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Jonathan Blackledge
Technological University Dublin, jonathan.blackledge@tudublin.ie

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Why is an Einstein Ring Blue?

Jonathan M Blackledge *

Abstract—**Albert Einstein predicted the existence of 'Einstein rings' as a consequence of his general theory of relativity. The phenomenon is a direct result of the idea that if a mass warps space-time then light (and other electromagnetic waves) will be 'lensed' by the strong gravitational field produced by a large cosmological body such as a galaxy. Since 1998, when the first complete Einstein ring was observed, many more complete or partially complete Einstein rings have been observed in the radio and infrared spectra, for example, and by the Hubble Space Telescope in the optical spectrum. However, in the latter case, it is observed that the rings are blue providing the light is not red shifted. The gravitational lensing equation does not include dispersion (i.e. wavelength dependent effects) and thus, can not account for this 'blue shift' and, to date, there has been no satisfactory explanation for this colour phenomenon. In this paper we provide an explanation for why Einstein rings are blue using a linear systems theory approach based on the idea that a gravitational field is generated by the scattering of very low frequency scalar waves in which the medium of propagation is space-time and that light waves can be both bent and diffracted by this field. The latter effect provides a quantitative result that explains why an Einstein ring is blue.**

Keywords: Einstein ring, Image analysis, Simulation and Modelling, Gravity

1 Introduction

The Hubble Space Telescope has provided a wealth of information of cosmological significance since it came into operation in 1993 [1], [2]. This has included optical images of Einstein rings, some typical examples being provided in Figure 1 [3]. While the existence of such rings is predicted by Einstein's general theory of relativity, to date, no reasonable explanation has been given as to why, apart from some 'red-shifted' cases [4] observed in the far-field (i.e. billions of light years away), they are blue as shown in Figure 1. This observation is in contrast to images of Einstein rings that are not in the optical spectrum but

*Dublin Institute of Technology, Kevin Street, Dublin 8, Ireland; http://eleceng.dit.ie/blackledge; jonathan.blackledge@dit.ie; +35 31 402 4707.

Figure 1: A gallary of Einstein rings in the optical spectrum obtained using the Hubble Space Telescope [3].

in the infrared [5] or radio spectra, where false colour coding is used for image display purposes only, i.e. the colours are of limited physical significance [6]. For example, Figure 2 show an example of a complete Einstein ring taken using the Hubble Space Telescope near-infrared camera [7] and Figure 3 shows a 5 GHz radio image of a compound lensing effect produced by two galaxies where neither the lensing galaxy nor the lensed object(s) has yet been identified optically [8].

Although there have been some minor comments with regard to the blueness of Einstein rings, they are clearly inadequate with regard to a physical explanation of the phenomenon. For example, the Astronomy Picture of the Day, July 28, 2008 given in Figure 4, is accompanied by the following text [9]: *What's large and blue and can wrap itself around an entire galaxy? A gravitation lens image. Pictured above on the left, the gravity of a normal white galaxy has gravitationally distorted the light from a much more distant blue galaxy.* **In [16] the phrase '...strongly lens background blue star-forming galaxies...' is referred to as the cause for the colour of the ring. It is clearly not conceivable that all Einstein rings observed to date (in the visible spectrum) are the coincidental result of gravitationally lensed light from 'blue galaxies' or 'blue star-forming galaxies'. Galaxies are certainly not blue, but rather, cosmological bod-**

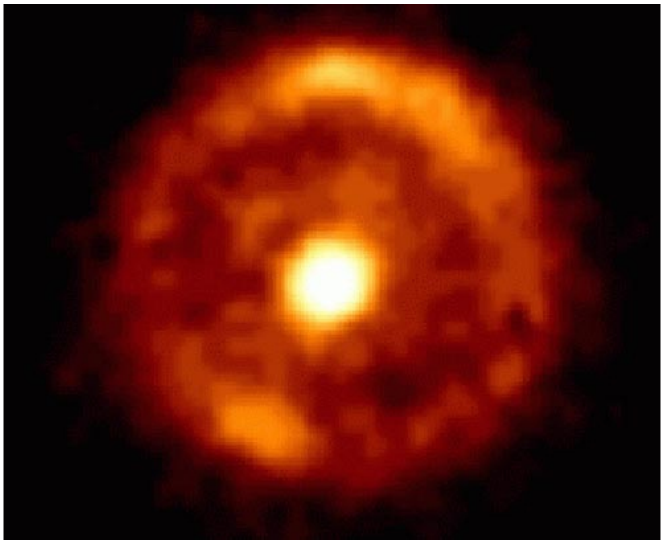

Figure 2: A near-infrared image of an Einstein rings obtained using the Hubble Space Telescope near-infrared camera [7].

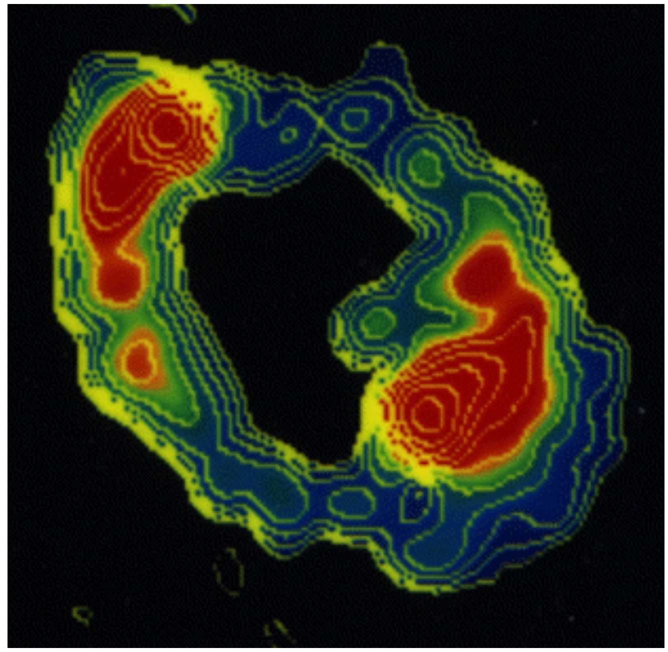

Figure 3: A 5 GHz radio image of a possible compound gravitational lens obtained using the MERlIN (Multi-Element Radio-Linked Interferometer Network) radio telescope [8].

ies that radiate electromagnetic waves over a vast spectrum.

Einstien rings were first predicted by Albert Einstein in 1936 [10] and although he originally judged that 'there is no hope of observing this phenomenon directly', the phenomena is now a relatively routine observation. There are many examples of Einstein rings with diameters up to an arcsecond as shown in Figure 1. However, because the mass distribution of the lenses is not perfectly axially symmetrical, or because the source, lens and observer are not perfectly aligned, many of the observation are of imperfect or partial Einstein rings. The study of Einstein rings has become an important aspect of cosmology in general. This includes the bending of starlight by the gravity of intervening foreground stars which is now commonly referred to as 'gravitational microlensing and has become a technique to detect planets orbiting stars [12].

Einstein rings are caused by the gravitational lensing which in turn is a consequence of Einstein's theory of General relativity. A fundamental consequence of this theory is that mass distorts space-time and thus, instead of a ray light from a source travelling in a straight line through a three-dimensional space, it is bent as a result of the distortion generated by the presence of a massive body. An Einstein Ring is a special case of gravitational lensing which is caused by the exact alignment of the source, lens and observer. This results in a symmetry around the lens, causing a ring-like structure with an 'Einstein Radius' in radians, given by [11]

$$\theta = \sqrt{\frac{4GM}{c_0^2}\frac{D_L d_S}{D_{LS}}}$$

where G is the gravitational constant M is the mass (of the lensing object) D_L is the angular diameter distance to the lens, D_S is the angular diameter distance to the source, D_{LS} is the angular diameter distance between the lens and the source and $D_{LS} \neq D_S - D_L$ over cosmological distances in general. The angular diameter distance to a cosmological object is defined in terms of an object's actual size divided by the angular size of the object as viewed from earth. This result is indicative of the theory of general relativity being a 'geometric' interpretation of gravity which does not include effects that depend on the wavelength of the light that is bent. Consequently, general relativity is not able to explain why an Einstein ring is blue. This paper attempts to explain the colour phenomena using a linear systems theory approach based on scalar Helmholtz scattering to evaluate the effect of light being not just bent but 'diffracted' from a 'thin' gravitational field of the

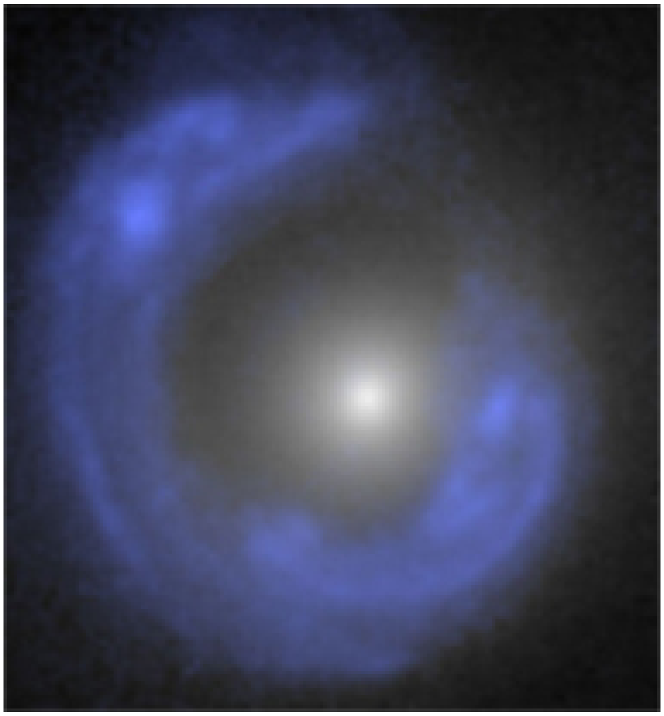

Figure 4: A near complete Einstein ring showing the distinctive blueness of the phenomenon [9].

type produced by a spiral galaxy, for example, where the gravitational field is taken to be due to the scattering of low frequency scalar waves. The diffraction effect generates an expression for the intensity of an Einstein ring that depends on the wavelength of light according to a λ^{-6} power law where λ is the wavelength. The paper then explores the consequences of this result in terms of a 'wavefield theory' of gravity.

2 Helmholtz Scattering

The three-dimensional inhomogeneous Helmholtz equation for a scalar wavefield u is given by [13]

$$(\nabla^2 + k^2)u(\mathbf{r}, k) = -k^2\gamma(\mathbf{r})u(\mathbf{r}, k) \qquad (1)$$

where ∇^2 is the Laplacian operator, $k = \omega/c_0$ is the wavenumber, ω is the angular frequency. Consider a scattering function γ which is of compact support, i.e. $\gamma(\mathbf{r}) \;\; \exists \;\; \forall \;\; \mathbf{r} \in V$ where V is a volume of arbitrary shape. The Green's function transformation of equation (1) [13] yields the solution

$$u = u_i + u_s$$

where u_i is the incident wavefield, u_s is the scattered wavefield given by

$$u_s(\mathbf{r}, k) = k^2 g(r, k) \otimes_3 \gamma(\mathbf{r})u(\mathbf{r}, k), \quad r = \mid \mathbf{r} \mid$$

and $\otimes_3$ denotes the three dimensional convolution integral over r. With regard to this integral equation, g is the 'out-going' Green's function given by [14]

$$g(r, k) = \frac{\exp(ikr)}{4\pi r}$$

which is the solution of

$$(\nabla^2 + k^2)g(r, k) = -\delta^3(\mathbf{r})$$

and it is assumed that

$$u(\mathbf{r}, k) = u_i(\mathbf{r}, k), \quad \forall \;\; \mathbf{r} \in S$$

where S is the surface of V and

$$u_i(\mathbf{r}, k) = \exp(ik\hat{\mathbf{n}}_i \cdot \mathbf{r})$$

satisfying the homogeneous Helmholtz equation

$$(\nabla^2 + k^2)u_i(\mathbf{r}, k) = 0$$

Note that

$$g(r, k) = \frac{1}{4\pi r}, \quad k \to 0$$

and thus,

$$\nabla^2 \left(\frac{1}{4\pi r} \right) = -\delta^3(\mathbf{r}) \qquad (2)$$

Let us now assuming that $u \sim u_i \forall \mathbf{r} \in V$ so that the scattered field is given by

$$u_s(\mathbf{r}_0, k) = k^2 g(r, k) \otimes_3 \gamma(\mathbf{r})u_i(\mathbf{r}, k)$$

This assumption provides an approximate solution (the Born approximation) for the scattered field which is valid if $k^2\|\gamma(\mathbf{r})\| << 1$. The result can be considered to be a first approximation to the (Born) series solution given by

$$u_s(\mathbf{r}, k) = u_i(\mathbf{r}, k) + k^2 g(r, k) \otimes_3 \gamma(\mathbf{r})u_i(\mathbf{r}, k)$$

$$+k^4 g(r, k) \otimes_3 \gamma(\mathbf{r})[g(r) \otimes_3 \gamma(\mathbf{r})u_i(\mathbf{r}, k)] + ...$$

which is valid under the condition $k^2\|\gamma(\mathbf{r})\| < 1$. Each term in this series expresses the effects due to single, double and triple etc. scattering events and because this series scales as k^2, k^4, k^6, ..., for $k << 1$ (i.e. low frequency wavefields), the Born approximation becomes an exact solution.

3 Low Frequency Scattering

If a Helmholtz wavefield oscillates at lower and lower frequencies, then we can consider an asymptotic solution of the form

$$u_s(\mathbf{r}, k) = \frac{k^2}{4\pi r} \otimes_3 \gamma(\mathbf{r})u_i(\mathbf{r}, k), \quad k \to 0.$$

This is a consequence of the fact that the higher order terms in the Born series can be ignored leaving just the first term as $k \to 0$ and because

$$\frac{\exp(ikr)}{4\pi r} = \frac{1}{4\pi r}, \quad k \to 0$$

giving an exact solution to the problem.

If the incident field is a unit plane wave, then

$$u(\mathbf{r}, k) = 1 + u_s(\mathbf{r}, k)$$

where

$$u_s(\mathbf{r}, k) = \frac{k^2}{4\pi r} \otimes_3 \gamma(\mathbf{r}), \quad k \to 0$$

Here, the wavelength of the incident plane wavefield is assumed to be significantly larger than the spatial extent V of the scatterer. For a given scattering function $\gamma(\mathbf{r})$ the wavefield is a 'weak field' because of the low values of k required to produce this (asymptotic) result. But this result is the general solution to Poisson's equation

$$\nabla^2 u_s(\mathbf{r}, k) = -k^2 \gamma(\mathbf{r})$$

since, from equation (2), we have

$$\nabla^2 u = \nabla^2 u_s = k^2 \nabla^2 \left(\frac{1}{4\pi r} \otimes_3 \gamma \right) = -k^2 \gamma.$$

By considering u_s to be a potential, we can write

$$\nabla \cdot \mathbf{U}_s(\mathbf{r}, k) = k^2 \gamma(\mathbf{r}), \quad \mathbf{U}_s(\mathbf{r}, k) = -\nabla u_s(\mathbf{r}, k).$$

Integrating over the volume of the scatterer V and using the divergence theorem, we can write

$$\oint\limits_S \mathbf{U}_s(\mathbf{r}, k) \cdot \hat{\mathbf{n}} d^2\mathbf{r} = k^2 \Gamma, \quad \Gamma = \int\limits_V \gamma(\mathbf{r}) d^3\mathbf{r}.$$

If we now consider a scatterer that is a sphere, then the field $\mathbf{U}$ will have radial symmetry, i.e. $\mathbf{U}_s = \hat{\mathbf{n}} U_s$. In this case, the surface integral becomes $4\pi r^2 U_s$ and we obtain

$$U_s = \frac{k^2 \Gamma}{4\pi r^2}, \quad k \to 0.$$

Hence, in the limit as $k \to 0$, Helmholtz scattering provides an exact solution for a weak scattered field whose gradient (for the radially symmetric case) is characterized by a $1/r^2$ scaling law.

4 Diffraction from a Low Frequency Scattered Field

For $k \to 0$, $u_s(\mathbf{r}, k)$, which we now denote by $u_s^0(\mathbf{r}, k_0)$, is the solution to

$$\nabla^2 u_s^0(\mathbf{r}, k_0) = -k_0^2 \gamma(\mathbf{r})$$

where k_0 denotes a value for k, $k \to 0$. Consider a Born scattered Helmholtz wavefield $u_s(\mathbf{r}, k)$ for $k >> 1$ given by

$$u_s(\mathbf{r}, k) = k^2 g(r, k) \otimes_3 \gamma(\mathbf{r}) u_i(\mathbf{r}, k)$$

We can then write

$$u_s(\mathbf{r}, k) = -\frac{k^2}{k_0^2} g(r, k) \otimes_3 u_i(\mathbf{r}, k) [\nabla^2 u_s^0(\mathbf{r}, k_0)]$$

from which we can derive an expression for the far field scattering amplitude generated by the field $\mathbf{U}_s^0$ given by

$$u_s(\mathbf{r}, k) = -\frac{k^2}{k_0^2} g(r, k) \otimes_3 u_i(\mathbf{r}, k) [\nabla \cdot \mathbf{U}_s^0(\mathbf{r}, k_0)]$$

$$= \frac{\exp(ikr_0)}{4\pi r_0} A(\hat{\mathbf{n}}_0, \hat{\mathbf{n}}_i), \quad \frac{r}{r_0} << 1$$

where, with $u_i(\mathbf{r}, k) = \exp(ik\hat{\mathbf{n}}_i \cdot \mathbf{r})$, writing $\hat{\mathbf{n}}_0 = \mathbf{r}_0 / \mid \mathbf{r}_0 \mid$ and with

$$\mathbf{U}_s^0 = \hat{\mathbf{n}} U_s^0 = \hat{\mathbf{n}} \frac{k_0^2 \Gamma}{4\pi r^2}$$

we obtain

$$A(\hat{\mathbf{n}}_0, \hat{\mathbf{n}}_i) =$$

$$-\frac{k^2 \Gamma}{4\pi} \int\limits_V \exp[-ik(\hat{\mathbf{n}}_0 - \hat{\mathbf{n}}_i) \cdot \mathbf{r}] \nabla \cdot \left(\frac{\hat{\mathbf{n}}}{r^2} \right) d^3\mathbf{r}$$

Hence, the wavefield $u_s(\mathbf{r}, k)$ (for $k >> 1$) generated by a scatterer that is simultaneously generating a scattered wavefield $u_s^0(\mathbf{r}, k_0)$ is, in the far field (under the Born approximation) determined by the Fourier transform of the scattering function (assuming radial symmetry) $f(r) = \nabla \cdot (\hat{\mathbf{n}} r^{-2})$. In other words, the weak field generated by very low frequency scattering will diffract a high frequency Helmholtz wavefield, the diffraction pattern (i.e. the far field scattering pattern) being determined by $f(r)$.

4.1 Diffraction by an Infinitely Thin Scatterer

Consider the case where an incident plane wavefield is travelling in the z-direction, i.e. $u_i = \exp(ikz)$ and is incident on an infinitely thin scatterer defined by the function $\gamma(\mathbf{r}) = \gamma(x, y)\delta(z)$. The scattered wavefield is then given by

$$u_s(x, y, z, k) = k^2 \frac{\exp(ik\sqrt{x^2 + y^2 + z^2}}{4\pi \sqrt{x^2 + y^2 + z^2}} \otimes_2 \gamma$$

where $\otimes_2$ denotes the two-dimensional convolution integral over area S and $\gamma(x,y) \ \exists \ \forall(x,y) \in S$. Writing out this result in the form

$$u_s(x_0,y_0,z_0,k) = k^2 \int\int dxdy$$

$$\frac{\exp[ik\sqrt{(x-x_0)^2+(y-y_0)^2+z_0^2}]}{4\pi\sqrt{(x-x_0)^2+(y-y_0)^2+z_0^2}}\gamma(x,y)$$

it is clear that if the scattered wavefield is now measured in the far field, i.e. for the case when $x/z_0 << 1$ and $y/z_0 << 1$, then

$$z_0\left(1+\frac{(x-x_0)^2}{z_0^2}+\frac{(y-y_0)^2}{z_0^2}\right)^{\frac{1}{2}}$$

$$\simeq z_0 - \frac{xx_0}{z_0} - \frac{yy_0}{z_0} + \frac{x_0^2}{2z_0} + \frac{y_0^2}{2z_0}$$

and thus,

$$u_s(x_0,y_0,z_0,k) =$$

$$\frac{\exp(ikz_0)}{4\pi z_0}\exp\left(ik\frac{x_0^2+y_0^2}{2z_0}\right)A(u,v)$$

where

$$A(u,v) = k^2\widetilde{\gamma}(u,v) = k^2\mathcal{F}_2[\gamma(x,y)]$$

$$= k^2\int\int \exp(-iux)\exp(-ivy)\gamma(x,y)dxdy$$

with spatial frequencies u and v being defined by

$$u = \frac{kx_0}{z_0} = \frac{2\pi x_0}{\lambda z_0} \quad \text{and} \quad v = \frac{ky_0}{z_0} = \frac{2\pi y_0}{\lambda z_0}$$

Here, $\mathcal{F}_2$ denotes the two-dimensional Fourier transform operator, the result being the standard expression for a diffraction pattern in the far field or Fraunhofer zone [13] and has been derived in preparation to the following section.

4.2 Diffraction by an Infinitely Thin Field

In the previous section, we derived the far field diffraction pattern for an infinitely thin scatterer. However, suppose this scatterer also radiates a field generated by low frequency Helmholtz scattering from the same scattering function. What is the contribution of this field to the diffraction of the same incident plane wave within and beyond the extent of the scatterer given that the scattered wavefield u_s^0 is taken to exist within and beyond the finite spatial extent of the scatterer $\gamma(\mathbf{r}), \mathbf{r} \in V$ (i.e. u_s^0 is not of compact support as it is given by the convolution of a function of compact support with r^{-1})? In this case, the scattered wavefield is given by (under the Born approximation)

$$u_s = -\frac{k^2}{k_0^2}g \otimes_3 u_i\nabla^2 u_s^0, \quad u_s^0 = \frac{k_0^2}{4\pi r}\otimes_3 \gamma.$$

For an infinitely thin scatterer given by $\gamma(x,y)\delta(z)$,

$$u_s^0(x,y,z,k_0) = \frac{k_0^2}{4\pi\sqrt{x^2+y^2+z^2}}\otimes_2 \gamma(x,y)$$

so that in the (x,y) plane located at $z=0$,

$$u_s^0(x,y,k_0) = \frac{k_0^2}{4\pi\sqrt{x^2+y^2}}\otimes_2 \gamma(x,y).$$

For an incident plane wave $u_i = \exp(ikz)$, the scattered wavefield u_s is thus, given by

$$u_s(x,y,z,k) = -k^2 g(r,k)\otimes_3 \exp(ikz)\times$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left(\frac{1}{4\pi\sqrt{x^2+y^2}}\otimes_2 \gamma(x,y)\right).$$

Repeating the calculation given in the previous section (for $z \to 0$), the diffracted wavefield now becomes

$$u_s(x_0,y_0,z_0,k) =$$

$$\frac{\exp(ikz_0)}{4\pi z_0}\exp\left(ik\frac{x_0^2+y_0^2}{2z_0}\right)A(u,v)$$

where

$$A(u,v) = -2zk^2$$

$$\times\mathcal{F}_2\left[\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\frac{1}{4\pi\sqrt{x^2+y^2}}\otimes_2 \gamma(x,y)\right].$$

Note that although the scatterer is taken to be 'infinitely thin' because $\gamma(\mathbf{r}) = \gamma(x,y)\delta(z)$, we still consider the physical thickness of the scatterer to be finite, i.e. $z \neq 0$ (z being taken to be a positive real 'infinitesimal' for all real k). Now, for an arbitrary function $f \leftrightarrow \widetilde{f}$, where $\leftrightarrow$ denotes the transform from real space to Fourier space,

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)f \leftrightarrow -(u^2+v^2)\widetilde{f},$$

$$\frac{1}{\sqrt{x^2+y^2}} \leftrightarrow \frac{2\pi}{\sqrt{u^2+v^2}},$$

and we obtain

$$A(u,v) = zk^2\sqrt{u^2+v^2}\widetilde{\gamma}(u,v).$$

Consider a Gaussian diffactor (a unit amplitude Gaussian function with standard deviation σ) given by

$$\gamma(r) = \exp(-r^2/\sigma^2), \quad r = \sqrt{x^2 + y^2}$$

Figure 5 shows numerical simulations of the diffraction patterns compounded in the (intensity) functions

$$\mid \widetilde{\gamma}(u,v) \mid^2 \quad \text{and} \quad (u^2 + v^2) \mid \widetilde{\gamma}(u,v) \mid^2$$

using a two-dimensional Discrete Fourier Transform. The analytical solutions for the intensity

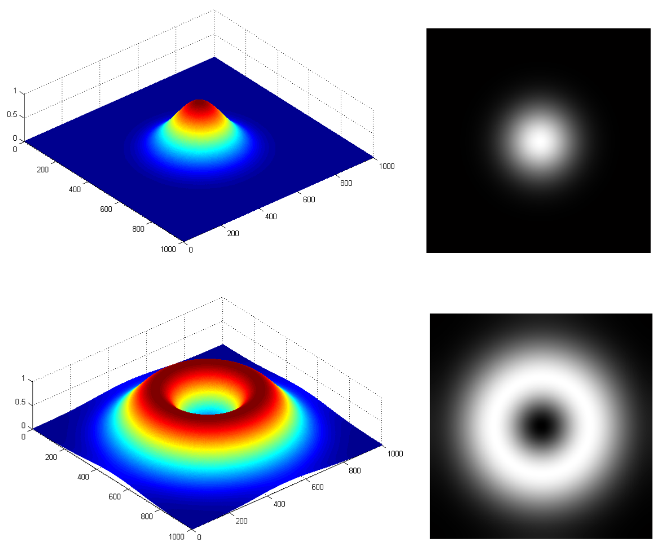

Figure 5: Numerical simulation of the intensity patterns generated by a Gaussian diffractor with $\sigma = 1.5$ for a 1000×1000 regular mesh. Top: Surface plot (left) and image (right) for $\mid \widetilde{\gamma}(u,v) \mid^2$; Bottom: Surface plot (left) and image (right) for $(u^2 + v^2) \mid \widetilde{\gamma}(u,v) \mid^2$

$\mid u_s \mid^2$ generated by diffraction from the scatterer γ and diffraction from the field $\nabla^2 u_s^0$ are by

$$I_1(r_0, \lambda) = \frac{\pi^4 \sigma^2}{z_0^2 \lambda^4} \exp\left[- \left(\frac{2\pi^2 \sigma^2 r_0^2}{\lambda^2 z_0^2} \right) \right]$$

and

$$I_2(r_0, \lambda) = z^2 \frac{4\pi^6 \sigma^2 r_0^2}{z_0^4 \lambda^6} \exp\left[- \left(\frac{2\pi^2 \sigma^2 r_0^2}{\lambda^2 z_0^2} \right) \right]$$

respectively. Note that the diffraction for a scattering function produces a pattern whose intensity peaks at the centre of the image plane (a standard result in Fourier optics) but that diffraction from a low frequency scattered field produces a pattern characterised by a ring which has a maximum when $r_0 = z_0 \lambda / (\sqrt{2} \pi \sigma)$ as illustrated in Figure 5. Also, observe that the magnitude of the intensity patterns generated by the field $\nabla^2 u_s^0$ is significantly less than that generated by scatterer γ, e.g.

$$\frac{I_2}{I_1} = \frac{4z^2 \pi^2 r_0^2}{z_0^2 \lambda^2}$$

and only if $r_0/\lambda \sim z/z_0$ will the magnitude become of the same order. However, with regard to the principal remit of this paper, note that the intensity generated by the scatterer γ scales as λ^{-4} whereas the intensity generated by the field $\nabla^2 u_s^0$ scales as λ^{-6}.

5 Colour Analysis

If we accept an Einstein ring to be a gravitational diffraction phenomena, then the intensity of the diffracted light scales as λ^{-6} which explains the colour of the rings (blue light having the shortest wavelength in the visible spectrum). This is analogous to the explanation of why the Earth's atmosphere is blue in colour. Under the Rayleigh scattering [17] condition in which the wavelength is significantly larger than the physical size of the scatterer (when the Born approximation is valid), the scattering amplitude becomes independent of the scattering angle and the intensity of the scattered field is proportional to λ^{-4}. Thus, the sky is blue, because sunlight is scattered by the electrons of air molecules in the terrestrial atmosphere generating blue light preferentially around in all directions [18]. Further, as the Sun approaches the horizon, we have to look more and more diagonally through the Earth's atmosphere. Our line of sight through the atmosphere is then longer and most of the blue light is scattered out before it reaches us, especially as the Sun gets very near the horizon. Relatively more red light reaches us, accounting for the reddish colour of sunsets.

The λ^{-6} scaling law associated with gravitational diffraction provides a method of validating or otherwise the theoretical model presented in this paper. We require a scenario in which the same Einstein ring is recorded simultaneously over a broad frequency spectrum (e.g. using radio, infrared, visible and ultraviolet imaging) in such a way that the intensities of each image (relative to a known source that can be used for calibration) can be compared on a quantitative basis. However, data available to undertake such an analysis are not yet available. Instead, another approach is considered based on the colour generated by light scattered under different conditions. For Tyndall scattering [19], [20] the intensity of light is proportional to λ^{-2} and for Rayleigh scattering , the light scattered intensity is proportional to λ^{-4}. Because of these wavelength scaling relationships, both Tyndall and Rayleigh scattering generate blue light. However, Tyndall scattering can be expected to generate a lighter blue than Rayleigh scattering. This is illustrated in Figure 6 which also shows, for comparison, the colour of the blue light scattered by a gravitational field which is proportionately darker because of the scaling relationship characterised by λ^{-6}. This comparison is quantified in Figure 7 which shows the differences in the lightness of blue using a Hue, Saturation and Lightness (HSL) colour model. The lightness factor associated with each image is characterised by the ratios 1:2:3 which is in agreement with the logarithmic scaling ratios $2\ln\lambda : 4\ln\lambda : 6\ln\lambda$.

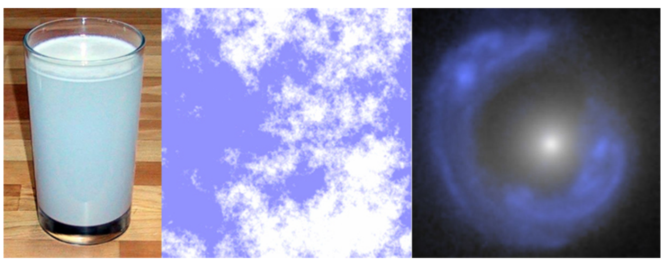

Figure 6: Examples of the differences in the lightness (for a HSL - Hue, Saturation and Lightness - colour model) of the blue light generated by Tyndall scattering (scattering of light by fine flour suspended in water - left), Rayleigh scattering (scattering of light by the atmosphere - centre) and 'gravitational scattering' (diffraction of light by the gravitation field generated by a galaxy - right).

Figure 7: The 'blues' associated with Tyndall (left), Rayleigh (centre) and gravitational (right) light scattering obtained by averaging over many images of each effect.

6 Gravitational Diffraction

An Einstein ring is an effect that is conventionally explained in terms of the bending of light through the curvature of space (and time) by a mass. This is a consequence of the field equations for a gravitational field. In order to obtain an Einstein ring, the magnitude of the gravitational field must be relatively high such as that generated by a spiral galaxy. Further, in order to generate a near perfect (complete) ring, the entire galaxy must be well aligned with regard to an observer in the 'object plane'. The bending of light by a gravitational field has an analogy with the geometrical interpretation of light interacting with a lens. At the edge of a lens, the light beam is 'bent' (discontinuously) by the change in refractive index from air to glass and from glass to air - the extreme edge of a lens acts like a prism. Like an optical lens, gravitational 'lensing' will produce distortions of the object plane when alignment of the 'earth-lens-object' is imperfect.

If we interpret an Einstein ring in terms of the results given in Section 4, then the ring is not due to light being bent (continuously) by the curvature of a space-time continuum but the result of the diffraction of a plane wave (i.e. light) by the field $\nabla^2 u_s^0$ which is taken to be in the plane of the galaxy and to extend beyond it. This requires the magnitude of the scattering function to be very large in order to compensate for $z \to 0$. If we model a (spiral) galaxy in terms of a Gaussian function, then the ring associated with the diffraction pattern given in Figure 5 is, in this sense, a simulation of a complete Einstein ring. The use of a Gaussian function to model the macroscopic gravitational field generated by a spiral galaxy is intuitive as the edges of a galaxy will not be discontinuous (especially on the scale of the wavelength of light!). However, in the case of a black hole, the event horizon defines an edge. In such a case, we might expect gravitational diffraction to produce a number of concentric rings, the black hole being modelled in terms of an opaque disc. Figure 8 shows a double ring pattern which is taken to be generated by the light from three galaxies at distances of 3, 6 and 11 billion light years [21]. The double ring structure is explained in terms of the light from two distant galaxies behind a foreground massive galaxy. The problem with this explanation is that there does not appear to be a foreground galaxy in the image! Multiple ring patterns associated with a black

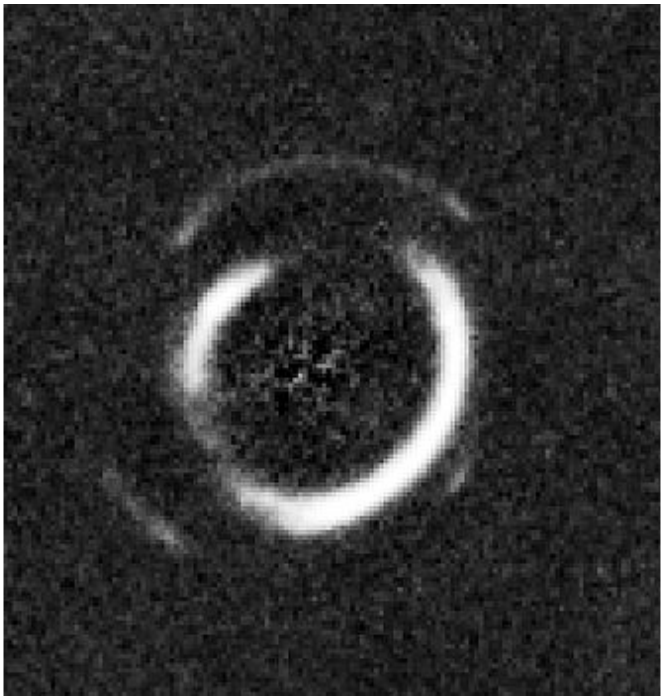

Figure 8: A double Einstein ring pattern taken to be caused by the complex bending of light from two distant galaxies behind a foreground massive galaxy [21].

hole are a prediction of the conventional bending of light by space-time curvature [22]. The idea is that, close to the event horizon, the gravitational field is so intense that light can be curved right around the black hole by 180 degrees or more to produce a ring associated with the light generated by an object that exists in alignment with, and behind, the image plane. These multiple Einstein ring predictions are based on arguments analogous to geometric optics whereas the multiple rings considered here are analogous to Fourier optics. In this sense, we are interpreting a gravitational field to be generated by the scattering of a long wavelength Helmholtz wavefield, i.e. the field $\mathbf{U}_s^0$ defines a 'gravitational field'. Figure 9 shows an example of a simulated double ring pattern based on the theory presented in Section 4.2 given by $(u^2+v^2) \mid \widetilde{\gamma}(u,v) \mid^2$ where γ is given by the opaque disc function

$$\gamma(r) = \begin{cases} 0, & r \leq a; \\ 1, & \text{otherwise.} \end{cases}$$

for $a = 6$ computed on a 1000×1000 element grid.

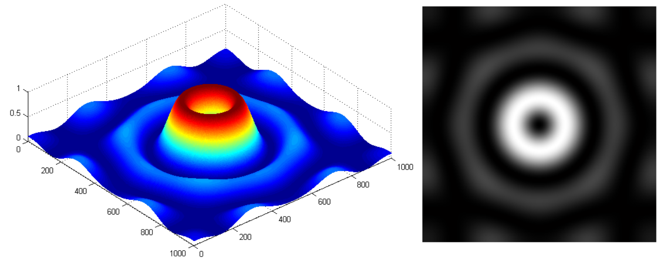

Figure 9: Simulation of a double Einstein ring based on the gravitational diffraction theory of light. Surface plot (left) and associated grey scale image (right).

7 Discussion

The results developed in this paper encapsulate a phenomenology where the Helmholtz equation is, in effect, being used in an attempt to develop a unified scalar wavefield theory where the wavefield u is taken to exist over a broad range of frequencies. At intermediate frequencies, u is taken to describe waves in the 'electromagnetic spectrum' and at low frequencies, u is taken to describe waves in the 'gravity wave spectrum'. Low frequency waves (gravity generating waves) are scattered by high frequency waves (matter waves) to produce a gravitational field; intermediate frequency waves (electromagnetic spectrum) are scattered by high frequency waves (e.g. a lens) but can also be scattered by the field generated from the scattering of low frequency waves to produce gravitational diffraction thus accounting for the blueness of Einstein rings observed in the visible spectrum.

In general relativity, the curvature of space-time bends light by the same amount irrespective of the frequency - there is no dispersion relation. The λ^{-6} scaling law associated with gravitational diffraction may be validated (or otherwise) from appropriate simultaneous observations of the same Einstein ring (complete or otherwise) at different wavelengths. Other consequences such as a gravitational field generating a repulsive force that is proportional to the mass squared in the relativistic case remain of theoretical consequence only. However, it is noted that inflation theory (the expansion of the early universe) requires gravity to be a repulsive force.

The model considered leads to the proposition that a gravity field is regenerative and exists through the continuous scattering of existing low frequency Helmholtz wavefields. This proposition may provide an answer to the following question: If nothing can escape the event horizon of a black hole because nothing can propagate faster than light then how does gravity get out of a black hole? The conventional answer to this question is that the field around a black hole is 'frozen' into the surrounding space-time prior to the collapse of the parent star behind the event horizon and remains in that state ever after. This implies that there is no need for continual regeneration of the external field by causal agents. In other words, the explanation defies causality. In the model presented here, the gravitational field generated by a black hole or any other body is the result of a causal effect - the scattering of low frequency scalar waves. In this sense, a black hole is just a stronger scatterer than other cosmological bodies and a gravitational field 'gets out of a black hole' because it was never 'in the black hole' to start with.

7.1 Propagative Theories

Propagative or wave theories of gravity have been proposed for many years. In 1805, Laplace proposed that gravity is a propagative effect and considered a correction to Newton's law to take into account the observation that gravity has no detectable aberration or propagation delay for its action. Laplace's ideas were advanced further by Weber, Riemann, Gauss and Maxwell in the Nineteenth Century using a variety of 'corrective terms'. In 1898, Gerber, developed a propagative theory that took into account the perihelion advance of mercury and in 1906 Poincaré showed that the Lorentz transform cancels out gravitational aberration. After the success of general relativity (1916) for explaining gravity in terms of a geometric effect, propagation theories were discarded. However, more recently, attempts at explaining gravity in terms of causal effects through a 'propagative' force have been revisited [23] as debate over the basic Einsteinian postulates (i.e. the invariance of the propagation of light in a vacuum for any observer which amounts to a presumed absence of any preferred reference frame) has intensified. Moreover, from Laplace to the present, propagation theories of gravity consider an object to be 'radiating' a field (in a passive sense). If general relativity considers gravity to be the result of an object warping space-time, then the proposition reported is that gravity is the result of an object scattering (long wavelength) waves that already exist as part of the low frequency component of a universal spectrum which is, itself, the by-product of the 'big-bang'.

7.2 Compatibility with General Relativity

The compatibility of this approach with general relativity can be realised if the wavefield as taken to warp space-time so that space-time is the medium of propagation. Only at very large wavelengths does the warping of space-time become so pronounced and over such a large scale that Einstein's field equations can then be used to describe the physics associated with the geometry of the field. In other words, if space-time is taken to be the medium of propagation of all (scalar) wavefields at all frequencies, then the theory of general relativity emerges naturally as $k \to 0$. A two-dimensional and qualitative illustration of this idea is given in Figure 10 which shows four frames of a simple two-dimensional wave function as $k \to 0$. It is assumed that the wavefunction is due to the scattering of a plane wave from a delta function located at the centre of the surface. If space is taken to be the medium of propagation which undergoes curvature as a wave propagates through it then Figure 10 can be taken to illustrate the curvature of a two-dimensional space into a three dimensional space at increasingly lower frequencies. As $k \to 0$ the wavefield is replaced by what appears to be a static curved space manifold within the locality of a low frequency scattering event. The curvature of this manifold is the taken to be responsible for generating a gravitation force which is attractive in terms of the influence of one mass upon another and is compounded in terms of Einstein's field equation, i.e.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu}\Lambda = \frac{8\pi}{c^4} T_{\mu\nu}$$

where $R_{\mu\nu}$ is the Ricci curvature tensor, R is the scalar curvature, $g_{\mu\nu}$ is the metric tensor, Λ is the consmological constant, G is the gravitational constant, c is the speed of light and $T_{\mu\nu}$ is the stress-energy tensor [15].

8 Conclusion

Any propagation/scattering theory of gravity must address some basic known observations:

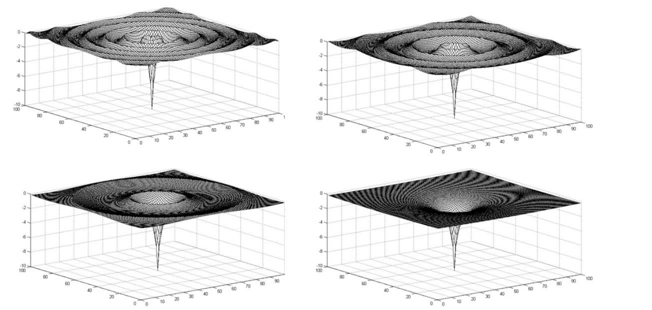

Figure 10: Qualitative illustration of the function $-\text{Re}[\cos(kr)/r], \quad r = \sqrt{x^2+y^2}$ for four frames as $k \to 0$ (from left to right and from top to bottom).

- **Gravity has no detectable aberration or propagation delay for its action leading to effects predicted by general relativity such a** *gravito-magnetism*;

- **the finite propagation of light causes radiation pressure for which gravity has no counterpart pressure.**

These results represent the most vital evidence with regard to gravity being a geometric and not a propagative effect. For example, in an eclipse of the Sun, the gravitational pull on the earth by this 3-body (Sun-Moon-Earth) configuration increases. By comparing the delay in time it takes to observe the visible maximum eclipse on Earth (which can be calculated from knowledge of the distance of the Moon from the Earth) with the equivalent gravitational maximum, then if gravity is a propagating force, it appears to propagates at least 20 times faster than light! [24] Irrespective of whether this value is valid or not, a fundamental issue remains, which is compounded in the question: what is the speed of gravity? If we consider gravity to be a propagation and/or a low frequency scattering effect, then in order to account for the lack of propagation delay, it must be assumed that the speed of gravity is greater than the speed of light. This is contrary to the Einsteinian postulates if these postulates are taken to apply to all wavefields irrespective of their wavelength. The model presented here assumes that the speed of gravity is the same as the speed of light c_0. However, the asymptotic result $k \to 0$ used to define a gravitational field yields, what will appears to be, an instantaneous effect from a wavefield that is taken to propagate at the speed of light. The wavelength is so long compared to the distances associated with a Sun-Moon-Earth system, for example, that the speed of gravity will appear to be significantly faster than the speed of light (i.e. $\mathbf{U}_s^0$ is observed to be an instantaneous field).

In general relativity, the curvature of space-time bends light by the same amount irrespective of the frequency, i.e. there is no dispersion relation. If general relativity considers gravity to be the result of an object warping space-time, then the proposition reported in this paper is that gravity is the result of an object scattering long wavelength waves. The compatibility of this idea with general relativity can be realised if the wavefield as taken to warp space-time so that space-time is the medium of propagation. Only at very large wavelengths does the warping of space-time become so pronounced and over such a large scale that Einstein's field equations can then be used to describe the 'geometry' of the field. In other words, if space-time is taken to be the medium of propagation of all (scalar) wavefields at all frequencies, then Einstein's equations 'emerge' as $k \to 0$. A two-dimensional and qualitative illustration of this idea is given in Figure 10 which shows four frames of a simple two-dimensional wave function as $k \to 0$. It is assumed that the wavefunction is due to the scattering of a plane wave from a two-dimensional delta function located at the centre of the surface. If space is taken to be the medium of propagation which undergoes curvature as a wave propagates through it then Figure 10 can be taken to illustrate the curvature of a two-dimensional space into a three dimensional space at increasingly lower frequencies. As $k \to 0$ the wavefield is replaced by what appears to be a static curved space manifold within the locality of a low frequency scattering event. The curvature of this manifold is the taken to be responsible for generating a apparent gravitational force. This idea is validated through the explanation as to why an Einstein ring is blue, a by-product of the proposition that gravity is a low frequency scattering effect in which space-time is the medium of propagation and scattering.

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