2010

# Estimating the Characteristic Vertical Response of a Flexible Footbridge Due to Crowd Loading

Joe Keogh
*Athlone Institute of Technology*, jkeogh@ait.ie

Colin C. Caprani
*Technological University Dublin*, colin.caprani@tudublin.ie

Paul Archbold
*Athlone Institute of Technology*, parchbold@ait.ie

*See next page for additional authors*


Funder: ABBEST Scholarship Programme

Authors
Joe Keogh, Colin C. Caprani, Paul Archbold, and Paul Fanning

## ESTIMATING THE CHARACTERISTIC VERTICAL RESPONSE OF A FLEXIBLE FOOTBRIDGE DUE TO CROWD LOADING

**J. KEOGH[1], P. ARCHBOLD[2], C. C. CAPRANI[1] & P. FANNING[3]**
[1]Dept. of Civil & Structural Engineering, Dublin Institute of Technology, Ireland
[2] Structures & Materials Research Group (STRAIT) Athlone Institute of Technology, Ireland
[3] School of Architecture Landscape & Civil Engineering, University College Dublin, Ireland

### Abstract
The issue of excessive vibration of footbridges due to pedestrian loading is now well documented. Bridge vibrations produced from a crowd of pedestrians have been estimated by modifying the effect caused by a single pedestrian by an enhancement factor to take crowd synchronization into account. In this paper this approach is extended to account for the fact that all pedestrians will not have the same pacing frequencies, and the effects of distributions of pacing frequency and other parameters on the enhancement factor are investigated. It is shown that this more faithful representation of pedestrian crowd walking behaviour gives reduced vibration response compared to the fully homogenous crowd case. Based on these results, enhancement factors for predicting the response due to a crowd from the predicted accelerations of a single pedestrian are proposed. Further, the results are compared with published test results to indicate that the model is reasonable.

**Keywords:** Bridges, Pacing frequency, Pedestrian, Stride, Synchronization, Vertical

### 1. Introduction

Recent developments in the design of structures, and increasing pressure on designers to deliver more aesthetically-pleasing structures, have led to longer and lighter footbridges. Increasingly, these structures are experiencing serviceability problems due to excessive vibration. This occurs when the natural frequency of the structure is within the range of pedestrian excitation frequencies. This can lead to discomfort for pedestrians traversing the bridge. Well known examples of footbridges that experienced vibrations due to the dynamic loading of pedestrians are, the Millennium Bridge, London (Dallard et al, 2001), the Pont du Solferino, Paris (Danbon and Grillaud, 2005) and the T-Bridge, Japan (Fujino et al, 1993).

### 1.1 Pedestrian Induced Vertical Loading

In this work, only the vertical vibrations induced by pedestrians are examined. Kala et al (2009) carried out research to investigate the vertical component of pedestrian force on a rigid surface using three sensors placed 0.9 m apart. They also examined the force transmitted by the heel to toe strike on impact with a solid walking surface and report that an increase in pacing velocity led to an increase in step length and peak force.

Pacing frequency is one of the most important parameters and corresponds to the application of vertical forces. It is classified as the inverse of time from the initial contact of the left foot with the surface to the initial contact of the right foot immediately thereafter (Archbold, 2008). Pacing frequency is often described using a normal distribution, and numerous parameter values have been published, Table 1. From these, 'meta-parameters' for the distribution have been derived. Where a range is given, a mean and standard deviation using a confidence interval (CI) of 95% are calculated.

**Table 1** – Parameters of Normal distribution of pacing frequency from the literature.

| Reference | Mean (Hz) | Standard Deviation (Hz) | Coefficient of variation |
|---|---|---|---|
| Dallard et al (2001) | 1.9 | 0.25 | 0.13 |
| Matsumoto (1978) | 2.0 | 0.173 | 0.087 |
| Grundmann and Schneider (1990) | 2.0 | 0.22 | 0.11 |
| Bachmann and Ammann (1987) | 2.0 | 0.13 | 0.065 |
| Pachi and Ji (2005) | 1.83 | -- | -- |
| Ebrahimpour et al (1996) | 1.8 | -- | -- |
| Kramer (1979) | 2.2 | 0.3 | 0.14 |
| Derived Meta Parameters | 1.96 | 0.209 | 0.1064 |

### 1.2 Crowd Loading

The dynamic loading from a crowd for low-frequency footbridges has not been researched extensively (Kala et al, 2009). In a crowd loading situation, vibrations produced by one pedestrian may be reduced or damped by the presence of others due to destructive interference. Conversely, constructive interference can also take place. Grundmann et al (1993) highlighted that under crowd loading, footbridges with a natural frequency close to 2 Hz are likely to experience higher levels of vibration than those induced by a single pedestrian. This is as a result of the synchronisation of the steps of some of the pedestrians in the group.

The level of synchronisation within a crowd is reported with respect to the number of pedestrians on the bridge, $N$. Grundmann et al (1993) use a value of $0.135N$, Fujino et al (1993) use $0.2N$, whilst Bachmann and Ammann (1987) use $\sqrt{N}$. However, none of the quoted values have been applied to structures other than those from which they were obtained. Further, the literature does not cover higher levels of synchronization. Recent tests carried out on the Sean O'Casey Bridge, Dublin, suggest that there is a threshold vibration response beyond which the vibration response levels off as the number of pedestrians increases (Fanning and Healy, 2008). This result highlights the need for further investigation into crowd loading on bridges.

## 2. Numerical Modelling

### 2.1 Problem Formulation

The work presented here is based on a moving force model, similar to those employed in current design standards. This model may be conservative, as it does not consider interaction between the pedestrian and the moving bridge surface (Archbold, 2004).

***Bridge***

The bridge is considered to be a simply-supported, 50 m long beam. The section properties used are: mass of 500 kg/m; width of 2 m, and; depth of 0.6 m. A flexural rigidity of $EI = 7.2\times10^9$ Nm$^2$ was used. Thus the fundamental natural frequency of the bridge is 2.38 Hz. This is similar to the bridge used by Archbold (2008). Damping is taken to be 0.5% for the first two modes, with Rayleigh damping assumed thereafter. It is acknowledged that this will dampen the influence of higher modes.

***Pedestrians***

Adult pedestrian weight was represented by a log-normal distribution with a mean of 4.28 kg and a standard deviation of 0.21 kg. This is equivalent to an average weight of 72.2 kgs (Portier et al, 2007). Although design codes prescribe a stride length of 0.90 m, the stride length is taken here to be normally distributed with a mean of 0.66 m (Barela et al, 2008), and based on a coefficient of variation of 10%, a standard deviation of 0.066 m. As reported in Table 1, the pacing frequency is considered as normally distributed with a mean of 1.96 Hz and standard deviation of 0.209 Hz.

***Crowd***

A crowd length of 100 m and a width of 2 m were used to establish a representative crowd on the bridge at any point in time. The phase angle of the pedestrians is uniformly random in the interval 0 to $2\pi$. Crowd densities of 0.75 p/m$^2$ (where 'p' is the number of pedestrians) and 1.5 p/m$^2$ are used, thus giving an average number of 150 and 75 pedestrians respectively on the bridge during the simulation. Pedestrians' starting locations are based on a Poisson arrival process and are described by the exponential distribution. The mean gap is a function of density and is 0.33 m/p and 0.66 m/p for the densities 0.75 p/m$^2$ and 1.5 p/m$^2$, respectively.

***Synchronization***

The proportion of pedestrians taken to be synchronized; that is, walking in step, is termed the level of synchronization and therefore ranges from 0 to 1. The synchronization levels quoted by Grundmann et al (1993), Fujino et al (1993) and Bachmann and Ammann (1987) given previously, are specifically examined in this work. More specifically, seven synchronization levels of 0, 0.135, 0.2, 0.5, 0.75 and 1.0 are considered, in addition to that of Bachmann and Ammann (1987), which depends on $N$. The pedestrians deemed to be synchronized are given the same pacing frequency and phase angle. These parameters are randomly selected according to their respective distributions previously given. The synchronized pedestrians are randomly distributed throughout the crowd.

## 2.2 Finite Element Modelling

A finite element model of the bridge was developed in Matlab. The beam was modelled using 10 one-dimensional beam elements, with lumped mass assumed. Transient solutions are obtained using the Newmark-$\beta$ method. Each pedestrian is described by a moving force which varies with time (Archbold, 2004) according to:

$$P(t) = W\left[1 + r\sin\left(2\pi f_p t\right)\right] \tag{1}$$

In which, $W$ is the pedestrian weight, $f_p$ is the pacing frequency, and $r$ is the dynamic force component, given by:

$$r = 0.25 f_p - 0.1 \quad (2)$$

Each moving force is distributed to the adjacent nodes according to the beam element shape functions as described in Wu et al (2000). The forces on the bridge due to the crowd at any point in time are taken as the superposition of the individual pedestrian forces.

The finite element model was verified using a closed form solution for a single moving force (Frýba, 1999) and for two moving pulsating forces using a corresponding finite element model in ANSYS.

### 2.3 Vibration Response

The vibration response in this work is assessed using a 5-second root-mean-square (RMS) moving average value from the acceleration history. The maximum of this RMS from any one particular scenario is taken as the response of the bridge to that particular loading scenario.

### 2.4 Enhancement Factor

Following investigations into the enhancement factors used by Grundmann et al (1993), Fujino et al (1993) and Bachmann and Ammann (1987), the crowd loading enhancement factor is defined as:

$$m = \frac{R_C}{R_{SP}} \quad (3)$$

In which $R_C$ is the response due to the crowd and $R_{SP}$ is the single pedestrian response. In this manner, the response due to a crowd can be estimated from that of a single pedestrian. Since the response due to a single pedestrian is easier to model, the idea of the enhancement factor has the potential to be used in codes of practice.

## 3. Results and Discussion

### 3.1 Single Pedestrian Response

***Critical Parameter for Single Pedestrian Excitation***

The response of the structure to a single pedestrian was investigated by considering permutations of randomly distributed and deterministic parameters. When each parameter is not varied according to its distribution, it is assigned the mean value, described previously. As expected, it was found that the response is most sensitive to the pacing frequency. The response function to pacing frequency was established by a pacing frequency sweep from 1.3 to 2.8 Hz, and is given in Figure 1(a). To estimate the distribution of RMS response to the population of pedestrians, varying only the pacing frequency, $10^6$ pacing frequency samples were taken, and the corresponding RMS noted. The resulting distribution of RMS is given in Figure 1(b).

From Figure 1(a), it can be seen that there is a significant increase in the response at 2.36 Hz, which is close to the natural frequency of the bridge (2.38 Hz), as may be expected. As the frequency of the beam is two standard deviations away from the mean pacing frequency, less than 5% of the pedestrians walk at this frequency. Figure 1(b) shows that there are a high number of incidences of low RMS. This is as a result of the mean pacing frequency being lower than that of the bridge.

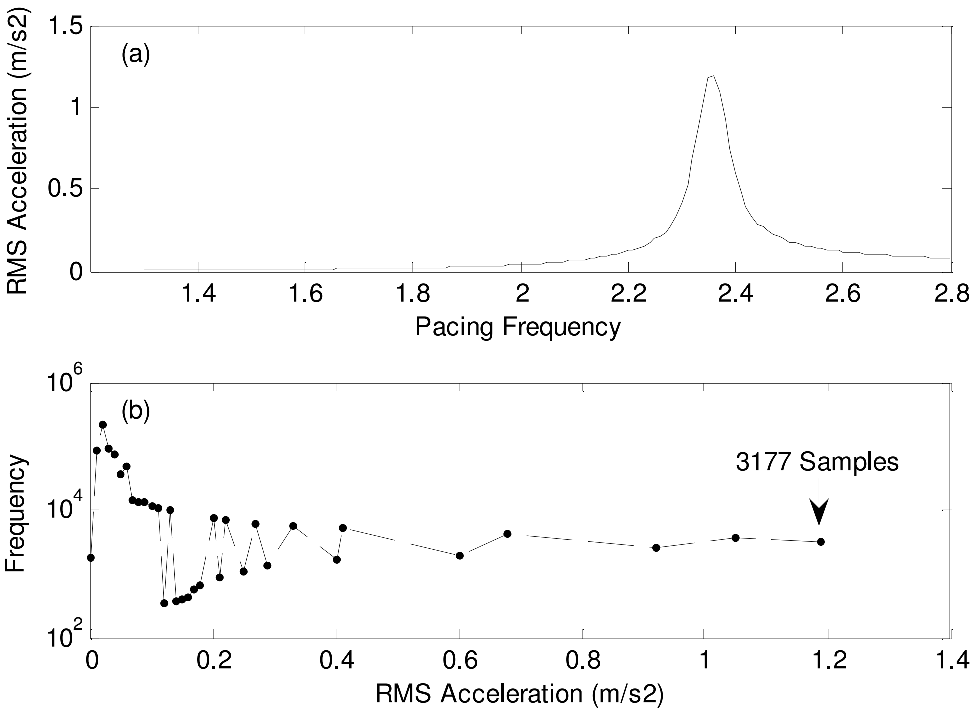


**Figure 1** – Single pedestrian: (a) Response function; (b) distribution of RMS accelerations from $10^6$ samples (only non-zero values shown).

***Characteristic Single Pedestrian Response***

Since there is not a single representative pedestrian, the response of the bridge for 1000 crossings of single pedestrians, with all parameters varied according to their representative statistical distributions, is determined. The characteristic response, $R_{SP}$, is defined as that below which 95% of samples are expected to fall, and is found in this case to have a value of 0.28 m/s$^2$. This is well below the limits prescribed in common design codes.

## 3.2 Crowd Loading Response

***Typical Crowd Response***

The response of the bridge to a typical crowd is given in Figure 2. This crowd has a density of 0.75 p/m$^2$ and 20% synchronization. From Figure 2, it can be seen that the peak acceleration response occurs when a cluster of synchronized pedestrians arrives onto the bridge at about 25 seconds. The midspan response then builds until it reaches a peak at about 38 seconds, when about 75 pedestrians are on the bridge. Consequently, the peak RMS is noted about 5 seconds later at 43 seconds.

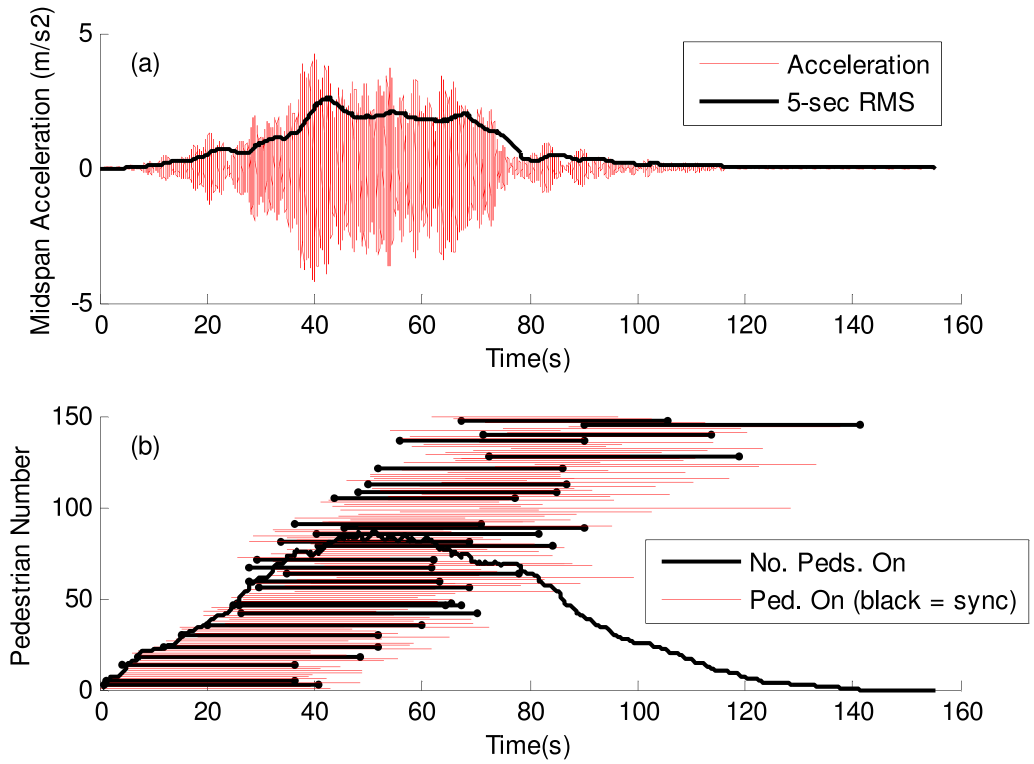


**Figure 2** – Typical crowd response for 20% synchronization and 0.75 p/m$^2$.

***Characteristic Crowd Response***

For the crowd densities of 0.75 p/m$^2$ and 1.5 p/m$^2$, and for each of the 7 levels of synchronization given earlier, 1000 sample crowd responses were determined. The characteristic response (the 95-percentile) was then determined for crowd scenario. The corresponding enhancement factors are determined from Equations (3) with the value of $R_{SP}$ found previously as 0.28 m/s$^2$. The results are given in Figure 3.

Figure 3 represents an improvement on existing enhancement factors which can state that if the synchronization is zero, the enhancement factor is zero. This implies that the response due to each pedestrian in a crowd is cancelled by that of another. Further, Figure 3 shows that the enhancement levels off for the lower crowd density. This may correspond to the threshold response identified in Fanning and Healy (2008).

Figure 4 compares the enhancement factors obtained in this work to those of:

- Bachmann and Ammann (1987), $m_B = \sqrt{N}$, with synchronization of $1/\sqrt{N}$;
- Grundmann et al (1993), $m_G = 0.135N$, with synchronization of 13.5%;
- Fujino et al (1993), $m_F = 0.2N$, with synchronization of 20%;

The results show a reasonable correlation with the work of these authors at the specified levels of synchronization. The enhancement factors for a density of 1.5 p/m$^2$ found here are higher than those reported by Bachmann and Ammann (1987) and Grundmann et al (1993), but lower than those reported by Fujino et al (1993). The present enhancement factors are higher than those of the other authors for the crowd density of 0.75 p/m$^2$.

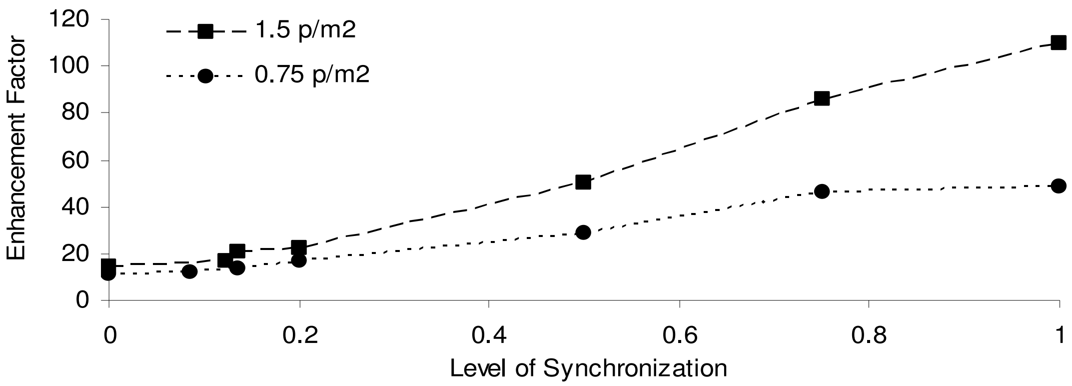


**Figure 3** – Crowd loading enhancement factors for two densities.

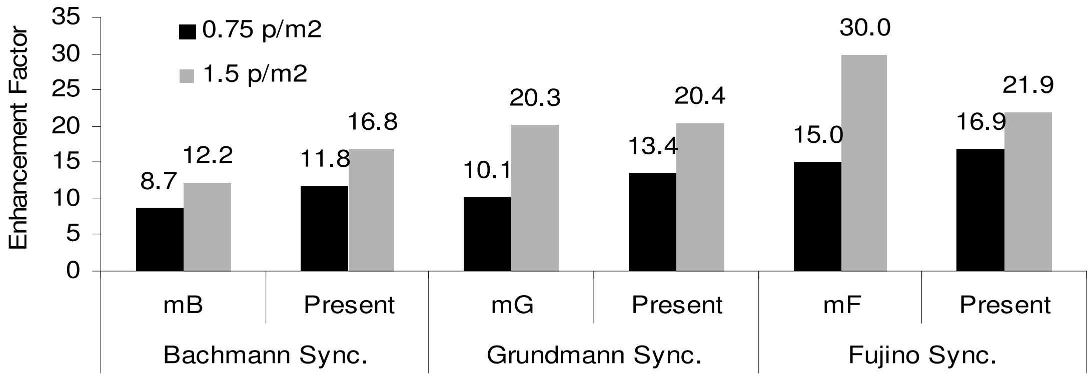


**Figure 4** – Comparison of enhancement factors with those from literature

## 4. Summary

This research used a moving force finite element formulation to determine the vertical response of a bridge due to pedestrian excitation. Statistical distributions of pedestrian parameters were used to derive characteristic responses, for various synchronization levels and crowd density. The response to a single pedestrian was examined in detail, and a characteristic response established. The enhancement factors to be applied to the single pedestrian response, to obtain the characteristic crowd response were derived.

It was found that the enhancement factor is not zero for zero synchronization. Further, the enhancement factors found compare reasonably well to those of the literature. Additional research is required to investigate the levelling off of the crowd response for the lower crowd density as found by Fanning and Healy (2008).

## 5. Acknowledgment

The authors would like to acknowledge the Dublin Institute of Technology ABBEST Scholarship Programme for funding this research.

## References

Archbold, P., (2004), *Novel Interactive load Models for Pedestrian Footbridges*, PhD Thesis, University College Dublin.

Archbold, P. (2008), 'Evaluation of Novel Interactive Load Models of Crowd Loading on Footbridges', *Proceedings of 4th Symposium on Bridge and Infrastructure Research in Ireland*, National University of Ireland, Galway, 35-44.

Bachmann, H. and Ammann (1987), 'Vibrations in Structures-Induced By Man and Machines', *IABSE*, Structural Engineering Document, Zurich.

Barela, A.M.F. and Duarte, M. (2008), 'Biomechanical Characteristics of Elderly individuals Walking on Land and on Water', *Journal of Electromyography and Kinesiology,* **18**(3), 446-454.

Dallard, P., Fitzpatrick, T., Flint, A., Low, A., Ridsdill Smith, R.M., Willford, M. and Roche, M. (2001), 'London Millennium Footbridge: pedestrian-induced lateral vibration', *ASCE Journal of Bridge Engineering*, **6**(6)**,** 412-417.

Danbon F. and Grillaud, G. (2005), 'Dynamic behaviour of a steel footbridge. Characterization and modelling of the dynamic loading induced by a moving crowd on the Solferino Footbridge in Paris', *Proceedings Footbridge 2005*.

Ebrahimpour, A., Hamam, A., Sack, R.L. and Patten, W.N. (1996), 'Measuring and Modeling Dynamic Loads Imposed by Moving Crowds', *ASCE Journal of Structural Engineering*, **122**(12), 1468 – 1474.

Fanning, P.J. and Healy, P. (2008), 'Sean O' Casey Bridge: Testing for Vibration Serviceability', *Proceedings of 4th Symposium on Bridge and Infrastructure Research in Ireland*, National University of Ireland, Galway, 61-69.

Frýba, L. (1999), *Vibration of Solids and Structures under Moving Loads*, 3rd Edn., Thomas Telford, London.

Fujino, Y., Pacheco, B.M., Nakamura, S. and Warnitchai, P. (1993), 'Synchronization of human walking observed during lateral vibration of a congested pedestrian bridge', *Earthquake Engineering and Structural Dynamics*, **22**(9), 741-758.

Grundmann, H. Kreuzinger, H., Schneider, M. (1993) 'Dynamic calculations of Footbridges*', Bauingenieur*, **68**(5), 215–225.

Grundmann, H and Schneider, M. (1990), 'Stochastic Representation of Footbridge Vibrations Taking into Account Feedback Effects', in W.B. Krätzig et al. (Eds.), *Proc. Eur. Conf. on Structural Dynamics*, *Eurodyn '90*, Bochum, Balkema, Rotterdam.

Kala, J., Salajka, V. and Hradil, P. (2009), 'Footbridge Response on Single Pedestrian Induced Vibration Analysis', *International Journal of Applied science, Engineering and Technology*, **5** (4), 269-280.

Kramer, H. and Kebe, H.W. (1980), 'Man-Induced Structural Vibrations', *Der Bauingenieur*, **54**(5), 195 – 199.

Matsumoto, Y., Nishioka, T., Shiojiri, H. and Matsuzaki, K. (1978), 'Dynamic design of footbridges', *IABSE Proceedings*, **2**, Paper P-17/78, Zurich.

Pachi, A. and Ji, T. (2005), 'Frequency and velocity of people walking', *The Structural Engineer*, **83**(3), 36–40.

Portier, K., Tolson, J.K., and Roberts, S.M. (2007), 'Body weight distributions for risk assessment', *Risk Analysis*, **27**(1), 11-26.

Wu, J.J., Whittaker, A.R. and Cartmell, M.P. (2000), 'Use of finite element techniques for calculating the dynamic response of structures to moving loads', *Computers and Structures*, **78**(6), 789-799.