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Performance of Utility Based Hedges

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John Cotter and Jim Hanly

Abstract

Hedgers as investors are concerned with both risk and return. However, when measuring hedging performance, the role of returns and investor risk aversion has generally been neglected in the literature by its focus on minimum variance hedging. In this paper we address this by using utility based performance metrics to evaluate the hedging effectiveness of utility based hedges for hedgers with both moderate and high risk aversion together with the more traditional minimum variance approach. To examine this for an energy hedger, we apply our approach to WTI Crude Oil, for three different hedging horizons, daily, weekly and monthly. We find significant differences between the minimum variance and utility based hedging strategies in-sample for all frequencies. However performance differentials between the different strategies are small and not economically significant.

Keywords: Hedging Performance; Utility, Energy, Risk Aversion.

JEL classification: G10, G12, G15.

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1.   INTRODUCTION

Recent uncertainty and the accompanying volatility in financial markets have highlighted the need for strategies that can address the risks associated with large and unexpected market movements. This is particularly important for energy consumers given that supply side shocks can arise from any number of sources, manmade or natural, and because of recent increases in the volatility of many energy commodities (Efimova and Serletis, 2014).

Hedging using futures has become an important way for investors to manage the risk of their exposures and a large literature has developed alongside the use of futures, detailing how to estimate an Optimal Hedge Ratio (OHR). This literature has in the main implicitly assumed that investors are infinitely risk averse and that therefore the optimal hedge is one that minimizes risk as measured by the variance[1]. A large number of papers (see for example, Lien and Tse, 2002, for an early survey) have used a utility maximisation approach but have assumed that futures prices follow a martingale in which case the utility maximising and variance minimising approaches are the same. A more limited number of papers have looked at hedging for other risk measures such as Value at Risk (Harris and Shen, 2006), however, only a relatively small number of papers (Cotter and Hanly, 2012) have followed a non-martingale utility based approach which implies that investors are concerned not just with risk, but also with the expected return. Also, where hedging performance has been examined, a variety of performance measures have been applied but in many cases, this has been done in a manner which fails to match the performance metric with the underlying optimization method for generating the hedge ratio[2].

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[1] One of the first papers to detail this approach is Ederington (1979). Papers have also used a conditional framework using GARCH models. See Brooks Henry and Persand (2002) for example.

[2] For example, Kroner and Sultan, 1993 assume that futures prices are a martingale and therefore effectively estimate a minimum variance hedge which they evaluate using average quadratic utility to measure hedging performance

Furthermore, papers that have looked at utility based hedges tend to do so only for a single time horizon.

A significant body of work has also examined hedging within energy markets. In terms of oil price hedging, Ripple and Moosa (2007) examined the impact of contract maturity on the effectiveness of crude oil hedges using a minimum variance approach. Their findings indicated that near month contracts yielded higher effectiveness due to the higher correlation between spot and futures. Switzer and El Khoury (2007) also use the minimum variance paradigm but employ an asymmetric GJR GARCH model and find that hedging efficiency can be improved by accounting for asymmetries within an oil hedging context. More recently, Chang, McAleer and Tansuchat, (2011) examine both West Texas Intermediate (WTI) and Brent Oil contracts. Applying a variety of different GARCH models, they find that OHR's are model dependent. They also find large differences in the hedging effectiveness of the different contracts, with Brent Crude Oil hedges showing variance reductions of the order of 57% as compared with around 80% for WTI hedges. We note however, that within the Oil hedging literature as with the more general hedging literature, utility based hedging incorporating utility based performance metrics has not been applied.

In this paper we contribute by addressing some of these issues. We estimate and compare optimal utility based hedges using performance metrics based on the underlying optimization criteria as well as an economic comparison between the different hedging models using Value at Risk (VaR). Given the potential impact of hedge horizon on optimal hedges (see Juhl, Kawaller and Koch, 2012) we estimate hedge strategies across three time horizons, daily, weekly and monthly. For our analysis we use two of the most broadly used characterisations of investor

utility; quadratic and exponential. We also recognise that hedgers are not homogenous in terms of their risk aversion. We therefore estimate utility based hedges allowing for investors who are infinitely risk averse to more realistic approximations including moderate and low risk aversion. This allows us to analyse the impact that different attitudes towards risk have on the OHR. Furthermore, while the literature has generally used a framework based on short hedgers, we apply our analysis to long hedgers who are short the underlying asset, since such hedgers are in many respects more susceptible to unexpected market movements[3]. Finally, we examine how utility based hedges perform in terms of their optimisation criteria in an out-of-sample setting. Our approach allows us to make a wide ranging comparison that encompasses different estimation methods and performance criteria across different time horizons, to see how well utility based hedges perform in a variety of settings as compared with more traditional approaches such as variance minimisation.

We examine these issues with a commonly used energy commodity, oil, as proxied by West Texas Intermediate Light Sweet Crude. Our empirical results show that there are significant differences between the quadratic and exponential hedge ratios and also between each of the utility based hedges and the variance minimising hedge. This relates to the non-normal characteristics of energy returns such as the presence of skewness and excess kurtosis. Since these characteristics are generally present for such data, it shows the importance of matching the utility function used to generate the hedge to the risk investor's preferences. There are also significant differences between the OHR's across time horizons with a clear implication that optimal hedges are frequency specific. We also find that using performance methods based on the optimization criteria will not yield significant performance differentials between the different

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[3] Demirer and Lien (2003) found that long hedgers tend to be more active in the futures market than short hedgers which may indicate that they felt more exposed to adverse price movements.

hedging models. The implication for oil market participants is that hedging performance is not significantly different enough to warrant using a more complicated utility based approach as compared with the simpler minimum variance based hedge.

The remainder of this paper proceeds as follows. In Section 2 we describe the utility based performance measures together with the associated models used for estimating optimal hedge ratios. Section 3 outlines our data set and estimation procedures. Section 4 presents our empirical results. Section 5 concludes.

2.   HEDGING MODELS AND PERFORMANCE

Optimal hedge strategies using futures have generally followed two distinct approaches. The first approach implicitly assumes that investors are essentially infinitely risk averse. This means that hedgers will ignore the return component of their investments and focus on minimising the risk via a minimum variance hedge ratio (MVHR). The problem with this approach is that it doesn't explicitly incorporate an investor's risk aversion, or the expected return in the estimation of the optimal hedge. The second approach focuses on maximizing expected utility. This framework incorporates risk, expected return and risk aversion as key elements in hedge ratio estimation. Many papers have avoided making a distinction between the utility maximizing and the variance minimising approaches by assuming that futures prices follow a martingale[4]. Under this assumption, the MVHR and the utility maximizing hedge ratios are equivalent, however, there is evidence that oil futures markets (Sadorsky, 2002) do not follow a martingale.

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[4] A majority of the literature on optimal hedging takes this approach despite some evidence to the contrary. See deVille deGoyet, Dhaene and Sercu (2008) for evidence and a discussion.

In this paper, we examine the hedging performance of four different hedging strategies with a focus on hedge strategies that are optimised based on attitudes towards risk and return via the investors utility function. This enables us to incorporate different attitudes towards risk within the hedging context and to allow the expected return to play a part. We use a rolling window model based hedge, incorporating three different approaches that reflect an infinitely risk averse investor, an investor that has a quadratic utility function[5] and an investor that has an exponential utility function. We will now introduce our hedging models and then detail the performance methods used.

2.1   Optimal Hedges

Assuming a fixed spot position let r_{st} and r_{ft} be logarithmic returns on the spot and futures series respectively, and β be the OHR. The return to the hedged portfolio is constructed as follows:

$$R_p = -r_s + \beta r_f \tag{1}$$

We define the OHR as the weight of the futures asset in the hedged portfolio that is chosen either to minimise risk or to maximize expected utility, depending on the underlying framework that is being applied. Assuming that the agent has a quadratic utility function, then the OHR that maximises quadratic utility can be calculated as[6]:

$$\beta = \frac{-E(r_{ft})}{2\lambda\sigma_{ft}^2} + \frac{\sigma_{sft}}{\sigma_{ft}^2} \tag{2}$$

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[5] We estimate both quadratic and exponential hedges using moderate and high values for risk aversion.
[6] See Duffie (1989) for more detail.

where $E\left(r_{ft}\right)$ is the expected return on futures, λ is the risk aversion parameter, σ_{ft}^{2} is the futures variance and σ_{sft} is the covariance between spot and futures. Equation (2) establishes the relationship between the risk aversion parameter λ and the OHR. As risk aversion increases, the individual hedges more and speculates less relative to the spot position, such that for extremely large levels of risk aversion, the first term in equation (2) will have a negligible influence on the hedge strategy. Therefore, the OHR for an investor with infinite risk aversion will be the minimum variance hedge which is given by:

$$\beta = \frac{\sigma_{sft}}{\sigma_{ft}^{2}} \qquad (3)$$

For an investor with exponential utility, their OHR will approximate the quadratic hedge where returns are normally distributed, however for non-normally distributed returns they will differ. The optimal exponential hedge can be estimated by choosing β to maximize the following expression:

$$\beta = \left\{\mu_{pt} - \frac{1}{2}\lambda\sigma_{pt}^{2} + \frac{\tau}{6}\lambda^{2}\sigma_{pt}^{3} - \frac{\kappa-3}{24}\lambda^{3}\sigma_{pt}^{4}\right\} \qquad (4)$$

where μ_{pt} is the expected return on the hedged portfolio, λ is the risk aversion parameter, σ_{pt}^{2} is the variance of the portfolio, τ is the skewness of the portfolio and κ is the kurtosis[7]. We use the following hedges.

2.2   No Hedge

This is a hedge ratio (HR) of zero, the exposure is left unhedged. This is included for use as a performance benchmark as energy hedgers and financial market participants in general choose

not to hedge their exposures since hedging reduces not only risk, but also the expected return to bearing that risk.

2.3   Model Based Hedge

GARCH models have been broadly applied in the hedging literature to estimate time varying OHR's, however, the performance of these models has been mixed and in particular their out-of-sample performance has been average at best (Brooks Cerny and Miffre, 2012). This suggests that there is little to be gained in terms of hedging efficiency from the use of GARCH models despite their additional complexity, as compared with rolling window OLS models (Miffre, 2004). We therefore use a rolling window approach to estimate our model based hedges given its relative simplicity and robust performance. This estimates the variance covariance matrix by conditioning on recent information on a period-by-period basis whereby the most recent observation is added and the oldest observation is removed from the sample, thus keeping the number of estimation observations unchanged. The advantage of this method from a hedging perspective is that by updating the information set we still obtain a more efficient estimate of the hedge ratio, which takes time variation in the return distribution into account. However, in common with other methods that require the hedge ratio to be time-varying, it may be expensive as changing the hedge ratio increases transactions costs.

To obtain the OHR's we require estimates of the variance and covariance's of the spot and futures for our crude oil contract. For the utility maximising hedges we also require risk aversion estimates. We use two values of the CRRA to reflect values for the risk aversion parameter that

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[7] See Alexander (2008) for more details.

are found in the asset pricing literature[8]. We use a value of five which reflects a moderately risk averse investor and a value of ten to reflect a more risk averse investor[9]. The MVHR is estimated using equation (3), and the quadratic utility maximizing HR (Quad) is estimated using equation (2). The exponential HR is obtained by maximizinq equation (4). Each of the utility maximizing hedges is estimated for a CRRA of 5 and a CRRA of 10. In effect this gives us a total of six hedges to consider. The procedure is outlined in more detail in Section 3.

2.4   Hedging Effectiveness

A key contribution of this paper is that we align the methods used for hedge ratio estimation with the performance methods used to measure hedging effectiveness. We focus on three different hedging models, each designed to optimise different criterion. These are variance minimisation, quadratic utility maximisation and exponential utility maximisation. Additionally, we evaluate each of the different OHR's across all three hedging effectiveness measures. This allows us to build up a profile, not only of how well a given hedge works within its own optimisation setting, but also how well that method transfers for broader application. This is important because hedgers may seek hedging strategies that perform well across a broad range of criteria (see Cotter and Hanly, 2006; who found diverse hedging performance using different evaluation methods).

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[8] A number of estimates for the CRRA have been used in the literature, but they generally fall within the range 1 - 10. See for example Mehra and Prescott (1985), Ghysels, Santa Clara, and Valkanov (2005), Guo and Whitelaw (2006). See also Cotter and Hanly (2010) for a more comprehensive discussion and derivation of the CRRA in the hedging context.

[9] We also estimated but do not report hedge ratios using a risk aversion value of 1 which reflects an investor with a log utility function. The resulting hedges were in excess of two and considered unrealistic.

The first performance metric we use to examine hedging performance is the variance. The variance metric (HE₁) measures the percentage reduction in the variance of a hedged portfolio as compared with the variance of an unhedged portfolio. The performance metric is:

$$HE_1 = 1 - \left[\frac{VARIANCE_{HedgedPorfolio}}{VARIANCE_{UnhedgedPortfolio}} \right] \qquad (5)$$

This gives us the percentage reduction in the variance of the hedged portfolio as compared with the unhedged portfolio. When the futures contract completely eliminates risk, we obtain $HE_1 = 1$ which indicates a 100% reduction in the variance, whereas we obtain $HE_1 = 0$ when hedging with the futures contract does not reduce risk. Therefore, a larger number indicates better hedging performance. As the standard measure of risk in finance, it is important to include the variance despite its shortcomings, chief of which is that it implicitly caters for investors who are infinitely risk averse. Since this is not a realistic proposition for real world investors we also use utility based measures. Therefore, the second metric we use to examine hedging performance is (HE₂) which is based on quadratic utility. This measures the percentage increase in the quadratic utility of a hedged portfolio as compared with the quadratic utility of an unhedged portfolio. It is worth noting that expected utility doesn't have a simple intuitive interpretation[10] in a way that other statistical measures do. The number of itself does not have an inherent meaning; rather it is the ordering of the utilities that matters. For this reason using the percentage change is important as it enables us to translate the utilities to provide a clear and unambiguous measure to allow us to compare utility based hedging effectiveness. The performance metric is:

$$HE_2 = 1 - \left[\frac{Quadratic\_Utility_{HedgedPorfolio}}{Quadratic\_Utility_{UnhedgedPortfolio}} \right] \qquad (6)$$

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[10] Ordinal utility shows that while the utility of an investment cannot be measured directly using a numerical scale a comparison can be made whereby investments can be ordered such that one is considered to be worse than, equal to, or better than the other. See Alexander (2008) page 228 for further discussion.

The third metric we use to examine hedging performance is (HE_3) which based on the exponential utility. This measures the percentage increase in the exponential utility of a hedged portfolio as compared with the exponential utility of an unhedged portfolio. The performance metric is:

$$HE_2 = 1 - \left[\frac{Exponential\_Utility_{HedgedPorfolio}}{Exponential\_Utility_{UnhedgedPortfolio}} \right] \tag{7}$$

Note that for the variance metric, hedging effectiveness relates to variance reduction whereas for the utility based measures it relates to an increase in utility.

3.   DATA

For the empirical analysis, we use spot and futures prices. The energy contract used is West Texas Intermediate (WTI) Light Sweet Crude. This energy product trades on CMEGROUP and is one of the most important pricing benchmarks for energy as it represents a highly liquid spot and futures market with a long and robust returns history[11]. Our full sample runs from 1st January 1990 until the 5th of September 2011 and includes data at daily (1-day), weekly (5-day) and monthly (20-day) frequencies. This large timeframe was chosen as it contains both tranquil and volatile periods, is of sufficient length to examine daily, weekly and monthly time horizons, and because it incorporates a number of key events for energy prices. This allows us to compare hedging scenarios that reflect the different holding periods of hedgers and includes periods of intense volatility which is useful for an examination of hedging performance. All data were

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[11] Contract details are available from http://www.cmegroup.com/trading/energy/crude-oil/light-sweet crude_contract_specifications.html.
http://www.cmegroup.com/trading/equity-index/us-index/sandp-500_contract_specifications.html

obtained from Datastream and returns were calculated as the differenced logarithmic prices. A continuous series was formed with the contract being rolled over by volume.

Figure 1 provides a time series plot and Table 1 provides descriptive statistics of the data for the full sample period. The characteristics of the data are in line with those commonly observed for energy assets, namely, the presence of both skewness and kurtosis. Jarque-Bera (J-B) statistics indicate non-normality at all frequencies[12]. We see a positive mean which is indicative of the strong price rises over the period. The strong price increase in the period up to 2008 has been linked to uncertainty about the supply of oil based products due to large increases in demand attributed to the BRIC countries. The series also exhibits periods of marked volatility, most notably associated with the financial crisis of 2008. The large changes in price and the rise in volatility for our sample period is useful as it allows us to examine whether hedging would be effective in circumstances where investors seek to reduce their exposures.

[FIGURE 1 HERE]

[TABLE I HERE]

3.1 Estimation Procedure

To estimate the utility maximizing hedges for use in the period $t-n$ to t, from equations (2) and (4) we require estimates of $E(r_{ft}), \sigma_{ft}^2, \sigma_{sft}, \tau$, and κ, as well as the estimated risk aversion parameter λ. The first hedge uses data from January 1990 until January 2003. To allow the hedge ratio to vary over time, we have adopted a rolling window approach with a window length

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[12] There are some differences in the distributional characteristics of the data at the different frequencies, This is of relevance to utility based hedgers in that where data is closer to normal (e.g. for the monthly frequency), hedging strategies may be more similar across different utility functions, whereas significant skewness and excess kurtosis will affect the hedging strategies and cause them to diverge for different utility functions such as the quadratic and exponential.

of 13 years[13] whereby the newest observation is added and the oldest observation is dropped therefore keeping the window length unchanged. In this way we generate 1560 in-sample hedges at the daily frequency, 312 at the weekly frequency and 170 hedges at the monthly frequency for the five year period from January 2003 to December 2008. The remaining three years of observations are used to generate out-of-sample hedges. We use two different values of the risk aversion coefficient, 5 and 10 to allow for different risk preferences. These are kept constant to allow the underlying changes in the variance covariance matrix to be the key driver of the OHR's[14] We also calculate a time-varying MVHR using the same rolling window methodology but employing (3). This doesn't incorporate the risk aversion parameter and is based on the variance covariance matrix alone.

The second set of OHR's we estimate are 1-step ahead forecast hedges for use in period t+1. To do this, we reserved a sub-period of three years of data to allow us to generate forecasted OHR's in a consistent manner. The time-varying hedges were forecast using a random walk process whereby we used the estimates from the t-period hedges to generate hedges for use in period t+1[15]. Using this methodology, we obtained 700 hedges for use in period t+1 at the daily frequency, 140 at the weekly frequency and 35 hedges at the monthly frequency. These covered the period from January 2009 to September 2011. The advantage of our approach is that it allows us to generate sufficiently large numbers of hedges for analysis using relatively low frequency data.

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[13] This allows for sufficient observations at the weekly and monthly frequency for robust estimation

[14] In related work, we estimated and applied a time varying risk aversion coefficient to match the time varying characteristic of the variance covariance matrix. See Cotter and Hanly (2010) for more detail.

[15] Although not reported, we carried out Stationarity tests on the OHR's using the Augmented Dickey Fuller Test. Results indicated that each of our model based hedges were non stationary. This finding motivated our use of a

4.   RESULTS

4.1   Optimal Hedges

Figure 2 plots a comparison of the OHR's for each of the different utility functions, quadratic and exponential together with the MVHR. Weekly frequency is chosen for illustration. Table 2 presents summary statistics for each of the hedges for the in-sample period.

[FIGURE 2 HERE]

[TABLE 2 HERE]

Figure 2 illustrates that there are large differences between the MVHR and both of the utility based hedges. More specifically we can see that the utility based hedges are larger than the MVHR. For example, from Table 2 reading across, we can see that mean values for the OHR for the weekly frequency using the quadratic and exponential utility functions with a CRRA of 5 are 1.044 and 1.103 respectively as compared with 0.983 for the MVHR. Similar differences are observed for each frequency. This finding shows that for non-normal data, there will be substantial differences between not just the MVHR and the quadratic utility hedge but also between the quadratic utility and exponential utility based hedges. This relates to the presence of skewness and kurtosis in oil returns data and supports Lien (2007) but differs from Brooks Cerny and Miffre (2012) who find only small differences between MVHR's vs. utility based hedges. The implication is that hedge strategies that focus only on risk reduction may not provide the best outcomes in terms of expected utility.

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random walk approach to forecast one step ahead hedge ratios for use out-of-sample. See Poon and Granger (2003) for further discussion.

We can also see that for the utility based hedges the risk aversion parameter has a clear impact with the spread between the hedges narrowing when we use the more risk averse CRRA value of 10 as compared with the moderate value of 5. For example, reading across Table 2 for the daily frequency, the mean OHR for the quadratic utility function with a CRRA of 5 is 0.990. This is significantly different than the equivalent value of 0.959 which is estimated using a CRRA of 10. Note also that both of the utility hedges are larger than one for the weekly and monthly but not the daily frequency. These findings indicate that allowing the expected return to play a part in determining the OHR via a utility function results in most cases in hedgers holding futures in excess of their initial spot position in the underlying asset. They also show the importance of using a risk aversion coefficient that reflects the hedgers individual risk preferences. These findings support earlier work by deVille deGoyet, Dhaene and Sercu (2008).

Table 3 further illustrates this by a comparison of the absolute differences between the mean of each of the OHR's. In all cases there are significant differences between the MVHR and the utility based hedge ratios, with differences smaller but still significant for the larger CRRA of 10. Using the daily frequency with CRRA 5 for example, the difference between the exponential and quadratic OHR's is 0.063 as compared with 0.031 for a similar comparison using CRRA 10. This reflects the fact that as risk aversion increases, the speculative component plays a smaller part in the utility based hedges and they then tend to converge towards the MVHR.

[TABLE 3 HERE]

We next compare the OHR's for hedgers with different hedging horizons. Again, from Table 2 we can see that there are significant differences between the hedge ratios for different frequencies in every single case across all three hedge ratios, both risk aversion values and for both assets. For example, reading down, mean HR values using the quadratic utility function with a CRRA of 5 ranges from an average of 0.990 for daily data to 1.044 and 1.084 for weekly

and monthly data respectively. This shows that using a hedge estimated at one hedging horizon, for example daily, which may be useful for an Oil commodities trader is inappropriate for a hedger such as an airline for whom a monthly hedging time horizon is more appropriate.

4.2 Hedging Performance

We now turn to hedging performance which we examine using three performance metrics, (HE_1) variance, (HE_2) quadratic utility and (HE_3) exponential utility which were chosen to match the underlying optimisation goals of the model based hedge ratios. Table 4 presents underlying values for these metrics which are calculated for each OHR; MVHR, quadratic utility hedge and exponential utility hedge together with No hedge which is used as a benchmark for comparison purposes. Values are presented for both CRRA of 5 and a CRRA of 10.

[TABLE 4 HERE]

From Table 4 we can observe the general effectiveness of hedging in that each model yields a large improvement in the underlying optimization criteria as compared with an unhedged position. We also note that the best hedging strategy for a given performance metric is the hedging model that is designed to optimize that metric; be it variance reduction or utility maximisation. For example, across frequencies and risk aversion values, we can see that in all cases the MVHR returns the minimum variance while the quadratic HR returns the best quadratic utility, as does the exponential hedge for the exponential utility. These results show that it is important to match the performance criterion with the process for estimating the hedge ratio. It also indicates that applying a MVHR which has been commonly done in the literature as an

approximation for all hedgers irrespective of their utility functions or attitudes towards risk may not be appropriate.

Turning next to the relative performance of the various hedging models, Table 5 presents performance measures (HE_1), (HE_2) and (HE_3) which show the percentage change for a given metric as compared with a No Hedge scenario.

[TABLE 5 HERE]

Looking first at daily hedges for a CRRA of 5, reading across Table 5 we can see that hedging using the MVHR reduces (HE_1), the variance criterion by 76.29% as compared with 75.87% for the quadratic HR and 74.72% for the exponential HR. The best performing hedge – the MVHR therefore outperforms the quadratic and exponential hedges in terms of variance reduction by just 0.4% and 1.5% respectively in absolute terms and just 0.1% and 0.3% respectively using a CRRA of 10. This illustrates that the relative performance differentials between the model based hedges are quite small irrespective of which performance metric is used. For weekly and monthly frequencies relative performance differentials are similarly small however the absolute hedging effectiveness increases dramatically from around 75% on average to 95% and 98% for weekly and monthly frequencies respectively. Taken together, these results show that in practice while different hedge strategies produce different hedge ratios, the performance differentials are not economically significant and therefore there is little point in adding additional complexity to the hedging decision by using the utility maximising as distinct from the variance minimising approach. It also shows that hedging oil is quite effective across a range of performance measures especially at lower frequencies.

Turning next to the out-of-sample performance of the different hedging strategies, Table 6 presents mean, variance, quadratic utility and exponential utility values while Table 7 presents hedging performance metrics. The first thing we note is that the out-of-sample hedging performance is good across each hedging model irrespective of the performance criteria used. A quick glance at Table 7 confirms this with improvements in performance in excess of 95% and in some cases of hedgers more than doubling their utility as compared with a No Hedge position. Looking at the relative performance of the difference HR's, from Table 7, when (HE_2) - the variance is used as the performance benchmark, we observe small differences. For example, using weekly data and a CRRA of 5 the difference between the best performing MVHR and the worst performing exponential HR is just 1.05%.

[TABLE 6 AND 7 HERE]

For the utility based performance criteria (HE_2), and (HE_3) some differences emerge when we compare results for the two different risk aversion values and across hedging frequency. Looking first at (HE_2), the quadratic utility, we can see that for the lower risk aversion value CRRA 5, there are some performance differences. For example, for daily hedgers with CRRA5, differences are of the order of 1.0 % to 3.0% using (HE_2) and 2.0% to 6.0% using (HE_3), however for weekly and monthly hedges the differences are quite small averaging between 1% and 2% respectively. Finally for the higher risk aversion of CRRA10 we again see a convergence in hedging performance across models and for each performance metric. An interesting observation is that the model based hedges don't uniformly perform best out-of-sample in terms of their given optimisation criterion. Using (HE_1) and daily data an example, the best performer

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at both risk aversion levels is the quadratic HR, and if we use (HE$_2$) and (HE$_3$), for the same data, the best performer is the exponential HR.

In order to illustrate these results in economic terms we also report Value at Risk (VaR) in Table 8 for each of the out-of-sample hedges based on the 95% confidence interval.[16] We assume the investment is $1million. VaR is estimated using the commonly applied Historical Simulation method. A quick glance confirms the overall effectiveness of hedging with each hedging model showing large reductions in excess of 80% in the VaR across different hedging horizons. We can see that VaR figures are broadly similar across each of the hedging models at for each of the different frequencies. For example, daily VaR ranges from $7,960 for the best performing quadratic model to $8,351 for the worst performer which is the exponential HR for CRRA 10, a difference of just $391 which is relatively small. For the monthly frequency however, the differences in the VaR from the different hedging models are slightly larger with the quadratic HR for CRRA 5 yielding the best performing monthly VaR of $17,748 which beats the worst performing exponential HR with a VaR of $20,759, a difference of $3,011.

In terms of which model provides the best economic performance, the results depend on frequency. The quadratic HR performs best for daily and monthly hedgers while the MVHR is the best at the weekly interval. These results seem to indicate that using performance metrics based on the optimisation criteria will yield only slight differentials for Oil hedgers, despite the differences in their optimal hedging strategies.

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[16] VaR has increasingly been used as a performance measure given its wide applicability in risk management and relative ease of understanding as compared with statistical measures.

5.   CONCLUSION

We estimate and compare optimal utility based hedges using performance metrics based on the underlying optimization criteria. We apply this approach for energy market participants across daily weekly and monthly hedging frequencies. By matching the performance appraisal metrics to the underlying methods used to generate the optimal hedges we can make a comprehensive comparison of the relative performance of utility based hedges and contrast these approaches with the more traditional variance minimising hedge. By focusing on the role of both utility and risk aversion in the hedging decision, we are reflecting real world investor concerns which encompass both risk and expected return.

Our findings indicate that there are significant differences between the quadratic and exponential hedge ratios and also between each of the utility based hedges and the variance minimising approach. The OHR's also differ significantly for each of the different hedging horizons, daily weekly and monthly, which implies that optimal hedge ratio estimation should be tailored to the specific hedging frequency of the individual hedger. We also find that using performance methods based on the optimization criteria will not yield significant performance differentials for oil hedgers in a comparison between different utility based hedging models.

Taken together these results suggest that for oil market participants hedging performance is not significantly different enough to warrant using a more complicated utility based approach as compared with the simpler minimum variance based hedge.

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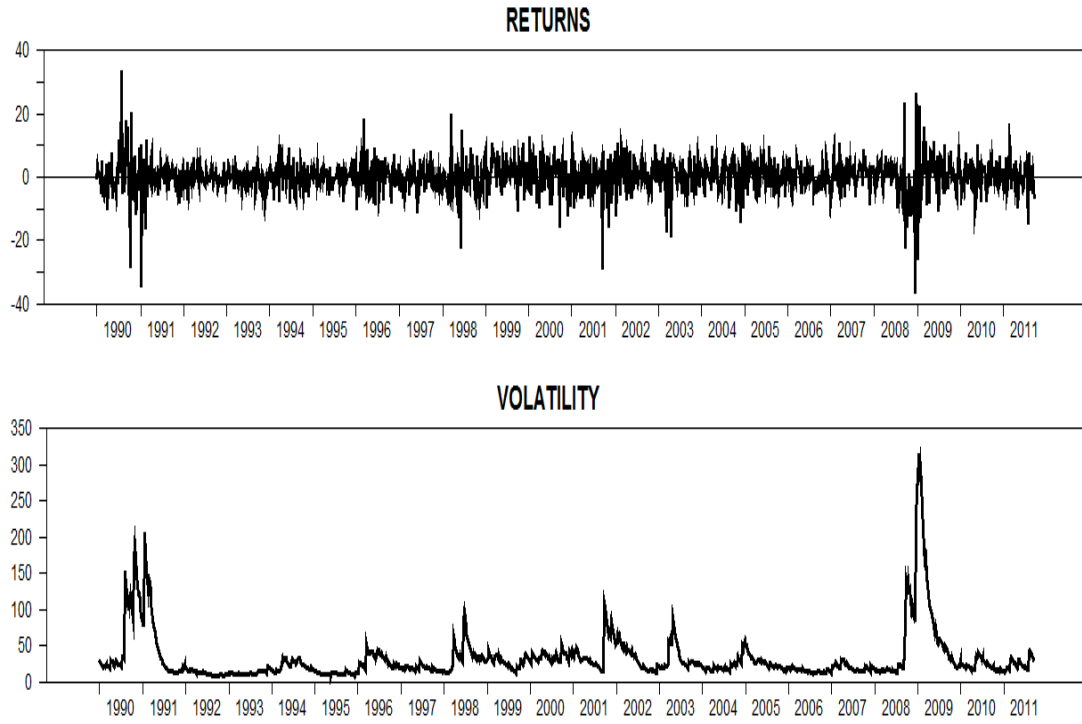
FIGURE 1   Time Series Plots of Weekly Returns and Volatility for Crude Oil

Returns and volatility are illustrated using Weekly data for Crude Oil (WTI). Volatility is obtained from fitting a GARCH (1, 1) model. Both Daily and Monthly data exhibit similar characteristics.

Optimal Hedge Ratios

WTI CRUDE OIL
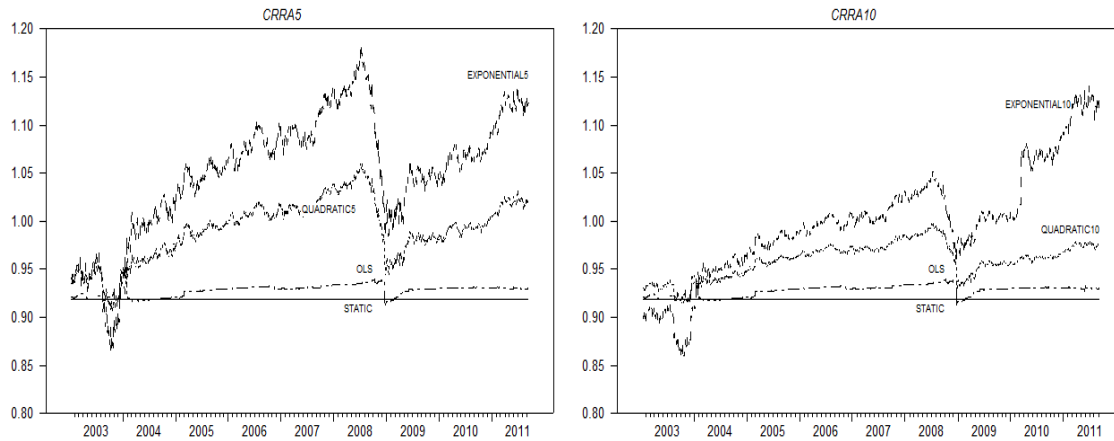
FIGURE 2   Time-Varying Optimal Hedge Ratios for Crude Oil

Notes: Time-series plots of the OHR's for in-sample and out-of-sample periods are illustrated using data at weekly frequency. Four different hedge ratios are presented: A Minimum Variance hedge (OLS), and two utility maximizing hedges based on Quadratic and Exponential utility. A static hedge ratio estimated using OLS is also included for comparison. Each of the utility based hedges is estimated using a risk aversion coefficient (CRRA) of 5 and a CRRA of 10. Each of the time-varying OHR's are estimated using a rolling window approach as detailed in the text. Note that for the higher CRRA of 10, the spread between the MVHR hedge and both the Quadratic and Exponential hedges is narrower. This is because at higher risk aversion levels, both of these utility functions will converge towards the MVHR hedge.

		Spot	Futures	Spot	Futures	Spot	Futures
		Daily		Weekly		Monthly	
Mean	%	-0.0229	-0.0229	-0.1235	-0.1234	-0.4938	-0.4935
Stdev	%	2.6386	2.4477	5.6426	5.4367	9.7802	9.5869
Skewness		-1.10*	-0.90*	-0.56*	-0.51*	-0.45*	-0.46*
Kurtosis		25.00*	17.28*	6.64*	5.84*	1.21*	1.33*
J-B		148754.48*	71273.40*	2135.88*	1658.94*	26.65*	30.82*

Table 1: Descriptive statistics for Spot and Futures returns series of Crude Oil

Notes: Statistics are presented for the full sample. The Mean and Standard Deviation (Stdev) for both series are given in percentages. The Jarque-Bera (J-B) statistic tests the null hypothesis that the distribution is normal and this hypothesis is rejected for both assets at all frequencies. Note that both the J-B statistic and excess kurtosis values fall as we move from the Daily to the Monthly frequency indicating that data at higher frequency are more parametric. Although not reported, we also tested for Stationarity using the Augmented Dickey-Fuller unit root test. The results indicated non-Stationarity in prices but Stationarity in returns. *Denotes Significance at the 1% level.

	(1)	(2)	(3)	(4)	(5)	(6)
	STATIC	MVHR	QUADRATIC	EXPONENTIAL	QUADRATIC	EXPONENTIAL
			CRRA 5		CRRA 10	
DAILY						
MEAN	0.909	$0.928^{\dagger *}$	$0.990^{\dagger *\lambda}$	$1.045^{\dagger *\lambda}$	$0.959^{\dagger *}$	$0.976^{\dagger *}$
MIN		0.911	0.906	0.866	0.914	0.857
MAX		0.940	1.060	1.178	0.998	1.051
STDEV		0.006	0.035	0.070	0.020	0.045
WEEKLY						
MEAN	0.991	0.983^{α}	$1.044^{\alpha\lambda}$	$1.103^{\alpha\lambda}$	1.013^{α}	1.042^{α}
MIN		0.978	0.968	0.954	0.974	0.966
MAX		0.998	1.107	1.226	1.045	1.104
STDEV		0.002	0.030	0.060	0.015	0.030
MONTHLY						
MEAN	1.004	1.008	1.084^{λ}	1.154^{λ}	1.046	1.078
MIN		0.996	0.985	0.965	0.990	0.981
MAX		1.019	1.152	1.281	1.081	1.142
STDEV		0.004	0.039	0.073	0.021	0.037

Table 2: Summary Statistics of Hedge Ratios for Crude Oil

Notes: Summary statistics are presented for the Hedge Ratioss for the In-Sample period at three different hedging intervals and for two risk aversion levels. Six different methods for estimating the hedge ratio are presented. Firstly we use a static hedge which is a single minimum variance hedge ratio estimated for the full in-sample period. We also estimate a minimum variance hedge (MVHR) based on OLS but adopting a rolling window approach as described in the text. We also estimate two utility maximizing hedges; namely Quadratic and Exponential using two different risk aversion CRRA values of 5 and 10. Two statistical comparisons are drawn. First we compare the Utility based HR's using the CRRA values. Reading across, if we look at the Daily frequency for example, the OHR with risk aversion CRRA 5 for the Quadratic utility function is 0.990. This is significantly different than the equivalent OHR using a risk aversion CRRA value of 10 which yields 0.959. We also compare the mean OHR's for daily, weekly and monthly intervals as we look down the table. Using risk aversion CRRA of 5 for for example, we find a significant difference between the Quadratic OHR (column 3) for a Daily hedger (0.990) and both the Weekly hedger (1.044) and monthly hedger (1.084). $^{\lambda}$ denotes significant at the 1% level for comparison of OHR's based on risk aversion. †, * and $^{\alpha}$ denotes significance at the 1% level for daily vs. weekly, daily vs. monthly and weekly vs. monthly comparisons respectively.

	(1) MVHR	(2) QUADRATIC	(3) EXPONENTIAL	(4) MVHR	(5) QUADRATIC	(6) EXPONENTIAL
		CRRA 5			CRRA 10	
DAILY						
MVHR	0.00	0.063*	0.117*	0.00	0.031*	0.048*
		(69.62)	(66.25)		(59.13)	(41.82)
QUADRATIC		0.00	0.055*		0.00	0.017*
			(27.65)			(13.34)
EXPONENTIAL			0.00			0.00
WEEKLY						
MVHR	0.00	0.061*	0.120*		0.031*	0.059*
		(35.42)	(35.55)		(34.62)	(34.44)
QUADRATIC		0.00	0.059*		0.00	0.029*
			(15.56)			(14.83)
EXPONENTIAL			0.00			0.00
MONTHLY						
MVHR	0.00	0.076*	0.147*	0.00	0.038*	0.070*
		(16.88)	(17.57)		(15.49)	(16.23)
QUADRATIC		0.00	0.070*		0.00	0.032*
			(7.43)			(6.48)
EXPONENTIAL			0.00			0.00

Table 3: Comparison of Differences between Optimal Hedge Strategies for Crude Oil

Notes: Mean values are presented for the Hedge Ratios for the In-Sample period at three different hedging intervals and for two risk aversion levels. The HR's presented are rolling window MVHR and the two utility maximizing hedges; namely Quadratic and Exponential using two different risk aversion CRRA values of 5 and 10. Table 3 presents a comparison of the absolute differences between the Mean OHR's using the rolling window MVHR as a benchmark. For example, using the Daily frequency with risk aversion of 5, the difference between the Quadratic and the MVHR HR's is 0.063. t-statistics are in parentheses. * Denotes a significant difference at the 1% level.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	%	%	%	%	%	%	%	%
	NO HEDGE	MVHR	QUAD	EXP	NO HEDGE	MVHR	QUAD	EXP
		CRRA 5				CRRA 10		
DAILY								
MEAN	-0.0307	-0.0021	0.0003	0.0025	-0.0307	-0.0021	-0.0009	-0.0001
VARIANCE	0.0564	**0.0134**	0.0136	0.0142	0.0564	**0.0134**	0.0134	0.0136
UTILITY - Quadratic	-0.3126	-0.0689	**-0.0677**	-0.0688	-0.5944	-0.1357	**-0.1351**	-0.1356
UTILITY - Exponential	-0.1680	-0.0352	-0.0335	**-0.0330**	-0.2983	-0.0676	-0.0668	**-0.0666**
WEEKLY								
MEAN	-0.1538	-0.0024	0.0092	0.0205	-0.1538	-0.0024	0.0034	0.0089
VARIANCE	0.2619	**0.0148**	0.0160	0.0194	0.2619	**0.0148**	0.0151	0.0159
UTILITY - Quadratic	-1.4635	-0.0767	**-0.0708**	-0.0763	-2.7732	-0.1509	**-0.1480**	-0.1506
UTILITY - Exponential	-0.7759	-0.0396	-0.0309	**-0.0281**	-1.3415	-0.0768	-0.0725	**-0.0712**
MONTHLY								
MEAN	-0.6133	0.0067	0.0640	0.1168	-0.6133	0.0067	0.0353	0.0597
VARIANCE	0.8497	**0.0190**	0.0247	0.0403	0.8497	**0.0190**	0.0204	0.0239
UTILITY - Quadratic	-4.8616	-0.0881	**-0.0596**	-0.0845	-9.1099	-0.1829	**-0.1686**	-0.1792
UTILITY - Exponential	-2.6007	-0.0422	-0.0002	**0.0123**	-4.3860	-0.0939	-0.0739	**-0.0686**

Table 4: In Sample Hedging: Return and Risk Measures for Crude Oil

Notes: Mean and Variance are in percentages. Utilities are x10^{-2}. We examine the hedging performance of Crude Oil for the In-Sample period at three different hedging intervals and for two risk aversion levels. Four hedge ratios are examined (Columns 1 - 8). They are a hedge ratio of zero (no hedge), minimum variance hedge (MVHR), quadratic utility maximising hedge (Quad), and exponential utility maximizing hedge (Exp). Three performance measures, the variance, quadratic utility and exponential utility are presented together with the Mean. The utility maximizing performance measures are estimated for both CRRA 5 and 10 using the relevant utility function. Using daily data for example, reading down, the MVHR (column 2) yields a hedged portfolio with a mean of -0.0021, a variance of 0.0134%, a quadratic utility of -0.000689 and an exponential utility of -0.000352 (both utilities estimated using CRRA value of 5). The best performing HR for a given metric is in bold text.

	(1)	(2)	(3)	(4)	(5)	(6)
	%	%	%	%	%	%
	MVHR **(HE1)**	**QUADRATIC** **(HE2)**	**EXPONENTIAL** **(HE3)**	**MVHR** **(HE1)**	**QUADRATIC** **(HE2)**	**EXPONENTIAL** **(HE3)**
			CRRA 5		CRRA 10	
DAILY						
HE1 - Variance	**76.29**	75.87	74.72	**76.29**	76.19	75.95
HE2 - Quadratic	77.95	**78.33**	78.00	77.16	**77.26**	77.18
HE3 - Exponential	79.06	80.08	**80.38**	77.35	77.60	**77.68**
WEEKLY						
HE1 - Variance	**94.33**	93.89	92.61	**94.33**	94.22	93.91
HE2 - Quadratic	94.76	**95.16**	94.78	94.56	**94.66**	94.57
HE3 - Exponential	94.90	96.02	**96.38**	94.28	94.60	**94.69**
MONTHLY						
HE1 - Variance	**97.77**	97.09	95.26	**97.77**	97.60	97.19
HE2 - Quadratic	98.19	**98.77**	98.26	97.99	**98.15**	98.03
HE3 - Exponential	98.38	99.99	**100.47**	97.86	98.31	**98.44**

Table 5: In Sample Hedging Performance for Crude Oil

Notes: Three performance measures are presented. HE1 - the Variance performance measure is measured by the percentage reduction in the variance as compared with no hedge. HE2 and HE3 - the Utility performance measures are measured by the percentage increase in the relevant utility as compared with no hedge. Each performance metric is applied to each of the different hedge strategies. The best performing strategy for a given performance metric is marked with a *. For example, for a Daily hedger with a risk aversion coefficient of 5, the best performing hedging strategy evaluated using HE1 is the MVHR. This reduces the variance by 76.29% as compared with a no hedge position whereas the Quadratic and Exponential Hedges yield variance reductions of 75.87% and 74.72% respectively. Similarly, when the evaluation criterion is HE2, the Quadratic hedge is the best performer showing a 78.33% increase in utility.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	%	%	%	%	%	%	%	%
	NO HEDGE	MVHR	QUAD	EXP	NO HEDGE	MVHR	QUAD	EXP
		CRRA 5				CRRA 10		
DAILY								
MEAN	-0.1100	-0.0081	-0.0016	0.0061	-0.1100	-0.0081	-0.0048	0.0015
VARIANCE	0.0673	0.0025	0.0023	0.0026	0.0673	0.0025	0.0023	0.0026
UTILITY - Quadratic	-0.4465	-0.0205	-0.0131	-0.0071	-0.7831	-0.0329	-0.0283	-0.0243
UTILITY - Exponential	-0.2793	-0.0143	-0.0074	-0.0005	-0.4503	-0.0207	-0.0167	-0.0115
WEEKLY								
MEAN	-0.4225	-0.0015	0.0089	0.0158	-0.4225	-0.0015	0.0037	0.0014
VARIANCE	0.3585	0.0060	0.0070	0.0097	0.3585	0.0060	0.0062	0.0064
UTILITY - Quadratic	-2.2152	-0.0313	-0.0262	-0.0327	-4.0078	-0.0610	-0.0581	-0.0627
UTILITY - Exponential	-1.2881	-0.0164	-0.0086	-0.0084	-2.1020	-0.0313	-0.0270	-0.0304
MONTHLY								
MEAN	-1.6902	0.0108	0.0822	0.1417	-1.6902	0.0108	0.0465	0.0626
VARIANCE	0.7286	0.0134	0.0116	0.0159	0.7286	0.0134	0.0116	0.0113
UTILITY - Quadratic	-5.3331	-0.0562	0.0240	0.0621	-8.9760	-0.1232	-0.0694	-0.0506
UTILITY - Exponential	-3.3860	-0.0228	0.0525	0.1010	-4.8916	-0.0563	-0.0128	0.0041

Table 6: Out of Sample Hedging: Return and Risk Measures for Crude Oil

Notes: Mean and Variance are in percentages. Utilities are x10^{-2}. We examine the hedging performance for the Out-of-Sample period at three different hedging intervals and for two risk aversion levels. Four hedge ratios are examined (Columns 1 - 8). They are a hedge ratio of zero (no hedge), minimum variance hedge (MVHR), quadratic utility maximising hedge (Quad), and exponential utility maximizing hedge (Exp). We use three performance measures, the variance, quadratic utility and exponential utility together with the Mean. The utility maximizing performance measures are estimated for both CRRA 5 and 10 using the relevant utility function. Using daily data for the for example, reading down, the Quadratic HR (column 3) has a mean of -0.0016%, a variance of 0.0023%, a quadratic utility of -0.0131% and an exponential utility of -0.0074% (both utilities estimated using CRRA value of 5).

	(1)	(2)	(3)	(4)	(5)	(6)
	%	%	%	%	%	%
	MVHR (HE1)	**QUADRATIC (HE2)**	**EXPONENTIAL (HE3)**	**MVHR (HE1)**	**QUADRATIC (HE2)**	**EXPONENTIAL (HE3)**
		CRRA 5			**CRRA 10**	
DAILY						
HE1 - Variance (MVHR)	96.31	**96.57**	96.09	96.31	**96.52**	96.17
HE2 - Quadratic	95.41	97.07	**98.41**	95.79	96.39	**96.90**
HE3 - Exponential	94.86	97.36	**99.82**	95.41	96.30	**97.45**
WEEKLY						
HE1 - Variance (MVHR)	**98.34**	98.04	97.29	**98.34**	98.28	98.21
HE2 - Quadratic	98.59	**98.82**	98.52	98.48	**98.55**	98.43
HE3 - Exponential	98.73	99.34	**99.35**	98.51	**98.71**	98.55
MONTHLY						
HE1 - Variance (MVHR)	98.16	**98.40**	97.81	98.16	98.41	**98.45**
HE2 - Quadratic	98.95	100.45	**101.16**	98.63	99.23	**99.44**
HE3 - Exponential	99.33	101.55	**102.98**	98.85	99.74	**100.08**

Table 7: Out of Sample Hedging Performance for Crude Oil

Notes: Three performance measures are presented. HE1 - the Variance performance measure is measured by the percentage reduction in the variance as compared with no hedge. HE2 and HE3 - the Utility performance measures are measured by the percentage increase in the relevant utility as compared with no hedge. Each performance metric is applied to each of the different hedge strategies. The best performing strategy for a given performance metric is in bold text.

	(1)	(2)	(3)	(4)	(5)	(6)
	NO HEDGE	MVHR	QUAD	EXP	QUAD	EXP
DAILY			CRRA 5		CRRA 10	
VaR 95%	$42,672	$8,197	$7,898	$8,439	**$7,960**	$8,351
		80.8%	81.5%	80.2%	**81.3%**	80.4%
WEEKLY						
VaR 95%	$98,488	**$12,689**	$13,777	$16,211	$12,933	$13,174
		87.1%	86.0%	83.5%	86.9%	86.6%
MONTHLY						
VaR 95%	$140,398	$19,038	$17,748	$20,759	**$17,703**	$17,497
		86.4%	87.4%	85.2%	**87.4%**	87.5%

Table 8: Economic Performance using Value at Risk for Crude Oil

Notes: This table presents 95% VaR values for each of the model based hedges together with a No Hedge position which is used as a benchmark. Both of the Utility based hedges have VaR estimates for both CRRA 5 and CRRA 10. Values are estimated based on an investment of $1,000,000. The percentage reduction in the VaR for each HR as compared with a No Hedge position is also given beneath the VaR value for the Hedge. The best performing strategy is in bold text.