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# EXERGETIC COST ANALYSIS OF A MECHANICAL EXERGY RECYCLE

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A mechanical plant that involves an exergy recycle is presented for analysis. It consists of gearboxes and a fluid coupling. There is one input of shaft power to the plant and there are three separate outputs of shaft power. An exergetic cost analysis approach is outlined and applied to the plant.

## NOMENCLATURE

| | |
|---|---|
| A, B, C,... | System identifiers (including subsystems, sources, and sinks) |
| $X$, $Y$ | Recirculating components of exergy rate interactions |
| $\dot{\Xi}_{\mathrm{A}\to\mathrm{B}}$ | Exergy rate interaction from system A to system B |
| $\dot{\Xi}^{*}_{\mathrm{A}\to\mathrm{B}}$ | Exergetic cost rate interaction from system A to system B |

## 1. INTRODUCTION

The concepts of exergy-based costing have been developed by various workers; including Beyer (1978), Tsatsaronis and Winhold (1985), and Valero et al. (1986). Difficulties have persisted and consensus has not yet been achieved among those active in this field. The object of writing this paper was to examine a simple plant that would focus attention on recycles in exergetic costing. A recycle is defined in Appendix A.

A mechanical plant that contains a recycle of energy and of exergy is described and examined in order to investigate the structure of its exergy interactions and exergetic cost interactions. All the exergy interactions at boundaries are in the form of shaft power. There are no exergy interactions associated with mass transfer, heat transfer, or displacement work. Hence the plant is a good subject for testing cost analysis procedures based on exergy.

## 2. DESCRIPTION OF THE PLANT

The external sources and sinks and the systems that comprise the plant are identified in Fig. 1. In Fig. 2 the net shaft power interactions between the sources, sinks, and component systems of the plant are shown. The corresponding exergy rate interactions are equal to these.

A source of mechanical power E provides 100 kW of shaft power to gearbox A. This gearbox also receives a shaft power input of 12 kW from gearbox D and has a shaft power output of 110 kW to gearbox B.

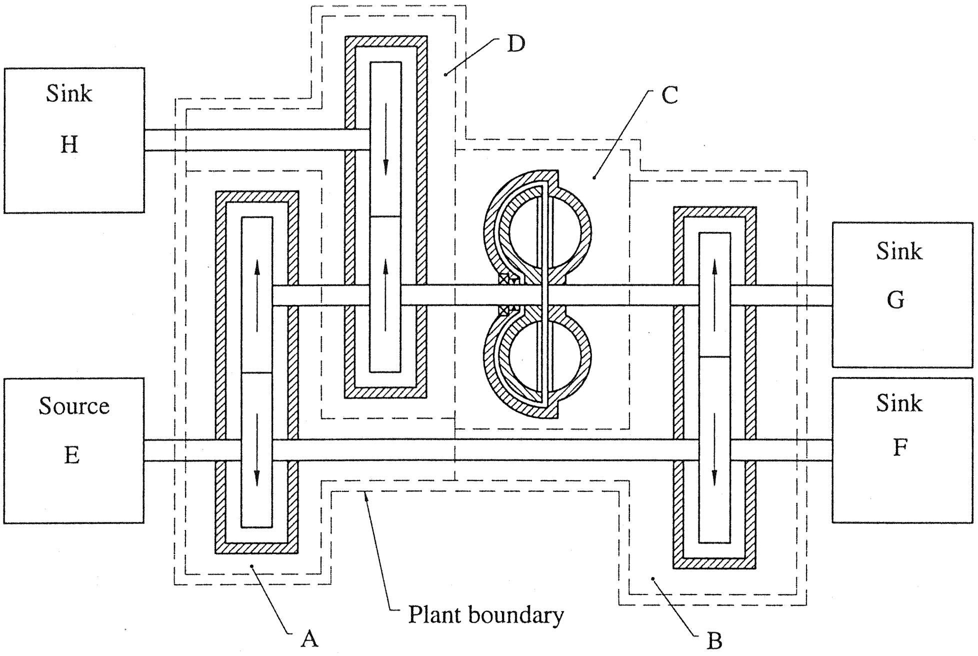

Fig. 1. Schematic representation of a plant, a source of mechanical power, and three sinks of mechanical power. The plant comprises three gearboxes and a fluid coupling. Arrows indicate the directions of rotation of the gears and shafts.

The difference between the total shaft power input to gearbox A and its shaft power output represents the rate at which heat transfer occurs from it to the environment. As the analysis boundaries of all the systems in Fig. 1 are drawn such that the temperature on the boundary is equal to the environmental temperature there is no exergy transfer associated with heat at any of the boundaries.

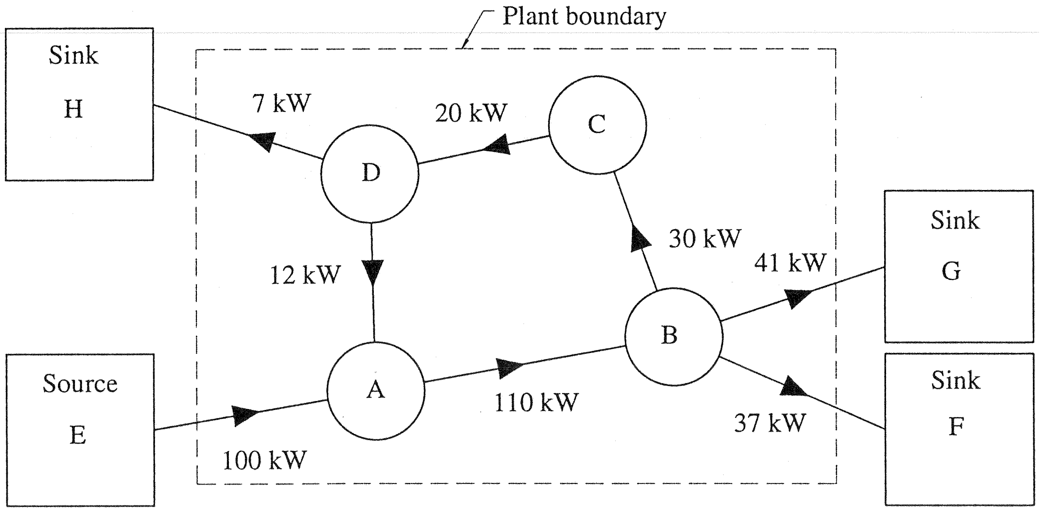

Fig. 2Shaft power interactions between the sources, sinks, and component systems of the plant. These are identical to the exergy rate interactions between the same systems.

Gearbox B receives 110 kW of shaft power from gearbox A and transfers 37 kW at the same shaft speed to the sink of mechanical power F. Gearbox B provides a shaft speed increase, in the ratio 2:3, from the lower to the upper shaft. The upper shaft of gearbox B transfers 41 kW of mechanical power to sink G and 30 kW to the fluid coupling C.

The input and output torques of fluid coupling C are equal, but the speed is reduced from the input to the output shaft in the ratio of 3:2. There is considerable heat transfer from the fluid coupling to the surroundings: 10 kW. However, the associated exergy transfer at the boundary is zero. Gearbox D receives a power input of 20 kW from the fluid coupling and transfers 12 kW to gearbox A and 7 kW to sink H. It should be noted that the upper gear in gearbox A applies a torque to the lower gear in the same direction as the torque provided by the mechanical power source E.

The exergy rate interactions linking systems A, B, C, and D in Fig. 2 all have the same sense. Some of the energy and exergy throughput of the plant is recycled through these four systems.

## 3. EXERGETIC COST ANALYSIS

The objective of an exergetic cost analysis is to determine the exergetic costs of the outputs of the plant: what exergy input from source E is associated with each of the exergy outputs to sinks F, G, and H? The exergetic costing approach to be applied is a cost accounting system based on the following tenets:

1. The appropriate measure of any energy transport or transfer across a boundary for costing purposes is the corresponding exergy transfer.
2. There may be multiple exergy interactions between any pair of systems, but only one net exergy interaction. Exergetic costing is based on the latter.
3. An exergetic cost balance applies for the plant as a whole and for each of its subsystems: the sum of the exergetic cost inputs equals the sum of the exergetic cost outputs.
4. Exergetic cost balances alone do not provide sufficient information to determine the exergetic costs corresponding to the exergy interactions between the sources, sinks, and systems comprising the plant. Valero et al. (1986) presented cost propositions that provided additional information so that the exergetic costs could be evaluated. McGovern and O'Toole (1992) presented alternative cost propositions, which were strongly influenced by those of Valero et al., for the same purpose. In this paper the cost propositions of McGovern and O'Toole have been changed significantly to overcome difficulties that arose with the previous versions. The new propositions are listed in Appendix B.

It is claimed that the output exergetic costs are fully and uniquely determined when the boundaries have been defined, the exergetic cost balances applied, and the cost propositions of Appendix B used.

There are eight exergy interactions in Fig. 2 and three of these are outputs of the plant. For each of the latter the corresponding exergetic cost must be found. Fig. 3 illustrates how the exergy interactions of the recycle can be regarded as a loop of recirculating interactions plus a set of non-recirculating interactions, at least one of which is zero. This is in accordance with the recycle proposition described in Appendix B. The non-recirculating component of $\dot{\Xi}_{\mathrm{D}\to\mathrm{A}}$ is zero in this case and so is not shown on the diagram. The recirculating interactions do not affect exergetic costs outside the recycle. In Appendix C a further example is given that shows how the recirculating and non-recirculating components of a more complex recycle that has multiple closed paths can be evaluated.

From the fuel proposition

$$\dot{\Xi}^{*}_{\mathrm{E}\to\mathrm{A}} = 100\ \mathrm{kW}. \tag{1}$$

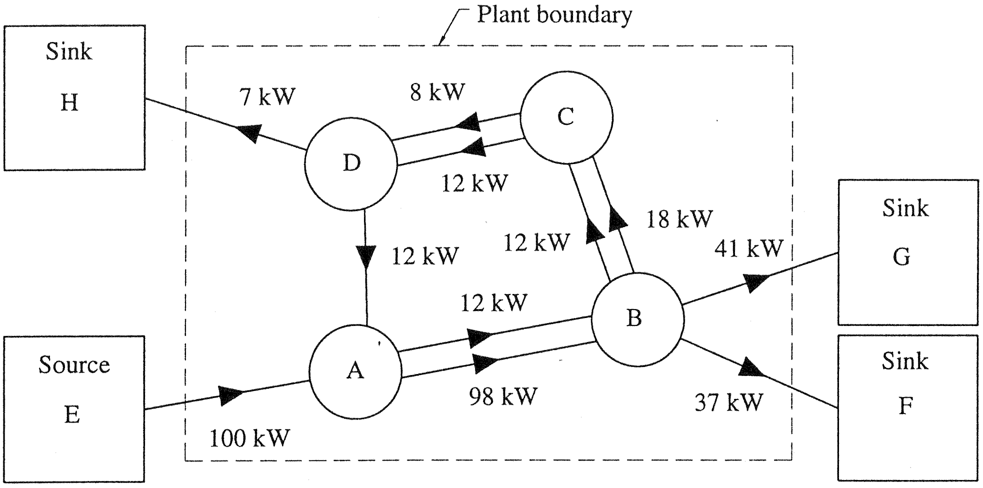

Fig. 3Exergy rate interactions between the sources, sinks, and component systems of the plant. These are shown as comprising a loop of recirculating interactions, each of which has a magnitude of 12 kW, and a set of three non-zero non-recirculating interactions: 98 kW, 18 kW, and 8 kW respectively.

From the recycle proposition

$$\dot{\Xi}^*_{D\to A} = 0 \text{ kW}. \tag{2}$$

From the exergetic cost balance at subsystem A

$$\dot{\Xi}^*_{A\to B} = \dot{\Xi}^*_{E\to A} = 100 \text{ kW}. \tag{3}$$

From the exergetic cost balance, the recycle proposition, and the fork proposition at B

$$\dot{\Xi}^*_{B\to C} = \frac{(30-12)}{(30-12)+41+37}\dot{\Xi}^*_{A\to B} \quad \text{or} \quad \dot{\Xi}^*_{B\to C} = 18.75 \text{ kW}. \tag{4}$$

Similarly,

$$\dot{\Xi}^*_{B\to G} = \frac{41}{(30-12)+41+37}\dot{\Xi}^*_{A\to B} \quad \text{or} \quad \dot{\Xi}^*_{B\to G} = 42.71 \text{ kW} \tag{5}$$

$$\dot{\Xi}^*_{B\to F} = \frac{37}{(30-12)+41+37}\dot{\Xi}^*_{A\to B} \quad \text{or} \quad \dot{\Xi}^*_{B\to F} = 38.54 \text{ kW}. \tag{6}$$

From the exergetic cost balance at subsystem C,

$$\dot{\Xi}^*_{C\to D} = \dot{\Xi}^*_{B\to C} = 18.75 \text{ kW}. \tag{7}$$

From the recycle proposition and the exergetic cost balance at subsystem D,

$$\dot{\Xi}^*_{D\to H} = \dot{\Xi}^*_{C\to D} = 18.75 \text{ kW}. \tag{8}$$

The three exergetic cost interactions to the external sinks have now been evaluated. Also, in Appendix C exergetic cost results are presented for a more complex recycle that has two closed head-to-tail paths.

## 4. CONCLUSION

The analysis approach that has been applied to the plant involving a mechanical exergy recycle is considered fundamental. If consensus can be reached on the exergetic cost analysis approach applied to this simple plant, considerable progress will have been made towards resolving problems in the exergoeconomic analysis of more complex plants. The principles involved are widely applicable. Net exergy interactions between systems that are equivalent to shaft power interactions also arise when exergy transfer occurs in association with heat or due to the transport of flow exergy (O'Toole and McGovern, 1990). The net exergy interactions that comprise a recycle can involve various forms of exergy transfer.

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## APPENDIX A—EXERGY RECYCLES

A recycle is a network of interactions between nodes such that every interaction is part of at least one closed path of interactions linked in a head-to-tail fashion. Fig. 4 illustrates an exergy recycle that has two such closed paths: ABCDEA and ADEA. It also has a closed path ABCDA where the interactions are not all linked in a head-to-tail fashion.

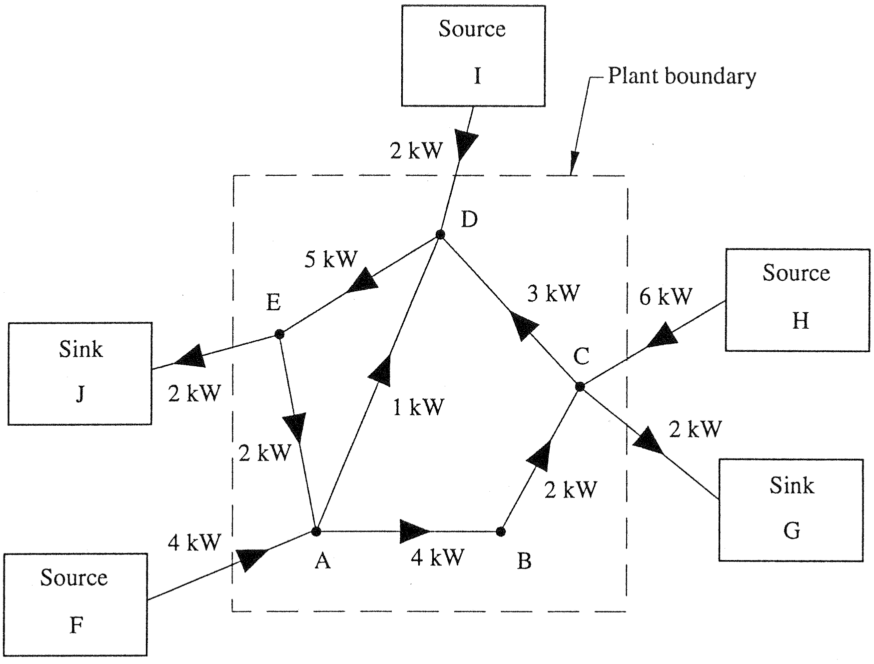

Fig. 4An exergy recycle. This consists of exergy rate interactions between systems as follows: A →B, B→C, C→D, D→E, E→A, and A→D.

## APPENDIX B—COST PROPOSITIONS

### B.1. Fuel Proposition

*The exergetic cost of an exergy interaction into the system enclosed by the plant boundary is equal to the exergy input.* This gives one equation for each net exergy input to the plant from an external system.

### B.2. Exergy Destruction-Sink Proposition

*The exergetic cost of an exergy interaction to an exergy destruction sink is zero.* It should be noted that for exergy destruction sinks within the plant boundary the exergy destruction-sink proposition is already incorporated in the cost balance equations. This proposition will yield one additional equation for each exergy interaction to an external exergy destruction sink.

### B.3. Recycle Proposition

*The exergy rate interactions of a closed path made up of interactions of a recycle that are linked in head-to-tail fashion can be considered as comprising a recirculating component with the same path and direction as the exergy interactions and a set of non-recirculating exergy rate interactions such that at least one of the non-recirculating exergy rate interactions is zero and all non-recirculating exergy rate interaction components are non-negative. Only the non-recirculating exergy rate interactions carry exergetic cost rate interactions. It follows that the exergetic costs assigned to the exergy interactions that comprise a recycle are such that all exegetic costs are non-negative* (i.e., have the same direction as the corresponding exergy interactions) *and at least one exergetic cost is zero for each closed path of interactions connected in head-to-tail fashion.* The proposition yields an additional independent equation for each closed path of head-to-tail interactions of each recycle of the plant: this is an equation in which one of the exergetic cost rate interactions of the closed path is set equal to zero.

### B.4. Fork Proposition

*Where a system that has no further subsystems in the context of the analysis boundaries that have been defined has more than one output exergetic cost interaction, the output exergetic costs are distributed in proportion to the exergy values; that is, the exergetic costs per unit of exergy are the same for all output exergetic cost interactions.* Valero et al. (1986) have categorised such systems as being at the "minimum level of aggregation." At a subsystem at the minimum aggregation level where multiple output exergetic cost interactions occur, the fork proposition yields a number of equations, which is one less than the number of output exergetic cost interactions of that subsystem. This proposition is subject to two preconditions, as follows:

*Precondition 1. Any output exergy interaction that has a zero exergetic cost due to the exergy destruction-sink proposition and exergetic cost balances for subsystems* (which are always "downstream" in this case) *or due to the recycle proposition is not involved in the fork proposition since it does not have an associated exergetic cost.* Therefore the cost balances, the exergy destruction-sink proposition, and the recycle proposition must be applied before the fork proposition.

*Precondition 2. Where an exergy interaction at a subsystem is part of a recycle, only the non-recirculating components of the interaction are involved in the fork proposition.* Therefore, for this reason too, the recycle proposition must be applied before the fork proposition.

## APPENDIX C—DETERMINATION OF THE RECIRCULATING AND NON-RECIRCULATING COMPONENTS WITHIN A RECYCLE

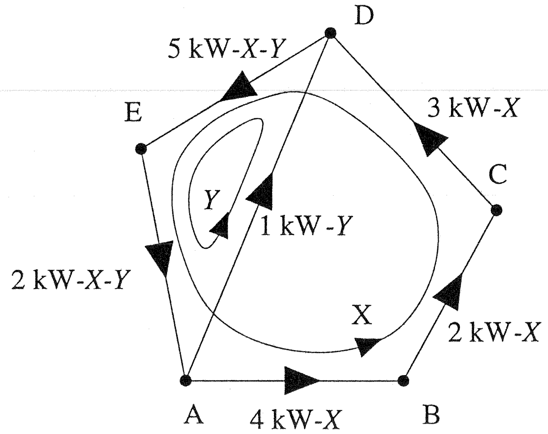

Fig. 5A diagram identifying the two recirculating exergy rate interactions, $X$ and $Y$, and the non-recirculating exergy rate interactions of the recycle shown in of Fig. 4.

Given the recycle cost proposition, the values of the two recirculating exergy rate interactions identified in Fig. 5 are unique. These can be determined using a search and elimination approach as follows. For the two head-to-tail paths of the recycle the recirculating exergy rate interactions $X$ and $Y$ will have values such that

$$5\ \text{kW} - X - Y \geq 0 \ \text{ and so } \ X + Y \leq 5\ \text{kW} \tag{9}$$

and $$2\ \text{kW} - X - Y \geq 0 \text{ and so } X + Y \leq 2\ \text{kW} \tag{10}$$

and $$4\ \text{kW} - X \geq 0 \text{ and so } X \leq 4\ \text{kW} \tag{11}$$

and $$2\ \text{kW} - X \geq 0 \text{ and so } X \leq 2\ \text{kW} \tag{12}$$

and $$3\ \text{kW} - X \geq 0 \text{ and so } X \leq 3\ \text{kW} \tag{13}$$

and $$1\ \text{kW} - Y \geq 0 \text{ and so } Y \leq 1\ \text{kW}. \tag{14}$$

Furthermore, at least one of the expressions (9) to (13) must be satisfied as an equality so that at least one of the non-recirculating exergy rate interactions of path ABCDEA will be zero. Likewise in order to meet the requirement that at least one of the recirculating exergy rate interactions of path ADEA must be zero, at least one of the expressions (9), (10), and (14) must be satisfied as an equality. It is found by inspection that all of the requirements are satisfied if and only if expressions (10) and (14) are satisfied as equalities. Therefore

$$X = 1\ \text{kW} \text{ and } Y = 1\ \text{kW}. \qquad (15)\ \&\ (16)$$

Hence, from the recycle cost proposition,

$$\dot{\Xi}^{*}_{\text{E}\to\text{A}} = 0 \text{ and } \dot{\Xi}^{*}_{\text{A}\to\text{D}} = 0. \qquad (17)\ \&\ (18)$$

All the resulting exergetic cost rate interactions are given in Table 1, together with the exergy rate interactions and the recirculating and non-recirculating exergy rate components.

Table 1

Summary of interactions for the plant shown in Fig. 4

| | **Exergy rate interaction /[kW]** | **Recirculating exergy component /[kW]** | **Non-recirculating exergy component /[kW]** | **Exergetic cost rate/[kW]** |
|---|---|---|---|---|
| **Inputs** | | | | |
| F → A | 4 | 0 | 4 | 4 |
| H → C | 6 | 0 | 6 | 6 |
| I → D | 2 | 0 | 2 | 2 |
| **Internal interactions** | | | | |
| A → B | 4 | 1 | 3 | 4 |
| B → C | 2 | 1 | 1 | 4 |
| C → D | 3 | 1 | 2 | 5 |
| D → E | 5 | 2 | 3 | 7 |
| E → A | 2 | 2 | 0 | 0 |
| A → D | 1 | 1 | 0 | 0 |
| **Outputs** | | | | |
| C → G | 2 | 0 | 2 | 5 |
| E → J | 2 | 0 | 2 | 7 |