

2011-09-30

## Rational Efficiency of a Heat Exchanger

Jim McGovern

*Technological University Dublin*, jim.mcgovern@tudublin.ie

Brian P. Smyth

*Institute of Technology Tallaght*, brian.smyth@it-tallaght.ie

Follow this and additional works at: <https://arrow.tudublin.ie/engschmecart>



Part of the [Heat Transfer, Combustion Commons](#)

---

### Recommended Citation

McGovern, Jim and Smyth, Brian P., "Rational Efficiency of a Heat Exchanger" (2011). *Articles*. 27.  
<https://arrow.tudublin.ie/engschmecart/27>

This Article is brought to you for free and open access by the School of Mechanical Engineering at ARROW@TU Dublin. It has been accepted for inclusion in Articles by an authorized administrator of ARROW@TU Dublin. For more information, please contact [arrow.admin@tudublin.ie](mailto:arrow.admin@tudublin.ie), [aisling.coyne@tudublin.ie](mailto:aisling.coyne@tudublin.ie), [vera.kilshaw@tudublin.ie](mailto:vera.kilshaw@tudublin.ie).

17/6/97

## RATIONAL EFFICIENCY OF A HEAT EXCHANGER

**James A. McGovern**

Department of Mechanical and Manufacturing Engineering, University of  
Dublin, Trinity College, Dublin, Ireland.

**Brian P. Smyth**

Department of Mechanical and Manufacturing Engineering, Regional  
Technical College Tallaght, Dublin, Ireland.

**Abstract** - The authors propose a *new* and *unique* definition for the rational efficiency of a heat exchanger. This new rational efficiency is defined in terms of its sub-rational efficiencies: a *heat transfer rational efficiency* and a *friction rational efficiency* for *each* of the fluid systems comprising the heat exchanger. The *heat transfer rational efficiency* is based on the definition of a *mean temperature for the heat source* and a *mean temperature for the heat sink* and reflects the exergy supplied and the exergy received, due to *heat transfer only*. The *friction rational efficiency* for *each* fluid system of the heat exchanger reflects the *internal friction irreversibility* of that fluid system. The authors also show how it is possible to *calculate* these rational and sub-rational efficiencies.

### **1 INTRODUCTION**

Upon careful examination of the published literature, the authors have noticed a marked absence of a clear, concise and unambiguous treatment of the *rational efficiency* of a heat exchanger. With heat exchanger design, there will always be a compromise between pressure drop and heat transfer performance. For example, improving the heat transfer coefficient of a heat exchanger by increasing the frictional pressure drop accompanying the movement of fluid through the device, will lessen the surface area requirements needed for heat transfer, thus resulting in the need for a smaller heat exchanger. Clearly there is a need to separate out the influence of the two competing irreversibility

mechanisms of heat transfer and fluid friction impacting on the thermodynamic performance of heat exchangers. A rational efficiency that manages to reflect the influence of these two competing mechanisms would be very useful. The definition of such a rational efficiency forms the subject of this paper. The work documented here develops and applies the concept of a *thermodynamic average temperature* reported by Bejan<sup>1</sup> et. al.

## 2 LITERATURE SURVEY

Many commercially published thermodynamic textbooks assess the performance of heat exchangers in terms of their *effectiveness*<sup>1-4</sup>. The *effectiveness*<sup>2</sup> (page 469) of a heat exchanger is defined as the ratio of the *actual* heat transfer rate to the *maximum* heat transfer rate. Whilst this definition is sufficiently general to be applied to any flow arrangement of a heat exchanger (e.g. counterflow, parallel flow, cross-flow etc.), it does not account for the fact that mechanical power is required to pump a fluid through its heat exchanger passage. In other words, it fails “to identify the exergy waste associated with the pressure drops of the heat exchanger working fluids”<sup>1</sup> (page 156).

The tendency for designers to maximise the ratio of heat transfer coefficient to pumping power is not necessarily sufficient in ensuring a better heat exchanger performance<sup>5</sup>. The use of a *number of entropy production units* ( $N_S$ ) as a general criterion for rating the performance of a heat exchanger has been proposed by Bejan<sup>5</sup>.  $N_S$  is defined as “the entropy production rate or irreversibility rate present in a heat exchanger passage divided by the stream-to-stream heat transfer rate to the passage”<sup>5</sup>. A large *number of entropy production units* would imply an excessive stream-to-stream temperature difference, or an excessive frictional pressure drop, or both. In other words, inherently reflected in the value of  $N_S$  is the *combined* contribution to entropy creation due to irreversible heat transfer *and* fluid friction.

Recent developments in second law analysis have allowed the formulation of new performance criteria for a variety of thermal plant to be defined. Such performance criteria are based on the concept of *exergy*. Kotas<sup>6</sup> for example, defines the *rational efficiency* of a heat exchanger to be equal to the *net* exergy supplied to the cold stream divided by the *net* exergy supplied by the hot stream. Heat transfer and fluid friction are primarily responsible for the creation of entropy in heat exchangers. The expression for the rational efficiency offered by Kotas<sup>6</sup> however, again reflects only the *combined* influence of these two competing irreversibility mechanisms.

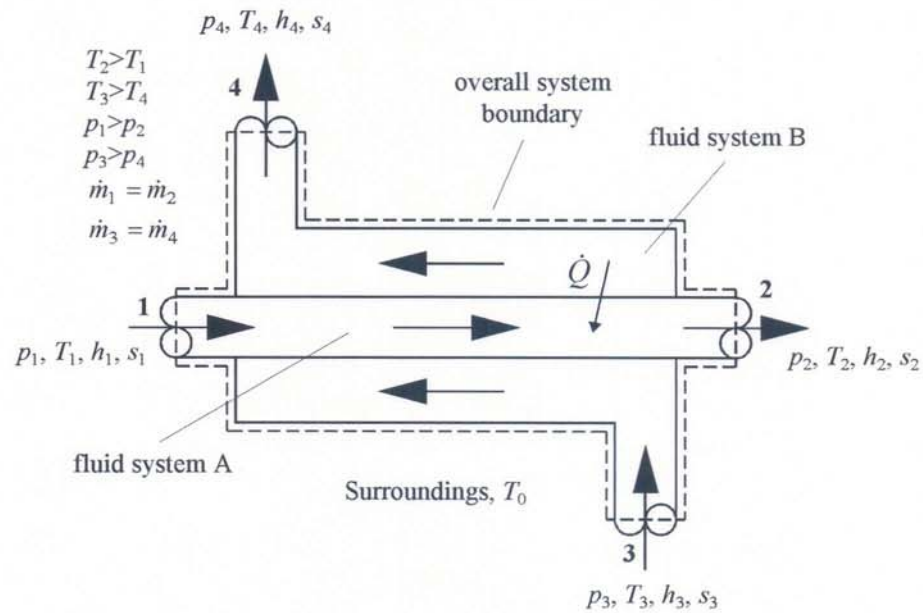
The various shortcomings associated with the expression of a *coefficient of exergy efficiency* for a heat exchanger are illustrated by Sorin<sup>7</sup> et. al. They attempt to improve this situation by introducing the concept of *transiting exergy*. Transiting exergy refers to the unaltered or conserved portion of the total exergy entering or leaving a system. Sorin<sup>7</sup> et. al. argue that focusing on the “transiting exergy to formulate the thermodynamic efficiency provides the grounds for the *non ambiguous* definition of useful exergy produced and exergy expended”. Whilst this may be so, their definition for the *coefficient of exergy efficiency* of a heat exchanger still reflects only the (non ambiguous) *combined* influence of heat transfer and fluid friction. Clearly, the definition of a much simpler parameter is required - one which explicitly *separates out* the contribution made towards exergy destruction by either mechanism.

### **3 RATIONAL EFFICIENCY OF A HEAT EXCHANGER**

Defining an overall rational efficiency for a heat exchanger might seem, on the face of it, to be relatively straightforward. For example, if a fluid enters and leaves a heat exchanger at known states (with negligible values of specific kinetic and potential energy), “and the temperature of the environment is also known, the net exergy interaction due to transport is fully defined - it equals the difference between the flow exergy function ( $h-T_0s$ ) at inlet and that at the outlet”<sup>8</sup>. This net exergy interaction is dependant only on the temperature of the reference environment,  $T_0$ . A heat exchanger therefore, that transfers heat

from one flowing fluid to another, can be viewed as a device with one net exergy input and one net exergy output interaction<sup>8</sup>. The rational efficiency for such a device could be simply defined as the ratio of the net exergy output to the net exergy input.

Consider, for example, a device where the temperature of fluid system A is increased from  $T_1$  to  $T_2$ , by passing it through one side of a counterflow heat exchanger (Fig. 1). The pressure of fluid A decreases from  $p_1$  (at inlet) to  $p_2$  (at exit), due to fluid friction. On the other side of the heat exchanger, fluid B gradually decreases in temperature from  $T_3$  (at inlet) to  $T_4$  (at exit), as a result of supplying heat transfer to fluid A. The pressure of fluid B decreases from  $p_3$  (at inlet) to  $p_4$  (at exit), due to fluid friction. Let us assume that heat transfer takes place at steady state and that there is no heat transfer across the overall system boundary. In other words, the surroundings remain completely unaffected by any changes undergone by either fluid system.



**Fig. 1** Counterflow Heat Exchanger

The rational efficiency for the heat exchanger defined by the analysis boundary in Fig. 1, could be stated as follows:

$$\begin{aligned}\psi &= \frac{\text{net rate of exergy output}}{\text{net rate of exergy input}} \\ &= \frac{\dot{m}_1[(h_2 - h_1) - T_0(s_2 - s_1)]}{\dot{m}_3[(h_3 - h_4) - T_0(s_3 - s_4)]}\end{aligned}\quad (1)$$

The *overall rate of entropy creation*,  $\dot{S}_{cr}$ , for the system defined in Fig. 1, is given by:

$$\begin{aligned}\dot{S}_{cr} &= \frac{\text{total exergy destruction rate}}{T_0} \\ &= \frac{(\text{net rate of exergy input} - \text{net rate of exergy output})}{T_0} \\ &= \frac{\dot{m}_3[(h_3 - h_4) - T_0(s_3 - s_4)] - \dot{m}_1[(h_2 - h_1) - T_0(s_2 - s_1)]}{T_0} \\ &= \dot{m}_1(s_2 - s_1) + \dot{m}_3(s_4 - s_3)\end{aligned}\quad (2)$$

since conservation of energy requires that  $\dot{m}_1(h_2 - h_1) = \dot{m}_3(h_3 - h_4)$

The *total exergy destruction rate* is given by:

$$\begin{aligned}\dot{I} &= T_0 \dot{S}_{cr} \\ &= T_0[\dot{m}_1(s_2 - s_1) + \dot{m}_3(s_4 - s_3)]\end{aligned}\quad (3)$$

However, upon closer inspection, there are some difficulties associated with the definition of the rational efficiency of a heat exchanger as defined in Eq. 1. Firstly, this definition *only* reflects the *combined* effect of the two irreversibility mechanisms of heat transfer and fluid friction (which act simultaneously). It offers no indication of the contribution made towards exergy destruction by any *one* of these two competing mechanisms. Secondly, if the processes and states comprising the heat exchanger were fixed and remained unchanged, the rational efficiency of the device *would change*, if the temperature of the environment,  $T_0$ , changed. Does this mean that every heat exchanger will require a whole series of rational efficiencies to be defined for

it, to reflect all of the conceivable environmental temperatures in which it might operate?

### 3.1 Heat Transfer Irreversibility

The authors endeavour to tackle the first difficulty here, by proposing a new and unique method for defining the rational efficiency of a heat exchanger that will reflect system irreversibilities associated with *heat transfer only*. If the rate of heat transfer per unit area (or heat flux rate) and the temperature both vary over the boundary surface of the heat exchanger, but both remain constant with time (steady state), then the *total rate of entropy transfer* into fluid system A,  $\dot{S}_{Q, IN, A}$ , due to *heat transfer only*, can be obtained by summing up the different heat flux rates entering the system,  $\delta\dot{Q}$ , divided by the boundary temperature,  $T_{bdry, A}$ , associated with each separate heat flux rate, over the entire boundary surface. Likewise, the *total rate of entropy transfer* out of fluid system B,  $\dot{S}_{Q, OUT, B}$ , due to *heat transfer only*, can be obtained by summing up the different rates of heat flux leaving the system,  $\delta\dot{Q}$ , divided by the boundary temperature,  $T_{bdry, B}$ , associated with each different heat flux rate, over the entire boundary surface. This can be stated mathematically, as follows:

$$\dot{S}_{Q, IN, A} = \sum_{\substack{\text{bdry} \\ \text{surface, A}}} \delta\dot{S}_{Q, IN, A} = \sum_{\substack{\text{bdry} \\ \text{surface, A}}} \frac{\delta\dot{Q}}{T_{bdry, A}}; \text{ and} \quad (4)$$

$$\dot{S}_{Q, OUT, B} = \sum_{\substack{\text{bdry} \\ \text{surface, B}}} \delta\dot{S}_{Q, OUT, B} = \sum_{\substack{\text{bdry} \\ \text{surface, B}}} \frac{\delta\dot{Q}}{T_{bdry, B}} \quad (5)$$

It is important to bear in mind that the boundary temperatures (i.e.  $T_{bdry, A}$  and  $T_{bdry, B}$ ) defined in Eqs. 4 and 5 are modelled as *bulk fluid temperatures* at each point on the fluid system boundary. In other words, the boundary for each fluid system (i.e. A and B) is drawn so as to *include* the *bulk fluid* of the system *only*. The *boundary layer* of each system at the heat transfer interface is *excluded*. If the flow of fluid through the heat exchanger is turbulent, then there will be a significant temperature variation within the boundary layer.

However, the bulk fluid temperature will be relatively uniform and hence, easier to determine.

We can now define a *mean temperature for the heat source* (i.e. for system B),  $T_{M, source}$ , and a *mean temperature for the heat sink* (i.e. for system A),  $T_{M, sink}$ , as follows:

$$T_{M, source} = \frac{\dot{Q}}{\dot{S}_{Q, OUT, B}}; \text{ and} \quad (6)$$

$$T_{M, sink} = \frac{\dot{Q}}{\dot{S}_{Q, IN, A}} \quad (7)$$

For analysis purposes therefore, fluid system B can now be viewed as a *thermal reservoir* of fixed temperature (i.e.  $T_{M, source}$ ) that provides heat transfer to fluid system A, without undergoing any change in its own temperature. Likewise, fluid system A can be viewed as a *thermal reservoir* of fixed temperature (i.e.  $T_{M, sink}$ ) that accepts heat transfer from fluid system B, without undergoing any change in its own temperature.

Based on these ‘fixed thermal reservoir temperatures’, the authors propose a new and unique definition for the *heat transfer rational efficiency* of a heat exchanger,  $\psi_Q$ , which will reflect the exergy supplied and the exergy received, due to *heat transfer only*, as follows:

$$\psi_Q = \frac{\text{exergy received as } \textit{heat transfer only}}{\text{exergy supplied as } \textit{heat transfer only}}$$

$$= \frac{\left(\frac{T_{M, sink} - T_0}{T_{M, sink}}\right)\dot{Q}}{\left(\frac{T_{M, source} - T_0}{T_{M, source}}\right)\dot{Q}}$$

$$\text{from which, } \psi_Q = \left(\frac{T_{M, sink} - T_0}{T_{M, source} - T_0}\right) \cdot \left(\frac{T_{M, source}}{T_{M, sink}}\right) \quad (8)$$



The *actual* rate of heat transferred from fluid system B to fluid system A,  $\dot{Q}$ , is given by Eq. 9 and thus can easily be evaluated from a knowledge of the state and mass flow rate of either fluid system at each point where it crosses the system boundary.

$$\dot{Q} = \dot{m}_1(h_2 - h_1) = \dot{m}_3(h_3 - h_4) \quad (9)$$

Note that all of the variables listed in Eqs. 1 to 9 inclusive, represent positive quantities.

The calculation of the total entropy transfer rates, as defined by Eqs. 4 and 5, requires a knowledge of the various heat flux rates and corresponding boundary temperatures across the entire boundary surface. Such information can be readily evaluated or approximated with good heat exchanger modelling software. Also, experimental measurement techniques can be employed to assist in the determination of such values.

For a *real* heat exchanger (i.e. one which facilitates *irreversible* heat transfer), the rate of entropy creation,  $\dot{S}_{cr, Q}$ , due to *heat transfer only*, can be evaluated as follows:

$$\dot{S}_{cr, Q} = (\dot{S}_{Q, IN, A} - \dot{S}_{Q, OUT, B}) > 0 \quad (10)$$

Eq. 10 accounts for the fact that the *rate of change of entropy* of the entire system is zero because the system is operating at steady state.  $\dot{S}_{cr, Q}$  approaches zero in the limit, as the difference between the boundary temperatures of fluid system A ( $T_{bdry, A}$ ) and fluid system B ( $T_{bdry, B}$ ), at each boundary patch of infinitesimal area, becomes infinitesimally small. In real heat exchangers,  $\dot{S}_{cr, Q}$  is sustained by the *thermal resistance* to heat transfer. The solid metal pipe separating both fluid systems and the fluid *boundary layers* existing on both sides of the pipe, are responsible for this thermal resistance to heat transfer<sup>3</sup>.

The exergy destruction rate due to *heat transfer only*, is given by:

$$\dot{I}_Q = T_0 \dot{S}_{cr, Q} \quad (11)$$

### 3.2 Fluid Friction Irreversibility

There is also exergy destruction associated with *frictional pressure drop* as each fluid system circulates through the heat exchanger. It is important that the design engineer is able to separate out the influence of this irreversibility mechanism in a typical analysis. For the case of turbulent flow through the heat exchanger, Bejan<sup>3</sup> (page 56), has indicated that frictional irreversibility is essentially a “*wall phenomenon*”, in that the wall region plays the dominant role in the creation of entropy.

The rate of entropy creation due to *frictional pressure drop only*, for fluid system A,  $\dot{S}_{cr, F, A}$ , is given by:

$$\dot{S}_{cr, F, A} = \dot{m}_1 (s_2 - s_1) - \dot{S}_{Q, IN, A} > 0 \quad (12)$$

The exergy destruction rate therefore, due to *frictional pressure drop only*, for fluid system A, is given by:

$$\dot{I}_{F, A} = T_0 \dot{S}_{cr, F, A} \quad (13)$$

Likewise, the rate of entropy creation due to *frictional pressure drop only*, for fluid system B,  $\dot{S}_{cr, F, B}$ , is given by:

$$\dot{S}_{cr, F, B} = \dot{m}_3 (s_4 - s_3) + \dot{S}_{Q, OUT, B} > 0 \quad (14)$$

The exergy destruction rate therefore, due to *frictional pressure drop only*, for fluid system B, is given by:

$$\dot{I}_{F, B} = T_0 \dot{S}_{cr, F, B} \quad (15)$$

The *total* rate of entropy creation due to *frictional pressure drop only*, for the overall system defined by Fig. 1,  $\dot{S}_{cr, F}$ , is given by:

$$\begin{aligned} \dot{S}_{cr, F} &= (\dot{S}_{cr, F, A} + \dot{S}_{cr, F, B}) > 0 \\ &= \dot{m}_1(s_2 - s_1) + \dot{m}_3(s_4 - s_3) - (\dot{S}_{Q, IN, A} - \dot{S}_{Q, OUT, B}) > 0 \\ &= (\dot{S}_{cr} - \dot{S}_{cr, Q}) > 0 \end{aligned} \quad (16)$$

$\dot{S}_{cr, F}$  approaches zero in the limit, as the frictional pressure drop for each fluid system (A and B), becomes infinitesimally small as it circulates through the heat exchanger. In real heat exchangers, friction will always contribute to the entropy creation rate.

The *total* exergy destruction rate due to *frictional pressure drop only*, for the system defined in Fig. 1, is given by:

$$\dot{I}_F = T_0 \dot{S}_{cr, F} \quad (17)$$

We can define a *friction rational efficiency* for each fluid system of the heat exchanger (i.e. fluid system A and fluid system B) which will reflect the *internal friction irreversibility* of the system, as follows:

$$\begin{aligned} \Psi_{F, A} &= \frac{\text{net rate of exergy output}}{\text{exergy received as heat transfer only}} \\ &= \frac{\dot{m}_1[(h_2 - h_1) - T_0(s_2 - s_1)]}{\left(\frac{T_{M, \text{sink}} - T_0}{T_{M, \text{sink}}}\right)\dot{Q}} \end{aligned} \quad (18)$$

$$\Psi_{F, B} = \frac{\text{exergy supplied as heat transfer only}}{\text{net rate of exergy input}}$$

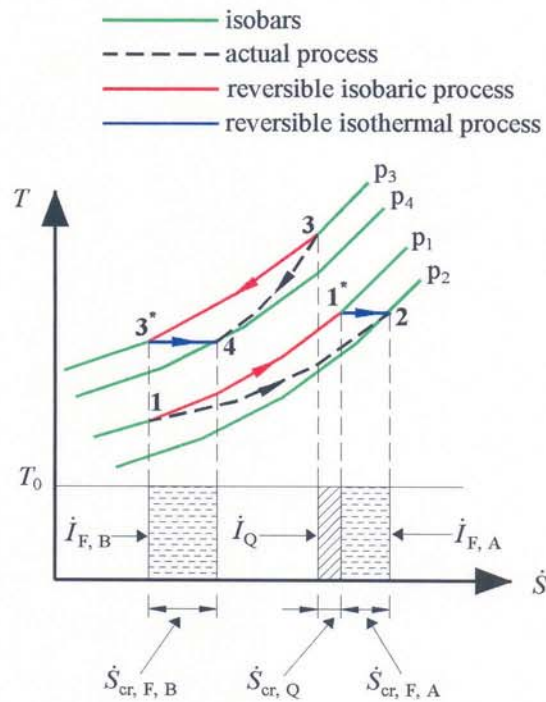
$$\begin{aligned} & \left( \frac{T_{M, \text{source}} - T_0}{T_{M, \text{source}}} \right) \dot{Q} \\ &= \frac{\dot{Q}}{\dot{m}_3 [(h_3 - h_4) - T_0 (s_3 - s_4)]} \end{aligned} \quad (19)$$

$\psi_{F, A}$  reflects the *internal friction irreversibility* associated with fluid system A and  $\psi_{F, B}$  reflects the *internal friction irreversibility* associated with fluid system B. By inspection of Eqs. 1, 8, 18 and 19, the overall rational efficiency of the heat exchanger,  $\psi$ , can be defined in terms of its three sub-rational efficiencies,  $\psi_Q$ ,  $\psi_{F, A}$  and  $\psi_{F, B}$ , as follows:

$$\psi = \psi_Q \cdot \psi_{F, A} \cdot \psi_{F, B} \quad (20)$$

#### 4 EVALUATION OF THE HEAT TRANSFER AND FLUID FRICTION IRREVERSIBILITIES FROM THERMODYNAMIC PROPERTY DIAGRAMS

Kotas<sup>6</sup> evaluates the heat transfer and friction irreversibilities (as defined in Eq.'s 11, 13 and 15) by plotting the heat transfer process on a  $T-\dot{S}$  diagram as shown in Fig. 2.



**Fig. 2** Heat Transfer Process in  $T-\dot{S}$  Co-ordinates

The actual processes in Fig. 2 are *non-equilibrium* processes. In other words, neither fluid system remains in equilibrium as its state changes. These processes, therefore, cannot be represented by process paths (indicated by solid lines) on state diagrams. Instead, they are represented by dashed lines.

The following analysis reproduces Kotas's<sup>6</sup> approach to the problem of separating out the irreversibility due to heat transfer and fluid friction. The initial and final states of the actual processes in Fig. 2 are assumed to be equilibrium states.

Eq. 3 can be written as:

$$\begin{aligned}\dot{I} &= T_0[\dot{m}_1(s_2 - s_1) + \dot{m}_3(s_4 - s_3)] \\ &= T_0[(\dot{S}_2 - \dot{S}_1) - (\dot{S}_3 - \dot{S}_4)]\end{aligned}\quad (21)$$

Kotas<sup>6</sup> replaces the actual processes with reversible isothermal and reversible isobaric processes as shown in Fig. 2. The entropy changes in Eq. 21, therefore, can be rewritten as:

$$\dot{S}_2 - \dot{S}_1 = (\dot{S}_{1*} - \dot{S}_1) + (\dot{S}_2 - \dot{S}_{1*}) \quad (22)$$

$$\dot{S}_3 - \dot{S}_4 = (\dot{S}_3 - \dot{S}_{3*}) - (\dot{S}_4 - \dot{S}_{3*}) \quad (23)$$

Substituting Eqs. 22 and 23 into Eq. 21, he defines the total exergy destruction rate as follows:

$$\dot{I} = T_0[\dot{S}_{cr, Q} + \dot{S}_{cr, F, A} + \dot{S}_{cr, F, B}] \quad (24)$$

where:

$$\dot{S}_{cr, Q} = (\dot{S}_{1*} - \dot{S}_1) - (\dot{S}_3 - \dot{S}_{3*}) \quad (25)$$

$$\dot{S}_{cr, F, A} = (\dot{S}_2 - \dot{S}_{1*}) \quad (26)$$

$$\dot{S}_{cr, F, B} = (\dot{S}_4 - \dot{S}_{3*}) \quad (27)$$

Both the entropy creation rates and the associated exergy destruction rates are represented graphically in Fig. 2.

This approach, whilst reflecting the state of the art, is not satisfactory in the author's opinion. The main problem with Kotas's<sup>6</sup> approach is that the *actual* processes can easily be replaced by *any number of different combinations* of internally reversible processes between the specified end states. Associated with each of these different combinations of reversible processes will be quantities of heat transfer that will, in general, bear no relation whatsoever to the *actual* heat transfer taking place. In other words, Eqs. 10, 12 and 14 define the *true* rate of entropy creation for the *actual* heat transfer process taking place. Eqs. 25, 26 and 27 define *fictional* rates of entropy creation that depend on the nature of the internally reversible path chosen between the specified end states and bear no relation to the actual process.

The authors believe that a more realistic and practical approach is needed to try to separate out the *real* entropy creation rates for the *real* heat transfer process taking place. Such an approach will now be considered which will allow the rational and sub-rational efficiencies, defined in Eqs. 1, 8, 18 and 19, to be evaluated.

For one dimensional flow, the properties of each fluid system over a given cross-section, can be assumed to be constant. Only two independent intensive thermodynamic properties, therefore, are needed to completely specify the thermodynamic state of each fluid system as it circulates through the heat exchanger. Attention therefore, could be focused on the locus of the *specific volumes* and *pressures* for each system.

Empirical relations exist for calculating the *pressure* (or pressure drop) at each point in a fluid as it flows through a section of pipe (e.g. see Ref. 2, pp. 97-105). For incompressible fluid flow, the *specific volume* will remain constant as the fluid circulates through the heat exchanger. For compressible flow however, another property such as the *bulk fluid temperature*, will be needed in

order to determine the *specific volume* at a particular cross section of the pipe. But, as stated earlier, if the flow of fluid through the heat exchanger is turbulent, then there will be a significant temperature variation within the boundary layer. However, the *bulk fluid temperature* will be relatively uniform and hence, easier to measure. In principle, therefore, it is possible to model the one dimensional flow of fluid through the heat exchanger as a *quasi-equilibrium process* and plot such a process on a *pressure versus volumetric flow rate*, or  $p - \dot{V}$  diagram. Each *quasi-equilibrium* state on the locus will be determined by the volumetric flow rate and pressure at that cross-section. The locus therefore, will also fix all other thermodynamic properties at each cross-section of the fluid flow.

Eq. 28 defines a useful property relation that is valid for both reversible and irreversible processes and for both closed and open systems.

$$dH = TdS + Vdp \quad (28)$$

With reference to Fig. 1, for a *finite reversible* process for fluid system A, Eq. 28 states that the *increase* in enthalpy of the system in going from state 1 to state 2 (i.e.  $H_2 - H_1$ ), is equal to the *reversible* heat transferred *to* the system

(i.e. the absolute value of  $\int_1^2 TdS$ ) *minus* the *reversible* work produced *by* the

system (i.e. the absolute value of  $\int_1^2 Vdp$ ). Likewise, for a *finite reversible*

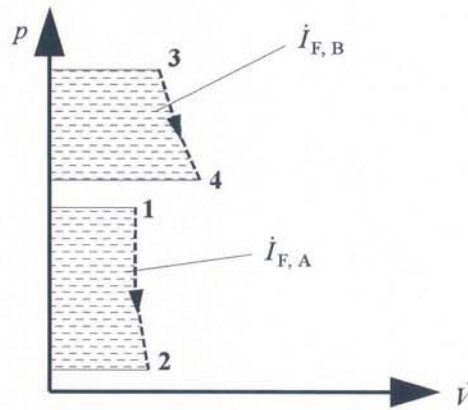
process for fluid system B, Eq. 28 states that the *decrease* in enthalpy of the system in going from state 3 to state 4 (i.e.  $H_3 - H_4$ ), is equal to the *reversible*

heat transferred *from* the system (i.e. the absolute value of  $\int_3^4 TdS$ ) *plus* the

*reversible* work produced *by* the system (i.e. the absolute value of  $\int_3^4 Vdp$ ).

Note that the *actual* rate of heat transferred from fluid system B to fluid system A is given by Eq. 9.

The *actual* useful work produced by either fluid system as it circulates through the heat exchanger, is zero. Any difference between the *reversible* work produced and the *actual* useful work produced, must be due to frictional irreversibilities present during the process. Therefore, the *reversible* work and the exergy destruction (or irreversibility) due to *frictional pressure drop only*, must be equal, since the *entire* work potential is destroyed during the process. Hence, for both fluid systems, the exergy destruction rate due to *frictional pressure drop only*, will be represented by the areas shown in Fig. 3.



**Fig. 3**  $p - \dot{V}$  Diagram for Fluid Systems A and B

Once the values of  $\dot{I}_{F,A}$  and  $\dot{I}_{F,B}$  are determined from the areas shown in Fig. 3, Eqs. 12, 13, 14 and 15 will allow the values of  $\dot{S}_{Q,IN,A}$  and  $\dot{S}_{Q,IN,B}$  to be derived. Eqs. 6 and 7 can then be used to determine the *mean temperature for the heat source*,  $T_{M,source}$ , and the *mean temperature for the heat sink*,  $T_{M,sink}$ , respectively. Finally, Eqs. 1, 8, 18 and 19 will allow the overall rational efficiency of the heat exchanger,  $\psi$ , to be *calculated* in terms of its three sub-rational efficiencies,  $\psi_Q$ ,  $\psi_{F,A}$  and  $\psi_{F,B}$ .

## **5 CONCLUSIONS**

The authors have proposed a *new* and *unique* definition for the rational efficiency of a heat exchanger. This new rational efficiency is defined in terms of its sub-rational efficiencies: a *heat transfer rational efficiency* and a *friction*



*rational efficiency* for *each* of the fluid systems comprising the heat exchanger. The definition hinges on the concept of a *mean temperature* for heat transfer, similar to the idea of a *thermodynamic average temperature* reported by Bejan<sup>1</sup> et. al. The analysis is developed for the case of a simple counterflow heat exchanger, but is sufficiently general to be applied to any other flow arrangement (e.g. counterflow, parallel flow, cross-flow etc.). The definition is simple and effective, and inherently allows the principal contributors to the thermodynamic inefficiency of a heat exchanger to be qualitatively understood. The authors have also indicated how it is possible to *calculate* the rational and sub-rational efficiencies defined in this paper.

### REFERENCES

- 1 **Bejan A., Tsatsaronis G., Moran M.** Thermal Design & Optimization, Wiley Publications, (1996).
- 2 **Bejan A.** Heat Transfer, John Wiley & Sons Publishers, (1993).
- 3 **Bejan A.** Entropy Generation Minimization - The Method of Thermodynamic Optimization of Finite-Size Systems and Finite - Time Processes, CRC Press, Boca Raton, FL, (1996).
- 4 **Burghardt M.D.** Engineering Thermodynamics with Applications, 3<sup>rd</sup> Edition, Harper & Row Publications, (1986).
- 5 **Bejan A.** General Criterion for Rating Heat-Exchanger Performance, Int. J. Heat Mass Transfer, Vol. 21, pp. 655-658 (1978).
- 6 **Kotas T.J.** The Exergy Method of Thermal Plant Analysis, Krieger Publishing Company, (1995).

- 7 **Sorin M.V., Brodyansky V.M., Paris J.** Observations on Exergy Efficiency Coefficients, Proceedings of the Florence World Energy Research Symposium, Florence, Italy, pp. 941-949, (1994).
- 8 **McGovern J.A., O' Toole F.** A Virtual System Concept for Exergy Analysis of Flow Network Plant; Part I: Principles; ECOS'92, Proceedings of the International on Symposium Efficiency, Costs, Optimization and Simulation of Energy Systems; American Society of Mechanical Engineers, pp. 155-160, (1992).

### NOMENCLATURE

$h$	specific enthalpy	$bdry$	boundary
$H$	enthalpy	$cr$	creation
$I$	irreversibility	$F$	friction
$m$	mass	$IN$	inwards direction with respect to system
$N_S$	number of entropy production units	$M$	mean
$p$	pressure	$OUT$	outwards direction with respect to system
$Q$	heat transfer	$0$	refers to the thermomechanical dead state of a system
$s$	specific entropy	$Q$	refers to heat transfer
$S$	entropy	$sink$	heat sink
$T$	absolute temperature	$source$	heat source
$V$	volume	$1, 2, \dots$	refers to a particular state of a fluid at a system boundary
<b><u>Greek</u></b>			
$\psi$	rational efficiency		

### Subscripts

A, B... refer to identified systems

### Conventions

$\dot{\phantom{x}}$  (dot above symbol) time rate of change of quantity