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Technical Note: Friction Factor Diagrams for Pipe Flow

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Abstract

This technical note describes diagrams of friction factor for pipe flow that have been prepared using, mainly, the equations that Lewis Moody used to prepare his famous diagram in 1944. The preparation of the new diagrams was prompted by the need for vector graphics versions that could be used for teaching purposes and that could be distributed freely to students and others under a Creative Commons Attribution-Share-Alike license.

Using a structure very similar to that of Moody's diagram, variants with the Darcy friction factor and the Fanning friction factor have been prepared. In addition, variants have been prepared that include not only monotonic roughness curves, but also inflectional roughness curves.

Keywords: friction factor, Moody diagram, vector graphic, Darcy friction factor, Fanning friction factor, pipe flow, relative roughness, Reynolds number, monotonic roughness, inflectional roughness.

Introduction

Since the mid 1940s, practicing engineers, engineering academics and students of engineering have made use of a diagram of friction factor for pipe flow of the type that was published by Lewis Moody in the Transactions of A.S.M.E. in 1944 [1]. Textbooks in such areas as Fluid Mechanics, Hydraulics, Heat Transfer or Unit Operations commonly include re-drawn or reproduced versions of Moody's diagram. The diagram is semi-empirical, based on some fundamental principles and the strong intuition of leading researchers up to 1944. At every stage since then the diagram could be regarded as a temporary solution, until sufficient further advance had been made. Although considerable progress has been made, notably in computation, in the measurement of surface roughness and in the measurement and

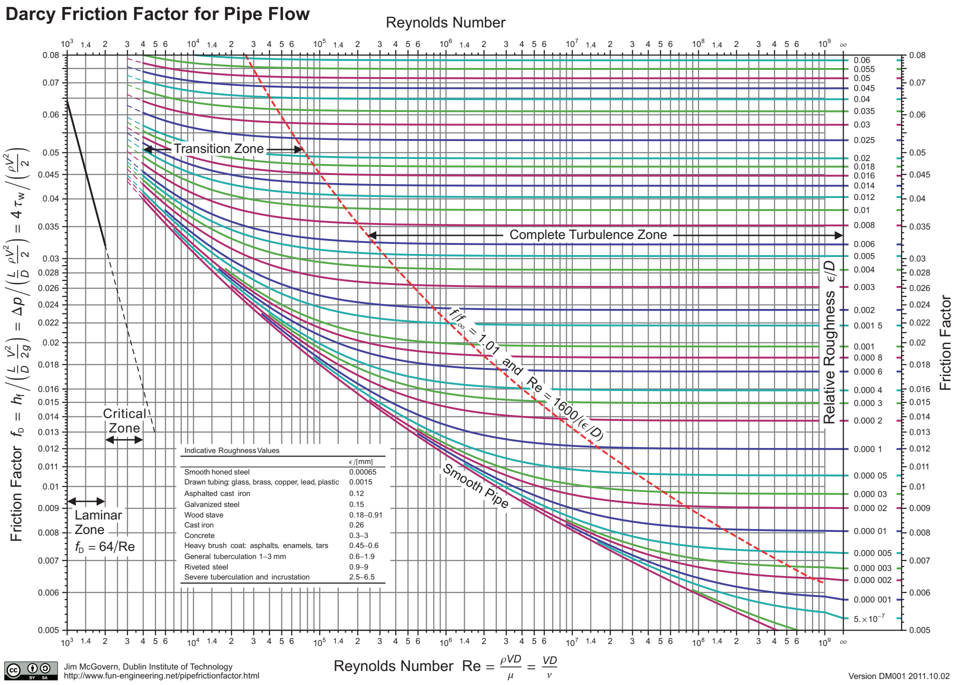

Figure 1: Newly-prepared diagram of friction factor for pipe flow.

understanding of velocity distribution within boundary layers, the diagram can still be regarded as 'temporary,' but continues to fulfill the function that Moody attributed to it: 'a simple means of estimating the friction factors.' It still has relevance for a very broad range of situations, from flow in micro tubes, e.g. reference [2], to flow in large pipelines or tunnels, e.g. references [3, 4].

The equations that Moody used to prepare his diagram had been developed by others, as cited by Moody, and were supported by published data. Figure 1 is a newly-prepared diagram of this type.

The friction factor f is a dimensionless term in the Darcy-Weisbach equation, Equation 1 or Equation 2. A concise history of the Darcy-Weisbach equation has been written by Brown [5]. Two variants of the friction factor are in common use: the Darcy friction factor f_{D}, which Moody plotted, and the Fanning friction factor f_{F}, which equals one quarter of the Darcy factor, as expressed in Equation 3. Versions of the diagram for both friction factors have been prepared.

$$h_{\mathrm{f}} = f_{\mathrm{D}} \left(\frac{L}{D} \frac{V^2}{2g} \right) \tag{1}$$

$$h_{\mathrm{f}} = 4 f_{\mathrm{F}} \left(\frac{L}{D} \frac{V^2}{2g} \right) \tag{2}$$

$$f_{\mathrm{F}} = f_{\mathrm{D}}/4 \tag{3}$$

The fact that the friction factor diagram in the form set out by Moody still endures is due to the use of dimensionless quantities and an effective consensus to continue to use the constants and underlying equations that were used by Moody. A consistent skeleton structure is provided. Moody and his contemporaries were under no illusions about its accuracy. Moody stated 'it must be recognized that any high degree of accuracy in determining f is not to be expected' and 'fairly reasonable estimates of friction loss can be made.'

The diagram is necessarily a simplification and a rough approximation: the conditions it describes (fully developed, isothermal, incompressible, dissipative, pseudo-steady-state flow) are never quite attained in practice. Conditions within pipelines are inherently non-uniform over the flow cross section and over a given length. Fluids are more or less compressible, rather than incompressible, and have finite thermal conductivity. The flowing fluid within pipelines is not isothermal in the radial direction because of the very dissipation that is quantified by the Darcy-Weisbach equation. Frictional dissipation and finite thermal conductivity together give rise to differences in temperature—and density and viscosity—between different parts of the flow. Inevitably too, any length of pipe for which the fully-developed flow requirement is approximated must be connected to entry and exit systems within which this is not the case. When the flow is turbulent, vortices form and collapse relentlessly over time. In the flow of liquids, vapour pressure may play a role. Nevertheless, the diagram is a very useful design tool. For computational purposes, it is easily represented as a data set of discrete points, or by the equations that define it along with appropriate solution algorithms.

The preparation of new diagrams by the present author was prompted by the need for vector graphics versions of the friction factor diagram that could be used for teaching purposes and could be distributed freely to students and others under a Creative Commons Attribution-Share-Alike license. The same four zones that were labeled by Moody (laminar, critical, transition and complete turbulence) are shown. Colour has been employed, although the diagram is also usable in grayscale form.

Preparation of the Diagrams

The diagrams were prepared using Wolfram Mathematica and exported (as layers in PDF format) to CorelDraw for minor adjustments to the layout. The nominal size of each of the diagrams is A3, in landscape format, and printed versions at this size are very convenient to use. As most users will not have a personal printer available for this size, care was taken to ensure usability at A4 or Letter size. Absolute size is of little relevance for use on a screen or tablet. The diagrams can be used conveniently on any device that can display a PDF document.

The mathematical curves have not been plotted directly. Rather, straight lines have been drawn between precisely calculated data points of the underlying curves. The underlying equations, such as the Colebrook White [6] equation, Equation 4, were solved or evaluated with a precision of about fifteen significant digits. The plotted points are sufficiently close together that visual smoothness is achieved and, as the diagram is a vector graphic, the file size is small.

$$\frac{1}{\sqrt{f_{\mathrm{D}}}} = -2\log_{10}\left(\frac{\epsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f_{\mathrm{D}}}}\right) \tag{4}$$

The horizontal axis in the diagram is for the Reynolds number Re, Equation 5, which is dimensionless. The range has been extended at the upper end to 10^9 (Moody's diagram went to 10^8) and then to infinity!

$$\mathrm{Re} = \frac{\rho VD}{\mu} = \frac{VD}{\nu} \tag{5}$$

The vertical axis of the diagram is for the friction factor. The Darcy and the Fanning friction factors are given by Equations 6 and 7 respectively.

$$f_{\mathrm{D}} = h_{\mathrm{f}}/\left(\frac{L}{D}\frac{V^2}{2g}\right) = \Delta p/\left(\frac{L}{D}\frac{\rho V^2}{2}\right) = 4\tau_{\mathrm{w}}/\left(\frac{\rho V^2}{2}\right) \tag{6}$$

$$f_{\mathrm{F}} = h_{\mathrm{f}}/\left(4\frac{L}{D}\frac{V^2}{2g}\right) = \Delta p/\left(4\frac{L}{D}\frac{\rho V^2}{2}\right) = \tau_{\mathrm{w}}/\left(\frac{\rho V^2}{2}\right) \tag{7}$$

Friction factors were in use prior to the publication of Moody's diagram. For example, in 1930 Fred Scobey [7] compiled 'Weisbach friction coefficients,' f_{D}, for riveted and analogous pipes under actual operating conditions. These friction factors were determined from measurements of flow rate and pressure drop or head loss in situ.

For turbulent flow in smooth pipes, the friction factor is calculated from the von Kármán, Prandtl, Nikuradse expression, Equation 8.

$$\frac{1}{\sqrt{f_{\mathrm{D}}}} = -2\log_{10}\left(\frac{2.51}{\mathrm{Re}\sqrt{f_{\mathrm{D}}}}\right) \tag{8}$$

The friction factor in rough pipes at infinite Reynolds number is calculated from von Kármán's expression, Equation 9.

$$\frac{1}{\sqrt{f_{\mathrm{D}}}} = -2\log_{10}\left(\frac{\epsilon/D}{3.7}\right) \tag{9}$$

where ϵ/D is the relative roughness of the pipe, ϵ being the surface absolute roughness and D being the inside diameter of the pipe. Equations 8 and 9 are both included in the Colebrook White equation, Equation 4, which achieves a blending of the two extremes to cover the full transitional and complete turbulence zones of the diagram. Friction factor curves are shown on the diagram for relative roughness values ranging from 1×10^{-7} to 0.06.

It can be noted that, in principle, the absolute roughness that is used is an experimentally determined 'equivalent sandgrain size' parameter for the particular pipe surface. It is referenced to tests carried out by J. Nikuradse [8].

Transition / Complete Turbulence Boundary

In his diagram, Moody showed a dashed line to mark the boundary between the transition zone and the complete turbulence zone. He plotted this by making use of an expression that H. Rouse had used, Equation 10.

$$\frac{1}{\sqrt{f_{\mathrm{D}}}} = \frac{\mathrm{Re}}{200} \frac{\epsilon}{D} \tag{10}$$

However, within the written discussion at the end of his paper [1] Moody mentioned that the boundary curve could be located 'so that it would correspond to some fixed percentage of excess in f over the f for complete turbulence.' This approach has been adopted by the present author: e.g. in Figure 1, the boundary curve is defined by Equation 11.

$$\frac{f}{f_\infty} = 1.01 \tag{11}$$

As part of the discussion, R.J.S. Pigott, who had published a friction factor diagram with basically the same structure as Moody's in 1933 [9], proposed Equation 12 for the boundary curve, while Moody responded with a suggestion that Equation 13 would approximate to a boundary at 1% above the friction factor for infinite Reynolds number. This has been verified by the present author. Therefore, Equation 13 also represents the boundary curve shown in Figure 1.

$$\frac{3500}{\epsilon/D} = \mathrm{Re} \tag{12}$$

$$\frac{1600}{\epsilon/D} = \mathrm{Re} \tag{13}$$

Hence, for a maximum positive correction factor of less than 1% in using the rough pipes expression (Equation 9) for the friction factor, the complete turbulence zone can be defined by the inequality labeled Equation 14.

$$\mathrm{Re} > \frac{1600}{\epsilon/D} \tag{14}$$

This can be generalized, with excellent precision, for an arbitrary positive maximum correction factor, η, as the expression labeled Equation 15.

$$\mathrm{Re} > \frac{16 \times 10^{-\mathrm{Log}_{10}(\eta)}}{\epsilon/D} \tag{15}$$

where $2.5 \times 10^{-7} \leq \eta \leq 0.02$, $10^4 \leq \mathrm{Re} \leq 10^9$ and $\epsilon/D > 0$. For example, setting $\eta = 0.01$ yields Equation 13. A practical application of this is to avoid solving the Colebrook White equation where it is unnecessary. The correction can be ignored, where it is insignificant, or applied (up to a maximum value of 0.02) according to Equations 16 and 17. This correction can be applied and has a value less than 0.01 in the entire 'Complete Turbulence Zone' of Figure 1.

$$f = (1 + \eta) f_\infty \tag{16}$$

$$\eta = 10^{-\mathrm{Log}_{10}\left(\mathrm{Re}\frac{\epsilon}{D}/0.16\right)} \tag{17}$$

Friction Factors for Smooth Pipes

Strong experimental data exist to support the friction factors in smooth pipes over the range $11 \leq \mathrm{Re} \leq 2.7 \times 10^7$ [8, 10, 2]. Laminar flow exists up to a Reynolds number close to 3,000 and the flow is turbulent from 3,000 upwards. Furthermore, finely honed steel pipe has been found to be hydraulically smooth at Reynolds numbers up to at least 2.7×10^7 [11], which is far higher than the equivalent sand grain roughness would predict according to the Colebrook White formula. Such pipe surfaces are said to have an inflectional transition curve, which was also the case in the pipes that were tested by Nikuradse [8] that had been artificially roughened with sand grains.

Alternative equations to Equation 8 (which is also embedded in Equation 4) for the friction factor of smooth pipes have been proposed [10, 12, 13] and there is some evidence to suggest that the smooth pipe friction factor curve should be a little higher at high Reynolds numbers. However, corroborating data are still scarce. Also, where friction factor has been measured at very high Reynolds numbers the fluid has been a gas, which is not incompressible. It is still possible that Equation 8 (based on the extrapolation of test data for water) is good for a hypothetical incompressible fluid at very high Reynolds numbers.

Determination of Absolute Roughness

The absolute roughness of a pipe could be measured by first determining friction factor values from measurements of head loss and flow rate where the flow is in the transition or complete turbulence zone and then solving for ϵ from Equation 4, as in Equation 18. Alternatively, the friction factor data points could be superimposed on a Moody diagram in order to read the relative roughness value and then calculate the roughness by multiplying by the diameter. This was done, for example, in reference [3].

$$\epsilon = 3.7D\left(10^{-1/2\sqrt{f_\mathrm{D}}} - \frac{2.51}{\mathrm{Re}\sqrt{f_\mathrm{D}}}\right) \tag{18}$$

However, as was well recognized by Colebrook and White [14], as well as by Nikuradse and Moody, the nature of the surface and not just its 'absolute roughness' influences the shape of the curves representing friction factor in the transition region. Strictly, therefore, a mapping of particular pipes to the relative roughness curves of the friction factor diagram should be based on measurements made in the fully turbulent region.

In choosing to base the friction factor curves on the Colebrook White equation, Moody choose to represent the characteristics of commercial pipes ('of the nature of cast iron, wrought iron or galvanized steel'—this phrase is from Colebrook and White [14]) rather than artificially roughened pipes, as had been tested by Nikuradse and by Colebrook and White. Recent work at Princeton University [11] has shown that a honed surface can have characteristics that are somewhat similar to those of Nikuradse's surfaces that were artificially roughened with sand.

Absolute and Relative Roughness Values

Moody [1] gave absolute roughness values and ranges for a selection of eight pipe surfaces, based on published data. In seven of the eight cases the same values and ranges had been shown on a friction factor diagram by Hunter Rouse (1943), which was also included as part of the written discussion at the end of Moody's paper [1], on p. 681. Similar values had been included in the 1938 paper by C.F. Colebrook [6].

In his 1933 friction factor diagram, Pigott [9] regarded pipes such as those of drawn brass, lead or tin, or glass tubes, as being 'as nearly dead smooth as we can get them.' Rouse's diagram had shown 'drawn' tubing (glass, brass, copper or lead) as 'smooth,' and Colebrook had also taken these as smooth 'at least for all ordinary velocities of flow,' whereas Moody gave a finite value, equivalent to 0.0015 mm, based on two cited references. A somewhat similar situation has arisen in relation to the smoothness of the original pipe of the 'SuperPipe' experiment at Princeton University [15]. This pipe was found to be smooth up to a very high Reynolds number.

Some indicative relative roughness values from the literature are shown in Table 1. Values such as these are commonly listed in textbooks and handbooks, e.g. [16]. Surface finishes can vary considerably and it is also important to take account of condition, corrosion or fowling. The summary table for inclusion on the printed diagram is shown as Table 2.

Relative roughness values depend on the pipe diameter and, for rough pipes, there could be some ambiguity about which 'diameter' to use—a volume based average would seem the most appropriate, as in Equation 19. 'Roughness' would thus extend inside and outside the 'diameter.'

$$D = \sqrt{\frac{4\mathcal{V}}{\pi L}} \tag{19}$$

Depending on the circumstances, different methodologies for measuring the diameter may be used, but the approach used by Nikuradse [8] was exemplary:

> 'The diameters of the pipe were determined from the weight of the water which could be contained in the pipe with closed ends and from the length of the pipe.'

Pipes with Inflectional Roughness

The roughness curves derived from the Colebrook White equation are said to be monotonic, i.e. the friction factor decreases continuously with increasing Reynolds number. In the tests carried out by Nikuradse on pipes that were artificially roughened with grains of sand the curves were inflectional in nature, i.e. the friction factor decreases to a minimum value with increasing Reynolds number and then rises again to reach a constant value for complete turbulence. Some modern pipes, e.g. honed steel pipe, demonstrate similar inflectional friction factor curves. Afzal [13] has provided an extension of the Colebrook White equation that is capable of representing Nikuradse's data and data from modern pipes that have inflectional friction factor curves with good accuracy, Equation 20. This is identical to the Colebrook White equation when the dimensionless roughness parameter j has a value of zero. Afzal found that a value of 11 for j provided a good fit to the data of Sletfjerding and Gudmundsson [17], Shockling [18, 11], and Nikuradse [8]. The version of Equation 20 given in [13] appears to omit a factor of two in the exponential term. The version given here has been checked by the present author against the experimental data to confirm good agreement.

$$\frac{1}{\sqrt{f_{\mathrm{D}}}} = -2\log_{10}\left(\frac{\epsilon/D}{3.7}\exp\left[-j\frac{2.83}{\mathrm{Re}(\epsilon/D)\sqrt{f}}\right] + \frac{2.51}{\mathrm{Re}\sqrt{f_{\mathrm{D}}}}\right) \tag{20}$$

Figure 2 is a is a variant of Figure 1 that includes inflectional roughness curves in accordance with Equation 20, where j has a value of 11.

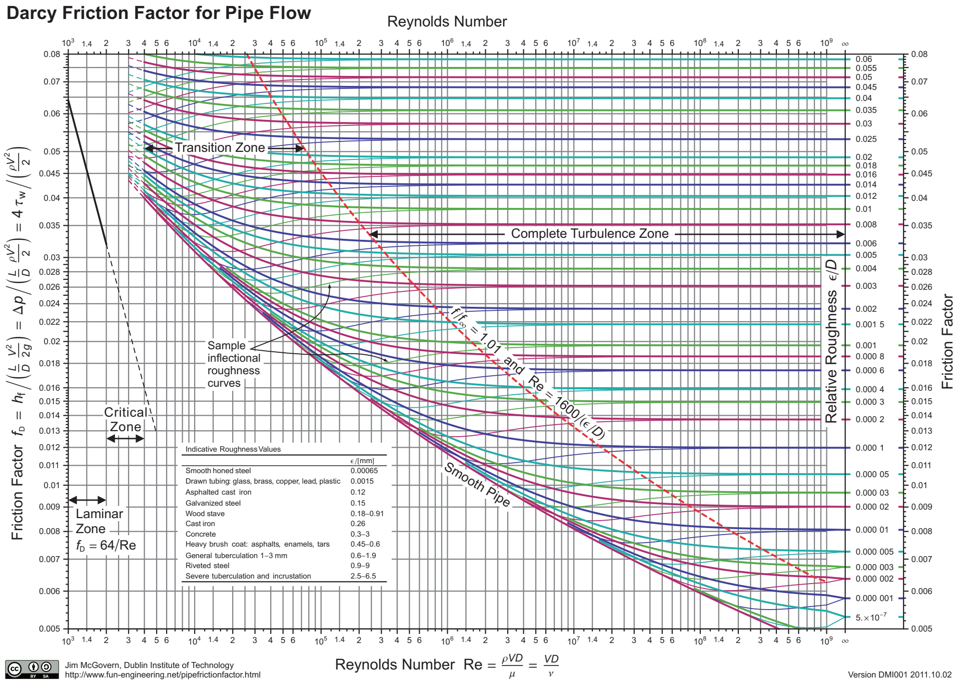

Figure 2: Diagram of friction factor for pipe flow, including sample inflectional roughness curves.

Absolute Roughness of Modern Pipe Surfaces

Precise information for friction factor versus Reynolds number is available for two specific pipe types from publications relating to the Princeton SuperPipe tests [15, 19, 11]. Of these, the 'rough' pipe displays an inflectional characteristic in the transition zone, which is similar to the inflectional characteristic of the sand-grain-roughened pipes that were tested by Nikuradse [8]. The data for the 'smooth' pipe all lie along the left-hand edge of the transitional zone and no inflectional friction factor characteristic is apparent up to the highest Reynolds number of 3.5×10^7.

Matching Equation 20 to the Princeton SuperPipe 'rough pipe' data suggests that the inner surface had an equivalent sand grain roughness of about 10 micron or 0.01 mm—the measured ϵ_{rms} was 2.5 micron or 0.0025 mm. On the same basis, the sand grain roughness of the Princeton SuperPipe 'smooth pipe' would have had to be less than about 0.65 micron, or 0.00065 mm—the measured root mean square roughness, ϵ_{rms}, was 0.15 micron.

Sletfjerding and Gudmundsson [17] provided test results (using dry natural gas) for pipes artificially roughened with glass beads and epoxy, which had inflectional friction factor curves, as well as for a honed steel pipe and an epoxy coated smooth pipe. An equivalent sand grain roughness of 21 micron was determined for the honed steel pipe, which had a measured ϵ_{rms}

of 3.66 micron. An equivalent sand grain roughness was not determined for the smooth coated pipe, which had a measured ϵ_{rms} of 1.41 micron.

There is still a scarcity of data on the absolute sand-grain-equivalent roughness of commercial pipes. The values provided by Farshad et al. [20, 21, 22] are illustrative. An examination of the data available in [21] indicates that flow measurements were made at a Reynolds number of about 6.5×10^5 and in all cases were within the transitional zone. Also, in [22] the authors seem to have misinterpreted Moody's chart (his Figure 2, p. 678) that related relative roughness to pipe diameter for various absolute roughness values—Moody's log-log chart of ϵ/D versus D was simply based on the inverse proportionality between relative roughness and pipe diameter.

Using the Diagram

When the diagram is printed-out, a straight edge can be used to assist in reading the value of the friction factor on the scales that are provided on the left and right edges. When using the diagram on a screen or tablet, users may find it helpful to use the rulers and movable guidelines that are usually available within a PDF reader program.

Conclusions

The diagrams have been prepared by the author for students of engineering and practicing engineers. He would welcome any suggestions for further adjustments.

Acknowledgement

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Nomenclature

Δp	pressure drop due to fluid friction
ϵ	absolute surface roughness (equivalent sand grain roughness)
ϵ_{rms}	root mean square surface roughness
η	correction factor
$\mathcal{V}$	volume

Re	Reynolds number
μ	absolute viscosity
ν	kinematic viscosity
ρ	density
τ_w	shear stress at the pipe wall
A	cross-sectional area of pipe
D	pipe diameter
f	friction factor
f_∞	friction factor at infinite Reynolds number
f_D	Darcy friction factor
f_F	Fanning friction factor
g	acceleration due to gravity
h_f	head loss due to fluid friction
j	dimensionless roughness parameter
L	pipe length
Q	volume flow rate
V	mean flow velocity $= Q/A$
$\rho V^2/2$	dynamic pressure corresponding to mean flow velocity
$V^2/2g$	velocity head corresponding to mean flow velocity
PDF	portable document format

Table 1: Indicative absolute roughness values

	Ref.	ϵ/[mm]
Finely honed smooth steel		
($\epsilon_{\mathrm{rms}} = 0.00015$ mm, inflectional)	11 (2007)	0.00065
Drawn tubing (e.g. glass, brass,		
copper, lead, plastic)	1 (1944)	0.0015
Butt-welded steel: new smooth pipe		
or with centrifugally applied enamels	3 (1977)	0.009 - 0.06
Honed steel		
($\epsilon_{\mathrm{rms}} = 0.0025$ mm, inflectional)	11 (2007)	0.01
($\epsilon_{\mathrm{rms}} = 0.0037$ mm, inflectional)	17 (2003)	0.021
Butt-welded steel: centrifugally		
applied concrete linings	3 (1977)	0.045 - 0.15
Wrought iron, steel	1 (1944)	0.046
Butt-welded steel:		
hot asphalt dipped	3 (1977)	0.06 - 0.15
Asphalted cast iron	1 (1944)	0.12
Galvanized iron	1 (1944)	0.15
Butt-welded steel: light rust	3 (1977)	0.15 - 0.35
Wood stave	1 (1944)	0.18 - 0.91
Cast iron	1 (1944)	0.26
Concrete	1 (1944)	0.3 - 3
Butt-welded steel: heavy brush coat,		
asphalts, enamels and tars	3 (1977)	0.45 - 0.6
Riveted steel	1 (1944)	0.9 - 9
Butt-welded steel:		
general tuberculation 1-3 mm	3 (1977)	1 - 1.85
severe tuberculation		
and incrustation	3 (1977)	2.5 - 6.5

Table 2: Summary table of indicative absolute roughness values

	ϵ/[mm]
Smooth honed steel	0.00065
Drawn tubing: glass, brass, copper, lead, plastic	0.0015
Asphalted cast iron	0.12
Galvanized steel	0.15
Wood stave	0.18 - 0.91
Cast iron	0.26
Concrete	0.3 - 3
Heavy brush coat: asphalts, enamels, tars	0.45 - 0.6
General tuberculation 1-3 mm	0.6 - 1.9
Riveted steel	0.9 - 9
Severe tuberculation and incrustation	2.5 - 6.5

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