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ABSTRACT: The issue of excessive vibrations of footbridges due to the passage of pedestrians has been well documented in the past decade. Despite this there still remains great uncertainty as to how to predict the acceleration response of a footbridge due to crowd loading. This paper investigates the vibration response of a flexible footbridge subjected to crowd loading. Using a statistical model which caters for the variability of pedestrians, the vibration response of the footbridge is obtained. In this work, the effect of social groups or clusters of pedestrians in a crowd is investigated. Herein a cluster is defined as two or more pedestrians walking together with the same velocity. The predictions of this model are compared to a model which uses only lone pedestrians walking within a crowd. None of the current design codes or guidelines considers the possibility of pedestrians walking together. The size of the clusters is found in literature to follow a Poisson distribution. In this paper variations of the probability of clusters appearing in the crowd are assessed. It is found that the response of a crowd with clusters present is similar to the predictions of the UK National Annex to Eurocode 1.

KEY WORDS: Bridge, Vibration, Pedestrian, Vertical, Pacing frequency, Cluster

1 INTRODUCTION

1.1 Background

Modern developments in the design of structures and progress in structural materials have led to longer and lighter footbridges. Increasingly, these typically low-frequency structures are experiencing serviceability problems.

Due to the dynamic nature of pedestrian loading, vibrations of the bridge deck can be expected if the bridge natural frequency is within, or close to, the typical pacing frequency range (1.5Hz to 2.5 Hz). Such vibrations are often magnified by the presence of a crowd of pedestrians crossing the structure. If these vibrations are large enough they can lead to discomfort for the pedestrians, resulting in failure of the serviceability limit state. Bridges that have experienced vibrations of this nature have been well documented in the last decade, including high profile bridges such as; the Millennium Bridge, London [1], the Pont du Solferion, Paris [2], and the T-Bridge, Japan [3].

1.2 Approach of this work

In this work the vertical vibrations induced by crowds on a flexible footbridge are examined. Typically, bridge vibrations produced from a crowd of pedestrians are estimated by using an enhancement factor applied to the effect caused by a single pedestrian. However, the models for the determination of the single pedestrian response are commonly deterministic and do not consider the sensitivity of bridge vibrations to slight changes in pacing frequency. The model presented here uses statistical distributions to model the variability of pedestrians in a crowd.

In a development of the model presented by Caprani et al [4] this model assesses the effect on the footbridge response of social groups or clusters of pedestrians walking within a crowd. For this work, a cluster is defined as a social group of two or more pedestrians who intentionally walk together. In order of the cluster to remain intact, each pedestrian within the cluster must have the same velocity.

The proposed cluster model of this work differs from the model presented by Caprani et al [4] which modelled a crowd as a collection of lone pedestrians. In this form of crowd the individual velocities are determined as the product of a random pacing frequency and step length, both chosen from predetermined statistical distributions. It was reported by Ebrahimpour et al [5] that pedestrians consciously make changes in their pacing frequency to synchronize their movements with those around them. This indicates that pedestrians tend to walk in phase while walking in a crowd. As a result, varying levels of synchronization were investigated by Caprani et al [4]. In the model presented here, no synchronization is forced between the pedestrians in order to assess the effect of clusters only.

2 CURRENT DESIGN CODES AND GUIDELINES

Many of the current design guidelines [6-10] for the prediction of crowd loading are based on different assumptions. As a result, their prediction of the response due to a typical crowd loading scenario on a low frequency footbridge was found to vary by as much as a factor of four [11].

Eurocode 5 [6] is a recent design code for the design of timber structures and includes recommendations for vibrations. The response model defined is not material-dependent and so can be used for the prediction of vibrations for a footbridge constructed from any material. To predict the response, a_1, of a footbridge with a natural frequency in the range 1.5 to 2.5 Hz resulting from single pedestrian loading, Eurocode 5 [6] uses the formula:

$$a_1 = \frac{200}{M\zeta} \quad (1)$$

Where M is the bridge mass and ζ is the damping ratio of the bridge. The pedestrian is assumed to be walking at the same natural frequency as the bridge and so no other parameters are required for the calculation. As a result, use of Equation (1) means that the single pedestrian response is found to be constant for any footbridge with a natural frequency within the given range. This approach neglects the sensitivity of vibrations of the deck to the pedestrian pacing frequency found by several authors including Keogh et al [12] and Pedersen and Frier [13], for example.

For the prediction of crowd loading, Eurocode 5 [6] multiplies the single pedestrian response by an enhancement factor to determine the response for N pedestrians, a_N (m/s^2):

$$a_N = 0.23 a_1 N k \quad (2)$$

where k is a reduction factor which reduces from 1 above and below the natural frequency range which is sensitive to vertical vibrations (1.5 Hz to 2.5 Hz).

ISO 10137 [7] uses a Fourier series with 5 harmonics to represent the force due to a single pedestrian given as:

$$F_1(t) = W\left[1 + \sum_{i=1}^{n} \alpha_i \sin\left(2\pi i f_p + \varphi_i\right)\right] \quad (3)$$

where W is the pedestrian weight, i is the harmonic number, $\alpha_1 = 0.37(f_p\text{-}1)$, $\alpha_2 = 0.1$, $\alpha_3 = \alpha_4 = \alpha_5 = 0.06$ and ϕ_i is the phase angle for the specific harmonic, and f_p is the pacing frequency. Inclusion of the pacing frequency in the equation means that this code considers changes in the force with variations in the pacing frequency. The guideline does not give guidance on what pacing frequency to use nor does it specify if the force is static or moving. It was assumed by Pavic [11] that the pacing frequency is chosen to match the bridge frequency and that the pulsating force given by Equation (3) is moving across the bridge. To obtain the total effective pedestrian load due to a crowd of N uncoordinated pedestrians, the dynamic load defined by Equation (3) is multiplied by $\sqrt{N}$. Although this is reported by Pavic [11] to be an improved method of prediction, work by Ingolfsson et al [14] found that the response due to crowd loading is overestimated using this approach.

The method applied in both the SETRA guideline [8] and the UK National Annex to Eurocode 1 [9] are similar in that both represent the mass of the pedestrians as a uniformly distributed load on the bridge which has the effect of reducing the natural frequency. The load applied by the crowd is defined as a load per unit area of the bridge deck. HIVOSS [10] uses a frequency domain response spectrum approach when calculating the response of footbridge to streams of pedestrians.

3 SOCIAL CLUSTERS IN CROWDS

3.1 Overview

The existence of clusters of pedestrians (two or more) walking in a crowd is typical of a real life situation. Moussaid et al [15] highlighted that there a high probability that small groups or clusters of pedestrians will be present in crowds. Despite this, none of the current guidelines mentioned in Section 2 make reference to this possibility. Moussaid et al [15] state that simulation of crowds with all the pedestrians walking individually, with their individual desired speed, is not representative of real life. If pedestrians intentionally walk in small social groups they will be travelling at the same velocity as the others in the group. Crowds of pedestrians were observed using video recordings, walking along a popular commercial walkway on two different days; population A (P_A) was observed at lunch time on a week day whilst population B (P_B) was observed on a Saturday afternoon [15]. It was found that a higher percentage of P_B walked in clusters of two or more pedestrians when compared to P_B. The higher percentage for P_B was expected due to a higher tendency for people to walk with friends on a Saturday [15]. It was also found that pedestrians in P_A also walked faster than those in P_B.

3.2 Cluster Size Distribution

Moussaid et al [15] find that a Poisson distribution, with a mean value (λ) of 0.83, could be used to represent the cluster sizes within the crowd of P_A, as shown in Figure 1. This shows that 33.2% (1 – 0.668) of the pedestrians walking on this day, during the video recording, were in a cluster.

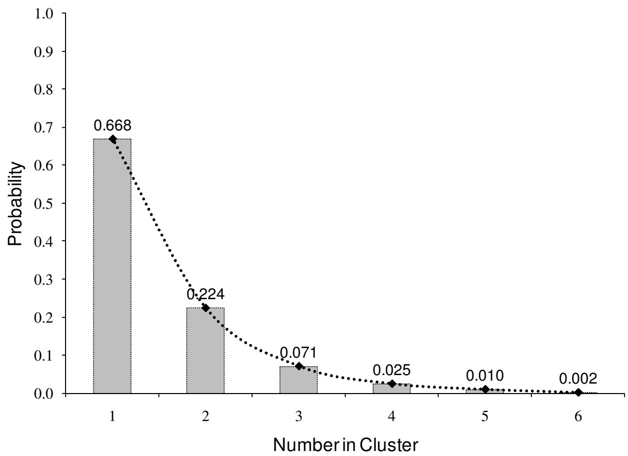

Figure 1. Cluster Size Distribution (after Moussaid et al [15]).

4 PEDESTRIAN AND BRIDGE MODELLING

4.1 Pedestrian parameters and model

In this work, pedestrians are considered to be non-homogeneous and so their individual parameters follow statistical distributions. The pedestrians in the model are considered to be healthy adults for the purpose of assigning pedestrian properties. The pedestrian mass is represented by a lognormal distribution with a mean of 73.9 kg and a coefficient of variation of 21.2% [16]. The pedestrian step length is taken to be normally distributed with a mean of 0.66 m and given a coefficient of variation of 10% [17]. The pacing frequency is taken to be normally distributed with a calculated mean of 1.96 Hz and a standard deviation of 0.209 Hz following a literature survey [5, 18-21]. The pedestrian velocity is calculated as the product of the pacing frequency and the step length, the mean velocity is found to be 1.29 m/s with a standard deviation of 0.19 m/s.

Brownjohn et al [22] reported on a phenomenon, termed intra-subject variability, that a pedestrian can never repeat exactly the same step twice. Despite this it is commonly assumed that the force applied by both feet of a pedestrian is of the same magnitude and periodic [23, 24]. Since there is constant contact between the pedestrian and the walking surface during walking, the ground reaction force (GRF) produced from consecutive footfalls (left and right) overlap in time (see Figure 2).

The total force applied to the structure is the sum of the forces applied at any point in time. This total GRF can be represented by a Fourier series (Equation (3)). The number of harmonics used in the representation varies in the literature [22]. However Fanning et al [25] found that using just the first harmonic did not significantly influence the accuracy of the results. As a result for this work just the first harmonic of Equation (3) is used. Therefore the walking force is given by the following sine wave approximation:

$$F(t) = W\left[1+\alpha\sin\left(2\pi f_p\right)\right] \quad (4)$$

where W is the pedestrian weight, f_p is the pacing frequency (Hz) and the Fourier coefficient, α, is given by [25]:

$$\alpha = 0.25 f_p - 0.1 \quad (5)$$

and is shown in Figure 2.

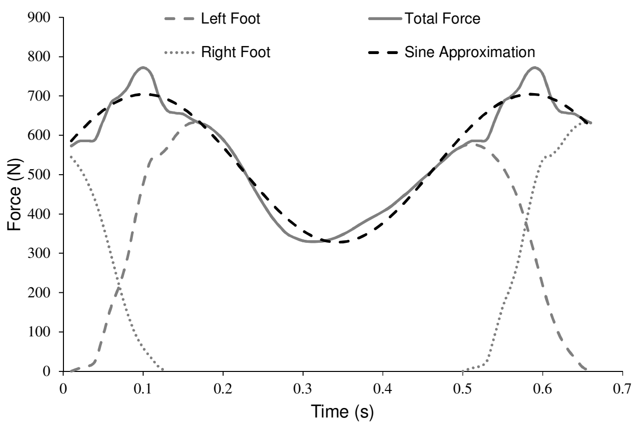

Figure 2. Typical shape of the ground reaction force due to a single pedestrian.

4.2 *Bridge parameters*

The bridge considering in this work is the 50 m long simply-supported beam with a mass of 500 kg/m, a width of 2 m and a natural frequency of 1.96 Hz. A modulus of elasticity of 200×10^{11} N/m^2 is used.

The damping ratio of the bridge is taken to be 0.5% with Rayleigh damping assumed thereafter [26]. This damping level is similar to a number of studies reported on low frequency structures (circa. 2 Hz) in the literature [8, 27-29].

The effect of humans on a structure's damping ratio is neglected in this paper. This is consistent with other researchers in the field, including the SETRA Guideline [8] and Pavic [11], who in his keynote address at the conference Footbridge 2011, used a bridge with a frequency of 2.16 Hz and a constant damping ratio of 0.6% in predicting the response for a non-stationary crowd (density of 0.5 p/m^2).

It should be noted that some authors indicate that the presence of pedestrians on a structure has a significant effect the damping ratio. Ellis and Ji [30] reported that this effect is dependent on whether the pedestrians are stationary or non-stationary. They report that standing or sitting people affect the damping of a structure but that people walking do not, and so should be represented as a load only. On the other hand, Zivanovic et al [31, 32] and Brownjohn et al [22] report that walking pedestrians can also increase the damping ratio of a bridge in the vertical direction. Zivanovic et al [31] in laboratory tests found an increase in damping for both standing and walking pedestrians (crowd density = 0.46 p/m^2), though the increase found for walking pedestrians was significantly lower than that for standing pedestrians. Zivanovic et al [32] and Brownjohn et al [22] also found an increase in damping due to walking pedestrians on as-built bridges; the Podgorica Bridge in Montenegro and a long span footbridge at Singapore Changi airport, respectively. However, further tests by Zivanovic et al [32] on the Reykjavik City footbridge in Iceland did not show an increase in damping.

4.3 *Finite element model*

The work presented here is based on a moving force model, similar to that used in the design standard BS 5400 [24]. It is acknowledged that this may be conservative as it does not consider the possible interaction between the pedestrian and the moving surface as the moving force is independent of the bridge movement [23].

A finite element model is used to establish the vibration response resulting from the passage of pedestrians across the bridge. The bridge is modelled with 10 Euler-Bernoulli beam elements, with lumped mass assumed. A sensitivity analysis was carried out and showed that 10 element was sufficiently accurate for comparison of the bridge vibrations. Transient solutions are obtained using the Newmark-β integration method. A one-dimensional model is used, and so torsional and lateral effects are ignored.

The vibration response of interest is taken as the mid-span acceleration and is assessed using a 5-second root-mean-square (RMS) moving average from the acceleration history of each simulation. To establish a characteristic response, 1000 simulations are carried out using randomly generated pedestrian parameters. The characteristic response is then defined as the response with a 5% probability of exceedance [8, 12, 22].

5 SIMULATIONS AND RESULTS

5.1 *Cluster model results*

In this model, no synchronization is considered between the pedestrians. Instead, those pedestrians deemed to be walking in a cluster were given the same velocity and thus they stayed together while crossing the bridge. This velocity is randomly chosen for each cluster from the statistical distribution given in Section 4. The pacing frequency for each pedestrian is also chosen from the statistical distribution and thus the step length is determined as velocity divided by the pacing frequency.

Figure 3 shows pedestrian location against time plot for ten pedestrians crossing the 50 m footbridge, following a single simulation. The time at which each pedestrian enters and leaves the bridge during the simulation is shown. Only 10 pedestrians are simulated in this instance to allow clarity in this figure. It can be seen that some of the pedestrians remain walking on their own (solid line) while others are walking in clusters (dotted lines). It is evident that the single pedestrians have different velocities as the time taken to cross the bridge varies; on the other hand, those deemed to be in a cluster have the same velocity and so remain together while crossing.

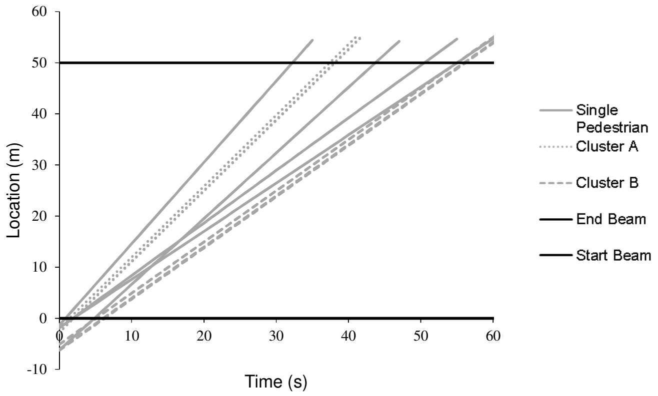

Figure 3. Analysis of pedestrian's (single and clustered) velocity whilst on the bridge

For all simulations, the bridge was subjected to a crowd of pedestrians with an average density of 0.5 p/m2 (persons per square metre). This is a typical crowd loading condition for unrestricted walking [11]. To investigate the effect of the size of a cluster on the bridge vibrations, simulations were carried out with a constant number of pedestrians in each cluster from one pedestrian (lone pedestrian crowd model [4]) up to five pedestrian in each cluster. No synchronization is forced between the pedestrian but each cluster has its own velocity. The results of this are shown in Figure 4. It can be seen that there is a gradual increase in the response despite a constant mean crowd density of 0.5 p/m^2.

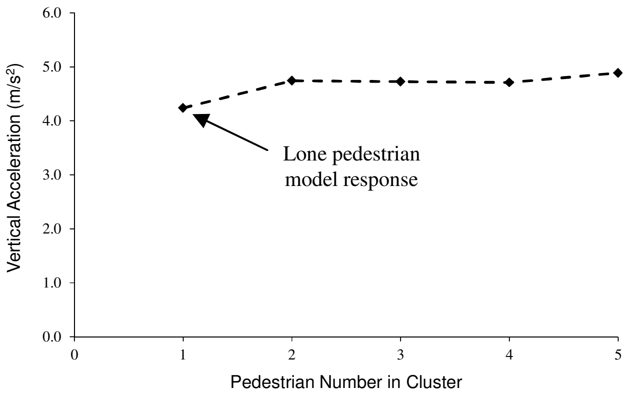

Figure 4. Increase in acceleration with the increase in the number of pedestrians in the clusters

To investigate a distribution of the probability of clusters being present in the crowd, different mean cluster sizes are considered within the range from 0.5 to 1.5, this resulted in simulations ranging from 60% clusters (λ = 0.5) to 23% clusters (λ = 1.5). Figure 5 shows that as the probability of a cluster appearing in the crowd increases, the acceleration response increases gradually. The result of the lone pedestrian crowd model is also shown, where the probability of a cluster is zero.

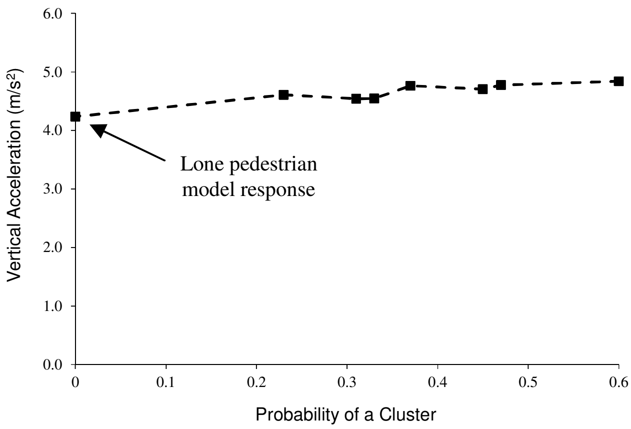

Figure 5. Increase in acceleration with the increase in cluster probability

5.2 *Comparison with design codes and guidelines*

The approach investigated here is compared to the predictions of some design guidelines [6, 8-10] and a lone pedestrian crowd model [4]. To allow direct comparison with published results, the bridge considered by Pavic [11] is analysed. The bridge is 38.85 m long, 2.5 m wide, has a mass of 1456 kg/m and has a natural frequency of 2.16 Hz. In the model presented here, similar to the SETRA Guideline [8] and the UK national Annex to Eurocode 1 [9], the mass of the crowd is taken to act as a uniformly distributed load on the bridge. This has the effect of reducing the unloaded natural frequency, f_n, to a loaded natural frequency, $f_n' = 2.10$ Hz :

$$f_n' = \frac{\pi}{2l^2}\sqrt{\frac{EI}{M + M_P}} \tag{6}$$

where l is the bridge length, EI is the flexural stiffness, M is the bridge mass per metre length, and M_P is the mass of the crowd per metre length.

The predicted characteristic response from the cluster model is shown compared to several design codes in Figure 6. It can be seen that the prediction is almost identical to that of the UK National Annex to Eurocode 1 [9]. It should be noted from this figure, as identified by Pavic [11], there is a large difference between the predictions of the design codes considered [6, 8-10]. The prediction by Eurocode 5 [6] is four times larger than the predictions of the UK National Annex to Eurocode 1 [9] for this particular crowd loading condition. UK National Annex to Eurocode 1 [9] is reported by Pavic [11] to give the most realistic response when compared to as built testing of bridges.

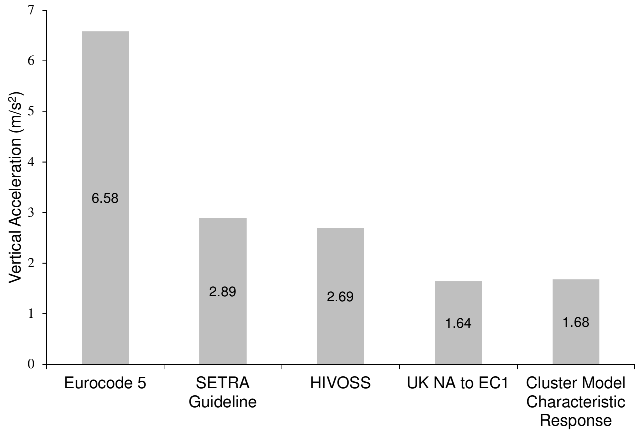

Figure 6. Comparison of the Cluster model characteristic response with those from current design codes and guidelines for the bridge used by Pavic [11].

Figure 7 shows the comparison of the prediction of the cluster model presented here to those of the lone crowd model [4] which allows for varying levels of synchronization. The lone pedestrian crowd model [4] is developed for pedestrians walking individually but allows for varying levels of synchronization within the crowd. Synchronization is enforced by assigning the pedestrians deemed to be synchronized the same pacing frequency and phase angle [24]. The pacing frequency assigned is randomly selected according to its distribution (mean 1.96 Hz and standard deviation of 0.209 Hz) while the phase angle of the pedestrians vertical harmonic force is taken to be uniformly random in the interval 0 to 2π. It is shown (Figure 7) that this lone pedestrian crowd model matches well with the predictions of the cluster model and UK NA to Eurocode 1 [9] at a synchronization of approximately 13%. This is similar to the findings of Grundmann et al [33] who reported that a synchronization level of 13.5% was typical in crowd loading.

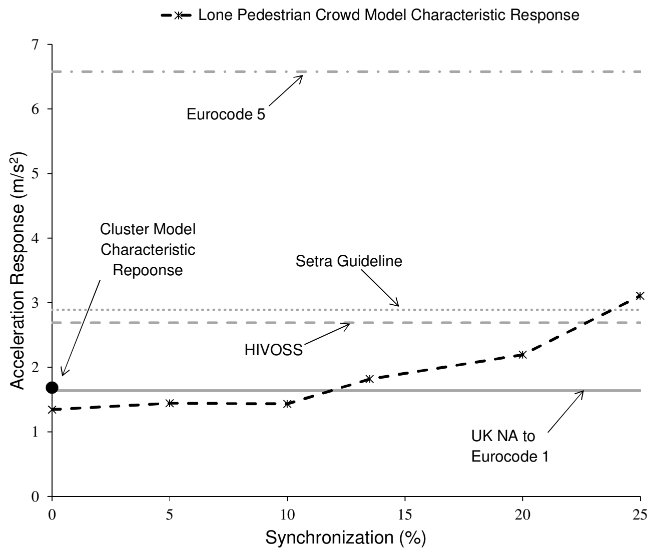

Figure 7. Comparison of the cluster model characteristic response with current design codes and guidelines and simulated response with varying levels of synchronization using the lone crowd model for the bridge used by Pavic [11].

6 SUMMARY AND CONCLUSIONS

In this work a model is presented for the prediction of footbridge vibrations resulting from clustered crowd loading. The clustered crowd used in this model allows for the possibility of clusters or social groups of pedestrians (two or more) being present within the crowd, as well as lone pedestrians. A Poisson distribution of cluster size taken from the literature is used. The model is compared to design codes and a published lone pedestrian crowd model.

It is shown that the clustered crowd model gives a good match with the predictions of the UK National Annex to Eurocode 1 [9]. The results also compare well with the predictions of a published lone pedestrian model in which synchronization is forced to cater for pedestrians walking instep.

The conclusion from this work is that it is possible to predict the response of a footbridge resulting from crowd loading by modelling the crowd as containing clusters of pedestrian, within which the pedestrians are walking at the same velocity. This is more typical of a real life situation.

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