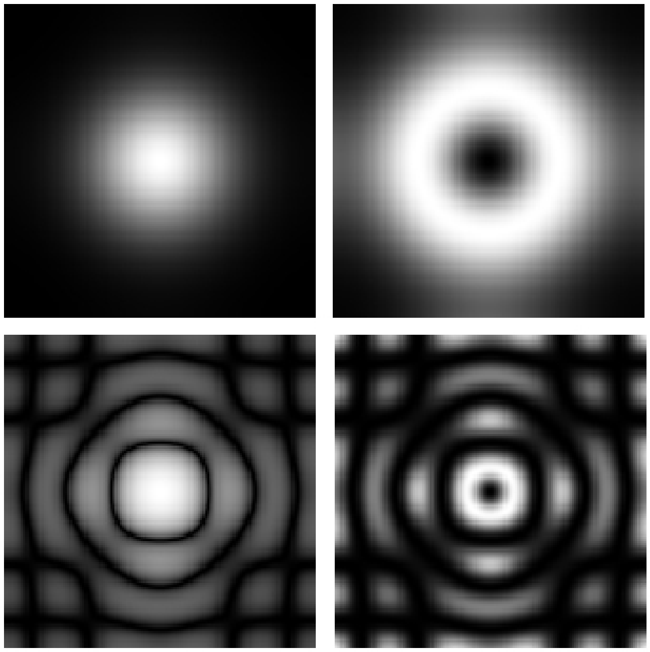

Fig. 1. Numerical simulation of the intensity patterns for an Gaussian function (top) and disc function (bottom) associated with the diffraction of a wavefield by an infinitely thin scatterer $\gamma(x,y)$ (left - plotted using a logarithmic scale) and the field $\nabla^2 u_s^0$ generated by the same scatterer.

generated by the field $\nabla^2 u_s^0$ are given by:

$$I_1(r_0,\lambda)=\frac{\pi^4\sigma^2}{z_0^2\lambda^4}\exp\left[-\left(\frac{2\pi^2\sigma^2r_0^2}{\lambda^2z_0^2}\right)\right]$$

and

$$I_2(r_0,\lambda)=z^2\frac{4\pi^6\sigma^2r_0^2}{z_0^4\lambda^6}\exp\left[-\left(\frac{2\pi^2\sigma^2r_0^2}{\lambda^2z_0^2}\right)\right]$$

for a Gaussian diffractor and, for a disc diffractor, with $\xi=\frac{2\pi a r_0}{\lambda z_0}$,

$$I_1(r_0,\lambda)=\frac{4\pi^4a^4}{z_0^2\lambda^4}\left(\frac{J_1(\xi)}{\xi}\right)^2$$

and

$$I_2(r_0,\lambda)=z^2\frac{16\pi^6a^4r_0^2}{z_0^4\lambda^6}\left(\frac{J_1(\xi)}{\xi}\right)^2.$$

Note that the Gaussian ring has a maximum when $r_0=z_0\lambda/(\sqrt{2}\pi\sigma)$ and that, in the latter case, the diffraction pattern is determined by the 'jinc' function $J_1(\xi)/\xi$ whose first minimum occurs when $\xi=3.83$, i.e. when

$$r_{\min}=1.22\frac{\lambda z_0}{a}$$

which is a classical result in (Fourier) optics - an Airy pattern [23]. Observe that the magnitude of the intensity patterns generated by the field $\nabla^2 u_s^0$ is significantly less than the scatterer γ, e.g. in the case of a Gaussian function

$$\frac{I_2}{I_1}=\frac{4z^2\pi^2r_0^2}{z_0^2\lambda^2}$$

and only if $r_0/\lambda\sim z/z_0$ will the magnitude become of the same order. Also observe that the intensity generated by the scatterer γ scales as λ^{-4} whereas the intensity generated by the field $\nabla^2 u_s^0$ scales as λ^{-6}. However, the most significant result is that diffraction for a scattering function produces a pattern whose intensity peaks at the centre of the image plane (a standard result in Fourier optics) but that diffraction from a low frequency scattered field produces a pattern characterised by a ring. The multiplicity of rings in either case is determined by whether or not the scattering function is discontinuous.

IX. The Poisson Spot and the Einstein Ring

Consider the images given in Figure 2 which show an example of a Poisson (or Arago) spot [25] and an Einstein ring [26]. The Poisson spot (named after Simeon Poisson who investigated the phenomenon in 1818) represents a landmark in the history of science in terms of validating whether or not light was a particle or a wave. The Poisson spot is a bright compact feature (a spot) that appears at the centre of the shadow of a circular opaque object. In Figure 2, the Poisson spot is the result of laser light diffracting from the edge of a ball-bearing. In a theoretical model of this effect, the ball-bearing can be replaced by an infinitely thin disc. However, because this disc is opaque, the scattering function must be defined by

$$\gamma(r)=\begin{cases}0, & r\leq a;\\ 1, & \text{otherwise}.\end{cases}$$

and the Fourier transform (assuming an incident plane wave $\exp(ikz)$ that is of infinite extent over the (x,y) plane) must be taken from $-\infty$ to $-a$ and from a to ∞. This is equivalent to computing the two-dimensional Fourier transform over all space and subtracting the Fourier transform over $r\leq a$. Since

$$\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}\exp(-iux)\exp(-ivy)dxdy=4\pi^2\delta(u)\delta(v)$$

the diffracted intensity for an opaque object is

$$I_1(r_0,\lambda)=\delta^2(r_0)+\frac{\pi^4a^4}{z_0^2\lambda^4}\left(\frac{2J_1(\xi)}{\xi}\right)^2.$$

The fact that the Poisson spot occurs within the geometrical shadow of an opaque object, is evidence that a particle and/or a geometrical theory of optics is invalid and that light must therefore be a wavefield. This deduction occurred some forty years before Faraday and Maxwell concluded that light was indeed a wave but one composed of electric and magnetic fields - a direct consequence of the fact that the field equations derived by Maxwell for an electric and magnetic field can be decoupled to yield a wave equation.

A. Gravitational Diffraction

The Einstein ring shown in Figure 2 is an effect that is conventionally explained in terms of the bending of light through the curvature of space (and time) by a mass. This is a consequence of the field equations for a gravitational field (the Einstein equations [5]). In order to obtain an Einstein ring, the magnitude of the gravitational field must be relatively high such as that generated by a spiral galaxy. Further, in order to generate a near perfect (complete) ring, the entire galaxy must be well aligned with regard to an observer in the 'object plane'. The bending of light by a gravitational field has an analogy with the geometrical interpretation of light interacting with a lens. At the edge of a lens, the light beam is 'bent' (discontinuously) by the change in refractive index from air to glass and from glass to air - the extreme edge of a lens acts like a prism. Like an optical lens, gravitational 'lensing' will produce distortions of the object plane when alignment of the 'earth-lens-object' is imperfect.

If we interpret an Einstein ring in terms of the results given in Section VIII(B), then the ring is not due to light being bent (continuously) by the curvature of a space-time continuum but the result of the diffraction of a plane wave (i.e. light) by the field $\nabla^2 u_s^0$ which is taken to be in the plane of the galaxy and to extend beyond it. This requires the magnitude of the scattering function to be very large in order to compensate for $z \rightarrow 0$. If we model a (spiral) galaxy in terms of a Gaussian function, then the ring associated with the diffraction pattern given in Figure 1 is, in this sense, a simulation of the Einstein ring given in Figure 2. The use of a Gaussian function to model the macroscopic gravitational field generated by a spiral galaxy is intuitive as the edges of a galaxy will not be discontinuous (especially on the scale of the wavelength of light!). However, in the case of a black hole, the event horizon defines an edge. In such a case, we might expect gravitational diffraction to produce a number of concentric rings similar to those associated with a Poisson spot, the black hole being modelled in terms of an opaque disc. Multiple ring patterns associated with a black hole are a prediction of the conventional bending of light by space-time curvature. The idea is that, close to the event horizon, the gravitational field is so intense that light can be curved right around the black hole by 180 degrees or more to produce a ring associated with the light generated by an object that exists in alignment with, and behind, the image plane [26]. These multiple Einstein ring predictions are based on arguments analogous to geometric optics whereas the multiple rings considered here are analogous to Fourier optics. In this sense, we are interpreting a gravitational field to be generated by the scattering of a long wavelength Helmholtz wavefield, i.e. the field $\mathbf{U}_s^0$ defines a 'gravitational field'.

B. Colour Analysis

Another feature of Einstein rings (complete or otherwise) is that, unless the source-galaxy system has been substantially red shifted (when both the galaxy and the ring appear red, e.g. [27]), the colour of the rings is blue (as in the example given in Figure 2) even, as in some cases that have been reported, when the galaxy itself is red [28]. If we accept an Einstein ring to be a gravitational diffraction phenomena, then the intensity of the diffracted light scales as λ^{-6} which explains the colour of the rings (blue light having the shortest wavelength in the visible spectrum). This is analogous to the explanation of why the Earth's atmosphere is blue in colour. Under the Rayleigh scattering condition in which the wavelength is significantly larger than the physical size of the scatterer (when the Born approximation is valid), the

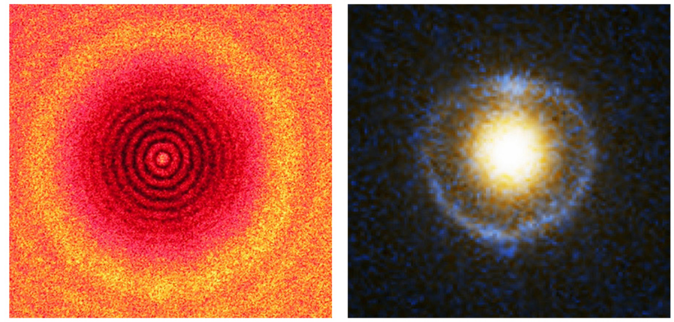

Fig. 2. Diffraction pattern from the incidence of laser light with a ball-bearing illustrating the Poisson spot (left) and an example of an Einstein ring generated by a spiral galaxy (central feature) observed with the Hubble Space Telescope (right).

scattering amplitude becomes independent of the scattering angle and the intensity of the scattered field is proportional to λ^{-4}. Thus, the sky is blue, because sunlight is scattered by the electrons of air molecules in the terrestrial atmosphere generating blue light preferentially around in all directions. Further, as the Sun approaches the horizon, we have to look more and more diagonally through the Earth's atmosphere. Our line of sight through the atmosphere is then longer and most of the blue light is scattered out before it reaches us, especially as the Sun gets very near the horizon. Relatively more red light reaches us, accounting for the reddish colour of sunsets. In other words, the λ^{-4} dependence of the scattered intensity implies that the atmosphere scatters green, blue and violet light photons more effectively than yellow, orange, and red photons. As the Sun approaches the horizon, the path of light through the atmosphere increases, so more of the short-wavelength photons get scattered away leaving the longer-wavelength photons and the Sun look progressively redder. Rayleigh scattering in the atmosphere also explains why the sun is yellow at mid-day. This is because the energy spectrum (i.e. Planck's radiation law [**?**]) for the Sun peaks at the point when the wavelength is that of green light (i.e. $\sim 4.7 \times 10^{-7}$metres). Since the atmosphere filters out blue light and since blue and yellow light combine to give green light, the Sun appears yellow.

Note that the λ^{-6} scaling dependency associated with gravitational diffraction provides a method of validating or otherwise the theoretical model presented in this paper. We require a scenario in which the same Einstein ring is recorded simultaneously over a broad frequency spectrum (e.g. using radio, infrared, visible and ultraviolet imaging) in such a way that the intensities of each image (relative to a known source that can be used for calibration) can be compared on a quantitative basis.

The theoretical ideas established so far and some of the implications that have been discussed are without reference to any physical significance of the scattering function. In the following sections we examine the characteristics of this scattering function by revisiting two wave equations in quantum mechanics, namely the Schrödinger equation (for the non-relativistic case) and the Klein-Gordon equation (for the relativistic case).

X. Schrödinger Scattering

If we consider the diffraction of light by a material object, then physically, the scattering function $\gamma(\mathbf{r})$ must describe some appropriate property of matter (the material properties) that is consistent with electromagnetic theory. On the macroscopic scale (i.e. many orders of wavelength) the relative permittivity, permeability and conductivity are the basis for defining Maxwell's macroscopic equations [**?**]. These material properties vary considerably from one application to the next. They may be isotropic or non-isotropic functions of space, time varying and field varying (non-linear optics), for example.

In electromagnetism, the use of the scalar Helmholtz equation to develop the results given so far, is compatible only with the case when the relative permeability is 1, the conductivity is zero and when the material is isotropic (i.e. the relative permittivity is a scalar function of space). However, in terms of a universal wavefield theory, matter is ultimately composed of matter waves which conform to matter wave equations such as the Schrödinger equation.

The fundamental postulates of quantum mechanics are that $E = \hbar\omega$ and $\mathbf{p} = \hbar\mathbf{k}$. Given that

$$E = \frac{p^2}{2m}$$

then

$$\frac{1}{c^2} = \frac{k^2}{\omega^2} = \frac{p^2}{E^2} = \frac{2m}{E}$$

and the wave equation

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)U(\mathbf{r},t) = 0, \quad \frac{1}{c^2} = \frac{1}{c_0^2}(1+\gamma)$$

can be written in terms of the Helmholtz equation

$$(\nabla^2 + k^2)u(\mathbf{r},k) = -k^2\gamma u(\mathbf{r},k), \quad \gamma = \frac{2mc_0^2}{E} - 1.$$

Note that for a potential energy function E_p when

$$E = \frac{p^2}{2m} + E_p,$$

the scattering function is given by

$$\gamma = \frac{2mc_0^2(E - E_p)}{E^2} - 1.$$

In either case, we note that Schrödinger's equation is obtained when the angular frequencies defining k and E are the same. Thus, the scattering function associated with the Helmholtz equation given above is, in this sense, a generalization of Schrödinger's equation where the wavefield $U(\mathbf{r},t)$ can oscillate at any frequency ω less than, or significantly less than the frequency, ω_1 say, associated with a matter wave of energy $E = \hbar\omega_1$. Schrödinger's equation is therefore taken to be a 'product' of the limiting case: $\omega \rightarrow \omega_1$[4].

Defining the scattering function in this way, we note that

$$U_s^0 = \frac{k_0^2\Gamma}{4\pi r^2}$$

where, for constant E and m,

$$\Gamma = Mm$$

and

$$M = \frac{V}{m}\left(\frac{2mc_0^2}{E} - 1\right), \quad V = \int\limits_V d^3\mathbf{r}.$$

Suppose that a mass m', placed in the vicinity of the field U_s^0, experiences a force F that is proportional to Um' so that

$$F = v^2 U m'$$

where v^2 is a constant of proportionality. Then

$$F = v^2 k_0^2 \frac{\Gamma m'}{4\pi r^2} = G\frac{mm'}{r^2}, \quad G = \frac{Mv^2k_0^2}{4\pi}$$

and v has the dimensions of velocity (i.e. $\text{length.second}^{-1}$). We can then derive an expression for the wavelength of the field U_s^0 in terms of the gravitational constant G, i.e.

$$\lambda_0 = \frac{2\pi}{k_0} = \frac{c_0}{\nu}$$

where ν is the frequency given by

$$\nu = r\frac{c_0}{v^2}\sqrt{\frac{Gm}{\pi V}}, \quad r = \sqrt{\frac{E}{2mc_0^2 - E}} \quad .$$

Note that for the frequency (and wavelength) to be a real positive quantity, we require that

$$2mc_0^2 > E$$

so that

$$\frac{2mc_0^2}{E} - 1 > 0 \quad \Longrightarrow \quad \gamma > 0.$$

Also note that because v has dimensions of velocity, the 'force field has an associated 'speed'.

The inhomogeneous Helmholtz equation

$$\left(\nabla^2 + \frac{\omega^2}{c_0^2}\right)u = -\frac{\omega^2}{c_0^2}\gamma u$$

where

$$\gamma = 2mc_0^2(E - E_p)/E^2 - 1$$

is the Schrödinger equation in 'disguise' in the sense that if $\omega \rightarrow \omega_1$ where $E = \hbar\omega_1$, then

$$(\nabla^2 + k_1^2)u = \gamma_1 u$$

where

$$k_1^2 = \frac{\omega_1^2}{c_0^2} = \frac{2mE}{\hbar^2} \quad \text{and} \quad \gamma_1 = \frac{2mE_p}{\hbar^2}.$$

Given that Proca's equations can be decoupled to produce inhomogeneous Klein-Gordon equations for ϕ and $\mathbf{A}$, we can adopt the same procedure to obtain the following inhomogeneous wave equations for the non-relativistic case, i.e.

$$\left(\nabla^2 - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)\phi(\mathbf{r},t) - \gamma\frac{1}{c_0^2}\frac{\partial^2\phi}{\partial t^2} = -\frac{\rho}{\epsilon_0}$$

and

$$\left(\nabla^2 - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{A}(\mathbf{r},t) - \gamma\frac{1}{c_0^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = -\mu_0\mathbf{j},$$

[4] An entirely phenomenological argument (like Schrödinger's equation itself!).

Maxwell's equations being modified to the form

$$\nabla\cdot\mathbf{E}=\frac{\rho}{\epsilon_0}-\gamma\frac{1}{c_0^2}\frac{\partial^2\phi}{\partial t^2},\quad \nabla\cdot\mathbf{B}=0$$

$$\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t},\quad \nabla\times\mathbf{B}=\mu_0\mathbf{j}+\epsilon_0\mu_0\frac{\partial\mathbf{E}}{\partial t}+\gamma\frac{1}{c_0^2}\frac{\partial^2\mathbf{A}}{\partial t^2}.$$

The fields ϕ_s^0 and $\mathbf{A}_s^0$ (the equivalent of U_s^0) are given by

$$\phi_s^0=\frac{k_0^2\Gamma}{4\pi r^2}+\frac{P}{4\pi\epsilon_0 r^2}$$

and

$$\mathbf{A}_s^0=\hat{\mathbf{n}}_0\frac{k_0^2\Gamma}{4\pi r^2}+\frac{\mu_0\mathbf{J}}{4\pi r^2},\quad \hat{\mathbf{n}}_0=\mathbf{A}_s^0/\mid\mathbf{A}_s^0\mid$$

where, for time-independent functions ρ and $\mathbf{J}$,

$$P=\int\limits_V\rho(\mathbf{r})d^3\mathbf{r}\quad\text{and}\quad \mathbf{J}=\int\limits_V\mathbf{j}(\mathbf{r})d^3\mathbf{r}.$$

Note that for the limiting case when $\omega\to\omega_1$ we obtain modified Schrödinger equations for ϕ and $\mathbf{A}$ given by

$$(\nabla^2+k_1^2)\phi=\gamma_1\phi-\frac{\rho}{\epsilon_0}$$

and

$$(\nabla^2+k_1^2)\mathbf{A}=\gamma_1\mathbf{A}-\mu_0\mathbf{j}.$$

In the context of the results above, we might interpret the field U_s^0 in terms of a low frequency electric scalar potential (in a charge free environment with $\rho=0$). In this sense, we could interpret the field U_s^0 as an ultra low frequency electromagnetic field in terms of an answer to the question: how long does a radio wave have to be before it becomes something else? However, in the universal wave model considered here, fields such as ϕ and $\mathbf{A}$ are subservient to the wavefield characterised by a governing wave equation in a similar sense to the rationale associated with the derivation of the Proca equations. Thus, the issue as to whether U_s is interpreted in terms of an electromagnetic, gravitational or quantum field is redundant, at least in the conventional sense. Rather, we consider all fields such as ϕ to be a characteristic of wavefields interacting over a broad frequency range. In this sense, the use of a scalar wavefield U in quantum mechanical equations such as the Schrödinger and Klein-Gordon equations is also being used in the interpretation of electromagnetism and gravitation. Field equations such as Maxwell's and Einstein equation's must be re-interpreted and derived from a universal wavefield approach alone, along with the physical interpretation of an electric and gravitational field.

XI. Klein-Gordon Scattering

For the relativistic case

$$E^2=p^2c_0^2+m^2c_0^4$$

and

$$\frac{1}{c^2}=\frac{k^2}{\omega^2}=\frac{p^2}{E^2}=\frac{1}{c_0^2}-\frac{m^2c_0^2}{E^2}.$$

The wave equation

$$\left(\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)U(\mathbf{r},t)=0$$

can thus be written in terms of the Helmholtz equation as

$$(\nabla^2+k^2)u(\mathbf{r},k)=-k^2\gamma u(\mathbf{r},k)$$

where γ is the 'Klein-Gordon scattering function' given by

$$\gamma=-\frac{m^2c_0^4}{E^2}$$

The field U_s^0 is then given by

$$U_s^0=-\frac{k_0^2\Gamma}{4\pi r^2}$$

where (for constant E and m)

$$\Gamma=Mm^2,\quad M=\frac{c_0^4V}{E^2}.$$

We note that in this case, U_s^0 is proportional to the square of the mass and is of negative polarity compared to the non-relativistic case, i.e. it will generate a repulsive force on a particle of mass m' given by

$$F=-G\frac{m^2m'}{r^2}.$$

XII. Intermediate Scattering

Since (for positive energies)

$$E=\sqrt{p^2c_0^2+m^2c_0^4}\simeq\frac{p^2}{2m}+mc_0^2,\quad \frac{p^2}{m^2c_0^2}<<1$$

we recover Schrödinger's equation

$$i\hbar\frac{\partial U}{\partial t}=-\frac{\hbar^2}{2m}\nabla^2U+mc_0^2U$$

which now includes the rest mass energy term mc_0^2U. In order to consider the intermediate scattering problem (intermediate between Schrödinger and Klein-Gordon scattering) we need to derive a wave equation that unifies both the Schrödinger and Klein-Gordon equations. One approach to this is through the introduction of a fractional time derivative $\partial^q/\partial t^q,\quad 1<q<2$ where $q=1$ provides Schrödinger's equation and $q=2$ yields the Klein-Gordon equation. A fractional partial differential equation that achieves this unification is (derived through induction)

$$\left(\nabla^2-\frac{1}{c^q}\frac{\partial^q}{\partial t^q}\right)U=K_nU$$

where (c having fractional dimension $L^{2/q}s^{-1}$)

$$\frac{1}{c^q}=\left(\frac{2m}{i\hbar}\right)^{2-q}\frac{1}{c_0^{2(q-1)}}$$

and

$$K_n=\begin{cases}2^{2-q}\kappa^2, & n=1;\\ a^{2-q}(q-1)\kappa^{2(q-1)}, & n=2.\end{cases}$$

The function K_n provides unification for the Schrödinger equation with $(n=1)$ and without $(n=2)$ the rest mass term, the constant a, with fractional dimension $L^{2(q-2)/(2-q)}$, being required to yield dimensional compatibility. With

$$\frac{1}{c^q}=\frac{1}{c_0^q}(1+\gamma)$$

we can then write

$$\left(\nabla^2 - \frac{1}{c_0^q}\frac{\partial^q}{\partial t^q}\right)U = \gamma\frac{1}{c_0^q}\frac{\partial^q U}{\partial t^q} + K_n U$$

where

$$\gamma = \left(\frac{2mc_0}{i\hbar}\right)^{2-q} - 1 = (-2i\kappa)^{2-q} - 1.$$

Defining a fractional differential in terms of the Fourier transform, i.e.

$$\frac{\partial^q}{\partial t^q}U(\mathbf{r},t) \Longleftrightarrow (i\omega)^q u(\mathbf{r},\omega),$$

we have

$$\left(\nabla^2 + \Omega^2\right)u = -\Omega^2\gamma u + K_n u$$

where

$$\Omega^2 = -\frac{(i\omega)^q}{c_0^q}, \quad \Omega = \pm i\frac{(i\omega)^{q/2}}{c_0^{q/2}}.$$

The Born scattered field is then given by

$$u_s = \Omega^2 g(r,\omega) \otimes_3 \gamma u_i - g(r,\omega) \otimes_3 K_n u_i$$

where

$$g(r,\omega) = \frac{\exp(i\Omega r)}{4\pi r}.$$

The time dependent Green's function can be evaluated using the series expression for the complex exponential term by term as follows (taking $\Omega = -i(i\omega/c_0)^{q/2}$ to give consistency with the 'outgoing free space' Green's function in the case when $q = 2$):

$$G(r,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega \exp(i\omega t)\frac{\exp[(i\omega/c_0)^{q/2}r]}{4\pi r}$$

$$= \frac{1}{4\pi r}\frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega \exp(i\omega t)[1 + (i\omega/c_0)^{q/2}r$$

$$+\frac{1}{2!}(i\omega/c_0)^q r^2 + ...] = \frac{\delta(t)}{4\pi r} + \frac{1}{4\pi}c_0^{-q/2}\frac{\partial^{q/2}}{\partial t^{q/2}}\delta(t)$$

$$+\frac{1}{4\pi}\sum_{n=1}^{\infty}\frac{1}{(n+1)!}r^n c_0^{-(n+1)q/2}\frac{\partial^{(n+1)q/2}}{\partial t^{(n+1)q/2}}\delta(t).$$

Inverse Fourier transforming and using the convolution theorem, the time-dependent scattered field is given by

$$U_s = -\frac{1}{c_0^q}\frac{\partial^q}{\partial t^q}G(r,t) \otimes_3 \otimes_t \gamma U_i - G(r,t) \otimes_3 \otimes_t K_n U_i$$

where $\otimes_t$ denotes the convolution integral over t and U_s and U_i are the time-dependent scattered and incident fields respectively (i.e. the inverse Fourier transforms of u_s and u_i, respectively[5]). We note that for $r \rightarrow 0$,

$$U_s = -\frac{1}{4\pi r} \otimes_3 \gamma\frac{1}{c_0^q}\frac{\partial^q U_i}{\partial t^q} - \frac{1}{4\pi r} \otimes_3 K_n U_i$$

$$-\frac{1}{4\pi c_0^{3q/2}}\frac{\partial^{3q/2}}{\partial t^{3q/2}}\int_V \gamma(\mathbf{r})U_i(\mathbf{r},t)d^3\mathbf{r}$$

$$-\frac{1}{4\pi c_0^{q/2}}\frac{\partial^{q/2}}{\partial t^{q/2}}\int_V K_n(\mathbf{r})U_i(\mathbf{r},t)d^3\mathbf{r}$$

and that in the ultra-low frequency range (i.e. in the limit as $\omega_0 \rightarrow 0$),

$$u_s^0 = \frac{\Omega_0^2}{4\pi r} \otimes_3 \gamma - \frac{1}{4\pi r} \otimes_3 K_n.$$

In this case, the field U_s^0 is given by (for constant γ and κ)

$$U_s^0 = \frac{V}{4\pi r^2}(\Omega_0^2\gamma - K_n)$$

which is zero when $\Omega_0^2\gamma = K_n$ or when

$$k_0 = \frac{(-2)^{(q-2)/q}\kappa^{(q-2)/q}K_n^{1/q}}{\left(1 - \frac{(i/2)^{2-q}}{\kappa^{2-q}}\right)^{\frac{1}{q}}}.$$

XIII. Interpretation

If we define a gravitational field (for a spherically symmetric scatterer) to be given by the field U_s^0 then the interpretation of what gravity is must change. According to the universal scalar wavefield model considered in this paper, a gravitational field is due to the scattering (by a material object composed of a spectrum of matter waves) of very low frequency scalar Helmholtz wavefields. Thus, if two bodies are in proximity, then each body will scatterer low frequency waves and each will interact with the scattered wavefield generated by the other, both experiencing an attractive (in the non-relativistic case) gravitational force given by $v^2 m' U_s^0$ where m' is the mass of the other body. In this sense, we define gravity as follows:

Two bodies are attracted to each other because each 'detects' the 'gravity waves' scattered by the other in the non-relativistic case.

However, the term 'gravity waves' used here is not the same as that used in general relativity. The term relates to the low frequency components of a scalar wave spectrum and must be interpreted within the context of the limiting condition $k \rightarrow 0$.

The model provides results that are compatible with observable characteristics of a gravitational field: (i) a gravitational field is a weak field; (ii) a gravitational field is characterized by an inverse square law; (iii) a gravitational field deflects light; (iv) gravity is an attractive only force. However, in this model, the 'deflection' of light is not taken to be due to the bending of light as it travels through a curved space-time manifold (Einstein's model) but through the diffraction of light (and other electromagnetic radiation) by a gravitational field. It should be noted that, according to this model, gravity waves (as understood in terms of Einstein's equations) can not be measured. The attempt to detect Einstein gravity waves (i.e. the gravity waves predicted by general reativity) is the equivalent of constructing a weighing machine to weigh itself! Rather,

[5]For notational convenience, we have used U_s to represent the time-dependent wavefield $U_s(\mathbf{r},t)$ which should not be confused with the use of $U_s(\mathbf{r},k), k \rightarrow 0$ in Section VII or U_s^0 as used in Section VIII.

we are ‘detecting’ gravity waves all the time, the effect of this ‘detection’ manifesting itself in terms of the ‘force of gravity’ we are all accustomed to.

The attractive only condition is valid for the non-relativistic case (i.e. for the Schrödinger scattering function). In the relativistic case, although the gravitational field U_s^0 is still weak, it depends on the square of the mass and generates a repulsive force. Note that in the case of the Schrödinger scattering function with potential energy E_p, then

$$\gamma > 0 \implies \frac{2mc_0^2(E-E_p)}{E^2} - 1 > 0$$

However, for any material characterised by a case when $E_p > E$, the scattering function is negative and the gravitational field defined by $\mathbf{U}_s^0$ will yield a repulsive force.

XIV. Principle of Eigenfield Tendency: Quantum Mechanics Revisited

Given the approach considered in this paper, an eigenfield tendency principle is required in order to explain the properties of matter as described by Schrödinger’s equation (in the non-relativistic case) as originally conceived by Schrödinger [6]. For different potential energy functions $E_p(\mathbf{r})$, it is well known that this equation describes eigenfield systems that can be used to model the properties of matter through the principles of quantum mechanics (in the full context of the subject). The original reason for deriving the Schrödinger scattering function was so that the asymptotic behaviour of a scattered Helmholtz wavefield (i.e. when $\omega \to 0$) could be examined. However, the consequence of this is that the Helmholtz equation is the governing wave equation only over a limited frequency band and that as the frequency of a wavefield increases (i.e. as $\omega \to \omega_1$) the Helmholtz equation reduces to the Schrödinger equation. If we consider the Schrödinger equation to represent eigenfields (at least in terms of its description of matter waves), then we can argue that at the higher end of the our universal spectrum, wavefields tend to behave more and more like eigenfields. Matter is thus taken to be composed of eigenfield systems at higher and higher frequencies; first the atom, then the nucleus, then the constituents of the nucleus (the quarks) and so on. Equations such as Schrödinger’s equation and Dirac’s equation are both descriptions for eigenfield systems at different energies (non-relativistic and relativistic energies respectively).

In the context of matter being an eigenfield described by solutions to Schrödinger’s equation, consider the case of a free electron and a free proton and the formation of hydrogen gas. In conventional (particle) terms, an electron and a proton have the same charge but of opposite polarity. This attracts the particles to form a neutral hydrogen atom, an effect which requires the introduction of a field, namely, an electric field. In terms of a wavefield theory, both the electron and proton are waves. In an ionised state, the electron is a free wave and the proton (relative to the electron) is a potential which is itself an eigenfield system (consisting of a higher frequency spectrum - the ‘nuclear spectrum’). The free wavefield requires greater energy to exist in a free state and hence, based on the principle of least energy, will ‘attempt to exist’ as an eigenfield. This ‘eigenfield’ may have a number of eigenstates, each with a specific energy level. The difference in energy between the free energy state and the available eigenstate(s) provides a residual energy, i.e. a free energy wavefield with frequency $E/\hbar$. Once formed, the eigenfield will not share its eigenstate(s) as this will require greater energy and hence, if another electron comes in to the vicinity of the neutral hydrogen atom, it will appear to undergo a repulsive force. On the other hand, since the combined eigenfields associated with two hydrogen atoms requires lower energy than two separate eigenfields (i.e. two hydrogen atoms) then the result is the diatomic Hydrogen molecule H_2 - the result of a covalent bond. In this sense, an electric field is not the product of a charge, rather it is that entity associated with the propensity for a free wavefield to become an eigen wavefield. A magnetic field is then a measure of the rate of change over which this propensity is satisfied, i.e. If $U(\mathbf{r},t)$ exists such that

$$\int\int \mid U(\mathbf{r},t)\mid^2 d^3\mathbf{r}dt$$

is a minimum, then

$$\text{Free Wavefield} \quad \xrightarrow[\text{Magnetic field } \frac{\partial \mathbf{E}}{\partial t}]{\text{Electric Field } \mathbf{E}} \quad \text{Eigen Wavefield}$$

Note that the transition described by Free Wavefield→ Eigen Wavefild may have both magnitude and direction since a free wavefield will attempt to find the shortest possible path in a three-dimensional space in order to become an eigen wavefield. An electric field will therefore appear to be a vector field. Further, if the transition has no directional preference, then an electric field will appear to have a Coulomb field strength characterised by an inverse square law.

The principle of eigenfield tendency is just the principle of least energy as applied to a universal wavefield model of the type attempted in this paper. It is, however, a principle which allows us to explain an electric field without having to refer to the concept of a field being ‘radiated’ by a charge! For example, ‘electron cloud’ repulsion theory (Valence Shell Electron Pair Repulsion) is used to predict shapes and bond angles of simple molecules in which the ‘electron cloud’ may be a single, double or triple bond, or a lone pair of electrons - a non-bonding pair of electrons. The ‘electron clouds’ are taken to be negatively charged since the electrons are negatively charged, so electron clouds repel one another and try to get as far away from each other as possible. Instead of considering the electron cloud to consist of negatively charged electrons, we consider the cloud to be a eigenfield which arranges itself in such a way that it can exist in a minimum energy state, a state that affects the geometry of the molecule. In a simple hydrogen atom, for example, the eigenfield will be distributed symmetrically because, in a three-dimensional space, spherical symmetry represents the most energy efficient configuration which is equivalent to the electron wavefield ‘experiencing’ a Coulomb potential.

The eigenfunctions that are the solutions to the Schrödinger equation for different materials will not necessarily be complete eigenfunctions. In some cases, solutions only allow for the existence of quasi-eigenfunctions. In conventional atomic physics, quasi-eigenfunctions are incomplete standing waves more commonly referred to a delocalised electrons. These are electrons that exist in the 'lattice' of a material but are free to move and provides a material with the property we refer to as conductivity. This includes materials such as various metals and chemicals (e.g. Benzene which is composed of a ring of delocalised electrons). The principle difference between an eigenfield and a quasi-eigenfield, is that a quasi-eigenfield has an energy spectrum, albeit a narrow one.

The Schrödinger scattering function for matter waves is

$$\gamma = \frac{2mc_0^2(E-E_p)}{E^2} - 1.$$

In a macroscopic sense, E_p is the total potential energy associated with all the nuclei from which a material of compact support is composed and E is the total energy associated with the electrons. In the case of elements such as gold, the arrangement of electrons around the nucleus is such that a single electron occupies the outermost shell and is an example of a quasi-eigenfield, i.e. a relatively free wavefield (a free electron) that is only loosely bound to the host atom. Successive energy levels are contained in a small energy range dE and are so close that, in effect, a continuous energy spectrum is formed. Each energy level in this spectrum can accommodate a left-travelling and right-travelling wave ('spin-up' and 'spin-down' electrons - Pauli's principle) and these free electrons will distribute themselves throughout the energy band from 0 to some value E. Irrespective of any particular system, the number of possible modes of oscillation per unit volume dn in a frequency range ν to $\nu + d\nu$ for waves with a propagation velocity of c is given by

$$dn = \frac{4\pi\nu^2 d\nu}{c^3}.$$

With $E = p^2/(2m) = \hbar\omega$ and $p = \hbar\omega/c = E/c$, then

$$dp = \frac{\hbar d\omega}{c} \quad \text{and} \quad dE = \frac{p}{m}dp = \hbar d\omega.$$

The number of states per unit volume in the energy interval dE is therefore

$$dn(E) = \frac{(2m^3)^{\frac{1}{2}}E^{\frac{1}{2}}}{2\pi^2\hbar^3}dE$$

and thus, the total number of electrons per unit volume in the energy spectrum $(0, E)$ is[6]

$$n(E) = 2\frac{(2m^3)^{\frac{1}{2}}}{2\pi^2\hbar^3}\int_0^E E^{\frac{1}{2}}dE = 2\frac{(2m^3)^{\frac{1}{2}}}{3\pi^2\hbar^3}E^{\frac{3}{2}}.$$

Here m is taken to be the mass of an electron. Note that if the material is in a 'ground state' then the available electrons will occupy the lowest possible energy level. Further, if the total number of electrons per unit volume is less than the total number of energy levels available in a band (the bandwidth of the material), then the electrons can occupy all energy states up to a maximum energy $E_{\max}$ - the Fermi Energy. In this sense, the Fermi energy defines the (energy) bandwidth of a (conductive) material composed of a quasi-eigenfield.

With an atomic number of 79, gold is the heaviest of the most conductive elements in the periodic table, i.e. the product of the conductivity with the atomic number ($\sim 3.57\times10^7 \text{cm}\Omega$) for gold is larger than any other element. If it were possible to reduce the total energy associated with the total quasi-eigenfield of gold such that $E < E_p$, then the result would be a scattering function that is negative. This requires the Fermi energy of gold to be reduced, the most influential factors being temperature and volume. Clearly, if the number of electrons per unit volume n is reduced then so is the Fermi energy. In terms of a physical material, n is determined by the number of atoms defining the physical extent of the material. This suggests an experimental investigation of the cryogenic properties of M-state (mono-atomic) gold. M-state gold is a white powder and is an example of a nano-material where each of the nano-metre size grains are clusters of a few hundred atoms. Like other M-state materials, the surface area is huge compared to the metallic (macro-crystalline) form. Thus, with the volume of each grain being small enough and the temperature of the material being low enough, it may be possibly to reduce the Fermi energy to an extent where $E < E_p$ for the material as a whole.

XV. Discussion

The results developed in this paper encapsulate a phenomenology where the Helmholtz equation is, in effect, being used in an attempt to develop a unified scalar wavefield theory where the wavefield $u(\mathbf{r}, \omega)$ is taken to exist over a broad range of frequencies limited only by the Planck frequency. At very high frequencies, u is taken to describe matter waves which are characterised by relativistic (Klein-Gordon and Dirac equations) and non-relativistic energies (Schrödinger equation) associated with nuclear and atomic physics respectively. At intermediate frequencies, u is taken to describe waves in the 'electromagnetic spectrum' and at low frequencies, u is taken to describe waves in the 'gravity wave spectrum'.

The structure of matter, the characteristics of light and other electromagnetic radiation and the properties of gravity become phenomenologically related via Helmholtz scattering over different frequency bands. Low frequency waves (gravity generating waves) are scattered by high frequency waves (matter waves) to produce a gravitational field; intermediate frequency waves (electromagnetic spectrum) are scattered by high frequency waves (e.g. a lens) but can also be scattered by the field generated from the scattering of low frequency waves to produce gravitational diffraction. In this sense, 'physics' becomes the study of waves interacting with waves at vastly different frequencies, the breadth of the spectrum 'reflecting' the instantaneous birth of the universe - the 'big-bang' - since it requires (noting that the Fourier transform of a δ-function is a constant over all frequency space) a short impulse to generate a broad frequency spectrum. However, in attempting to derive a 'wavefield theory of everything' we must re-interpret the nature of an electric field using the principle

[6] The factor of 2 is because of Pauli's principle.

of eigenfield tendency. Thus, instead of contemplating an electron in terms of a particle with a negative charge that 'radiates' an electric field and is attracted to particles with a positive charge (which also 'radiate' an electric field), we can visualise an electron in terms of a wave which is 'attracted' by the 'requirement' (through the minimum energy principle) of becoming an eigenfunction (a standing wave with lower energy than a free wave) whose properties are determined by the potential energy associated with the atomic nucleus which is itself, a higher (nuclear) frequency eigenfield system (quarks).

The form of the wave equation

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)U(\mathbf{r},t) = 0$$

dictates that c must be of finite value. If a wavefield (whatever the wavefield may be) was to convey information from one point in space to another instantaneously, then the second term of the above equation would be zero; the 'wave equation' would be reduced to 'Laplace's equation' $\nabla^2 u = 0$. Einstein's principal postulate is that the upper limit at which any wavefield can propagate is the speed of light c_0 in a perfect vacuum and thus $c \leq c_0$. In a more general perspective, the rationale associated with the fact that c must have a finite upper bound is that the influence of any physical wavefield on any measurable entity can only occur in a finite period of time and that there can be no such thing as instantaneous 'action at a distance', i.e. as Issac Newton put it: *That one body may act upon another at a distance through a vacuum, without the mediation of anything else, by and through which their action and force may be conveyed from one to the other, is to me so great an absurdity, that I believe no man who has in philosophical matters a competent faculty of thinking, can ever fall into it.* Taking Newton's own term, *mediation* requires the propagation (of information), but propagation at infinite speeds is not propagation and thus, we postulate that instantaneous fields are not possible, i.e. the speed at which a wavefield propagates must be finite for a wavefield to exist. In this context, the results developed for this paper highlight the idea that the 'physics' of a wavefield is more fundamental than the 'physics' of a field. This principle should be considered in light of the fact that the one property common to the principal field equation of physics (e.g. Einstein's equations, Maxwell's equations, Proca's equations), is that they all describe wave phenomena - at least in an 'indirect' sense. In the case of Proca's equations, the field equations are derived with the singular aim of ensuring that they can be decoupled to yield the inhomogeneous Klein-Gordon (wave) equation.

The underlying philosophy associated with the approach considered, is based on a 'waves within waves' model, i.e. to quote an old Chinese proverb *'In every way, one can see the shape of the sea'*. This is a universal self-affine or fractal model in which the 'fractal field' is a scalar wavefield, a symbolic representation of the idea being given in Figure 3. As the frequency increases, a wavefield tends to become an eigenfield. This principle is required to explain the structure of matter and much of the discussion given in Section XIII is quantum mechanics revisited without the need to define an electric field in terms of a charge. If we consider the structure of matter at the atomic, nuclear and sub-nuclear scales (indeed at all scales down to the scale of the Planck length) to be determined by eigenfields, then the question remains as to why eigenfield systems should 'kick-in' at the atomic scale? If the principle of eigenfield tendency applies at all frequencies then why do we not observe equivalent naturally occurring eigenfield systems in the electromagnetic spectrum? Perhaps we do under special circumstances, e.g. ball-lightning.

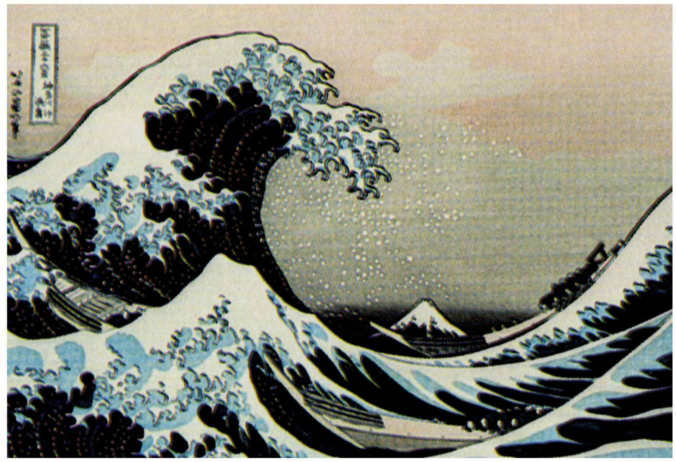

Fig. 3. Example of fractal waves by the Japanese artist K Hokusai from the 1800s illustrating waves of different scale in both amplitude and wavelength.

The approach to unification considered in this paper has yielded a number of questionable and speculative results. The only experimental evidence offered in confirmation to our model for a gravitational field is a possible explanation as to why the Einstein rings associated with near field galaxies observed by the Hubble Space Telescope are blue. However, it should be noted that this 'evidence' is most typical of Carl Popper's principle that all observation statements are 'theory laden' and that other explanations may be possible that are more appropriate in terms of established physical models.

In general relativity, the curvature of space-time bends light by the same amount irrespective of the frequency - there is no dispersion relation. The λ^{-6} scaling law associated with gravitational diffraction may be validated (or otherwise) from appropriate simultaneous observations of the same Einstein ring (complete or otherwise) at different wavelengths. Other consequences such as a gravitational field generating a repulsive force that is proportional to the mass squared in the relativistic case remain of theoretical consequence only. However, it is noted that inflation theory (the expansion of the early universe) requires gravity to be a repulsive force.

The model considered in this paper leads to the proposition that a gravity field is regenerative and exists through the continuous scattering of existing low frequency Helmholtz wavefields. This proposition may provide an answer to the following question: If nothing can escape the event horizon of a black hole because nothing can propagate faster than light then how does gravity get out of a black hole? The conventional answer to this question is that the field around a black hole is 'frozen' into the surrounding space-time prior to the collapse of the parent star behind the event horizon and remains in that state ever after. This implies that there is no need for continual regeneration of the external field by causal agents. In other words, the explanation defies causality. In the model presented here, the gravitational field generated by a black hole or any other body is the result of a causal effect - the scattering of low frequency scalar waves. In this sense, a black hole is just a stronger scatterer than other cosmological bodies and a gravitational field 'gets out of a black hole' because it was never 'in the black hole' to start with.

Propagative or wave theories of gravity have been proposed for many years. In 1805, Laplace proposed that gravity is a propagative effect and considered a correction to Newton's law to take into account the observation that gravity has no detectable aberration or propagation delay for its action. Laplace's ideas were advanced further by Weber, Riemann, Gauss and Maxwell in the Nineteenth Century using a variety of 'corrective terms'. In 1898, Gerber, developed a propagative theory that took into account the perihelion advance of mercury and in 1906 Poincaré showed that the Lorentz transform cancels out gravitational aberration. After the success of general relativity (1916) for explaining gravity in terms of a geometric effect, propagation theories were discarded. However, more recently, attempts at explaining gravity in terms of causal effects through a 'propagative' force have been revisited [29] as debate over the basic Einsteinian postulates[7] has intensified. Moreover, from Laplace to the present, propagation theories of gravity consider an object to be 'radiating' a field (in a passive sense). If general relativity considers gravity to be the result of an object warping space-time, then the proposition reported in this paper is that gravity is the result of an object scattering (long wavelength) waves that already exist as part of the low frequency component of a universal spectrum which is, itself, the by-product of the 'big-bang'. The compatibility of this approach with general relativity might be realised if the wavefield as taken to warp space-time so that space-time is the medium of propagation.

Any propagation theory of gravity must address some basic known observations: (i) Gravity has no detectable aberration or propagation delay for its action leading to effects predicted by general relativity such a *gravitomagnetism*; (ii) the finite propagation of light causes radiation pressure for which gravity has no counterpart pressure. These results represent the most vital evidence with regard to gravity being a geometric and not a propagative effect. For example, in an eclipse of the Sun, the gravitational pull on the earth by this 3-body (Sun-Moon-Earth) configuration increases. By comparing the delay in time it takes to observe the visible maximum eclipse on Earth (which can be calculated from knowledge of the distance of the Moon from the Earth) with the equivalent gravitational maximum, then if gravity is a propagating force, it appears to propagates at least 20 times faster than light! [30] Irrespective of whether this value is valid or not, a fundamental issue remains, which is compounded in the question: what is the speed of gravity? If we consider gravity to be a propagation and/or a low frequency scattering effect, then in order to account for the lack of propagation delay, it must be assumed that the speed of gravity is greater than the speed of light. This is contrary to the Einsteinian postulates if these postulates are taken to apply to all wavefields irrespective of their wavelength. The model presented here assumes that the speed of gravity is the same as the speed of light c_0. However, the asymptotic result $k \to 0$ used to define a gravitational field yields, what will appears to be, an instantaneous effect from a wavefield that is taken to propagate at the speed of light. The wavelength is so long compared to the distances associated with a Sun-Moon-Earth system, for example, that the speed of gravity will appear to be significantly faster than the speed of light (i.e. $\mathbf{U}_s^0$ is observed to be an instantaneous field).

[7]The invariance of the propagation of light in a vacuum for any observer which amounts to a presumed absence of any preferred reference frame.

XVI. Final Comments

In terms of the fractal wavefield model considered here, the gravitational force is a consequence of very long wavelength waves and is therefore a long range force. Electromagnetism is a consequence of intermediate wavelength waves which exist as both free wavefields and eigen wavefields at the atomic scale, the transition from one to the other creating an 'electric field'. The strong force is a consequence of a nuclear eigen wavefield where the values of $E = \hbar\omega$ and $p = \hbar k$ are in the relativistic energy limit. The weak force (associated with radioactive decay, for example) is explained in terms of the transformation of a nuclear eigen wavefield to a more stable form allowing for the emission of a free wavefield (quantum 'tunneling effect' when the potential barrier is low). For example, Rutherford scattering (the scattering of alpha particles from gold nuclei which historically provided the basic model for the atom) is an example of a free (nuclear) wavefield, interacting with a stable eigenfield system which consequently appears to exert a repulsive Coulomb force. At this frequency range the governing equation is Schrödinger's equation which has a far field scattering amplitude determined by the three-dimensional Fourier transform of a Coulomb potential. Thus, as a function of the scattering angle θ

$$A(\theta) = \frac{2\pi}{k\sin\left(\frac{\theta}{2}\right)}\int_0^\infty \sin\left[2kr\sin\left(\frac{\theta}{2}\right)\right]\gamma(r)rdr$$

and for the screened Coulomb potential[8]

$$\gamma(r) = \frac{\exp(-ar)}{r}, \quad a > 0$$

we obtain (for $a \to 0$)

$$A(\theta) = \frac{\pi}{k^2\sin^2\left(\frac{\theta}{2}\right)}\left(1 + \frac{a^2}{\left[2k\sin\left(\frac{\theta}{2}\right)\right]^2}\right)^{-1} = \frac{\pi}{k^2\sin^2\left(\frac{\theta}{2}\right)}.$$

The intensity (scattering cross-section) is therefore inversely proportional to $\sin^4(\theta/2)$ which is the basic 'signature' of Rutherford scattering. In terms of neutron scattering, a neutron is a free nuclear wavefield which, during its life time, is unable to combine with an existing nuclear eigen wavefield until it does, in some cases producing unstable nuclear eigen

[8]Required in order evaluate the integral over r.

wavefield systems which transform into new stable systems involving the emission of free wavefields, i.e. nuclear fission.

Note that the principle of eigenfield tendency in which free wavefields tend to become eigen wavefield in order to achieve a minimum energy is equivalent to the least action principle. In field theory - in this case, the wavefield $U(\mathbf{r}, t)$ - the Lagrangian density $\mathcal{L}$ is a functional that is integrated over all space-time, i.e.

$$\mathcal{S}[U] = \int\int \mathcal{L}[U, \partial_\mu U] d^3\mathbf{r}dt$$

where, using 'relativistic notation',

$$\partial_\mu = (\partial^0; \nabla), \quad \partial^\mu = (\partial^0; -\nabla),$$

$$\partial^0 = \frac{1}{c}\frac{\partial}{\partial t} \quad \text{and} \quad \partial_\mu \partial^\mu = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2.$$

The Lagrangian is the spatial integral of the density and application of the least action principle yields the Euler-Lagrange equations

$$\frac{\delta \mathcal{S}}{\delta U} = -\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu U)} \right) + \frac{\partial \mathcal{L}}{\partial U} = 0$$

which are then solved for U.

The wavefield approach adopted in this paper is consistent with the basic concepts associated with the *Grand Unified Theories* of C H Tejman [31] and in one sense, we have attempted to explain the example images given in Figure 2 using a single phenomenological model. Just as Poisson used a wave model to explain the Poisson spot without reference to light being an electromagnetic wave (Maxwell's equations for an electric and magnetic field which Poisson did not know of at the time), so we have attempted to explain both a Poisson spot and an Einstein ring without reference to general relativity (Einstein's equation for a gravitational field). The problem then remains of how to formally 'recover' Maxwell's equations and Einstein's equations from a single wave theoretic model.

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