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ABSTRACT: Footbridge vibration has received much attention in recent years. However, stochastic models for crowd loading are not common, and estimation of crowd-induced vibration is typically done through enhancement factors applied to single pedestrian loading models. This work compares two such models, a moving force model and a spring-mass-damper model (SMD). Typical ranges for various pedestrian parameters are examined, and it is found that the pacing frequency has by far the greatest influence on bridge vibration response. It is also found that the magnitude of the response for pacing frequencies near the bridge natural frequency is lower for the SMD model, but otherwise the results prove similar. This suggests that moving SMD models may be more suitable than moving force models when the bridge natural frequency is in the critical frequency range.

1 INTRODUCTION

1.1 Background

Recent developments in the design of structures and increasing pressure on structural designers to deliver more aesthetically-pleasing structures have led to longer and lighter footbridges, stadium stands, lightweight office floors, etc. As a result these structures may have a low natural frequency, which can lead to them presenting serviceability performance issues. In all cases, these serviceability problems are due to excessively large vibrations which can occur when the dynamic nature of pedestrian load application – related to the individual's pacing frequency – causes resonant vibration effects to occur in the structure. Such vibrations are quite often magnified when a large crowd of pedestrians is present. A further difficulty is that human response to vibrations is complicated and it is often difficult to establish a comfort criterion that satisfies all (Rasmussen & von Scholten, 2010). In addition, it seems that people are becoming more sensitive to vibrations and, therefore, are quicker to complain (Bachmann, 2002); the same author suggested that this could be partly a reaction to increasing environmental influences.

Currently there are two principal approaches in design guidelines for dealing with the issue of pedestrian-induced vibrations in footbridges. The first is to ensure that lower frequencies of vibrations of a bridge are outside those frequency ranges associated with typical pedestrian pacing rates while the second is to limit induced structural accelerations to levels below prescribed acceptable limits (Mullarney & Archbold, 2011). Here, the latter of the two approaches is considered, to improve methods to estimate the magnitude of the vibrations induced by pedestrians.

Currently the response generated from a crowd of pedestrians is most commonly derived by applying an enhancement factor to the estimated response caused by an individual person walking. This single pedestrian is typically modelled using a deterministic moving force model which does not consider interaction between the bridge and the pedestrian. Moreover, Pavic

(2011) noted that several recently published design guidelines, while adopting a similar general approach, all differed in the input parameters used even for these moving force models and that the results derived from applying each of the guides to a benchmark structure varied by a factor of approximately four.

It has further been acknowledged for a long time that there is a need for a probabilistic approach to pedestrian loading (Matsumoto et al, 1978; Wheeler, 1982). Despite this, design codes such as BS 5400 (1978, 2006) and Eurocode 5 (EN 1995-2:2004) use deterministic moving force models to predict the response of a single pedestrian. Zivanovic (2006) stated that these models are commonly unable to accurately predict the response of a bridge due to a single pedestrian and usually overestimate it significantly. This inevitably leads to overestimation of the crowd response if enhancement factors are used in conjunction with moving force models.

Archbold (2008) also found the moving force model to be conservative as it does not consider interaction between the pedestrian and the moving bridge surface, while Clemente et al (2010) added that if the interaction between the pedestrian and the bridge is to be considered, the pedestrians should be modelled as '*biological oscillators*'.

1.2 Approach of this work

In order to model the interaction between a pedestrian and the bridge surface, the authors have employed a moving spring mass damper (SMD) model to represent a single pedestrian (Caprani et al, 2011). This single degree of freedom SMD model captures changes in vertical forces applied to vibrating bridge surfaces. This SMD model is then moved across an idealized bridge at a velocity derived from a combination of pacing frequency and step length. The bridge used in the model is a simply-supported beam, chosen to be susceptible to excitation from typical pedestrian pacing rates. To model the footfall force, a time-varying harmonic force is applied to the pedestrian mass, as proposed by Fanning et al (2005). Input parameters for this model include pedestrian mass, step length, pacing frequency, pedestrian stiffness and damping properties associated with the pedestrian model.

The aim of the work reported herein is to determine the sensitivity of the numerical models and estimated acceleration levels induced in the bridge to a range of input parameters related to the movement of the crossing pedestrians. The results from the SMD pedestrian model are thus compared to those estimated using the conventional moving force model.

2 USE OF SMD MODELS TO REPRESENT HUMAN LOADING

2.1 Human leg stiffness

Rapoport et al (2003) stated that in repetitive physical activity, such as running, hopping and trotting, a subject bounces on the ground in a spring-like manner. Geyer et al (2006) state that walking is also a bouncing gait. This is due to knee, ankle and hip flexure throughout the gait cycle (Rapoport et al, 2003; Lebiedowska et al, 2009). Blum et al (2009) stated that leg stiffness is a key parameter of modelling legged locomotion. Lee and Farley (1998) and Geyer et al (2006) represented the human leg as a compliant spring-mass model while running and walking, respectively. Rapoport et al (2003) stated that constant mechanical stiffness may not be applicable to the human leg as joint stiffness is nonlinear in nature as damping may be present and as a result, a model which accounts for this damping may improve the model predictions

Lee and Farley (1998) acknowledged that spring and damping elements have been incorporated into the legs of some models of walking in order to match ground reaction force (GRF) patterns observed in human walking. They report that the values used in these models are generally higher (k_P=12-35.5 kN/m) than the leg stiffness values reported for normal walking ($k_P \approx 11$ kN/m). Geyer et al (2006) stated that these models are too complex to serve as conceptual models. The authors reference Dickinson et al (2000) and Srinivasan and Ruina (2005) as stating that, despite being inaccurate, the stiff-legged motion remains the mechanical concept for a walking gait. Geyer et al (2006) themselves state that not stiff but compliant legs are fundamental to the walking gait.

According to literature, the stiffness of the leg varies depending on what the person is doing. For example research carried out by Zhang et al (2000) on test subjects bouncing with their feet in a stationary position found a leg stiffness value of 28.5 kN/m and a damping value of 950 Ns/m, which equates to a damping ratio of 0.3 for a mass of 78 kg. In the case of hopping, Rapoport et al (2003) found that the leg stiffness increased from 9.8 to 14.6 to 20.9 kN/m with an increase in hopping frequency from 1.53 to 1.87 to 2.20 Hz respectively. This highlights that the joint stiffness increases proportionally with increasing impact frequency, due to the reduced stance time, leading to an increase in the overall leg stiffness.

2.2 Human leg damping

To account for the damping of the human leg while walking, Bertos et al (2005) incorporated a shock absorber into a rocker based inverted pendulum model which resulted in a reduction in the movement of the COM. Using a test subject with a range of velocities, they verified their model and from this graphs of walking speed against damping ratio and stiffness were developed. The damping ratios were found to range between 0.40 and 0.70 across the velocity range of 0.8-2.2m/s and the stiffness from 2 kN/m at 0.75 m/s up to 13 kN/m at 2 m/s. However, Zajac et al (2003) and Geyer et al (2006) stated that the inverted pendulum cannot reproduce the characteristic M-shaped GRF and so does not represent the stance phase of a pedestrian correctly. Also Lee and Farley (1998) found that the inverted pendulum cannot reproduce accurately the trajectory of the COM as it overestimates its height at mid-stance. As a result, the damping values quoted by Bertos et al (2005) may be excessive in order to match the displacement of the test subject's COM, as the model used uses the trajectory of the standard inverted pendulum.

Gayer et al (2006) used a bipedal spring mass model to represent five test subjects walking. The point mass was placed on two massless spring elements. They investigated the angle of attack, made with the leg and the ground before touchdown. They reported an increase in stiffness with an increase in attack angle from 14 kN/m at 69° to 20 kN/m at 76°.

2.3 Adopted model

The model used in this work employs statistical distributions of these pedestrian model input parameters and multiple simulations of pedestrians traversing the bridge are used to derive characteristic responses due to a single pedestrian. The following distributions are used to represent the possible variations in the characteristics of a pedestrian on the bridge: pedestrian weight is represented by a log-normal distribution, while stride length, pacing frequency and leg stiffness respectively are represented by a normal distribution. Pedestrians' starting locations are based on a Poisson arrival process and are thus given gaps described by the exponential distribution.

3 PEDESTRIAN AND BRIDGE MODELLING

3.1 Bridge model

The bridge considered is a simply-supported, 50 m long beam with a mass of 500 kg/m and width of 2 m. The flexural rigidity was adjusted to provide a first vertical natural frequency in the range susceptible to pedestrian excitation. Damping is taken to be 0.5% for the first two modes, with Rayleigh damping assumed thereafter. It is acknowledged that this will dampen the influence of higher modes.

For this work, the bridge examined is modeled in two ways. Firstly, modal analysis is used for both single pedestrian moving force and single pedestrian spring-mass-damper models. In both cases, 5 modes are used to estimate the bridge response. Further details on the model formulation have previously been presented by the authors (Caprani et al, 2011). The response of interest in this study is taken as the vertical midspan acceleration. The vibration response is assessed using a 5-second root-mean-square (RMS) moving average from the acceleration history of each simulation (Archbold, 2008). The maximum of the RMS from any one particular scenario is taken as the response of the bridge to that particular loading scenario (da Silva et al, 2007).

3.2 Pedestrian crowd model

The pedestrians in this work are considered to be non homogeneous and so their parameters follow statistical distributions. The pedestrian mass is taking to follow a log normal distribution (Portier et al, 2007) with a mean of 73.9 kg and a coefficient of variation of 21.2 %. The stride length is taken to be normally distributed with a mean of 0.66 m (Barela and Duarte, 2008) and a coefficient of variation of 10% is assumed. The pacing frequency is also considered to be normally distributed with a calculated mean of 1.96 Hz with a standard deviation of 0.209 Hz following a literature survey (Matsumoto et al, 1978; Grundmann and Schneider, 1990; Pachi and Ji, 2005; Ebrahimpour et al, 1996; Karmer and Kebe, 1980). The pedestrian stiffness is taken to be normally distributed with a mean of 22.5 kN/m and a standard deviation of 2.25 kN/m, this was chosen as it is the midpoint of the values quoted by Lee and Farley (1998).

3.3 Numerical model

The moving force model employed in this work is that proposed by Archbold (2004) and is also presented by the authors elsewhere (Caprani et al, 2011). The SMD model, meanwhile, represents the pedestrian mass, supported by a massless spring and damper which represent the stiffness and damping of the human body, (specifically those acting between the centre of gravity and contact surface of the bridge). A pulsating force identical to that used in the moving force simulations is also applied to the bridge surface at the pedestrian location, as shown in Figure 1.

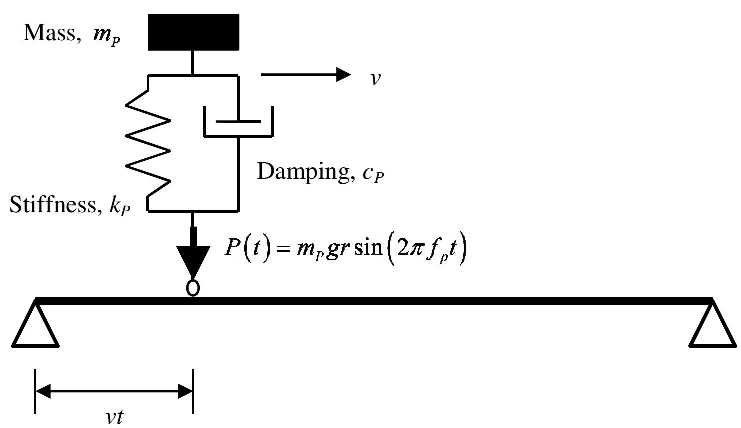

Figure 1. Mixed spring-mass-damper (SMD) and pulsating force model of pedestrian-bridge interaction.

Further details on the numerical formulation of these models have been previously reported by the authors (Caprani et al, 2011). Comparison between the moving force models and the moving SMD models is presented through use of a non-dimensional value, μ, which is the ratio of the response generated from the moving SMD model to the moving force model.

4 BRIDGE RESPONSE TO SINGLE PEDESTRIAN

4.1 Critical Parameter for Single Pedestrian Excitation

The response of the structure to a single pedestrian was investigated by using the moving force model and the SMD model. The response function to pacing frequency was established by a pacing frequency sweep from 1.3 to 2.8 Hz, and is given in Figure 2. All other parameters including mass, step length and stiffness (where appropriate) are assigned the mean values of 73.9 kg, 0.66 m and 22.5 kN/m, respectively.

As expected, it was found that the vibration response is most sensitive to the pacing frequency. Figure 2 shows that there is a significant increase in the vibration response for both models close to the natural frequency of the bridge (1.94 Hz), as may be expected. It is however evident that there is a reduction in the response of the SMD model when compared to the MF model.

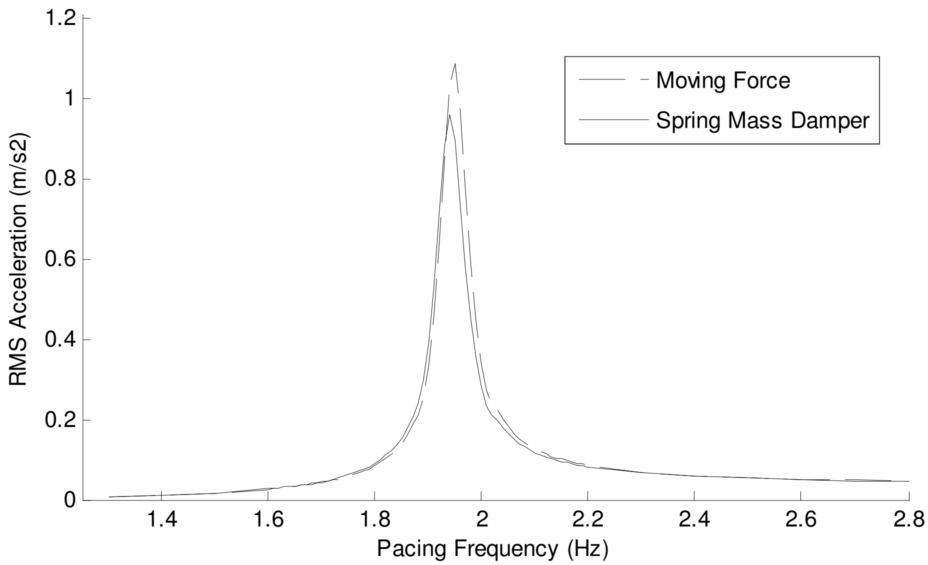

Figure 2. Single pedestrian response function.

5 OPPOSING CROWD LOADING PARAMETRIC SENSITIVITY STUDY

5.1 Basis for comparison

To determine the effect of the improved representation of the pedestrians with the SMD model, a range of parameters were varied, and the 5-second RMS vibration response (R) noted. The results are examined through the non-dimensional ratio of the spring-mass-damper to moving force RMS results:

$$\mu = \frac{R_P^{SMD}}{R_P^{MF}} \tag{1}$$

5.2 Pacing frequency, leg stiffness and mass

From Figure 2, it is evident that if the pacing frequency is removed from the natural frequency of the bridge, the moving force model gives similar result to that of the SMD model. To further investigate the effect of the pacing frequency on the modification factor (μ), the pacing frequency was varied within the extremes of the statistical range (5% and 95%) and included the mean value, thus pacing frequency values of 1.617, 1.96 and 2.303 Hz were investigated. In this the step length was set constant at the mean value of 0.66 m. To allow creation of a surface plot of the μ values, the pedestrian spring stiffness was varied from 10 to 35 kN/m and the pedestrian mass was varied from 30 to 130 kg, thus fully exploring the possible values. The pedestrian damping ratio in this case was kept constant at 0.3, the effect of variations in this value have been investigated in a previous study by the current authors (Caprani et al, 2011).

Figure 3 show the results of these simulations. The extreme values, removed from the mean show similar results for the modification factor, however in the case of the mean value, there is a significant reduction in the response with high mass and in particular in the case of spring stiffness in the range 20-30 kN/m. This extends the finding of Figure 2, that if the pacing frequency is removed from the bridge natural frequency there is little difference between the moving force and SMD model results.

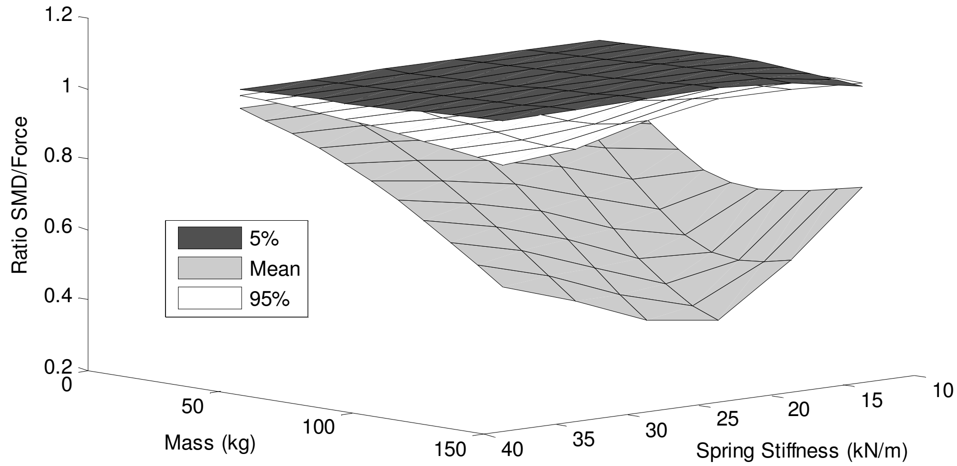

Figure 3. Modification factor results for pacing frequency extremes

Further investigations highlighted that the modification factor is extremely sensitive to even slight variations in pacing frequency. To allow comparison of the results of the modification factor, a frequency sweep of 1.9 to 2.02 Hz in steps of 0.02, as shown in Figure 4.

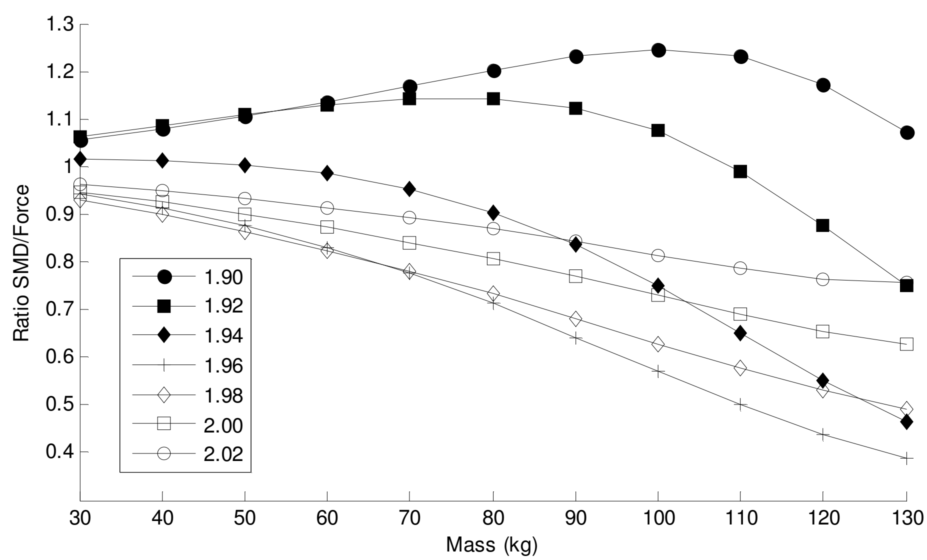

Figure 4. Comparison of modification factors with constant stiffness

In Figure 4, the pedestrian stiffness was set constant at the mean value (22.5 kN/m) and the full range of masses was simulated as before. This highlights that the greatest difference between the SMD and the moving force is at a frequency of 1.94 Hz. It would be expected that this would be at the bridge frequency but it is thought that the frequency is slightly shifted due to the mass influence of the pedestrian on the bridge.

5.3 Step length

To investigate the effect of the step length on the modification factor (μ) values, the pacing frequency was kept constant at the mean value (1.96 Hz) and the step length was varied in the same manner as with the pacing frequency to include the mean value and the extremes of the statistical range (5% and 95%). The results showed that variations in the step length had little effect on the modification factor (μ). As a result, in subsequent tests, the step length was kept constant at the mean value. However, it was evident that there was a slight increase in the values with the increase in step length.

6 SUMMARY AND CONCLUSIONS

6.1 Summary

This paper presents the results of a parametric comparison between moving force and spring-mass-damper models of pedestrian loading on footbridges. A statistical population of pedestrians was considered. The differences in the vibration undergone by the bridge due to the two models were compared through a model ratio, μ. The parameters investigated were pacing frequency, leg stiffness, pedestrian mass, and step length.

6.2 Conclusions

It was found in this work that significant differences between the moving force and spring-mass-damper models only emerge when the pacing frequency is close to the bridge natural frequency. When this is the case, the SMD model gives lower vibrations than the moving force model. Of all the parameters considered, it was found that the pacing frequency has by far the most influence over the vibration response of the bridge, for both models.

It can be concluded from this work that bridge with natural frequencies outside of the narrow range of 1.8-2.2 Hz (typical pacing frequency) can be readily assess using a moving force model. Within this range, the moving force model is conservative and an SMD model is more appropriate. It is clear from this work that more information on the mechanical properties of human legs (stiffness and damping) is required, due to the importance of the SMD model.

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