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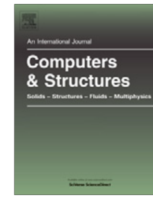

Enhancement factors for the vertical response of footbridges subjected to stochastic crowd loading

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ABSTRACT

The vertical acceleration response of a hypothetical footbridge is predicted for a sample of single pedestrians and a crowd of pedestrians using a probabilistic approach. This approach uses statistical distributions to account for the fact that pedestrian parameters are not identical for all pedestrians. Enhancement factors are proposed for predicting the response due to a crowd based on the predicted accelerations of a single pedestrian. The significant contribution of this work is the generation of response curves identifying enhancement factors for a range of crowd densities and synchronization levels.

1. Introduction

Recent developments in the design of structures, and increasing pressure on structural designers to deliver more aesthetically-pleasing structures, have led to longer and lighter footbridges. Increasingly, these structures are experiencing serviceability problems due to excessive vibration. This occurs when a natural frequency of the structure is within the range of pedestrian pacing frequencies. This can lead to discomfort for pedestrians traversing the bridge. Well known examples of footbridges that experienced vibrations due to the dynamic loading of pedestrians include the Millennium Bridge, London [1], the Pont du Solferino, Paris [2] and the T-Bridge, Japan [3]. This however is not a new phenomenon and is not limited to lightweight structures. For example, in 1975 the Auckland Harbour Bridge in New Zealand, which is an 8-lane motorway bridge, suffered from lateral vibrations as a result of a crowd of pedestrians traversing the bridge [4].

The main contribution of the work described in this paper is the proposal of new enhancement factors which can be used to predict the response of a typical crowd crossing a simply supported footbridge. These factors are obtained using the predicted response of a non-homogeneous sample of single pedestrians and a sample of non-homogeneous crowds. Based upon these results, crowd loading enhancement factors are proposed. In addition, different levels of synchronization between pedestrians are accounted for, as well as a range of crowd densities. This also facilitates a comparison of the proposed enhancement factors with those proposed by previous researchers which were carried out for specific bridge frequencies and crowd densities. The work offered here results in a much wider range of enhancement factors than heretofore available, within the limitations of the study with regard to the numerical models examined.

1.1. Pedestrian induced vertical loading

A pedestrian produces a dynamic time varying force which has components in all three directions [5]. These periodic forces are in the vertical, horizontal-lateral and horizontal-longitudinal directions. In this work, only the vertical vibrations induced by pedestrians are examined. The vertical force imparted due to walking is a periodic force and is regarded as the largest of the three forces [3] as it has the highest amplitude and as a result has been studied most widely in the past [6]. Recently, Kala et al. [7] investigated this vertical component of pedestrian force on a rigid surface using three sensors placed 0.9 m apart. They examined the force transmitted by the heel to toe strike on impact with the walking surface and found the force produced by a single pedestrian taking one step was of the kind shown in Fig. 1. It was found that the forces

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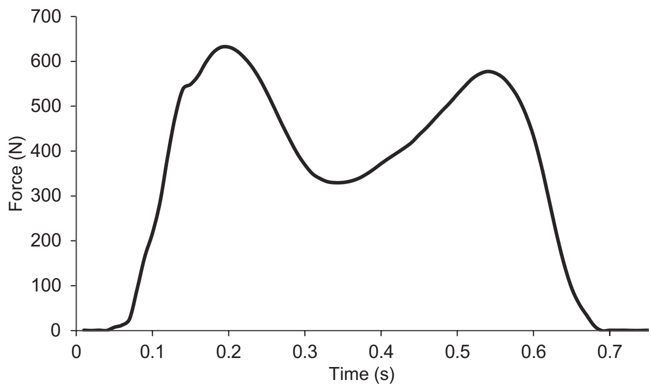

Fig. 1. Typical shape of single step vertical force.

from the left and right foot respectively overlap in time while walking as there is always one foot on the ground, as was previously reported by Wheeler [8]. Zivanovic et al. [6] discussed other authors who found the same general shape and conclusions. Kala et al. [7] and Wheeler [8] found that an increase in pacing velocity led to an increase in step length and peak force, and thus a change in the shape of the walking force time plot.

Pacing frequency is one of the most important parameters of human locomotion and corresponds to the rate of application of vertical forces. It is classified as the inverse of time from the initial contact of the left foot with the walking surface to the initial contact of the right foot immediately thereafter, or more simply as the number of footfalls per second [5,8]. Pacing frequency is often described using a normal distribution, and numerous parameter values have been published. One of the first notable works on the subject was by Matsumoto et al. [9], who investigated a sample of 505 persons and found that their pacing frequency had a mean of 2 Hz and a standard deviation of 0.178 Hz.

For this work, a review of published values of pacing distributions is carried out as shown in Table 1. The values presented are all based on experimental results, from which an average is obtained for the mean and standard deviation. The coefficient of variation (COV) of the results is also presented in the table.

1.2. Crowd loading

The dynamic loading from a crowd on low-frequency footbridges has not been researched extensively [7]. In a crowd loading situation, vibrations produced by one pedestrian may be reduced or damped by the presence of others due to destructive interference. Conversely, constructive interference can also take place, amplifying the bridge response. This means that the vibration induced by a crowd is not simply the sum of the responses caused by each individual pedestrian.

Wheeler [8] found, following simulations of a number of bridges, that the crowd effect was not significant unless the frequency was close to 2 Hz. The same author also found that a crowd walking on a bridge with a natural frequency removed from the typical pacing rate (2 Hz) would generate less response than a single pedestrian walking with the same frequency as the bridge. As a result of this work it was suggested that the ‘single test pedestrian remains the most appropriate excitation model’ [8]. Grundmann et al. [14] on the other hand found that, under crowd loading, footbridges with a natural frequency close to 2 Hz are likely to experience higher levels of vibration than those induced by a single pedestrian. This is as a result of the synchronization of the steps of some of the pedestrians in the crowd.

In the pedestrian crowd-bridge interaction problem there are two types of synchronization: there is pedestrian-bridge synchronization, in which the pedestrian’s (or pedestrians’) pacing frequency (frequencies) matches the natural frequency of the bridge (studied by Grundmann et al. [14], for example). There is also inter-pedestrian synchronization where pedestrians in a crowd are walking in-step with each other, but not necessarily at the natural frequency of the bridge [6]. It is this second form of synchronization that is referred to in this paper.

Zivanovic et al. [15] stated that, although synchronization within a crowd takes place, the force peak amplitude per person decreased with increasing numbers of people. Recent tests carried out on the Sean O’Casey Bridge, Dublin, also suggested a threshold (or limit) of vibration response beyond which the vibration response levels off as the number of pedestrians increases [16].

Matsumoto et al. [9] found following tests on the Shibuya West Exit Bridge in Tokyo, that pedestrian arrivals to a bridge tend to follow a Poisson distribution, typical of arrival-type phenomena. Subsequently, the vibration response to a crowd was determined by superimposing stochastically the response of the bridge due to one pedestrian crossing. Matsumoto et al. [9] concluded that the response of the bridge due to crowd loading, with N people, can be found by multiplying the single pedestrian response by $\sqrt{N}$. The authors stated that this is true for a bridge with a natural frequency within the range 1.8–2.2 Hz. Outside of this range, 1.6–1.8 Hz and 2.2–2.4 Hz, this factor reduces linearly to 2.0, which is equivalent to two people marching in step [5]. Bachmann and Ammann [5] went onto verify this factor for a crowd density (pedestrians per unit area) of 0.55 p/m^2 against crowd simulations of the same density carried out by Wheeler [8]. From this work, the level of synchronization within a crowd is reported with respect to the number of pedestrians on the bridge, N. However, Blanco et al. [17] pointed out that the relationship described by Matsumoto et al. [9] is only valid for simply supported bridges. Equally these studies relate only to single crowd densities and whether the relationship between pedestrian numbers and enhancement factors can be applied confidently for all crowd densities is not proven.

Fujino et al. [3] studied a footbridge that connects a bus terminal and a sports stadium which periodically caters for very high crowd densities of up to 2.11 p/m^2. It was found in this study that up to 20% of the crowd was synchronized with the bridge in the lateral direction. This implies that 20% of the crowd was synchronized with each other, and this is represented in this report as 0.2N.

Grundmann et al. [14] studied a simply supported footbridge near Munich which had a natural frequency of 1.94 Hz and a crowd density of 0.44 p/m^2. It was found that if the pacing frequency of the pedestrians in the crowd matched that of the bridge, the level of synchronization between the crowd and the bridge can be given as 0.135N for bridges within a frequency range of 1.5–2.5 Hz. It is evident that if a number of pedestrians are synchronized with the bridge, they are also synchronized with each other. If the pacing frequency and natural frequency do not coincide, there is a reduction factor provided.

EC5 [18] uses a similar approach to that described here. With a crowd density of 0.6 p/m^2 on a bridge with a natural frequency which is susceptible to excitation from pedestrians (1.5–2.5 Hz) the formula used in the code can be simplified to 0.23N times the response of a single pedestrian. The current literature does not cover higher levels of synchronization which are included in this study – the most obvious example of which is troops marching in step (close to 100% synchronization).

1.3. Probabilistic design approach

The need for a probabilistic approach to pedestrian loading has been acknowledged for a long time [8,9]. Despite this, most current design codes [18–20] continue to use deterministic load models. As

Table 1
Parameters of normal distribution of pacing frequency from the literature.

Reference	Mean (Hz)	Standard deviation (Hz)	Coefficient of variation
Matsumoto et al. [9]	2.00	0.178	0.089
Grundmann and Schneider [10]	2.00	0.22	0.11
Pachi and Ji [11]	1.83	0.11	0.06
Ebrahimpour et al. [12]	1.80	–	–
Kramer and Kebe [13]	2.20	0.30	0.136
Derived parameters	1.96	0.20	0.10

discussed by Zivanovic [21], these models are commonly unable to accurately predict the response due to a single pedestrian, and usually overestimate it significantly; furthermore they cannot account for the non-homogenous nature of crowds of pedestrians and their individual gait patterns.

A number of researchers, in recent years, have begun using probabilistic methods rather than deterministic methods which use only mean values for the important parameters associated with pedestrian loading [22–27]. Pedersen and Frier [22] developed a single pedestrian response model using a normal distribution for the pacing frequency and the step length to find the statistical distributions of vibrations on a simply supported bridge beam. Zivanovic et al. [25] also presented a single pedestrian model which was further developed by Zivanovic et al. [27] to account for crowd loading. This was done by assuming the crowd to be a number of single pedestrians in a stream along the centre line of a bridge. In this crowd model, which did not include any statistical distribution to account for varying pedestrian weight, they used a Poisson arrival process, as per Matsumoto et al. [9]. The authors attempted to verify the model against measured results from two pedestrian footbridges. The results from one bridge were promising with an overestimation of only 8% for the peak response and root mean square (RMS) values were almost the same. However, for the second footbridge predictions using the model were out by as much as 65%, it was acknowledged by the authors that further refinement of the model was required.

In this paper a probabilistic model, including normal distributions for pacing frequency, step length and pedestrian mass, for a single pedestrian is used. For varying crowd densities, and different levels of synchronization, enhancements factors relative to the response due to a characteristic pedestrian are determined. These enhancement factors are compared to enhancement factors previously reported for specific crowd densities to good effect. The significant contribution of this paper is the development of enhancement factors for crowds, with a range of levels of synchronization and a range of crowd densities up to a limit of 2.11 p/m^2. These enhancement factors can then be applied to a single characteristic pedestrian response, which can be used to determine the peak vibration response due to the corresponding crowd.

2. Numerical modelling

2.1. Problem formulation

The work presented here is based on a moving force model, similar to those employed in the current standards [18,20]. It is acknowledged that this model may be conservative, as it does not consider mass or stiffness interaction between the pedestrian and the moving bridge surface [15,28] but this degree of conservatism is offset by its use probabilistically rather than deterministically. In addition, the damping ratio of the bridge is increased in this work to represent the pedestrian-bridge interaction that was found to occur by Zivanovic et al. [27].

The bridge considered in this work is a simply-supported 50 m long beam. The mass is 500 kg/m, the width is 2 m and the depth was varied according to Table 2, to achieve different natural frequencies. A modulus of elasticity of 200×10^{11} N/m^2 was used for the beam.

2.1.1. Bridge damping

Damping in pedestrian bridges is typically very light. Heinemeyer et al. [29] review damping ratios according to construction material for serviceability conditions and found an average damping ratio for a steel bridge of 0.4%. Comparing damping ratios for a number of steel bridges, of different frequencies and span lengths, they report that for bridges with spans of the order of 50 m and a frequency ca. 2.0 Hz a damping ratio of 0.5% would be typical. This is borne out by a number of studies reported in the literature. The Solferino footbridge in Paris has a natural frequency of 1.94 Hz and a damping ratio of 0.5% (prior to the addition of dampers) in the vertical direction [30]. Experimental tests carried out by Fanning et al. [16] on the Sean ‘O Casey footbridge in Dublin found a natural frequency of 2.14 Hz and a damping ratio of 0.5% for the first vertical mode. Caetano et al. [31] found similar damping ratios, 0.53% and 0.58%, for the first and second mode shapes of the Pedro e Ines footbridge in Portugal. As a result, for this work, the damping ratio of the structure alone was taken to be 0.5% for the first two modes, with Rayleigh damping assumed thereafter [32].

To reflect the possible contributions to damping of stationary (non-moving) and non-stationary (moving) crowds two different levels of damping ratios for the crowded bridge are considered. There is some evidence in the literature that the contribution made by humans to the damping of a system, is dependent on whether they are stationary or non-stationary. In tests to determine the damping ratio of the bridge with a crowd, Fanning et al. [16] prompted a crowd (density of 0.15 p/m^2) randomly walking on the bridge to stop at once, and found that there was a small increase in damping when compared to the empty footbridge due to the standing pedestrians. They also carried out tests with one pedestrian jumping with up to 30 stationary pedestrians on the bridge and found that the damping increased from 0.5% to a range between 1.1% and 1.6%. Ellis and Ji [33] found that standing or sitting people affect the damping of a structure but that people walking do not, and so should be represented as a load only.

On the other hand, Zivanovic et al. [27,35] and Brownjohn et al. [36] reported that walking pedestrians as well as stationary pedestrians can increase the damping ratio of a bridge in the vertical direction. Zivanovic et al. [35] carried out laboratory experiments on a simply supported prestressed reinforced concrete footbridge which had a natural frequency of 4.44 Hz and a damping ratio of 0.72%. The tests were carried out using up to 10 standing or walking pedestrians, which equates to an average of 0.46 persons/m^2. In the tests with 10 standing pedestrians, similar to the findings of Ellis and Ji [33], the damping ratio was found to increase significantly to 3.62%. A slight reduction in natural frequency to 4.21 Hz was also noted. In the case of the tests with walking pedestrians, an increase in damping ratio was also apparent and varied approximately linearly from 0 pedestrians to 10 pedestrians (0.72–2.86%). There was also a slight increase in natural frequency to 4.51 Hz. Zivanovic et al. [27] also found an increase in damping due to crowd loading in experiments on the Podgorica Bridge in Montenegro. Calibration of a finite element model to match the bridge and crowd loading conditions showed that the damping

Table 2
Bridges considered.

Natural frequency (Hz)	Depth (m)	Reference
1.94	0.523	Grundmann et al. [14]
2.00	0.535	Fujino et al. [3]
2.10	0.552	Bachmann and Ammann [5]

Table 3
Damping ratios for both damping models (DM) considered.

Crowd density (pedestrians/m^2)	DM 1 (%)	DM 2 (%)
0.44	0.5	2.53
0.55	0.5	3.04
0.75	0.5	3.97
1.50	0.5	7.43
2.11	0.5	10.25

ratio increased from 0.26% (empty) to 0.67% under crowd loading. Further tests on the Reykjavik City footbridge in Iceland did not show an increase in damping, but this was attributed to lower bridge acceleration levels and a relatively short period of exposure to loading. Brownjohn et al. [36] also found an increase in damping in the vertical direction due to the presence of the walking pedestrian on the bridge from tests on a long span footbridge at Singapore Changi airport.

Based on the above inconclusive findings in the literature, two different damping models are used in this work. Damping Model 1 (DM 1) uses a damping ratio of 0.5% for all simulations, regardless of the presence a crowd. This is consistent with other researchers in the field, including Pavic [34], who in his keynote address at the conference Footbridge 2011, used a bridge of frequency 2.17 Hz and a constant damping ratio of 0.6% in predicting the response for a non-stationary crowd (density of 0.5 p/m^2). In light of the findings of Zivanovic et al. [35], Damping Model 2 incorporates an increase in damping dependent on the crowd density. The pedestrian crowd-bridge system, or total damping (ζ_T) is assumed here to be of the following form:

$$\zeta_T = \zeta_B + \zeta_C \tag{1}$$

where ζ_B is the bridge damping (0.5%) and ζ_C is the extra damping induced by the crowd. Zivanovic et al. [35] found the increase in damping from 0 to 10 pedestrians is approximately linear, and in this work it is assumed that this trend continues for further increases in crowd density. Hence the crowd damping is expressed as a linear relationship between the crowd density, ρ, and a crowd-damping factor, γ as follows:

$$\zeta_C = \rho\gamma \tag{2}$$

Following this formulation, the total damping (2.86%) found by Zivanovic et al. [35] with 10 pedestrians walking on the bridge is separated into the bridge damping (0.72%) and the damping due to the crowd ($\zeta_C = 2.14\%$ for a crowd density of 0.46 p/m^2). The crowd damping factor γ found by Zivanovic et al. [35] is thus 4.65%/p/m^2, and this value is used in this work for DM 2. The damping ratios taken for both damping models are given in Table 3.

2.1.2. Pedestrian properties

The pedestrians in this work are deemed to be healthy adults for the purposes of assigning pedestrian properties. Adult pedestrian weight is represented by a log-normal distribution with a mean of 73.85 kg and a standard deviation of 15.68 kg [37]. The stride length is taken here to be normally distributed with a mean of 0.66 m [38] and assuming a coefficient of variation of 10%, a standard deviation of 0.066 m is used. As reported in Table 1, the pacing frequency is considered as normally distributed with a mean of 1.96 Hz and standard deviation of 0.209 Hz. The phase angle, φ, of a pedestrian's vertical harmonic force is taken to be uniformly random in the interval 0–2π.

2.1.3. Crowd properties

A crowd with an initial length of 100 m and a width of 2 m is used to establish a representative crowd on the bridge at any point in time. Crowd densities considered are given in Table 4, along with reference studies where applicable. In addition to crowd densities reported in the literature, densities of 0.75 and 1.5 p/m^2 are also included to provide a more complete spectrum of crowd densities. Based on the starting crowd length of 100 m, and the bridge length of 50 m, the average number of pedestrians on the bridge during the simulations is also given in Table 4. Pedestrian arrival is considered as a Poisson process [9] and gaps are thus described by the exponential distribution. The mean gap is a function of density and the mean arrival gaps are also given in Table 4.

2.1.4. Synchronization

The proportion of pedestrians taken to be synchronized with each other (that is, walking in phase at the same frequency) ranges from 0 to 1. Seven synchronization proportions of 0, 0.135 [14], 0.2 [3], 0.5, 0.75 and 1.0 are considered, in addition to that of Matsumoto et al. [9], which depends on *N*. Synchronization in the crowd is enforced by giving the pedestrians deemed to be synchronized the same pacing frequency and phase angle. These parameters are randomly selected according to their respective distributions previously given. Also, the synchronized pedestrians are randomly distributed throughout the crowd. It is acknowledged that this is a simplification as some clusters of synchronized pedestrians may occur, but this is not considered here. For the case of no enforced synchronization, it is still statistically possible to have some pedestrians with similar properties, and thus it may be expected that very low levels of synchronization may yield similar results to zero synchronization results.

2.2. Finite element modelling

To establish the vibration response under the crowds defined previously, a finite element model of the bridge was developed in Matlab. The beam was modelled using 10 Euler–Bernoulli beam elements, with lumped mass assumed. Transient solutions were obtained using the Newmark-β method.

While walking, the vertical force induced by both human feet is assumed to be of the same magnitude and to be periodic [6,39]. As reported by numerous authors, including Bachmann and Amman [5] and Kala et al. [7], the force from successive footfalls can be represented by the Fourier series:

$$F_P(t) = G + \sum_{i=1}^{n} G\alpha_i \sin(2\pi i f_p t - \varphi_i) \tag{3}$$

where: $F(t)$ is the time-varying vertical force, G the pedestrian weight, α_i the Fourier's coefficient of the ith harmonic i.e. dynamic load factor (DLF), f_p is the pacing frequency (Hz), t the time (s), φ_i the Phase shift of ith harmonic, i the order number of the harmonic, and n is the total number of contributing harmonics.

The number of harmonics used in the Fourier series for the vertical force varies between authors. Fanning et al. [40] found that the response of a bridge due to a crossing pedestrian can be accurately predicted with a single harmonic and hence, in this work, each pedestrian is described by a moving force which varies with time according to:

$$F_P(t) = G\left[1 + \alpha \sin(2\pi f_p t)\right] \tag{4}$$

Fanning et al. [40] also determined the linear relationship between the Fourier coefficient α and the pacing frequency to be:

$$\alpha = 0.25 f_p - 0.1 \tag{5}$$

which completes the single pedestrian load model definition used in this work.

Each moving force is distributed to the adjacent nodes according to the beam element shape functions [41]. The forces on the bridge due to the crowd at any point in time are taken as the super-

Table 4
Crowd densities considered.

Density (pedestrians/m^2)	Mean number on bridge	Mean arrival gap (m)	Reference
0.44	44	0.568	Grundmann et al. [14]
0.55	55	0.454	Bachmann and Ammann [5]
0.75	75	0.333	–
1.50	150	0.166	–
2.11	211	0.118	Fujino et al. [3]

position of the individual pedestrian forces. Inherent to the use of a force model is the assumption that the crowd mass is not sufficient to change the natural frequency significantly.

The finite element model was verified using a closed form solution for a single moving force [42] and for two moving pulsating forces using a corresponding finite element model in ANSYS.

2.3. Vibration response

The response of interest in this study is taken as the mid-span acceleration. The vibration response is assessed using a 5-s root-mean-square (RMS) moving average value from the acceleration history of each simulation [28]. The maximum of this RMS from any one particular scenario is taken as the response of the bridge to that particular loading scenario [43].

2.4. Enhancement factor

The crowd loading enhancement factor, m, is defined as the ratio of the characteristic response due to the crowd, R_C, to the characteristic response due to a single pedestrian, R_{SP}:

$$m = \frac{R_C}{R_{SP}} \tag{6}$$

In this manner, the response due to a crowd can be estimated from that of a single pedestrian. Since the response due to a single pedestrian is easier to model, the idea of the enhancement factor has good potential to be used in codes of practice. Notably, in this work, the crowd and single pedestrian response will be determined statistically, leading to a more appropriate enhancement factor suitable for design and assessment.

3. Results and discussion

3.1. Single pedestrian response

3.1.1. Critical parameter for single pedestrian excitation

The response of the bridge to a single pedestrian is investigated by considering permutations of randomly distributed and deterministic parameters. When each parameter is not varied according to its distribution, it is assigned the mean value, described previously. Consistent with the literature, it is found that the bridge vibration response is most sensitive to the pacing frequency. The response function to varying pacing frequency alone, Fig. 2(a), is established using a pacing frequency sweep from 1.3 to 2.8 Hz. To estimate the distribution of RMS response to the population of pedestrians, varying only the pacing frequency, 10^6 pacing frequency samples were taken, and the corresponding RMS noted. The resulting distribution of RMS accelerations is given in Fig. 2(b). This figure highlights that occurrences of RMS accelerations above 0.3 m/s^2 for a single pedestrian are relatively few, with the majority of cases being below this value. In particular, 18 880 of the 10^6 (1.88%) simulations were found to have an RMS acceleration of approximately 1.0 m/s^2.

From Fig. 2(a), it can be seen that there is a significant increase in the response at 1.98 Hz, which is close to the natural frequency of the bridge (2.0 Hz), as may be expected. Fig. 2(b) shows that there are a relatively high number of incidences of low RMS. For bridges with natural frequencies removed from the mean of the pedestrian pacing frequency, the number of high responses is found to reduce, as may be expected. It was found also that using the reduced step length of 0.66 m, as opposed to the codified value of 0.9 m [20], increased the response of the bridge, due to the increase in applications of the load in crossing the bridge.

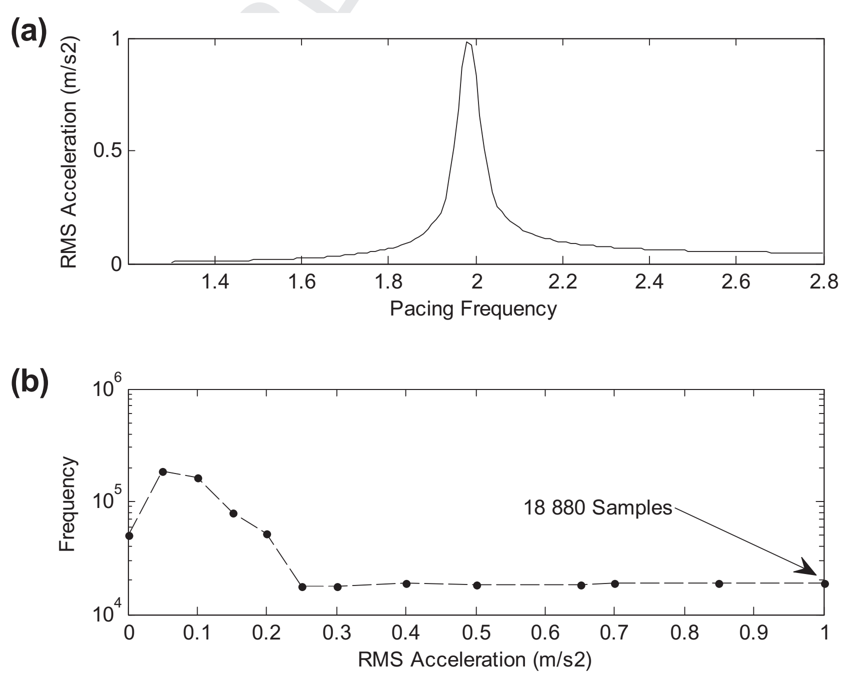

Fig. 2. Single pedestrian: (a) response function, (b) distribution of RMS accelerations from 10^6 samples (only non-zero values shown).

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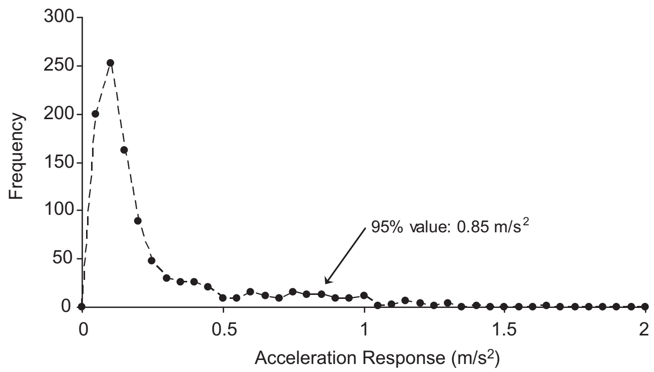

Fig. 3. Distribution of 2 Hz bridge response for random single pedestrians.

3.1.2. Characteristic single pedestrian response

Since there is not a single representative pedestrian, the response of the bridge for 1000 crossings of single pedestrians, with all parameters varied according to their representative statistical distributions, is determined. The distribution of responses is given in Fig. 3. The characteristic response, R_{SP}, is defined here as that response below which 95% of samples are expected to fall, and is found in this case to have a value of 0.85 m/s^2 for the bridge with the natural frequency of 2.0 Hz. This is above the common basic rule used in BS 5400 [19,20] of $0.5\sqrt{f_p}$ (which gives 0.7 m/s^2 in this case). However, it was found that over 90% of the values fell below this lower limit from the design code. Values of 0.76 and 0.84 m/s^2 were obtained for the bridges with a natural frequency of 1.94 and 2.1 Hz, respectively. In another test with a modelled bridge of natural frequency 2.38 Hz, it was found that the single pedestrian response reduces significantly to 0.27 m/s^2 due to it remoteness from the mean pacing frequency of 1.96 Hz.

3.2. Crowd loading response

3.2.1. Typical crowd response

The acceleration response of the bridge to a typical crowd is given in Fig. 4(a), while Fig. 4(b) and (c) give the crowd diagnostics for this particular crowd which has a density of 0.55 p/m^2 with 20% synchronization. Fig. 4(b) gives the total number of pedestrians on the bridge with respect to time and the number of whom is synchronized. Fig. 4(c) shows the time at which each pedestrian (synchronized and unsynchronized) enters and leaves the bridge. From Fig. 4(a), it can be seen that the peak acceleration response occurs at about 52 s and corresponds to two clusters of synchro-

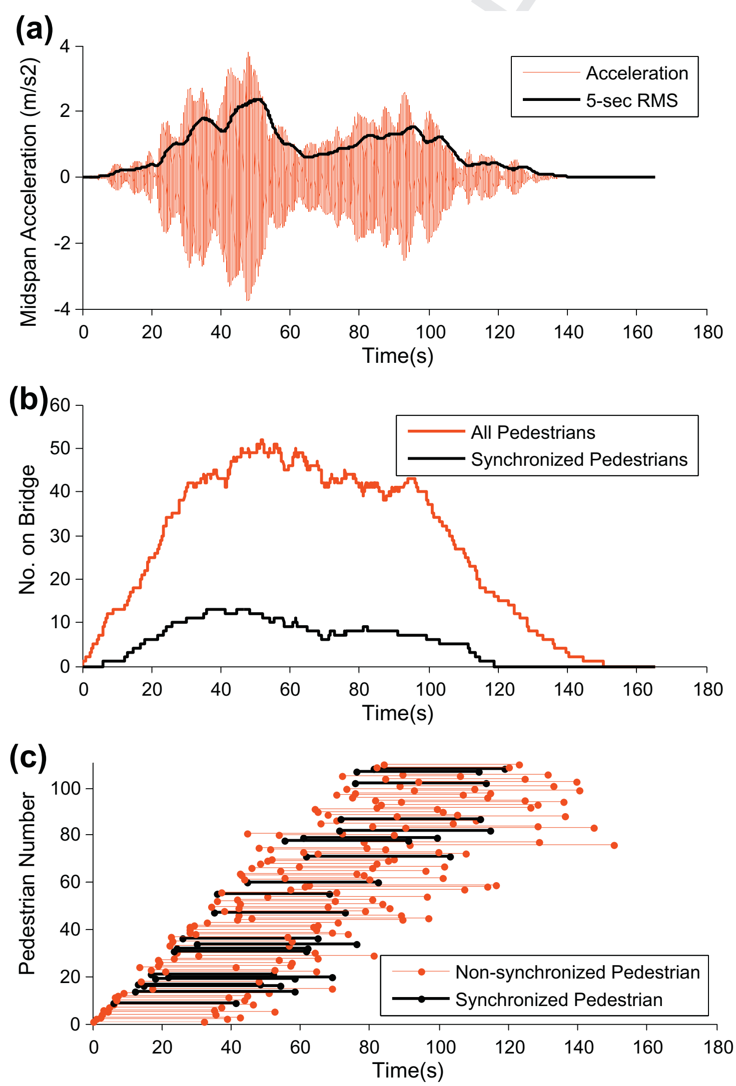

Fig. 4. Typical crowd response for 20% synchronization and 0.75 p/m^2.

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nized pedestrians which arrive onto the bridge at about 18 and 22 s. The mid-span response then builds until it reaches the peak, when about 52 pedestrians are on the bridge. Consequently, the peak RMS of 2.33 m/s^2 is noted.

3.2.2. Characteristic crowd response

For each of the crowd densities considered in this study (See Table 3), and for each of the levels of synchronization (given earlier), 1000 sample crowd responses were determined. The

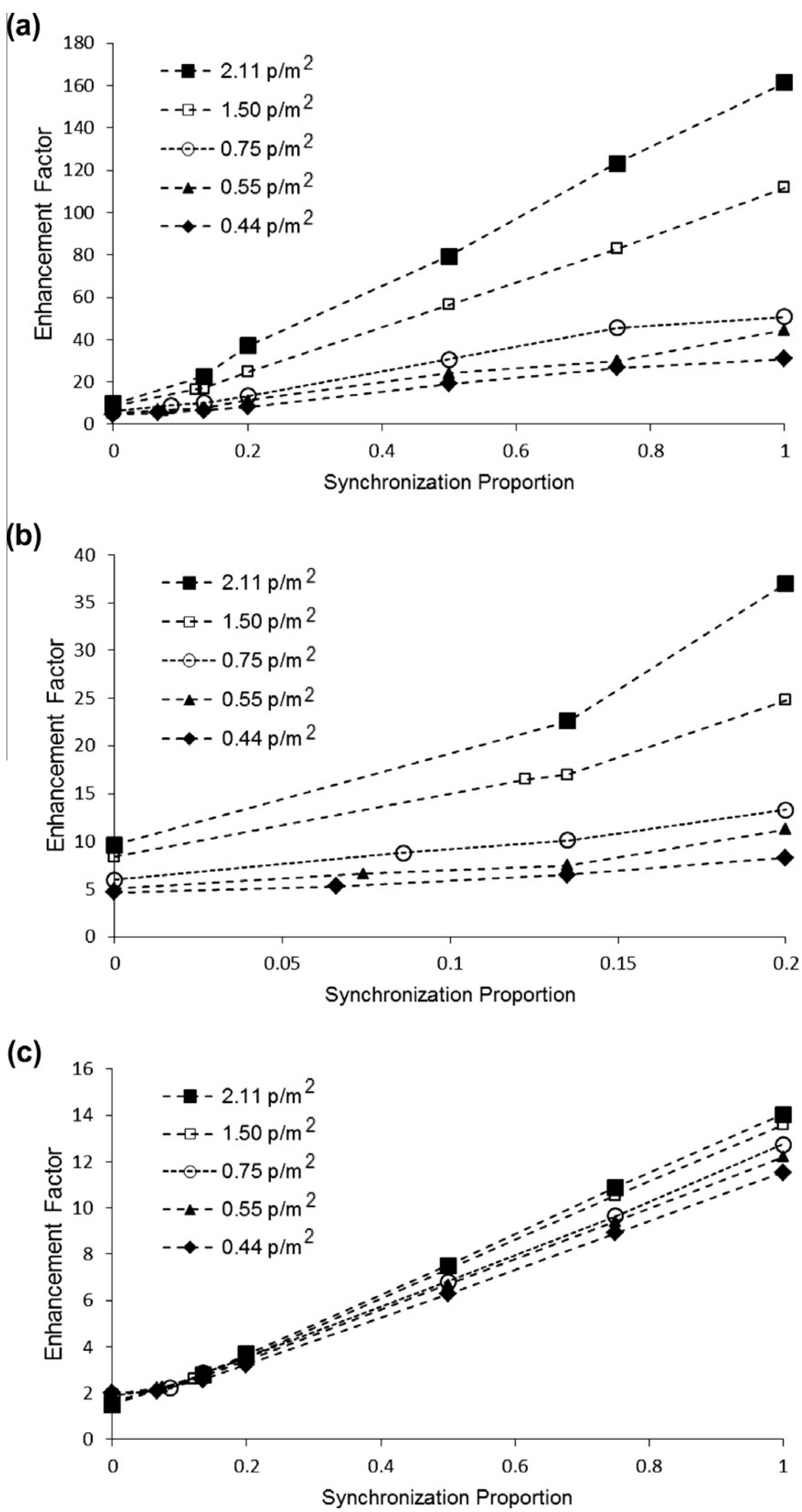

Fig. 5. Crowd loading enhancement factors: (a) showing all synchronization proportions, (b) showing only those levels at or under 20% synchronization proportions, and (c) showing results for Damping Model 2.

Table 5
Enhancement factors for all crowd densities and synchronization proportions for Damping Model 1 and Damping Model 2.
Q2

Synchronization proportion	Density (p/m^2)				
	0.44	0.55	0.75	1.50	2.11
DM 1					
0.000	4.7	5.1	5.9	8.4	9.6
Matsumoto et al.[a]	5.3	6.6	8.7	16.4	**
0.135	6.5	7.5	10.1	17.0	22.6
0.200	8.3	11.3	13.3	24.8	37.0
0.500	19.0	23.9	30.6	56.3	79.3
0.750	26.5	29.9	45.5	82.9	123.4
1.000	30.9	44.3	50.7	111.7	161.3
DM 2					
0.000	1.99	1.93	1.90	1.64	1.51
Matsumoto et al.[a]	2.10	2.24	2.21	2.60	**
0.135	2.57	2.71	2.87	2.83	2.80
0.200	3.23	3.43	3.56	3.56	3.67
0.500	6.27	6.69	6.81	7.36	7.51
0.750	8.92	9.40	9.65	10.54	10.88
1.000	11.53	12.21	12.74	13.58	14.05
	0.066	0.074	0.086	0.1225	[b]

[a] Synchronization proportion.
[b] The formula given by Matsumoto et al. [9] does not extend to this high density.

characteristic response, R_C, (the 95-percentile) was then determined for each crowd scenario. The corresponding enhancement factors are determined from Eq. (6)9 with the corresponding value of R_{SP} (characteristic single pedestrian response). The results are given in Fig. 5 and Table 5.

Fig. 5(a) shows the results found using Damping Model 1. It can be seen that the enhancement factor is a function of crowd density and the proportion of the crowd that are synchronized. Furthermore it demonstrates that the enhancement factor can become unrealistically large for high crowd densities and synchronization proportions. It is thought that in practice this will not be reached because as the vibrations become excessive, pedestrians will tend to stop, thus damping the vibrations [15]. Fig. 5(b) gives a closer view of the enhancement factors for lower synchronization proportions, more typical of a random crowd, and more representative of proportions previously studied, again for DM 1. For crowd densities of 0.75 p/m^2, and lower, there is a levelling off of enhancement factors; this is consistent with the limiting responses observed by Fanning et al. [16] and Zivanovic et al. [15] in crowd loading tests on two separate bridges. Note that there is no enhancement factor quoted for the Matsumoto et al. [9] synchronization level for a density of 2.11 p/m^2. This is because Bachmann and Ammann [5] report that this enhancement factor is limited to mean flow rates (persons/s over the width of the deck) below 1.5 persons/s/m, whereas the flow rate for a density of 2.11 p/m^2, given the distributions of pedestrian and crowd parameters in this work, is 2.6 p/s/m on average (the minimum is 1.6 p/s/m). Fig. 5(c) gives the results of DM 2 and it can be seen that regardless of the increase in crowd density, the enhancement factors remain similar due to the corresponding increase in damping.

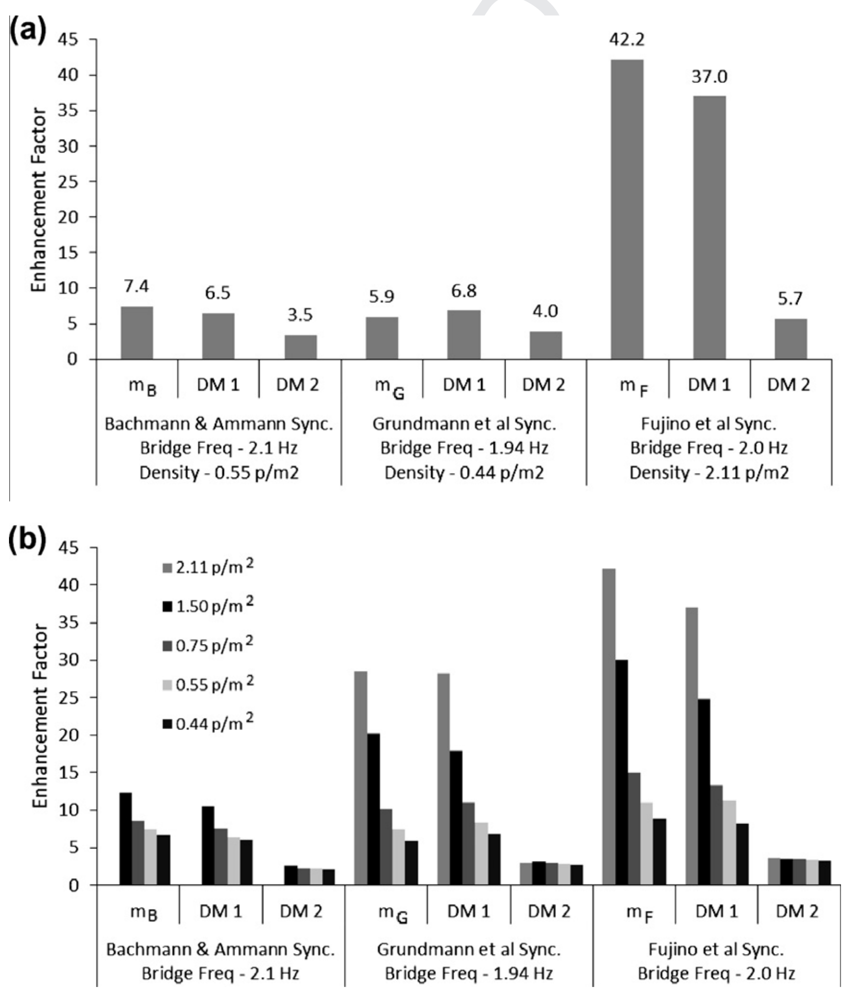

Fig. 6. Comparison of enhancement factors with those from literature for specific synchronization proportions: (a) for only those densities considered in the literature, (b) for all crowd densities.

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3.2.3. Relation to past work and current guidelines

To relate the findings of this work to existing literature, the enhancement factors (m) found here (Fig. 5) are compared to the enhancement factors for specific synchronization proportions, crowd densities, and bridge frequencies given by previous authors as follows:

- Bachmann and Ammann [5]: enhancement factor, $m_B = \sqrt{N}$, at a synchronization of $(\sqrt{N})\%$, for a crowd density of 0.55 p/m^2 and a bridge natural frequency of 2.1 Hz;
- Grundmann et al. [14]: enhancement factor, $m_G = 0.135N$, for a crowd density of 0.44 p/m^2 with synchronization of 13.5%, for a bridge natural frequency of 1.94 Hz;
- Fujino et al. [3]: enhancement factor $m_F = 0.2N$, for a crowd density of 2.11 p/m^2, synchronization of 20%, and a bridge natural frequency of 2.0 Hz.

The comparison of the results of the present work with those of the above authors is given is Fig. 6(a). It can be seen that the results are in reasonable agreement for DM 1. However when the damping ratio is increased with increasing crowd density (DM 2) the results no longer match those presented in the literature. However, it still may be that DM 2 is more suitable as some authors conclude that the constant damping assumption of DM 1 is overly conservative [35,36].

For the full range of crowd densities considered here, we further compare the enhancement factors of the previous authors considered above to the present results. The results are given in Fig. 6(b), and there can be seen to be a good comparison for DM 1. In the case of DM 2, the results are significantly lower than those presented by the cited authors.

Across the range of crowd densities and synchronization proportions reported by [3,5,10] there is close agreement with the method advanced here in DM 1. The significance of this close agreement is that it confirms the validity of each but only for the specific crowd density and synchronization proportion from which they are derived. For example, for 44 pedestrians (density of 0.44 p/m^2 on a 50×2 m wide bridge), the enhancement factor (m) derived by Bachmann and Amman [5] is based on a synchronization level of $\sqrt{N}\%$, giving $m_B = \sqrt{N} = \sqrt{44} = 6.6$, while Grundmann et al. [14] had 13.5% synchronization, giving an enhancement factor of $m_G = 0.135\,N = 0.135 \times 44 = 5.9$, as shown in Fig. 6(a). The difference is due to these projections being based on specific values for crowd density and synchronization proportions – comparisons with the probabilistic approach advanced in this paper are shown to be accurate for both, but for their specific cases only.

In Fig. 6(a) the sensitivity of each enhancement factor projection method to crowd density is assessed. The trends in predictions for the method advanced here compared to the alternative approaches discussed are consistent. This implies that the main reason for the difference in values of enhancement factors achieved using previous approaches is due to the level of synchronization rather than the crowd densities.

Current guidelines set out in EC1 [44] state that if the forces applied to the structure by pedestrians are at a frequency identical to the natural frequency of the bridge, special consideration should be given to the acceleration of the bridge deck. The standard states that an appropriate dynamic model of the pedestrian load should be defined. The methods for modelling the pedestrian loads are however left to the designer. The vertical acceleration of a bridge at any part of the deck should be limited to 0.7 m/s^2, thus giving a similar value to that quoted in BS 5400 [19,20] for which the max acceleration is given as $0.5\sqrt{f}$, where f is the pacing frequency of the pedestrian. For all bridges with a natural frequency less than 5 Hz in the vertical direction, EC5 [18] also requires calculation of the acceleration response caused by small groups and streams of pedestrians with the same limiting value of 0.7 m/s^2 in the vertical direction. A simplified method for calculating vibrations of the bridge deck of a simply supported bridge, made from any material, due to crowd loading is given in EC 5: Annex B [18]. However, it states in the code that results of the calculations are subject to very high uncertainties and as a result if the comfort criteria (max response of 0.7 m/s^2) is not satisfied with a “significant margin” the installation of dampers may be required. This leaves designers with great uncertainly and highlights the requirement for a more accurate method of predicting the acceleration response of a bridge to crowd loading.

4. Conclusions

The work presented here uses a moving force finite element model to determine the vertical response of a footbridge due to pedestrian excitation. Statistical distributions of pedestrian parameters determined from the literature were used to derive characteristic responses, for various synchronization proportions and crowd densities. The damping ratio of the structure is increased to account for the effect of a crowd of pedestrians. Characteristic responses to a single pedestrian and to crowd loading scenarios were obtained. Enhancement factors, defined as the ratio of characteristic crowd response to characteristic single pedestrian response were derived and presented graphically.

The significant conclusion is that enhancement factors were found to be a function of both crowd density and synchronization proportion. A limitation of currently available methods for estimating enhancement factors is that they are founded on single synchronization levels and are thus not suitable for capturing the sensitivity of enhancement factors to synchronization proportion. The enhancement factors determined using the probabilistic approach derived match each of the specific cases, thereby unifying them, and also enable selection of appropriate enhancement factors for varying crowd densities and synchronization proportions. In respect of the scope of existing methods, it was found that their effectiveness is good for varying crowd densities provided they are applied only at synchronization proportions from which they were derived. The simulations which ignored increased damping due to the crowd also identified a levelling off of enhancement factors, a feature previously observed in pedestrian loading tests on two different bridges by different authors, at crowd densities lower than about 0.75 p/m^2.

The enhancement factors derived in this work are represented by a series of curves, which represent a range of crowd densities and synchronization levels. These could prove to be very beneficial tools to designers and researchers in studying the effects of vertical crowd loading on flexible footbridges. This will in turn eliminate the uncertainty in the use of the Eurocodes for predicting the acceleration response of a crowd of people.

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