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Microtonal Systems and Guitar Composition

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Dublin Institute of Technology

School of Physics

MICROTONAL SYSTEMS AND GUITAR COMPOSITION

MICHAEL NIELSEN LTCL

OCTOBER 2003

Thesis submitted for the degree of MPhil

Supervisors: **Patrick Healy** BSc MSc HDip (Ed)
Ita Beausang BMus MA PhD LRAM

I certify that this thesis which I now submit for examination for the award of MPhil, is entirely my own work and has not been taken from the work of others save and to the extent that such work has been cited and acknowledged within the text of my work.

This thesis was prepared according to the regulations for postgraduate studies by research of the Dublin Institute of Technology and has not been submitted in whole or in part for an award in any other Institute or University.

Signature Michael Nielsen Date 9/2/2004

Table of Contents

Acknowledgements

I would like to thank my supervisors at DIT Kevin St., Physics Lab., Mr. Paddy Healy and Dr. James Walsh, my supervisor at DIT Conservatory of Music and Drama, Dr. Ita Beausang, and Mr. Paul McGettrick, my initial supervisor at DIT Rathmines, Music Technology Centre, for their interest, support and knowledge throughout the duration of this project.

I am grateful to the Head of Physics, Dr. Vincent Toal, and the previous Head of Music, Dr. Eibhlis Farrell, for their support and encouragement. I wish also to thank instrument-maker Fitz Howard for his excellent guitar work and information. I am indebted to Ciarán O'Connell, lecturer in flute at the DIT Conservatory of Music and Drama, Chatham Row, who initially proposed the idea of postgraduate study to me.

Abstract

This thesis focuses on the use of microtones in guitar composition. In the course of this study the physical aspects of musical sound (pitch, loudness and quality/timbre), vibrational frequencies of strings, and the theory of musical scales and tuning systems are discussed. Whole-number ratios corresponding to musical intervals within the twelve-note/equal temperament scale are calculated in relation to the overtone series for the purpose of surveying consonance and dissonance within the tempered system. These are compared to the ratios for corresponding intervals within the overtone series. Whole-number ratios are also calculated for intervals involving the microtones referred to in this study.

The frequencies of the microtones are theoretically predicted and experimentally measured using a high-precision sound level meter and frequency analyser. A brief analysis of the consonant and dissonant quality of five different intervals involving tempered and microtonal notes played on both electric and acoustic guitars is carried out on a computer using a Fast Fourier Transform algorithm. Twenty-three graphs were produced as a result of this experiment. A brief analysis of the consonant and dissonant quality of a pitch interval involving two microtones was also taken. Whole-number ratios were calculated for each experiment and compared to recognized consonant whole-number ratios within the overtone series.

Microtonal musical compositions, *Guitar Opus 1* and *Guitar Opus 2*, show how the measured microtones can be utilized in musical composition. The microtonal parts of the composition are discussed and illustrated in tabular format using the information gathered from the physical measurements. Whole-number ratios are calculated for part of the composition *Guitar Opus 1* and the consonant and dissonant quality of the intervals used are discussed. *Etude for Amplified Classical Guitar* illustrates how microtonal notes can be utilized in composition along with tempered notes of similar loudness. In *Etude for Electric Guitar with added frets,* for modified electric guitar (11 extra frets added in between original frets), a melody is first played using tempered notes and then played as a microtonal melody by moving up or down a fret utilizing the added microtonal frets. The thesis concludes with a discussion of the use of different tuning systems in composition, and their relationship to the microtonal system used in the project.

Summary of Thesis

The search for a system of frequencies outside twelve-note/equal temperament and its application to guitar composition is the basis of this thesis. By the beginning of the twentieth century composers were looking beyond the limitations of the 'closed' twelve-note/equal temperament system of tuning and began to divide the semitone into smaller tempered divisions, for example, quarter tones. The tuning system of just intonation in which all of the intervals can be represented by whole-number frequency ratios began to appear through the work of Béla Bartók (1881-1945), Charles Ives (1874-1954), Harry Partch (1901-1974) and Lou Harrison (b 1917). This tuning system gives the composer an infinite number of frequencies to choose from and is based on the overtone series.

A microtone refers to any frequency that is not contained in the twelve-note/equal temperament system of tuning. Microtones feature in many types of folk music throughout the world. In ancient Greek music, micro-intervals called *'chroai'* were used. In Indian music *'srutis'* or intervals smaller than a semitone are used. Béla Bartók found divergences from the intervals of the diatonic scale in his study of folk-song in Hungary, Romania and Bulgaria.

In Irish traditional music microtonal changes in pitch are a normal part of the technique of instrumentalists and sean-nós singers who slide up to an important note through an interval that may be greater or less than a semitone. The seventh degree of the scale

with its varying pitch (either natural or sharp or between the two) often figures in slides.

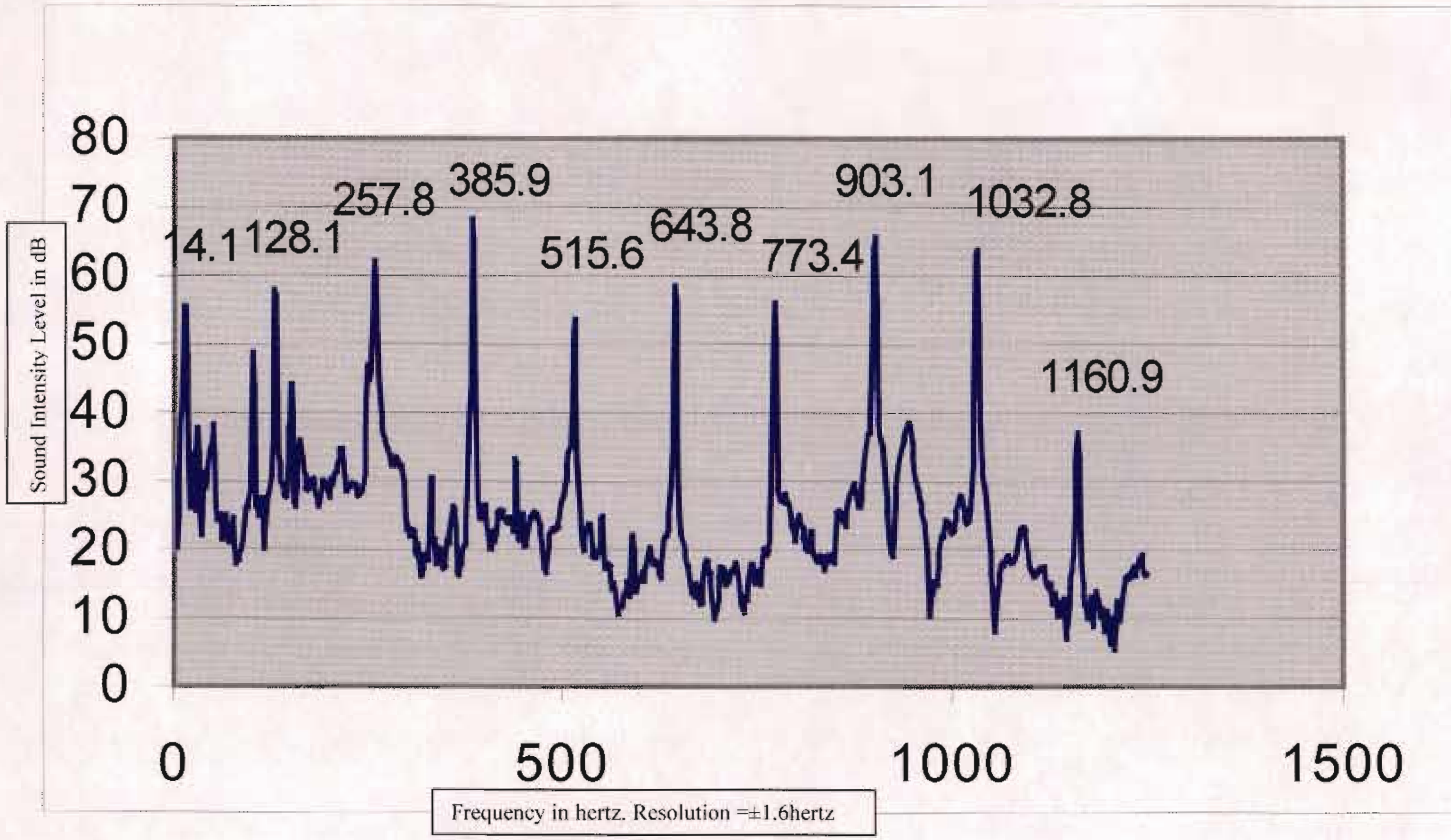
Microtonal tuning systems can be extracted from a traditionally built classical guitar and used in guitar composition by plucking the opposite side of a stopped string. The microtonal frequencies used in the project have been predicted mathematically and measured using a high precision sound level meter and frequency analyser. Accurate measurements to within two cents are essential as the ear can hear a change in pitch of 1/50 of a semitone and smaller. A frequency analysis of a single microtonal note is shown below in Figure 1.

The graph below is a Fast Fourier Transform (FFT) of a measured microtone in Hz. using a resolution of ± 1.6 hertz. The Fast Fourier Transform was carried out using a frequency analyser [Larson•Davis SLM and Frequency Analyzer, model 2800] and the data was imported into a Microsoft Excel Spreadsheet from which the graph was printed.

The graph shows peaks corresponding to the fundamental frequency (128.1 HZ) of the microtone and its harmonics. The harmonics are found by the doubling, trebling, et cetera of the fundamental. For example the 2nd harmonic (1st partial) is 128.1 hertz + 128.1 hertz and equals 256.2 hertz. The departure from exact harmonicity in the higher partials is expected due to string stiffness.
As in all FFT plots in this thesis, Sound Intensity Level is shown on the vertical axis. As explained in 1.6, p.17, this does not correspond accurately to the sensation of loudness.

This is particularly so at low frequencies such as the fundamental frequency of 128 hertz. In fact such maybe heard mainly through it's harmonics as explained in Chapter 4 of this thesis.

Figure1 Microtone frequency 128.3Hz as produced by a nylon string on acoustic guitar (19th fret, 6th string). The opposite side of a stopped note on the sixth string at the 19th fret was sounded. (A resolution of ± 1.6 hertz is used)



The twelve-note/equal temperament scale is the tempering of a system of pitch intervals obtained within the overtone series. There are twelve equidistant pitch intervals in the system. Whole-number ratios which reflect each pitch interval of the system, were calculated. The information is compared to 'special relationship' intervals-consonant pitch intervals within the overtone series, which are compared to the consonant/dissonant quality of the equally-tempered scale. Whole-number ratios were also calculated for the microtones used in the project.

A brief survey of the consonant and dissonant quality of five different intervals involving tempered and microtonal notes on electric guitar and acoustic guitar is carried out. Whole-number ratios are calculated for each pitch interval and twenty-three graphs are produced as a result of this experiment. Also, a brief survey of the consonant and dissonant quality of a pitch interval containing two microtones is carried out. Whole-number ratios are calculated and two graphs are produced.

The frequencies of the microtonal notes were measured on a Larson•Davis Sound Level Meter and Frequency Analyser. An electrical signal was produced from the pick-up under the nut, fed into an amplifier, and thence into the frequency analyzer. This allowed accurate measurements of the microtonal frequencies. The microtones were then employed in musical experimentation using three different compositional techniques as follows:

- *Guitar Opus 1, Guitar Opus 2* for classical guitar, utlizing plucked microtones from the opposite sides of a stopped string and tempered notes.
- *Etude for Amplified Classical Guitar* with pick-ups under the saddle and

nut, using hammer-on technique and utilizing both microtones and tempered notes

- *Etude for Electric Guitar with added frets* utilizing pentatonic tempered melody and microtonal melody as a basis for improvisation

The microtonal parts of each composition are discussed and illustrated in tabular format using the information gathered from the physical and mathematical measurements.

Three different guitars are used in this project. The initial microtonal system was extracted from a normal classical guitar by plucking the nut-side of stopped strings. A second classical guitar was fitted with two pick-ups, one under the bridge and one under the nut, allowing both normal pitches and microtonal pitches to be amplified. A third guitar, a 21-fret electric guitar, was fitted with eleven extra frets. Nine extra frets were added between the first 12 normal frets of the guitar and two were added above the 12th fret. These extra frets were measured by reversing the fret measurements of the fingerboard, i.e. the distance between the 21st and the 20th fret becomes the distance between the nut and the first fret, the distance between the 20th and the 19th fret is added as the second fret, the third fret is the original first fret, the 18th fret is added as the fourth fret, and so on. The frets were added in addition to the original frets and those frets that were too close to function, were omitted.

Whole-number ratios for pitch intervals containing a tempered pitch and microtone from part of the composition *Guitar Opus 1* are discussed in relation to the "special relationship" whole-numbers of the overtone series. Guitarists are normally restricted to using the twelve-note/equal temperament scale because of fixed frets. However, plucking the stopped string on the nut-side can produce microtonal notes. These microtones, which are related to the guitar's measurements, are readily available on a normal guitar. When a certain technique of 'hammering' a finger on to an open string is applied in normal guitar playing, both the bridge-side (normal frequency) and nut-side (microtonal frequency) notes sound together. Guitarists are therefore familiar with these sounds and adapt more easily to a microtonal system than would other instrumentalists.

The four compositions were recorded on CD and have also been performed live. A number of concerts have been given using the electric guitar with added frets in a quartet with saxophone, double bass and drums, within the framework of 1950's jazz music. The music of Charlie Parker was used to show that the playing technique was not hampered by the added frets and that microtones could be used in improvisation within this jazz style. *Etude for Electric Guitar with added frets* and *Etude for Amplified Classical Guitar* were arranged for quartet with saxophone, double bass and drums and recorded on CD.

Chapter 1 Theory

1.1 Sound

When longitudinal waves in air strike the ear, the listener experiences the sensation of *sound.* The human ear is sensitive to waves in the frequency range from about 20 hertz (vibrations per second) to 20,000 hertz, although the term *sound* is sometimes applied also to similar waves with frequencies outside the range of human audibility.[1] If a vibration is regular, the resulting sound is a musical note of definite pitch; if it is irregular the result is noise. Musical sounds are distinguished using three broad characteristics: *pitch, loudness* and *quality.*

1.2 Pitch

Highness and lowness of pitch depends on the frequency (number of vibrations per second) of the vibrating source. Frequency of vibration is measured in cycles per second called hertz. Distinguishing between the highness and lowness of a musical sound is called pitch perception. The pitch interval or musical interval between two vibrations, each of a single frequency depends on the ratio of the frequencies. All frequency ratios of 2:1 correspond to a pitch interval of one octave.

In equal temperament there are twelve notes or semitones in an octave. The ratio between all successive notes is $2^{\frac{1}{12}}$: 1 or 1.05946:1. The pitch intervals between all notes are identical in this scale. The pitch interval between successive notes or semitones is taken to be 100 cents on the equally-tempered scale.

1.3 Definition of Cents

If the note of frequency f_2 is n cents above the note of frequency f_1, then by definition

$$f_2 = f_1 \times \left(2^{\frac{1}{12}}\right)^{\frac{n}{100}}$$

$$f_2 = f_1 \times 1.05946^{\frac{n}{100}} \qquad \textbf{(1.1)}$$

$$f_2 = f_1 \times \left\{(1.05946)^{\frac{1}{100}}\right\}^n$$

$$log_{10} f_2 = log_{10} f_1 + log_{10} 1.05946^{\frac{n}{100}}$$

$$log_{10} f_2 = log_{10} f_1 + \frac{n}{100} log_{10} 1.05946$$

$$\frac{n}{100} log\, 1.05946 = log_{10} f_2 - log_{10} f_1$$

$$\frac{n}{100} = \frac{log_{10} f_2 - log_{10} f_1}{log_{10} 1.05946}$$

$$n = 100\left\{\frac{log_{10} f_2 - log_{10} f_1}{log_{10} 1.05946}\right\}$$

$$n = 100\left\{\frac{log_{10} f_2 - log_{10} f_1}{0.02508}\right\} \qquad \textbf{(1.2)}$$

For example -:

If $f_1 = 323.4Hz \quad f_2 = 329.6Hz$

$$n = \frac{100(2.51798 - 2.50974)}{0.02508} = \frac{0.8247}{0.02508}$$

= 32.9 *cents*

1.4 Vibrational Frequencies of Strings

The frequencies of the open strings of a guitar in hertz are: $6^{th}/E_2 = 82.4Hz$, $5^{th}/A_2 = 110Hz$, $4^{th}/D_3 = 146.8Hz$, $3^{rd}/G_3 = 196Hz$, $2^{nd}/B_3 = 246.9Hz$ and $1^{st}/E_4 = 329.6Hz$. The first string frequency is two octaves higher than the 6th string. The arrangement of these strings is illustrated in figure 1.1 below.

Figure 1.2 below illustrates a stopped string on the 6th fret. When the string is plucked on the nut-side a non-standard note or microtone is sounded. When a string is stopped at the 6th fret and plucked on the nut-side, the string leans against the fret wire of the 5th fret.

Figure 1.1 Guitar String Names and Frequencies in hertz

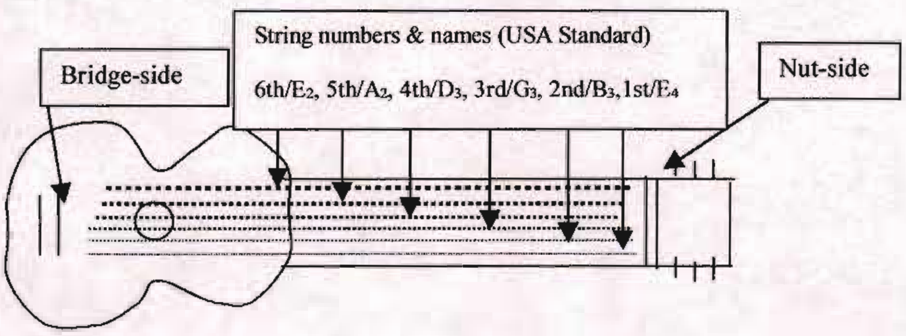
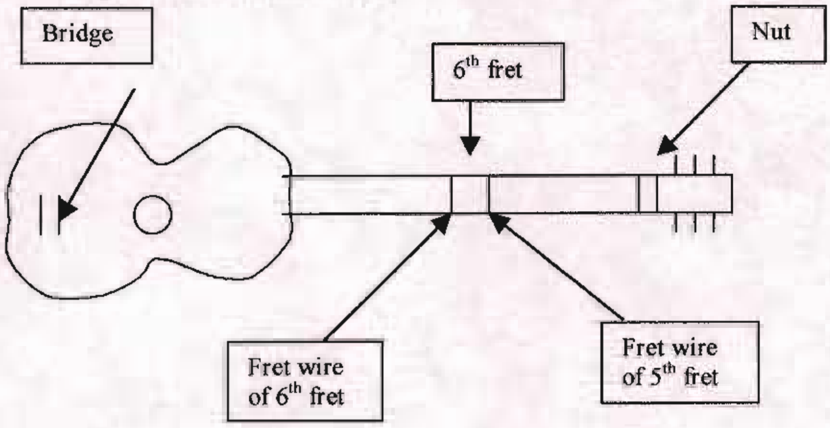


Figure 1.2 A Stopped Microtone at the Sixth Fret



1.5 Inverse Proportion Formula

A microtone is a note outside twelve-note/equal temperament scale. Such notes can be produced by plucking stopped strings on the nut-side rather than on the conventional bridge-side.

An idealized string, with constant length and tension and having no stiffness, has a fundamental frequency of vibration:

$$f_1 = \frac{1}{2l_1}\sqrt{\frac{T}{\mu}}$$ Where

f_1 = Fundamental frequency in hertz

l_1 = Length of string between the two fixed points in metres

T = Tension of string in newtons.

= Mass per unit length of string in kg/m

Let f'_1 = Fundamental frequency of the microtone played on the nut-side of the stopped string.

$$f'_1 = \frac{1}{2l'_1}\sqrt{\frac{T}{\mu}}$$

Where l'_1 is the vibrating length of the string producing the microtone.

T and μ are the same for both the bridge-side note and the nut-side note.

$$\frac{f'_1}{f_1} = \frac{l_1}{l'_1}$$

For example for the bridge-side note of frequency 116.5Hz

l_1 = 459.63mm

$l'_1 = 163.04\text{mm}$

$$f'_1 = f_1 \times \frac{l_1}{l'_1}$$

$$f'_1 = 116.5 \times \frac{459.63}{163.04}$$

$= 328.5\text{Hz}$

This is the predicted frequency of the nut-side note or microtone.

In this manner, the microtonal frequencies in Table 1.1 below were calculated.

Table 1.1 Predicted Microtonal Frequencies

Strings ▶	6th	5th	4th	3rd	2nd	1st
Frets no.* ▼	Frequency /Hz					
19	127.5	170.2	227.1	303.2	381.9	509.9
18	131.8	175.9	234.7	313.4	395	527
17	136.5	182.2	243.1	324.7	409	546
16	142.2	190	253.3	338.2	426	568.7
15	148.6	198.4	264.7	353.5	445.2	594.4
14	156	208.3	278	371.2	467.6	624.2
13	164.8	220	293.6	392	493.8	659.2
12	175.2	234	312.2	416.8	525	700.9
11	187.8	250.7	334.6	446.7	562.7	751.2
10	203.3	271.3	362.1	483.5	609.1	813.1
9	222.7	297.3	396.7	529.7	667.3	890.8
8	247.8	330.8	441.4	589.4	742.4	991.1
7	281.3	375.6	501.2	669.2	843	1125.4
6	328.5	438.5	585.2	781.4	984.3	1314
5	399.4	533.2	711.6	950.1	1196.9	1597.7
4	518	691.4	922.7	1232	1551.9	2464
3	755.3	1008.3	1345.6	1796.6	2263.1	3021.2
2	1468.21	1960	2615.5	3492.3	4399.3	5872.8

* The frets are numbered in accordance with normal fret numbering

In Table 1.1 above the first row corresponds to predicted microtonal frequencies produced by stopping strings on the 19th fret and plucking on the nut-side. The portion of the string on the nut-side leans on the 18th fret. The frequencies of the microtones were calculated by reference to the frequencies of the bridge-side notes on the equal temperament scale.

This calculation assumes that the guitar string when vibrating has constant length and tension and zero stiffness. Because these assumptions are not exactly true in practice, the measured frequencies of the microtones will deviate from the predicted values.

The plucked string, in fact, simultaneously produces notes of a number of frequencies given approximately by:

$$f_n = \frac{n}{2l}\sqrt{\frac{T}{\mu}}$$

Where n = 1,2,3,4,5,6, etc.

The notes of higher frequency than the fundamental are called harmonics.

1.6 Loudness

The loudness of a note at a place depends on the amplitude of vibration of a layer of air at that place and also on the frequency of the vibration. Loudness usually increases with intensity, but because of the varying sensitivity of the ear with frequency, there is no simple relation between the two.[2] Intensity is the quantity of sound energy crossing unit area perpendicular to the direction of propagation in one second. It is measured in Watts/m^2. The Sound Intensity Level of a sound, which is related to loudness, is defined as

$$\text{Sound Intensity Level} = 10 \log \frac{I}{I_0} \text{ dB}$$

I = intensity of the sound

I_0 = A reference intensity of 10^{-6} WM^{-2}

A sound reaching a point causes a variation in the pressure of the air at that point. The change in the total pressure at that point due to the sound is called the sound pressure. It is well established that at audience distances the intensity $\propto (P_{rms})^2$ where P_{rms} is the root mean square sound pressure. Here Sound Intensity Level =

$$10\log \frac{P_{rms}^{\ 2}}{P_0^{\ 2}}$$

$$= 20\log \frac{P_{rms}}{P_0}$$

P_0 is the reference sound pressure level corresponding to I_0

$P_0 = 2x10^{-5}$ WM^{-2}

When the definition 20log P_{rms} is used, the quantity is called the Sound Pressure Level (SPL).

At audience distances the Sound Intensity Level and the Sound Pressure Level are the same.

Table 1.2 Typical sound levels [3]

Jet takeoff (60 m)	120 dB	
Construction site	110 dB	Intolerable
Shout (1.5 m)	100 dB	
Heavy truck (15 m)	90 dB	Very noisy
Urban street	80 dB	
Automobile interior	70 dB	Noisy
Normal conversation (1 m)	60 dB	
Office, classroom	50 dB	Moderate
Living room	40 dB	
Bedroom at night	30 dB	Quiet
Broadcast studio	20 dB	
Rustling leaves	10 dB	Barely audible
	0 dB	

The energy involved in any vibration process is proportional to the square of the amplitude of the vibration. Since intensity is energy per second per m^2, intensity is proportional to amplitude squared. Because of this, a 2-fold increase in amplitude involves a 2 x 2 = 4-fold increase in the intensity, while a tripling of the amplitude is associated with a 9-fold increase. It naturally follows that a 10-fold intensity change calls for a $\sqrt{10}$ = 3.162-fold increase in amplitude.[4] The amplitude of vibration of the air particles at the ear can be increased, by increasing the amplitude of vibration of the plucked string.

The actual sensation of loudness is also effected by frequency. Above 20,000Hz and below 20Hz, a vibration will not be heard no matter how great it's intensity. In addition, the sensitivity of the ear varies within the frequency range (20Hz-20,000Hz). In particular, the loudness decreases sharply at very low frequencies in the region of 100Hz and below. To take account of this a Loudness Level is defined. The Loudness Level, measured in Phons, of a given sound is equal to the number of Decibles (dB) of a 1000Hz tone that is judged by an average listener to be equally loud. A sound intensity level of 80dB, for example, will give rise to a Loudness Level of 80 Phons at a frequency of 1000Hz but to a much lower Loudness Level at 100Hz. Sound Intensity Levels in Decibels and Loudness Level in Phons due to a number of sounds heard simultaneously cannot be added arithmetically. A scale of Loudness which is linear is defined as follows:

$$S = 2 \frac{(P-40)}{10}$$

If P is the Loudness Level in Phons, S is the Loudness in Sones. If sounds of Loudness S_1, S_2, S_3 etc. Sones are heard simultaneously the combined Loudness S is given by

$$S = S_1 + S_2 + S_3 \text{ etc.}$$

1.7 Quality/Timbre

A sounded string vibrates with many frequencies simultaneously as mentioned in Section 1.5, p. 17. The term *harmonics* refers to modes of vibration of a system that are whole-number multiples of the fundamental mode, and also to the sound that they generate. It is customary to stretch the definition so that it includes modes that are *nearly* whole-number multiples of the fundamental: 2.005 times the fundamental rather than 2 times, for example. The modes of vibration of an ideal vibrating string are harmonics of the fundamental. The modes of a real string are usually so close to being whole-number multiples of the fundamental that they are also spoken of as harmonics. Note that the term "first harmonic" refers to the fundamental.

Many vibrators do not have modes that are whole-number multiples of the fundamental frequency. However, the term *overtone* is used to denote their higher modes of vibration. Harmonics are therefore described as overtones whose frequencies are whole-number multiples of the fundamental frequency. The second harmonic is the first overtone and the third harmonic is the second overtone etc. Partial is another term in common use that refers to modes of vibration of a system or components of a sound. Partials include all the modes or components, the fundamental plus all the overtones, whether they are harmonic or not. The term *upper partials* excludes the fundamental and is thus a synonym of overtones.

If the harmonics of a note have different intensity distributions, the resulting sound will differ in *quality* or *timbre*, but the perceived pitch of the note will be the same. The main reason that the same fundamental note played on different instruments sounds differently is because of this phenomenon.[5] The first thirty-five partials of the overtone series, from which the just intonation tuning system is derived, are shown in Table 1.3 below:

Column 1 shows each partial number. Column 2 shows the frequency ratio between adjacent partials. Column 3 shows the interval between the partials in cents. Column 4 shows the nearest tempered interval to each partial ratio. Column 5 shows the tempered intervals in cents. Column 6 shows the difference between the intervals corresponding to the partial ratios and the tempered intervals. The frequency of upper partial 3 is exactly double the frequency of upper partial 1. The frequency of upper partial 7 is exactly double the frequency of upper partial 3. The doubling of frequency corresponds to a pitch interval of 1200 cents or one octave.

Table 1.3 Frequency Ratios and Nearest Tempered Intervals

Upper Partial no.	Frequency Ratio between successive partials	Pitch intervals between successive partials (Cents)	Name of nearest tempered interval	Tempered interval (Cents)	Difference between Columns (3)-(5) (Cents)
1	2:1	1200	Perf. octave	1200	0
2	3:2	702	Perf. 5th	700	+2
3	4:3	498	Perf. 4th	500	-2
4	5:4	386	Maj.3rd	400	-14
5	6:5	315.6	Min.3rd	300	+15.6
6	7:6	266.8	Min.3rd	300	-33
7	8:7	231.2	Maj.2nd	200	+31.2
8	9:8	204	Maj.2nd	200	+4
9	10:9	182.4	Maj.2nd	200	-17.6
10	11:10	165	Maj.2nd	200	-35
11	12:11	150.6	Maj.2rd	200	-49
12	13:12	138.6	Min.2nd	100	+38.6
13	14:13	128.3	Min.2nd	100	+28.3
14	15:14	119.4	Min.2nd	100	+19.4
15	16:15	111.7	Min.2nd	100	+11.7
16	17:16	105	Min.2nd	100	+5
17	18:17	99	Min.2nd	100	-1
18	19:18	93.6	Min.2nd	100	-6.4
19	20:19	88.8	Min.2nd	100	-11.2
20	21:20	84.5	Min.2nd	100	-15.5
21	22:21	80.5	Min.2nd	100	-19.5
22	23:22	77	Min.2nd	100	-23
23	24:23	73.7	Min.2nd	100	-26.3
24	25:24	70.7	Min.2nd	100	-29.3
25	26:25	68	Min.2nd	100	-32
26	27:28	65.3	Min 2nd	100	-34.7
27	28:27	63	Min.2nd	100	-37
28	29:28	60.7	Min.2nd	100	-39.3
29	30:29	58.7	Min.2nd	100	-41.3
30	31:30	56.8	Min.2nd	100	-43.2
31	32:31	55	Min.2nd	100	-45
32	33:32	53.3	Min.2nd	100	-46.7
33	34:33	51.7	Min.2nd	100	-48.3
34	35:34	50.2	Min.2nd	100	-49.8
35	36:35	48.7	Q. tone	50	-1.3

Figure 1.3 The first 16 harmonics of the Overtone Series when a C_2 (65.4 Hz) frequency is sounded

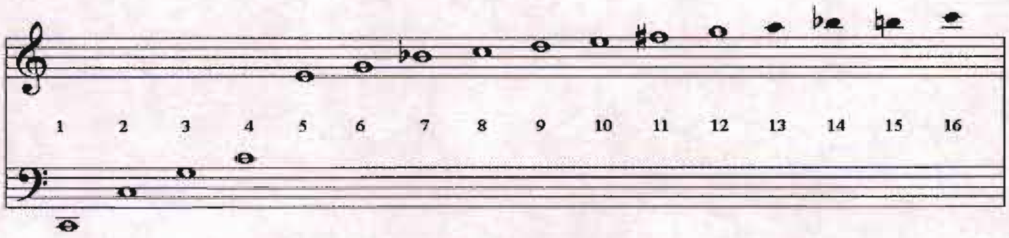
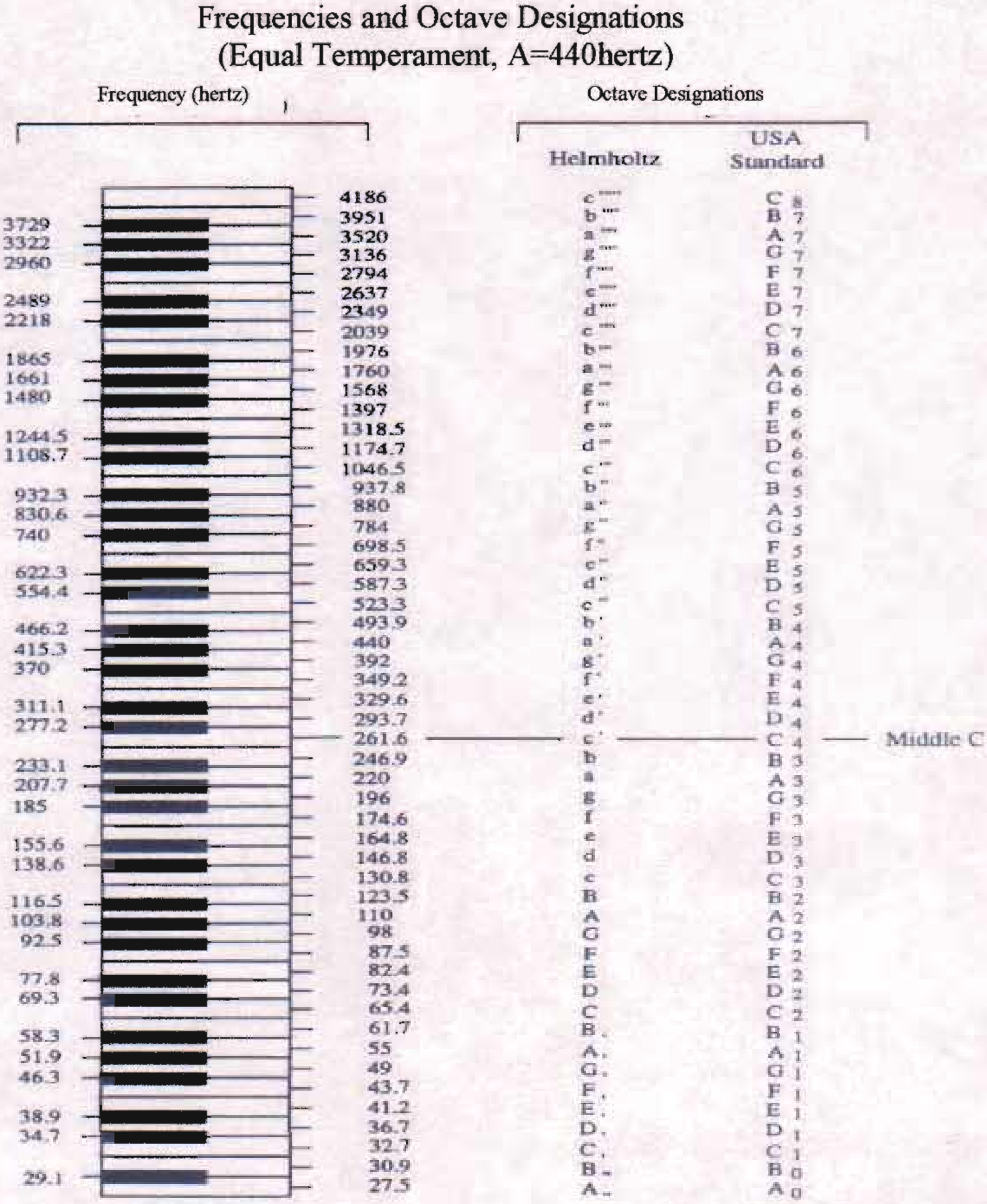


Figure1.3 above shows the first 16 harmonics that are present when a C frequency is sounded After the seventh harmonic, traditional notation is not adequate for notating these frequencies, and completely inadequate for the frequencies after the twelfth harmonic which are shown in black.

In Table 1.4 below the frequencies and octave designations on the equal temperament scale are shown, A=440 hertz (American standard-A_4, Helmholtz-a').

Table 1.4 Frequencies and Octave Designations (Equal Temperament , A=440hertz)



1.8 Evolution of Pitch Measurement

In the baroque period pitch levels for musical performance varied by as much as 400cents (four semitones). The first declaration of standard pitch was made in France in 1859, when A_4 was set at 435Hz by ministerial decree (today A_4 = 440Hz). A similar standard was adopted by several other countries but international agreement was not reached until 1939 when A_4 = 440Hz became standard, although today many symphony orchestras exceed this pitch.[6]

In the 17th century both pitch and the relative tuning system varied considerably. Even within the same city, church organs were tuned to different pitches, and music for the opera or for other secular uses was also performed at different pitches. Players adapted their instrument with either crooks, extra joints or different strings in order to perform in different surroundings. String and keyboard instruments were the most flexible in pitch as this depended on the density and thickness of their strings.

During J.S. Bach's lifetime (1685-1750), two pitch levels were in use: Chor-ton or 'Choir pitch' and Cammer-ton or 'Chamber pitch'. Cammer-ton was lower than Chor-ton by a whole step (200 cents) or sometimes by a minor third (300 cents). Organs and brass instruments were constructed to play in Chor-ton, while woodwind and strings were often tuned to Cammer-ton. Scholars have differed on the pitch levels of A_4. Some put A_4 at a pitch of approx. 490Hz (almost B_4) while others suggested that the pitch A_4 should be 460Hz (almost Bb_4), or even between 445Hz and 460Hz.

The common pitches that were present in Bach's day can be summarized as follows:

Chor-ton	A_4 = 445 - 460?
High Cammer-ton	a tone lower
Low Cammer-ton	a minor third lower

In France A_4 was pitched between 392Hz and 420Hz (The modern equally-tempered frequency G_4 = 392Hz). The English musician, John Playford (b.1623), directed viol players to tune the top string as high as it "conveniently would bear" without breaking, and then to tune the other strings to it.[7]

The following Table 1.5 presents a chronological outline of musical pitch.[8] Column 1 shows the variation of pitch from 0.0 semitones to 7.4 semitones. Column 2 shows the different A note frequencies in hertz. Column 3 gives information on the uses of the different A frequencies. Column 4 shows the different musical idioms in which particular 'A' pitches were used.

Table 1.5 An Outline of Musical Pitch

Diff. in semitones	A note in Hz	Uses of the different A frequencies	The idioms where the diff. A notes were used
0.0	370	Ideal Lowest or zero-point.	Church Pitch Lowest
0.2	374	Hospice Comtesse, 1700.	
0.3	377	Schlick low, 1511; Bedos, 1766.	
1.0	392	Euler's Clavicord, 1739	Church Pitch Low
1.1	395	R.Smith, 1759; Roman pitch pipes, 1720.	
1.2	396	De Caus, 1615; Versailles Chapelle, 1789.	
1.4	403	Mersenne Spinet, 1648.	
1.6	407	Sauveur, 1713.	Chamber Pitch Low
1.7	408	Mattheson, Hamburg, 1762.	
1.7	409	Pascal Taskin, court tuner, 1783.	
2.0	415	Dresden chained fork, 1722.	European Mean Pitch for two centuries
2.2	420	Freiburg, 1714; Seville, 1722.	
2.3	422	Mozart, 1780.	
2.3	423	Handel, 1751.	
2.4	424	Praetorius' suitable pitch. 1619; original Philharmonic,	
2.5	428	1813.	
		R. Harris, 1696; Opera Comique, 1823.	
2.7	433	Sir George Smart's fork, 1820-26.	Compromise Pitch
2.8	435	French Diapason Normal, 1859.	
3.0	440	Scheibler's Stuttgart Standard, 1834.	Modern Orchestral Pitch, and *Ancient Medium Church Pitch.
3.1	442	*Bernhardt Schmidt, low, 1690.	
3.2	445	Madrid, 1858; San Carlo, Naples, 1857.	
3.2	446	Broadwood's Medium, 1849; French Opera, 1856; Griesbach's A, 1860= C534.	
3.4	449	Griesbach's C 528, 1860.	
3.5	451	Lille Opera, 1848; British and Belgian Army,1879.	
3.5	453	Mean Philharmonic, 1846-54.	
3.6	455	Highest Philharmonic, 1874; Broadwood, Erard, and (English) Steinway, 1879.	
3.6	456	Vienna, high, 1859.	
3.7	457	(American) Steinway, 1879.	
3.8	458	Great Franciscan Organ, Vienna, 1640.	Church Pitch high
4.0	466		
4.3	474	Tomkins, 1668; B. Schmidt, high, 1683.	
4.5	481	St. Catherine's, Hamburg, 1543.	
4.8	489	St.James's, Hamburg, 1688.	
5.0	494	St.James's, Hamburg, 1879.	Church Pitch highest
5.1	496	Rendsburg, 1668.	
5.3	504	Schlick, high, 1511; Mersenne, ton de chapelle, 1636.	
5.4	506	Halberstadt Cathedral, 1361.	
6.0	523		
7.0	554		Chamber Pitch highest
7.3	563	Mersenne, ton de chambre, 1636	
7.4	567	Praetorius, North German, very old.	

Table 1.6 below gives some insight into why there were pitch differences throughout Europe.[9] Organs were constructed using the 'foot-rule' of each

country.

Table 1.6 The Different Foot Measurements found in Organ Building.

Foot Measurements	mm
Long old French foot, or *pied de roi*	325
Long German, or Rhenish foot	314
Long Austrian foot	316
English foot	305
Old Nurnberg foot	304
Old Roman foot (medieval)	295
Old Augsburg foot	296
Bavarian foot	292
Short Saxon foot	283
Short Brunswick foot and Frankfort foot	285
Short Hamburg and Danish foot	286
Very short old Brabant foot of 11 inches	278
13 Rhenish inches	340
13 Saxon inches	307
12 old Brabant inches	303

A difference of 12 per cent in the lengths of two pipes will, for the same scale, make a difference in pitch of nearly a whole meantone in their pitch. Thus a pipe of length equal to the short Saxon foot was a meantone sharper than the long Austrian or Rhenish foot. The percentage difference in lengths converted to pitch difference is as follows: 6 per cent, corresponds to an equal semitone, 3 per cent to an equal quartertone, 5.75 per cent, to a semi-meantone, 4.5 per cent, to a small meantone semitone and 7 per cent to a great meantone semitone.

Thus it can be seen from the Table 1.6 above that a pipe an English foot long is nearly a great semitone sharper than a French foot and about an equal quarter of a tone sharper than a Rhenish foot, while a pipe a Rhenish foot long is about a small semitone sharper than a French foot. Throughout Germany there was a wide variety of foot measurements which accounted for extensive variation in organ pitch.[10]

1.9 Musical Scales and Tuning Systems in Western Music

A musical scale is a succession of notes arranged in ascending or descending order. The number of notes contained in a scale can vary. The most common numbers of notes in a scale are 5 and 7, (see Appendix B). The white notes played on a piano from the notes C_4 to C_5 (C,D,E,F,G,A,B,C) form a C major scale. A C major pentatonic is equal to a C major scale minus the 4^{th} and 7^{th} notes, (C,D,E,G,A,C). The chromatic scale contains all of the 12 notes used in western music (the black and white notes on a piano between two similar notes an octave apart) for example, from C_4 to C_5. (C,C#,D,D#,E,F,F#,G,G#,A,A#,B,C)

Table 1.7 shows the ascending chromatic scale. 'S' shows an equal pitch difference of a semitone (the smallest pitch interval in western music).

Table 1.7 The Chromatic Scale

1	2	3	4	5	6	7	8	9	10	11	12	2/1
C_4	C#/Db	D	D#/Eb	E	F	F#/Gb	G	G#/Ab	A	A#/Bb	B	C_5
S S	S	S S	S	S S	S	S S	S	S S	S	S		

Moving up a semitone from the lower C in Table 1.7 above, each movement is equivalent to a pitch interval of 100 cents. Arriving at the higher C_5 again, up an octave, the total pitch interval is 1200 cents above the lower C_4.

Table 1.8 below shows the C major scale (middle row), 'tonic-solfa' syllables which are used when singing a major scale (top row), and the distances between each note (bottom row).

Table 1.8 C Major Scale

Do	Re	Mi	Fa	So	La	Ti	Do
C_4	D	E	F	G	A	B	C_5
t	t	s	t	t	t	s	

In the above scale "t" is a tone (two semitones) and "s" is a semitone. The movement in cents from C_4 to the other notes of the scale is illustrated in Table 1.9.

Table 1.9 below shows the movement in cents from C_4 to the other notes of the scale, the intervals between the notes in cents of the major scale, the nearest equivalent just intonation intervals and the corresponding whole-number ratios.

Table 1.9 C Major Scale and Equivalent Just Intonation Pitches

C_4	D	E	F	G	A	B	C_5
200	400	500	700	900	1100	1200	

Just Intonation (ratio and cents):

1	9:8	5:4	4:3	3:2	5:3	15:8	2:1
C_4	D	E	F	G	A	B	C_5
204	386	498	702	884	1088	1200	

The distances or *intervals* from the first note C_4 to the other notes of the chromatic scale are named in Table 1.10 below. The nearest whole-number ratios (converted to cents) of the just intonation tuning system are also shown for comparison.

Table 1.10 The Chromatic Scale and Equivalent Just Intonation Pitches

Interval Name	**Equal Temperament**	**Just Intonation**
	Cents	Cents
Min. 2nd	100	112 (16:15)
Maj. 2nd	200	204 (9:8)
Min. 3rd	300	316 (6:5)
Maj. 3rd	400	386 (5:4)
Perf. 4th	500	498 (4:3)
Aug 4th/Dim 5th	600	590 (45:32)
Perf. 5th	700	705 (3:2)
Min. 6th	800	814 (8:5)
Maj. 6th	900	884 (5:3)
Min. 7th	1000	996 (16:9)
Maj. 7th	1100	1088 (15:8)
Perf. octave	1200	1200 (2:1)

From Table 1.10 above it can be seen that there is a considerable difference in pitch between the notes derived from the ratios of the overtone series (the template for just intonation) and the notes of equal temperament. The ratios, 16:15, 15:14 and 14:13 are three different pitch intervals for a semitone. The ratio 16:15 or 112 cents is the closest to the equal temperament semitone of 100 cents. The ear is able to distinguish a pitch difference of 2 cents (and lower), therefore, 12 cents is a substantial difference.

The most important scale systems are the Pythagorean scale system, the tempered scale systems (equal and meantone temperament), and just intonation. In the sixth century B.C. Pythagoras is credited with introducing whole-number ratio tunings for the octave, perfect fourth and perfect fifth (based on the ratios of the numbers 1, 2, 3 and 4) into Greek music theory. The octave, perfect fourth and perfect fifth were deemed *consonant* or pleasing to the ear, all other intervals were deemed *dissonant* or not pleasing to the ear. In the early Middle Ages Western music was based on Pythagorean intonation (see Appendix B p.170). In the later Middle Ages, and early Renaissance period as music became more complex, and aural perception developed, thirds and sixths were deemed consonant intervals. With the development of independent instrumental music based on fixed pitch instruments this tuning became inadequate because of modulation difficulties. Eventually theorists were forced to partially abandon the Pythagorean framework in order to explain the existence of consonant thirds and sixths, because the most consonant possible thirds and sixths were based on ratios involving the number five. [11]

When two or more notes are sounded and their frequencies are in the ratio of small whole-numbers, our ears perceive them as consonant. For instance: if an A is sounded at 440Hz along with an E at 660Hz, the frequencies are in a ratio of 660 to 440 or 3 to 2 (3/2) and the human ear will perceive this as consonant. They have a common factor (220 Hz) in the musical range of frequencies. In Table 1.11 below a list of small whole-number ratios commonly considered consonant are listed.

Table 1.11 Consonant Whole-number Ratios

Ratio	Cents	Note/difference in Cents	Interval	Tempered (Cents)
1/1	0.00	C +0	Unison	0
6/5	316	Eb +16	Min. 3rd	300
5/4	386	E –13.8	Maj. 3rd	400
4/3	498	F –2	Perf. 4th	500
3/2	702	G +2	Perf 5th	700
8/5	814	Ab +14	Min 6th	800
5/3	884	A –16	Maj 6th	900
2/1	1200	C +0	Perf. octave	1200

The ratio 7:4 is 31cents narrower than a tempered minor seventh and is deemed consonant whereas a tempered minor seventh is deemed a mild dissonant pitch interval. The ratio 7:4 is not contained in Table 1.11 above as it deviates substantially from equal temperament and demands a departure from common practice.[12]

The concept of equal temperament provided the solution to the problem of intonation on fixed pitch instruments. The basic advantage of equal temperament is that the number of pitches required to play in different keys can be reduced by compromising the tuning of certain tones so that they can perform different functions in different keys, whereas in just intonation, a slightly different pitch would be required for each function. Equal temperament compromises the quality of intervals and chords in the interest of simplifying instrumental design and construction and playing technique.[13] It can also be seen as the slightly lessening or enlarging of musical intervals away from the 'natural scale' in order to fit them for practical performance, as shown in Table 1.9, p. 33.

Meantone temperament is based on a succession of thirds (5/4 ratio), which leaves an out-of-tune octave. In the equally tempered tuning system a movement of three consecutive third pitch intervals (400 cents) span an octave (1200 cents). Meantone temperament is based on pure thirds which are in ratio 5/4 and equal to 386 cents. A succession of three pure thirds (386 cents) will span 1158 cents leaving an out-of-tune octave. The difference is 48 cents nearly a quarter of a semitone less than a just or tempered octave (2/1 or 1200 cents).

Meantone temperament was the preferred tuning system in the building of keyboard instruments. This tuning system was based on perfect major thirds but it presented difficulties when modulating to non-related keys and as a result the system of equal temperament was adopted. Twelve-note/equal temperament tuning, unlike meantone tuning, alters all the intervals except the octave. Meantone temperament gave a nearer approximation to natural tuning (just intonation) than equal temperament for C major and keys related to it.

If pitch is increased within equal temperament in the cycle of fifths starting at C, a higher C will be reached after 12 equal fifths. In meantone temperament, an increase of approximately 31 perfect fifths would lead back to a similar higher C. According to the Pythagorean Laws of Acoustics, 12 perfect fifths are equivalent to seven octaves. But actually the distance of seven octaves is a little larger. For example: the ratio for a fifth is 3:2 and when multiplied by itself twelve times a figure of 129.746 is reached. The ratio for an octave is 2:1 and when multiplied by it self seven times a figure of 128 is reached. The ratio difference is 129.746:128, which is 1.01364 or 23.5 cents larger. This difference is called the *Pythagorean comma.*

Meantone temperament, as stated above, is based on a succession of pure thirds which leaves an out-of-tune octave. The difference between third intervals in Pythagorean tuning and pure thirds (thirds, which are strictly based on the appropriate whole-number ratios) is called a *syntonic comma*. A Pythagorean third has a ratio of 81:64 which is equal to 1.265625. A pure third is 5:4 and is equal to

1.25. The difference is a frequency of 1.0125 (a syntonic comma). This syntonic comma when expressed in cents is equal to 21.506 cents or rounded off to 22 cents, and is shown by the Greek letter δ.

A syntonic comma is slightly smaller than a Pythagorean comma. All the notes of the Pythagorean scale are raised or lowered by fractions of this syntonic comma to form a meantone tempered scale called the quarter-comma meantone temperament scale. These fractions are 1/4, 3/4, 1/2, 5/4 of a syntonic comma.

See Table 1.12 below.

Table 1.12 Quarter-comma Meantone Scale

Equivalent note names in Pythagorean Intonation	C	D	E	F	G	A	B	C
Quarter-commaMean tone Scale	C	D -1/2 δ	E - δ	F +1/4 δ	G -1/4 δ	A -3/4 δ	B -5/4 δ	C

Table 1.13 below shows the equal temperament scale, just intonation scale, Pythagorean intonation scale and the quarter-comma meantone temperament scale. The bottom row of each scale shows the difference in cents in relation to the just intonation scale (2nd scale below) except for the quarter-comma meantone scale where the information is in the third row. The fourth row of the quarter-comma meantone scale shows the difference in cents between it and the Pythagorean scale and illustrates the information given in Table 1.12 above.

Table 1.13 Comparing Scales in Cents

Equal Temperament Scale

Note names	C	D	E	F	G	A	B	C
Pitch intervals between C and the other notes in this scale in cents		200	400	500	700	900	1100	1200
Difference in cents from the just intonation scale	-4	+14	+2	-2	+16	+12	0	

Nearest notes in Just Intonation (ratio and cents)

Just intonation whole no. ratios	1	9:8	5:4	4:3	3:2	5:3	15:8	2:1
Equivalent note names	C	D	E	F	G	A	B	C
Pitch intervals between C and the other notes in this scale in cents		204	386	498	702	884	1088	1200

Pythagorean Intonation (ratio and cents)

Pythagorean intonation whole no. ratios	1	9:8	81:61	4:3	3:2	27:16	243:128	2:1
Equivalent note names	C	D	E	F	G	A	B	C
Pitch intervals between C and the other notes in this scale in cents		204	408	498	702	906	1110	1200
Difference in cents from the just intonation scale	0	+22	0	0	+22	+22	0	

Quarter-comma Meantone Intonation

Equivalent note names	C	D	E	F	G	A	B	C
Pitch intervals between C and the other notes in this scale in cents		193	386	503.5	696.5	889.5	1082.5	1200
Difference in cents from the just intonation scale	-11	0	+5.5	-5.5	+5.5	-5.5	0	
Difference in cents from Pythagorean intonation	-11	-22	+5.5	-5.5	-16.5	-27.5	0	

Just intonation is a tuning system based on the overtone series, which is a set of frequencies present in the overall sound of a note when played. This series of frequencies is written as whole-number ratios as shown in Table 1.3 p. 25.

Strictly speaking, just intonation is any system of tuning in which all of the intervals can be represented by whole-number frequency ratios, with a strongly implied preference for the simplest ratios compatible with a given musical purpose.[14]

Hermann Helmholtz[15] (1821-1894) was a strong advocate of just intonation as a tuning system. By the beginning of the twentieth century composers had exhausted all the possible combinations of twelve-note/equal temperament (12TET) and began to divide the twelve tones into smaller mathematical divisions (microtones), for example third-tones (18TET), quarter-tones (24TET), sixth-tones (36TET), eighth-tones (48TET) etc. As stated earlier, microtones are musical notes, which are higher or lower in frequency than the notes of the twelve-note/equal temperament scale. 'Tempering' is when a musical interval is slightly lessened or enlarged away from the 'natural' scale . From a physical point of view, the previous two statements suggest that the twelve-note/equal temperament scale, could be regarded as a microtonal scale, relative to the natural scale from which it deviates (see Table 1.10, p. 34). Clearly, twelve-note/equal temperament has shown its limitations since composers have begun to divide the 12 tones into smaller equally tempered intervals so as to find new compositional ideas.

Harry Partch[16] (1901-1974) was the first twentieth-century composer to use just intonation as a basis for composition and was responsible for the revival of this system. He devised a system of tuning with 43 tones per octave and directly influenced many contemporary composers who use this system today. The Czech composer Alois Haba[17] (1893-1972) pioneered the use of quarter and sixth-tones in composition. Charles Lucy[18] (b 1917) is the inventor of Lucy Tuning, a hybrid of just intonation and equal temperament. A number of interesting re-fretting ideas for the guitar have been developed to accommodate tunings such as:

* The 62 tone Just
* 12-Tone Plus
* Sixth-Tone
* Just Multi-tonic
* 19 per octave
* 24 quarter-tones per octave

Just intonation gives composers an infinite source of notes to choose from and enables them to meet their compositional/improvisational needs. Consonant intervals in the system of just intonation are more pure than in the system of equal temperament and there are more strikingly dissonant intervals in just intonation than in equal temperament.

More dissonance is available in the just intonation tuning system because of the infinite number of frequencies possible within that system and more consonance is available because in equal temperament the pitch intervals of the scale have been tempered and deviate from the recognized consonant whole-number ratios of just intonation.

The limitations of the twelve-note/equal temperament system have prompted composers to divide the 12 tones into smaller equally tempered intervals in order to find new compositional ideas. Just intonation provides the obvious template for composers moving outside the limits of other tuning systems.

1.10 Microtonal System in Composition

As stated in the Introduction the object of this work is to facilitate the use of microtones in composition and in improvisation. When a guitarist hammers a finger (a common guitar technique) on to the open sixth string, for example at the twelfth fret, the sound produced will be a combination of the tempered E note (164.8 Hz) and the microtone 171.1Hz sounded on the lower fret (11th fret). This microtone is nearest in pitch to a tempered F note (174.6Hz). The difference is 35.1 cents, which is 14.9 cents less than a quarter-tone.

For this project a pick-up has also been placed under the nut of the guitar so that the microtones can be amplified and measured for use in composition. They can also be used in composition without being amplified. 11 extra frets have been added to an electric guitar. These frets represent the maximum amount of microtones available (functional added frets) using the reverse fingerboard measurements from a 21-fret electric guitar. The remaining 12 frets from a possible addition of 21, were either too close to or matched existing frets.

Guitarists are aurally familiar with these microtonal sounds - even if they are not aware of their origin. These microtones contribute to the overall sound of a note especially when using the hammering technique described above, regardless of whether the guitar is acoustic or electric. Therefore guitarists should adapt more easily to these sounds than players of other instruments.

Chapter 2 Methodology

2.1 Overview

It is desired to measure the exact pitch of each of the 108 microtones found on a classical guitar. The predicted pitches were found using a mathematical formula. The frequencies of the microtonal notes were measured on a Larson•Davis Sound Level Meter and Frequency Analyser. An electrical signal was produced from the pick-up under nut-side, fed into an amplifier, and thence into the frequency analyzer. This allowed accurate measurements of the microtonal frequencies. The microtones were then experimented with using three different compositional techniques and improvisation ideas.

2.2 Methodology

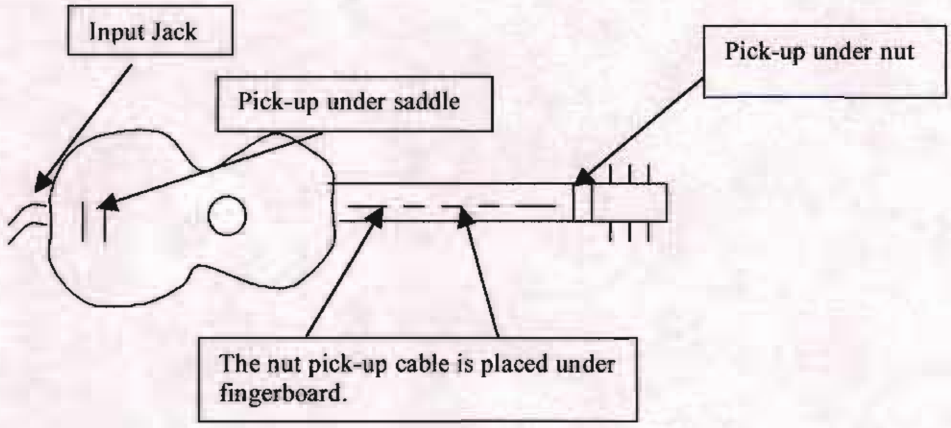
The predicted frequencies of the microtones were initially calculated using the method outlined in Section 1.5 of this thesis. These frequencies are set out in Table 1.1, p.18. The frequencies of the microtones were calculated by reference to the frequencies of the bridge-side notes on the equal temperament scale.

The frequencies of the same microtones were measured using a *Larson•Davis model 2800 Real-Time Sound level Meter.* This meter performs two measurements simultaneously; that of a Precision Sound level meter and that of a real-time frequency analyser. As a single channel real-time analyser, it can perform frequency analysis using digital 1/1 and 1/3 octave bandwidths and FTT analysis using, 100, 200, 400 and 800 line resolution. For example, if a base frequency is set at 0-2500 Hz and 800 line resolution is used, the resolution will be $\frac{2500}{800}$ = 3.125Hz. The meter also has zoom capabilities with real-time zoom: X256 and when operating on a non-real-time frequency range (buffered) it is capable of X64 (1 channel), X32 (2 channels). A microphone compatible with this instrument was used (model: 2541 free-field microphone, and a pre-amp, model: 900B) to capture the frequency of a single microtone plucked acoustically (un-amplified) from the opposite side of a stopped string. The microtone was Fourier analyzed in the Larson/Davis SLM and the data was imported into a Microsoft Excel Spreadsheet. Graphs such as that shown in Figure 1, p.9, were produced.

For the recording of all the microtones on the nut-side, a pick-up was placed under the nut of the guitar. A pick-up was also placed under the saddle [piece of ivory/wood fixed to the bridge where the strings sit]. Two pre-amplifiers are used so as to achieve a similar sound from each pick-up. Both pick-ups are then fed into two regular guitar amplifiers via two volume pedals. The volume from the pick-up under the saddle is turned off so that only the nut-side notes are sounding. The airborne sound is then picked up by the microphone and transferred to the Larson•Davis meter.

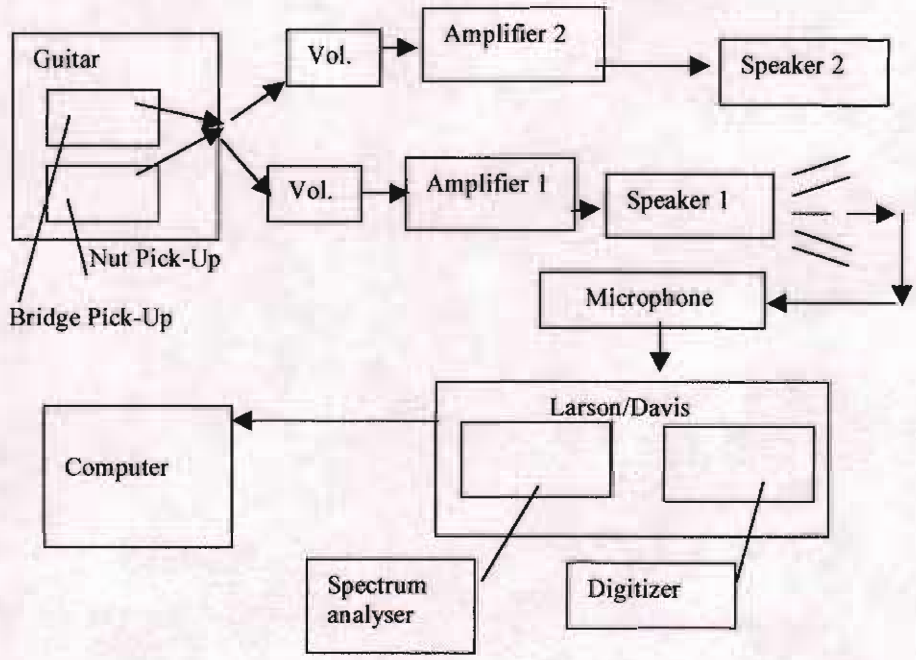
Experimental arrangements are illustrated in Figure 2.1, and Figure 2.2. The fundamental frequencies of the microtones are read from the graphs and are presented in Table 3.2, p.53.

Figure 2.1 Placement of Bridge and Nut Pick-Ups



The output signal of the nut-side pick-up gives out approximately four times the voltage of the pick-up under the saddle. This arrangement is designed to compensate for the lower resonance amplifications on the nut-side. A stereo lead is plugged into a four-pole end jack socket, which gives a stereo output from both pick-ups.

Figure 2.2 Process of Measuring a Microtone



The above schematic diagram shows a guitar signal from both the bridge-side and nut-side pick-ups being transferred into two separate amplifiers, via two volume pedals using a stereo lead. Amplifier 1 is fed by the nut-side pick-up and amplifier 2 is fed by the bridge-side pick-up. The bridge-side pick-up is turned off using the volume pedal, so that only the microtones from the nut-side pick-up are sounded and picked up by the microphone. The sound is then transferred into the Larson•Davis meter. The sound is digitized and analysed. This information is then transferred to a computer, translated and analysed in Microsoft Excel.

2.3 Re-fretting an Electric Guitar

A solid-body electric guitar has been modified by adding 11 more frets to the existing 21. Electric guitars usually have more frets than acoustic guitars. The classical acoustic guitar used for measurement above has 19 frets while the electric guitar has 21. To find the positions of the new frets on the electric guitar, a template was made of the frets on the fingerboard of the guitar. It was then reversed (21st fret beginning at the nut of the guitar). The frets that were far enough away from the existing frets (those able to function properly) were marked and added to the fingerboard. Ten of the reversed 21 frets were either too near or matching the existing frets. The result gives 32 frets in total (a 32-fret guitar within the measured distance of twenty-one frets). See Figure 3.1, p.54.

When adding frets to an electric guitar using a reversed fingerboard, the placing of the added frets depends on how many original frets the guitar to be modified contains. The guitar used here contains 21 original frets. If a guitar contains, for example, 24 frets then a reversed fingerboard would place the added frets in different positions. The displaced difference between the 21-fret guitar used here and a 24-fret guitar would be the distance from fret 21 to fret 24 based on a similar scale length. The first 3 added frets of a 24-fret guitar would be frets 23, 22 and 21 which is the first added fret of a 21-fret guitar. The result of adding frets to a 24-fret guitar may well result in more added frets being available because of the different placement of the added frets. The microtonal frequencies based on added frets were measured in the same manner as in previous experiments. The results are presented in Table 3.4, p.57.

Chapter 3 Physical Results

3.1 Introduction

Table 1.1, p.18 shows the theoretically predicted nut-side notes in hertz, which are calculated using the formula for inverse proportion outlined in Section 1.5, p17. These figures do not take into account the fact that a stopped string is a little stretched or that the string has some stiffness. Therefore, it is important to measure the different frequencies using a frequency analyser, as the actual frequencies may differ significantly from those predicted earlier.

The first row of Table 1.1, p. 18 contains the lowest microtonal frequency for each string of a standard classical guitar (this is where a string is stopped on the 19th fret and plucked on the opposite side, sounding the 18th fret microtone).

3.2 Measurement of Microtonal Frequencies on Sixth string

The frequencies of the microtones on the sixth string are measured using the Larson•Davis sound level meter in Fourier analysis mode. The frequency resolution was in all cases ±1.6Hz or better. The guitar was tuned with a reliable chromatic tuner. The results of the measurements presented in Table 3.1 below show the microtonal frequencies and the difference in cents between the microtones and the nearest tempered notes.

The first microtone is measured at 171.1Hz and the nearest tempered note is

174.6Hz (F_3) the pitch difference in cents is 35.1. This microtonal pitch can also be seen as 164.9cents situated above D#$_3$ (155.6Hz), See fret numbers 14, 13 and 12 in Table 3.1 below. The pitch interval corresponds to the just intonation whole-number ratio 11:10, which is 165cents. The interval is also 65 cents narrower then the tempered interval of a major second.

Row 2 of Table 3.1 contains a list of the measured microtonal frequencies in hertz measured on the sixth string using a resolution of ±1.6Hz. The microtones are numbered 19-2, counting the frets normally. A list of the nearest tempered notes in hertz, and the USA standard Octave Designations, is shown in Row 3. Row 4 shows the difference in cents between the microtonal frequencies and the nearest tempered frequencies.

Table 3.1 Measured Microtonal Frequencies on the sixth string , (Resolution = ±1.6Hz)

Fret No.	**19**	**18**	**17**	**16**	**15**	**14**	**13**	**12**	**11**
Measured Freq/Hz	124.6	128.5	133.6	139.1	145.7	153.1	161.3	171.1	183.6
Nearest Note Freq/Hz	123.5 B2	130.8 C3	130.8 C3	138.6 C#3	146.8 D3	155.6 D#3	164.9 E3	174.6 F3	185 F#3
Difference in Cents	15.4	-30.7	36.7	6.2	-13	-28	-37.2	-35.1	-13.2

10	**9**	**8**	**7**	**6**	**5**	**4**	**3**	**2**
199.2	216	241.4	275.8	323.4	392.2	508.6	745.3	1462.5
196 1G3	220 A3	246.9 B3	277.2 C#4	329.6 E4	392 G4	523.3 C5	740 F#5	1480 F#6
-28	-31.8	-39	-8.8	-32.9	0.9	-49.3	12.4	-20.6

A smaller resolution is needed for more accurate measurements and measurements for all the microtones on all strings using a resolution of 2 cents are shown in Table 3.2, p.53. Using the measurements in Table 3.2 the first microtone in *Guitar Opus 1* (see Table 5.1, p.122) is measured at 176.6Hz with a resolution of 2 cents, and the nearest tempered note is 174.6Hz (F_3), the pitch difference is 19.7 cents. This microtonal pitch can also be seen as 119.7 cents situated above E_3 (164.8Hz), this pitch interval of 119.7 cents corresponds to the just intonation whole-number ratio 15:14. See Table 1.3, p.25.

3.3 Measurement of all Microtonal Notes

Using the Larson•Davis sound level meter in Fourier Analysis Mode, measurements were made of the fundamental frequencies of the microtonal notes on all strings. These frequencies together with the frequency resolutions used are set out in Table 3.2, p.53. The measured frequencies are compared with the predicted frequencies set out in Table 1.1, p.18.

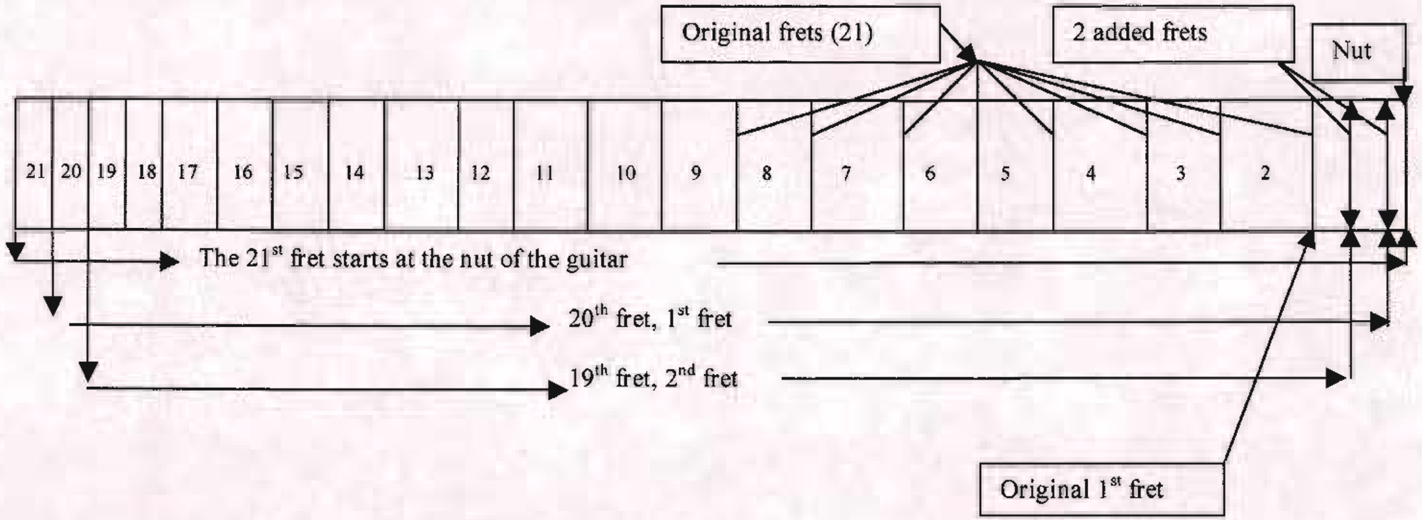
The frets are numbered normally. By stopping the strings at the nineteenth fret the lowest microtonal pitches are produced on each string. The human ear can detect pitch differences of 2 cents or less. The microtonal frequencies were measured using frequency resolutions corresponding to pitch intervals less than 2 cents. The data is set out in Table 3.2 below. From the lowest nut-side note 124.6Hz to 220Hz a resolution of ± 0.1Hz is needed. The limit of resolution can be then doubled for the frequencies 220Hz-440Hz so as to achieve the same accuracy. The limit of resolution can be increased in a similar manner for measurements in the higher frequency region.

Table 3.2 Measured Microtonal Frequencies on All Strings, (Pitch resolutions are under two cents).

Strings ►	6th	5th	4th	3rd	2nd	1st
Frets ▼	Frequency/Hz					
Resolution	**±0.1Hz**	**±0.1Hz**	**±0.2Hz**	**±0.2Hz**	**±0.2Hz**	**±0.4Hz**
19 (normal)	128.3 (C_3)	170.9 (F_3)	228.5($A\#_3$)	306.8($D\#_4$)	384.6 (G_4)	512.5 (C_5)
18	132.6 (C_3)	176.5 (F_3)	235.9($A\#_3$)	317.2($D\#_4$)	397.9 (G_4)	530.5 (C_5)
17	137.5 ($C\#_3$)	183.1($F\#_3$)	244.7 (B_3)	329.3 (E_4)	412.1($G\#_4$)	549.2($C\#_5$)
16	143.1 (D_3)	190.3($F\#_3$)	255.3 (C_4)	343.2 (F_4)	429.3($G\#_4$)	570.7($C\#_5$)
15	149.6(D_3)	199 (G_3)	266.2 (C_4)	358.8 (F_4)	447.9 (A_4)	596.5 (D_5)
Resolution	**±0.1Hz**	**±0.1Hz**	**±0.2Hz**	**±0.2Hz**	**±0.4Hz**	**±0.4Hz**
14	156.8($D\#_3$)	209.2($G\#_3$)	280.1($C\#_4$)	376.8($F\#_4$)	471.1($A\#_4$)	627.3($D\#_5$)
13	165.8 (E_3)	221 (A_3)	296.7 (D_4)	398.4 (G_4)	498.8 (B_4)	662.9 (E_5)
Resolution	**±0.1Hz**	**±0.2Hz**	**±0.2Hz**	**±0.2Hz**	**±0.4Hz**	**±0.4Hz**
12	176.6 (F_3)	234.4($A\#_3$)	315.4($D\#_4$)	423.8($G\#_4$)	529.3 (C_5)	706.3 (F_5)
11	189.5($F\#_3$)	253.1 (B_3)	337.7 (E_4)	454.7($A\#_4$)	569.5($C\#_5$)	756.6($F\#_5$)
Resolution	**±0.1Hz**	**±0.2Hz**	**±0.2Hz**	**±0.4Hz**	**±0.4Hz**	**±0.4Hz**
10	205.2($G\#_3$)	273.4($C\#_4$)	364.3($F\#_4$)	490.6 (B_4)	613.7($D\#_5$)	818.8($G\#_5$)
9	224 (A_3)	299.2 (D_4)	400.4 (G_4)	538.3 (C_5)	675 (E_5)	897.3 (A_5)
Resolution	**±0.2Hz**	**±0.2Hz**	**±0.2Hz**	**±0.4Hz**	**±0.4Hz**	**±0.8Hz**
8	248.8 (B_3)	333.2 (E_4)	444.5 (A_4)	601.2 (D_5)	750.8($F\#_5$)	999 (B_5)
Resolution	**±0.2Hz**	**±0.2Hz**	**±0.4Hz**	**±0.4Hz**	**±0.4Hz**	**±0.8Hz**
7	281.8($C\#_4$)	377.7($F\#_4$)	505.9 (B_4)	684 (F_5)	853.5($G\#_5$)	1135.9 ($C\#_6$)
Resolution	**±0.2Hz**	**±0.2Hz**	**±0.4Hz**	**±0.4Hz**	**±0.8Hz**	**±0.8Hz**
6	331.6 (E_4)	441.8 (A_4)	592.2 (D_5)	801.2 (G_5)	995 (B_5)	1325 (E_6)
Resolution	**±0.2Hz**	**±0.4Hz**	**±0.4Hz**	**±0.8Hz**	**±0.8Hz**	**±0.8Hz**
5	403.1 (G_4)	537 (C_5)	718 (F_5)	977 (B_5)	1214.8 ($D\#_5$)	1617 ($G\#_6$)
Resolution	**±0.4Hz**	**±0.4Hz**	**±0.8Hz**	**±0.8Hz**	**±0.8Hz**	**±01.6Hz**
4	521 (C_5)	695 (F_5)	930.5 ($A\#_5$)	1275.8 ($D\#_6$)	1578.9(G_6)	2104 (C_7)
Resolution	**±0.4Hz**	**±0.8Hz**	**±0.8Hz**	**±01.6Hz**	**±01.6Hz**	**±01.6Hz**
3	771.1 (G_5)	1017.2(C_6)	1351.6(E_6)	1873 (A_6)	2305 (D_7)	3054 (G_7)
Resolution	**±0.8Hz**	**±01.6Hz**	**±01.6Hz**	**±3.2Hz**	**±3.2Hz**	**±3.2Hz**
2	1492 ($F\#_6$)	1977 (B_6)	2641 (E_7)	3825 (A_7)	4591	6006

Table 3.2 above includes the USA standard octave designations in brackets (see USA standard octave designation frequencies in hertz in Table 1.4, p.27). Figure 3.1 below shows the fingerboard of an electric guitar with added frets numbered 21-2. Two of eleven added frets are shown (lines with arrows at each end) in the diagram and are accommodated within the normal first fret. The two frets correspond to frets 20 and 19 indicated by the arrows underneath the fingerboard.

Figure 3.1 The Electric Guitar Fingerboard with the first two added frets shown using a reversed fingerboard



The positions of the added frets are shown in Table 3.3 below.

Table 3.3 Positions of the added frets on the fingerboard

Positions of the added frets.	**Number of added frets**	**Note names on each string were frets have been added in between**					
		6th String	5th String	4th String	3rd String	2nd String	1st String
Between 0 and 1st	2	E and F	A and Bb	D and Eb	G and Ab	B and C	E and F
Between 1st and 2nd	1	F and F#	Bb and B	Eb and E	Ab and A	C and Db	F and F#
Between 2nd and 3rd	2	F# and G	B and C	E and F	A and Bb	Db and D	F# and G
Between 3rd and 4th	1	G and Ab	C and Db	F and F#	Bb and B	D and Eb	G and Ab
Between 4th and 5th	1	Ab and A	Db and D	F# and G	B and C	Eb and E	Ab and A
Between 5th and 6th	1	A and Bb	D and Eb	G and Ab	C and Db	E and F	A and Bb
Between 10th and 11th	1	D and Eb	G and Ab	C and Db	F and F#	A and Bb	D and Eb
Between 13th and 14th	1	F and F#	Bb and B	Eb and E	Ab and A	C and C#	F and F#
Between 15th and 16th	1	G and Ab	C and Db	F and F#	Bb and B	D and D#	G and Ab

Table 3.4 below shows the note frequencies that are produced by the added frets. The normal frequencies are in italics. There are sixty-six in total (eleven on each string). The measurements were made using a Larson•Davis Sound Level Meter as in previous SLM experiments. The differences in cents between the notes produced by using the added frets on the sixth string, and the notes produced by using the normal frets were calculated and are shown below each microtone in brackets. The differences will be approximately the same for the other five strings.

Table 3.4 Measured Microtonal Frequencies produced by the added frets

Strings ▶	6^{th}	5^{th}	4^{th}	3^{rd}	2^{nd}	1^{st}
Frets ▼	Frequency /Hz					
	±0.4Hz	±0.4Hz	±0.8Hz	±0.8Hz	±0.8Hz	±1.6
Open String Frequencies ▶	82.4 (E_2)	110 (A_2)	146.8 (D_3)	196 (G_3)	246.9 (B_3)	329.6 (E_4)
1 (Between 0&1)	84 (E_2 +33 cents)	112.1	150	200.9	251.6	334.4
2 (Between 0&1)	85.6 (F_2 −38 cents)	114.1	153.1	203.9	257.0	340.6
Normal 1^{st} fret	87.5 (F_2)	116.5 (Bb_2)	155.6 (Eb_3)	207.7 (Ab_3)	261.6 (C_4)	349.2 (F_4)
3 (Between 1&2)	89.1 (F_2 +31.4 cents)	119.1	159.4	213.3	268.8	356.3
Normal 2^{nd} fret	92.5 ($F\#_2$)	123.5 (B_2)	164.8 (E_3)	220 (A_3)	277.2($C\#_4$)	370 ($F\#_4$)
4 (Between 2&3)	94.2 ($F\#_2$ +31.5 cents)	125	168	224.2	282.8	375
5 (Between 2&3)	96.9 (G_2 −21.3 cents)	129.3	172.7	230.5	290.6	385.9
Normal 3^{rd} fret	98 (G_2)	130.8 (C_3)	174.6 (F_3)	233.1 (Bb_3)	293.7 (D_4)	392 (G_4)
6 (Between 3&4)	100.4 (G_2 +41.8 cents)	133.2	178.1	238.3	300	398.4
Normal 4^{th} fret	103.8 (Ab_2)	138.6 (Db_3)	185 (Gb_3)	246.9 (B_3)	311.1 (Eb_4)	415.3 (Ab_4)
7 (Between 4&5)	107.8 (A_2 -35 cents)	143.4	192.4	256.3	322.7	428.1
Normal 5^{th} fret	110 (A_2)	146.8 (D_3)	196 (G_3)	261.6 (C_4)	329.6 (E_4)	440 (A_4)
8 (Between 5&6)	112.5 (A_2 +38.9 cents)	149.2	200	267.2	336.7	446.9
Normal 6^{th} fret	116.5 (Bb_2)	155.6 (Eb_3)	207.7 (Ab_3)	277.2(Db_4)	349.2 (F_4)	466.2 (Bb_4)
Normal 7^{th} fret	123.5 (B_2)	164.8 (E_3)	220 (A_3)	293.7 (D_4)	370 ($F\#_4$)	493.9 (B_4)
Normal 8^{th} fret	130.8 (C_3)	174.6 (F_3)	233.1 (Bb_3)	311.1 (Eb_4)	392 (G_4)	523.3 (C_5)
Normal 9^{th} fret	138.6 (Db_3)	185 (Gb_3)	246.9 (B_3)	329.6 (E_4)	415.3 (Ab_4)	554.4 (Db_5)

Normal 10th fret	***146.8 (D_3)***	***196 (G_3)***	**261.6 (*C_4*)**	**349.2 (*F_4*)**	**440 (*A_4*)**	**587.3(*D_5*)**
9 (Between 10&11)	151.6 (Eb_3 –45.1cents)	201.2	268	357.9	450.8	596.9
Normal 11th fret	**155.6 (*Eb_3*)**	**207.7 (*Ab_3*)**	**277.2(*Db_4*)**	**370 (*Gb_4*)**	**466.2 (*Bb_4*)**	**622.3(*Eb_5*)**
Normal 12th fret	**164.8 (*E_3*)**	**220 (*A_3*)**	**293.7 (*D_4*)**	**392 (*G_4*)**	**493.9 (*B_4*)**	**659.6(*E_5*)**
Normal 13th fret	**174.6 (*F_3*)**	**233.1 (*Bb_3*)**	**311.1 (*Eb_4*)**	**415.3 (*Ab_4*)**	**523.3 (*C_5*)**	**698.5(*F_5*)**
10 (Between 13&14)	182.4 (Gb_3 +24.5cents)	242.2	322.7	432	545.3	721.9
Normal 14th fret	**185 (*Gb_3*)**	**246.9 (*B_3*)**	***329.6 (E_4)***	**440 (*A_4*)**	**554.4 (*Db_5*)**	**740(*Gb_5*)**
Normal 15th fret	***196 (G_3)***	**261.6 (*C_4*)**	**349.2 (*F_4*)**	**466.2 (*Bb_4*)**	**587.3(*D_5*)**	**784(*G_5*)**
11 (Between 15&16)	205.1 (Ab_3 –21.8 cents)	205.9	362.5	484.4	614.8	810.9
Normal 16th fret	**207.7 (*Ab_3*)**	**277.2(*Db_4*)**	**370 (*Gb_4*)**	**493.9 (*B_4*)**	**622.3(*Eb_5*)**	**830.6(*Ab_5*)**

Table 3.5 below shows the outcome when E_5 (659.3 Hz) is played twenty times using normal pressure. The experiment was taken to see if there is a marked variation in pitch.

Table 3.5 Recording E_5 (659.3 Hz)

	E_5 (659.3 Hz)-1st string on 12th fret
	Resolution - ±0.39Hz (1.03 cents)
1	659.8
2	659.2
3	658.8
4	657.2
5	660.6
6	658.8
7	658.8
8	659.6
9	659.8
10	658.8
11	659.8
12	658.8
13	658.4
14	658.8
15	659.4
16	659.0
17	661.2
18	658.8
19	659.8
20	660.0
21	659.3Hz (mean)
22	0.87Hz (2.3 cents) (1 Std Deviation)
23	1.74Hz (4.6 cents) (2 Std Deviations)

The frequencies in Rows 1 – 20 in Table 3.5 above were measured on a Larson•Davis Sound Level Meter in Fourier analysis mode. For each pluck, frequencies less than 3dB below the peak frequency power were recorded. The mean was found, and the resulting figures are illustrated in rows 1 – 20. Row 21 shows the average (mean) of Rows 1 – 20. Row 22 shows one standard deviation, and Row 23 shows two standard deviations. The notes played appeared to be identical in pitch. It is evident that random pitch differences up to 4.6 cents can occur while playing. It was necessary to tune the string during this experiment as it would be during a performance.

3.4 Calculating Whole-number Ratios for Tempered Note Frequencies

Finding the tempered pitches as whole-number ratios within the overtone series is important because the results show that the tempered pitches are not contained within the recognized consonant small whole-number ratios except in the case of the octave, perfect fourth and perfect fifth.

Table 3.6 Calculating the 12 tempered pitch intervals as whole-number ratios in the overtone series (±2 cents).

Tempered interval names	**Tempered intervals in cents**	**Lowest equivalent whole no. ratios (±2 cents)**	**Equivalent whole no. ratios in cents (±2 cents)**
Min. 2nd	100	18:17	99
Maj. 2nd	200	**37:33** (9x4)+1:(8x4)+1	198.1
Min. 3rd	300	**25:21** (6x4)+1:(5x4)+1	301.9
Maj. 3rd	400	**29:23**	401.3
Perf. 4th	500	**4:3**	498
Dim. 5th	600	**41:29**	599.5
Perf. 5th	700	**3:2**	702
Min. 6th	800	**27:17**	800.9
Maj. 6th	900	**37:22**	900
Min. 7th	1000	**41:23**	1000.8
Maj. 7th	1100	**17:9** (inversion of 18:17)	1101
Perf. octave	1200	**2:1**	1200

Table 3.6 above shows where the 12 tempered intervals, as whole-number ratios, are found in the overtone series. An accuracy of ±2 cents is used because the ear can hear a change of two cents or less. The lowest whole-number ratios with a resolution of ±2 cents are shown in column 3 above, and the equivalent cents are shown in column 4.The main ratios above are 2:1, 3:2, 4:3 and 18:17 and when possible these ratios are added or divided to find the other 8 ratios. The method of finding some of the higher figured ratios is shown after each ratio.

It is concluded that apart from the perfect fourth, the perfect fifth and the perfect octave, the intervals of the twelve-note/equal temperament tuning system will be expressed as large whole-number ratios. They therefore exhibit considerable dissonance.

The physicist Arthur Benade[19] tested professional musicians on their ability to distinguish between three different third pitch intervals, (a) pure third, (b) tempered third and (c) Pythagorean third. An interesting response to the dissonance quality of the tempered third and Pythagorean third compared to the pure third was found. Musicians found that the tempered third was more dissonant than the pure third and the Pythagorean third more dissonant than the tempered third.

3.5 Calculating Whole-number Ratios for Microtonal Frequencies

To find the measured microtonal frequencies within the overtone series, the microtones in ratio with the sixth string frequency are calculated within two cents. The ratios calculated in Table 3.6 above were used to find the following ratios in Table 3.7 below.

Table 3.7 Calculating the Measured Microtones as Whole-Number Ratios in the Overtone Series (±2 cents).

Normal fret numbers	Measured microtonal frequencies in hertz and nearest tempered note (in brackets)	Sixth string frequency 82.4Hz	Interval in cents (±2 cents)	Ratio and calculation method used to obtain the lowest whole-number ratio between each microtonal frequency in column 2, and the sixth string frequency (82.4Hz) in column 3 (±2 cents)
19	128.32 (C_3)	82.4	766.8	**14:9** (764.9 cents)
18	132.62 (C_3)	82.4	823.9	**29:18** (825.7 cents)
17	137.5 ($C\#_3$)	82.4	886.45	**45:27** (884.4 cents)
16	143.07 (D_3)	82.4	955.2	**33:19** (955.8 cents)
15	149.61 (D_3)	82.4	1032.58	**49:27** (1031.8 cents)
14	156.84 ($D\#_3$)	82.4	1114.28	**59:31** (1114.2 cents)
13	165.82 (E_3)	82.4	1210.67	**145:72** (1212 cents)
12	176.56 (F_3)	82.4	1319.3	**15:7** (1319.5 cents)
11	189.45 ($F\#_3$)	82.4	1441.31	**85:37** (1440 cents)
10	205.18 ($G\#_3$)	82.4	1579.4	**97:39** (1577.5 cents)
9	224.02 (A_3)	82.4	1731.49	**106:39** 2:1+4:3=8:3, (8x13)+2:(3x13) = (1731.02 cents)
8	248.83 (B_3)	82.4	1913.32	**151:51** 2:1+3:2 = 3:1, (3x100)+2:(1x100) = (1913.45 cents)
7	281.84 ($C\#_4$)	82.4	2128.98	**89:26** (2130.46 cents)
6	331.64 (E_4)	82.4	2410.67	**145:36** (2412:1 cents)
5	403.13 (G_4)	82.4	2744.33	**39:8** 4:1+120:101=480:101, (480x2)+15:(101x2)-2 = (2742.47 cents)
4	521 (C_5)	82.4	3192.67	**215:34** 4:1+27:17=108:17, (108x2)-1:(17x2) = (3192.87 cents)
3	771.09 (G_5)	82.4	3871.41	**215:23**(3869.6 cents)
2	1492 ($F\#_6$)	82.4	5014.13	**181:10** 16:1+55:49=880:49, 880+25:49+1 = (5013.48 cents)

The tuning of the guitar strings from the lowest E string frequency (82.4) hertz to the highest E string frequency (329.6) is perfect 4th (4:3), perfect 4th (4:3), perfect 4th (4:3), major 3rd (29:23) and perfect 4th (4:3). These ratios correspond to those in Table 3.6, p.61. To find the corresponding microtonal notes on the fifth string, multiply the ratios in column 5, Table 3.7 above by 4:3, this gives a pitch interval on the fifth string. For example, the interval 14:9 + 4:3= 1.56+1.33 = 1263.6 cents corresponds to the pitch interval between the open 6th string and the frequency on the 5th string in the 19th fret.

Chapter 4 Analysis

4.1 Beats

It is well established in elementary physics that when an object vibrates with two nearly equal frequencies simultaneously, the resultant vibration occurs at the mean of the two frequencies and the intensity varies at a 'beat' frequency equal to the difference between the two frequencies. In sound, loudness is mainly affected by intensity which is proportional to the square of the resultant amplitude. The more nearly equal the two original amplitudes, the bigger the variation in loudness. When two sound waves of nearly equal frequency fall on the ear drum simultaneously beats are heard if the amplitudes are comparable. This can occur due to two sound sources, such as two vibrating strings amplified by the instrument body, sounding together. But, very importantly for musical performance, it also occurs for notes played consecutively when there is an overlap in the time intervals for which both impinge on the ear. This will occur in any room or auditorium with a significant reverberation time, provided the two notes are sounded at a sufficiently small time interval. As the interval between the frequencies of two pure tones increases, the beat frequency increases until separate beats can no longer be heard. This takes place at a beat frequency of 15-20 Hz. However a sensation of roughness and musical unpleasantness continues until a frequency separation of the pure tones called *the critical band* is reached. Over most of the musical range this interval falls between a minor third and a whole tone. The presence of roughness arising from interference beats is generally recognized as the principal cause of musical dissonance.

Musical notes normally contain a fundamental and harmonics whose frequencies are nearly whole-number multiples of the frequency of the fundamental. Consequently, when two musical notes of nearly the same fundamental frequency overlap, for example, the frequencies 600Hz and 601Hz (difference of 2.8 cents) there will be considerable roughness. Both fundamental frequencies will beat at one beat a second but upper partial number 1 will beat at two beats a second and upper partial number 2 will beat at three beats a second, et cetera. This will be musically unpleasant.

4.2 Implied Tone or Periodicity Pitch

If sinusoids representing a fundamental frequency of vibration and all its harmonics are added together by the principle of superposition, the resultant will be a vibration of the same frequency as the fundamental. This resultant frequency will remain the same despite alterations to the amplitudes and phases of the sinusoids. Further, *the resultant frequency will remain the same even if the fundamental frequency is omitted from the superposition.* This can be demonstrated by superposing electrical signals and displaying them on an oscilloscope. Indeed, it can be shown mathematically[20] that if any two components of a harmonic series are superposed, the resultant frequency will be the highest common factor of the component frequencies. Thus, if vibrations of frequencies 300Hz and 200Hz (ratio 3:2) were superposed, the resultant frequency would be 100Hz. In the case of component frequencies 600Hz and 400Hz, the resultant frequency would be 200Hz.

When a musical note is sounded, the fundamental and the low-numbered harmonics are normally audible. When the human auditory system encounters a number of harmonically related pure tones with comparable amplitudes sounding simultaneously, it assigns the composite sound a single pitch. The pitch assigned is in accordance with the resultant frequency outlined above. This *pitch* will be unchanged whether the fundamental or indeed any particular harmonic is missing. This pitch is called the *periodicity pitch* of the sound and the phenomenon is sometimes called the *missing fundamental*.

While the periodicity pitch corresponds to the overall repetition rate of vibration of the ear drum, no sinusoidal component of this frequency can be detected in the ear , if the fundamental is missing from the source of sound. Also for example, if a pure tone of frequency 300Hz is applied to one ear and a pure tone of frequency 200Hz is applied to the other ear, the auditory system will assign a pitch corresponding to 100Hz, the highest common factor, to the sound perceived. It is important to note that even though 100Hz is the difference between 300Hz and 200Hz, this is not the same phenomenon as the sum and difference tones which arise due to non-linearity of the response of the ear drum which is discussed below. Periodicity pitch is the subject of continuing research and controversy. This can be illustrated by the perception of the chimes of bells.[21] The human auditory system appears to attempt to arrange and identify incoming complex sounds as portions of harmonic series, supplying the fundamental when it is absent. Though this is not fully understood, it is as if the auditory system compares incoming sounds with a template containing harmonic series.

Consonance of musical notes is associated with frequencies in the ratio of small whole-numbers and consequently high periodicity pitch. Above a base note of 256Hz, let us consider the intervals corresponding to ratios 2/1, 3/2, 4/3, 5/3, 5/4, 7/4, 6/5, 7/5, 8/5, 7/6-----------13/11--.These are in ascending order of integers. The periodicity pitch is found by dividing 256Hz by the denominators which generally tend to increase. In the case of the ratio 2/1 (1 octave), the periodicity pitch is 256Hz, whereas in the case of the ratio 13/11 the periodicity pitch is approximately 23Hz. This frequency is at the lower limit of the sensitivity of the human ear. In general for two pure tones with the frequencies f_1 and f_2, where $f_2 = m/n \times f_1$ and m and n are integers, the result is a wave pattern with a period t_0, that is equal to $t_1 \times n$, where t_1 is the period of the lower tone; and a frequency, f_0, that is equal to f_1/n. Two extreme examples are set out below.

Example 1: when,

f_2 = 304Hz, f_1 =256Hz

$$304 = \frac{19}{16} \times 256$$

$$t_1 = \frac{1}{256}$$

$$t_0 = \frac{1}{256} \times 16$$

$$f_0 = \frac{256}{16} = 16\text{Hz}$$

This frequency is below the human auditory range. The periodicity pitch is low therefore it produces a dissonant pitch interval.

Example 2: when,

f_2 = 384Hz, f_1 =256Hz

$$384 = \frac{3}{2} \times 256$$

$$t_1 = \frac{1}{256}$$

$$t_0 = \frac{1}{256} \times 2$$

$$f_0 = \frac{256}{2} = 128\text{Hz}$$

This frequency is significantly above the lower limit of the human auditory range. The periodicity pitch in Example 2 above is high therefore it produces a consonant pitch interval.

Although any pair of tones having a frequency ratio corresponding to consecutive degrees of a harmonic series has a repetition rate corresponding to the fundamental of that harmonic series, not all such pairs produce an *unambiguous sensation* of periodicity pitch corresponding to that repetition rate. Above denominators of 8 or 9 it is difficult for the human auditory system to unambiguously identify periodicity pitch. Such unambiguous identification is necessary for musical consonance.[22]

4.3 Heterodyne Components and Specially related Tone Pairs

When a single sinusoid is sounded at normal musical amplitude the ear drum vibrates not only with the frequency of the sinusoid but also at frequencies which are integer multiples of the frequency of the sinusoid. A harmonic series with the incoming sinusoid as fundamental is physically created in the ear. This is due to the non-linearity of the response of the ear above low amplitudes of incoming sound. When two or more tones are supplied simultaneously, further vibrational frequencies called *sum and difference tones* are generated. This effect is well known in other branches of science and engineering. The components generated due to non-linear response of a system are called *heterodyne components.* These can be found in the ear by the use of a search tone.[23]

When two musical notes, each with its fundamental and partials, enter the ear simultaneously several additional vibrational frequencies are created by this mechanism. Beating can take place between any pair of frequency components whether they be heterodyne components or supplied components. The existence of heterodyne components has, therefore, important consequences for the consonance/dissonance of musical intervals. The calculations of the simplest heterodyne components when two sinusoidal components are sounded are shown below. Experiments using surgical techniques to probe into the fleshy parts of the middle and inner ear have found this phenomenon to be the case. The fourth row of heterodyne components was added to Arthur Benade's original schema.[24]

Table .4.1 Heterodyne component schema

Original Components	Simplest Heterodyne Components	Next – Appearing Heterodyne Components	Next – Appearing Heterodyne Components
(P)	(2P)	(3P)	(4P)
	(P+Q), (P-Q)	(2P+Q), (2P-Q)	(3P+Q), (3P-Q)
		(2Q+P), (2Q-P)	(3Q+P), (3Q-P)
(Q)	(2Q)	(3Q)	(4Q)

The above schema shows that the net response due to the two stimuli is not the sum of the responses due to each separately. Pitches such as (P+Q), (P-Q), (2P+Q), (2P-Q) etc. are also heard.

The amplitudes of the heterodyne components depend on the amplitudes original input signals.

4.4 Pitch Matching

The following experiment titled *Beat Phenomenon and the "Almost unison" between two musical tones,* was conducted by Arthur Benade.[25]

If two tones are sounded labeled J and K, one having its four harmonic partials (J_1, J_2, J_3, and J_4) based on a 250Hz fundamental frequency, the other having its four harmonic partials (K_1, etc) based on 252Hz. Beats can take place in the neighborhood of four frequencies. The beating pairs (leaving aside heterodyne components) are as follows:

Tone J	250	500	750	1000 Hz
Tone K	252	504	756	1008 Hz
Beat frequency	2	4	6	8 Hz

As tone J is moved closer to tone K, the strongly marked beat between the fundamental components eventually become so slow as not to be easily heard. The second and higher harmonics are still beating vigorously, however, so that our attention is drawn to them as the next guide to the tuning process. This successive transfer of auditory attention to beats of the higher partials is very useful because it provides even finer indications as we approach an exact match. For instance, an almost unhearable ¼Hz beat between the fundamental components is associated with an easily detected rate of 1 Hz at the fourth harmonic. The presence of heterodyne components adds to the complexity of the beating phenomenon.

For example:

$(J_2 - K_1) = (248)$ Hz

$(K_2 - J_1) = (254)$ Hz

$(J_3 - K_1) = (498)$ Hz

$(K_3 - J_1) = (506)$ Hz

Two of the above heterodyne components beat near the fundamental and two near the second harmonic. Table 4.2 below shows a list of special relationship pitch intervals and the grouped (clumped) heterodyne components that occur. The specially related tone pairs decrease in definition in the table as the ratios move up the overtone series. The ear automatically clumps together heterodyne components which are nearly similar in frequency (when the tones are out of tune), for example, in Table 4.2 below, 1 quintuple stands for five partials in the first clump of heterodyne components, the second clump of heterodyne components contained in a unison is labelled 1 sextuple-a clump of six partials. These groups illustrate the amount of partials that will beat if the tones are out of tune. The more partials there are in each clump the finer the tuning (zero-beat) will be achieved. The ratio 8/5 and 7/6 contain only 3 clumps of only two heterodyne components. It can be deduced that the special relationship diminishes the further up the overtone series a ratio is found.

Table 4.2 Specially related tone pairs and indicators (amount of clumps and clump sizes)

Frequency Ratios for Specially Related Tone Pairs			
Ratios	Musical Names	Cents	Indicators
1/1	Unison	000	1 quintuple 1 sextuple 1 septuple 1 octuple
2/1	Perf. octave	1200	1 triple 4 quadruple 3 quintuple
3/2	Perf. 5th	702	3 double 9 triple
4/3	Perf. 4th	498	12 double 1 triple
5/3	Maj. 6th	884	14 double
5/4	Maj. 3rd	386	10 double
6/5	Min. 3rd	316	6 double
7/4	-	969	6 double
7/5	-	583	4 double
8/5	Min. 6th	814	3 double
7/6	-	267	3 double

Table 4.3 below shows four different groups of two tones. The first group in a whole-number ratio 3/2 which equals a 'special relationship' ratio produced by the frequencies 200 and 100Hz. Group 2 contains two frequencies that are not in special relationship and form a whole-number ratio 45/33. Group 3 contains two tempered frequencies produced by the frequencies 329.6 and 261.6Hz spanning a tempered pitch interval of a major third. The frequencies are in a whole-number ratio 29/23, and group 4 contains a microtonal frequency 137.5Hz in relation to the open E_2 string 82.4Hz. The whole-number ratio relationship produced by the pitch interval is 45/27.

The first column contains the simple mathematical devices for working out harmonics and heterodyne components. The heterodyne components are shown under each group.

Table 4.3 Heterodyne components for four whole-number ratios

Heterodyne Components	Group 1 3/2 = whole-number ratio relationship produced by the frequencies 200 and 100 Hz		Group 2 45/33 = whole-number ratio for frequencies 273 and 200 Hz as used by A.Benade[26]		Group 3 29/23 =whole-number ratio relationship produced by the frequencies 329.6 and 261.6 Hz spanning a tempered pitch interval of a major third		Group 4 45/27 = whole-number ratio relationship produced by the microtonal frequency 137.5 in relation to the open E_2 string 82.4 Hz	
FIRST FOUR HARMONICS OF EACH FREQUENCY GROUP								
	P = 200 Hz	Q = 100 Hz	P = 273 Hz	Q = 200 Hz	P = 329.6 Hz (E_4)	Q = 261.6 Hz (C_4)	P = 137.5 Hz	Q = 82.4 Hz (E_2)
2P	400		546		659.2		275	
2Q		200		400		523.2		164.8
3P	600		819		988.8		412.5	
3Q		300		600		784.8		247.2
4P	800		1092		1318.4		550	
4Q		400		800		1046.4		329.6
HETERODYNE COMPONENTS OF EACH FREQUENCY GROUP								
P+Q = summation tone	300		473		591.2		219.9	
P-Q = difference tone or implied tone and harmonic partials	100 *200* *300* *400*		73 *146* *219* *292*		68 *136* *340* *408*		55.1 *110.2* *165.3* *220.4*	
2P+Q	500		646		920.8		357.4	
2P-Q	300		246		397.6		192.6	
2Q+P	400		673		852.8		302.3	
2Q-P	0		127		193.6		27.3	
3P+Q	700		1019		1250.4		494.9	
3P-Q	500		619		727.2		467.6	
3Q+P	500		873		1114.4		384.7	
3Q-P	100		327		452.2		109.7	

When two tones having a special relationship (zero-beat), for example, the 3/2 (Group 1 above), are sounded, the auditory system assigns an implied tone of frequency equal to the highest common factor of the two supplied frequencies. In the case of two frequencies with an octave relationship such as 100Hz and 200Hz, the implied tone coincides with the difference tone (P-Q).The implied tone is only unambiguously assigned when frequencies are in special relationship. As two frequencies come into special relationship (ratio of small integers) all the harmonics, the heterodyne components and the implied tone become aligned in a single harmonic series.

The first four harmonic partials for (P-Q) are shown in italics. The calculated tone of 100 Hz is in octave relationship with the lower tone. If the tones are not in special relationship of this sort as shown by (P-Q) in Groups 2, 3 and 4 in Table 4.3 above the sub-collection of harmonics will be heard as a rough sounding low-pitched 'difference tone' along with the original tones P and Q. The simplified heterodyne components for group 2 are (19 [4Q-3P], 54 [2P-2Q], 73, 127, 146 Hz). Three components are harmonic multiples of the 73Hz component. Another tone normally inaudible is the summation tone (P+Q) as shown in Table 4.3 above and it can be found (including harmonics) by using a search tone. It is not necessarily heard by the ear but is present in the overall outcome of the tones make-up. If the tones, for example, in Group 2 were progressively moved towards a special relationship with one another, the difference tone, the summation tone, the unclassified other components, and the partials of the original tones all align themselves into a single harmonic relationship–the one associated with the implied tone.

4.5 Dissonant and Consonant Qualities of Combined Microtonal/ Tempered Frequencies

Table 4.5 p. 79 and Table 4.6, p. 80 below, give information on measurements taken to produce the first twenty-one graphs-Figure 4.2-Figure 4.23.Measurements are of single and combined frequencies. The experiment shows that a combined microtone and tempered pitch interval can be more consonant than a tempered pitch interval. The measurements were taken on an electric guitar with added frets and amplified acoustic guitar in relation to the normal tempered guitar frequencies. Frets were added in between the normal frets on an electric guitar using a reversed fingerboard as shown in Figure 3.1. Whole-number ratios were found and compared to the calculated whole-number ratios for the equally-tempered system in Table 3.6, p.61.

A similar experiment was done on an acoustic guitar with added pick-ups under both bridge and nut shown in Figure 2.1, p.46. The nearest equivalent frequencies to those measured on the electric guitar were found by plucking the opposite side of a stopped string and are illustrated in Table 4.6, p. 80. This experiment was taken to find whole-number ratios for pitch intervals involving microtones only which are compared to pitch intervals involving tempered frequencies.

Five notes were measured on the electric guitar spanning from the second fret, fifth string (B_2) to the added fret above the semitone C_3 (normal third fret). Between B_2 and C_3 two frets are added and one above C_3. The five notes are: (1) B_2, (2) microtone, (3) microtone, (4) C_3, (5) microtone. These five notes were measured singly and as pitch intervals with B_3.

The following accurate measurements were taken on an instrument that records the fundamental and harmonic partials only as it is processing airborne sound.

The produced graphs are visual aids to show the harmonics from a single tone, and the closeness of the upper partial of two specific measured notes when sounded together. The graphs give information on upper partials beating phenomenon. Heterodyne components are calculated using the calculations on Table .4.1, p.70. The calculations were taken in the light that a major 7^{th} tempered pitch interval is the single most dissonant pitch interval after a minor 2^{nd} pitch interval. Each calculated pitch interval was compared to the ratio of the major 7^{th}. The major 7^{th} pitch interval is shown in Table 3.6, p. 61 as the whole-number ratio (within 2 cents) 17/9-an intervallic inversion or complement of the minor 2^{nd} pitch interval ratio 18/17, the most dissonant pitch interval in the equally tempered system. An interval of a major seventh, 130.8 (C_3) - 246.9 (B_3), 1100 cents, was used as an example of a tempered dissonant interval in, Table 4.5, p.79. The lower frequency of the pitch interval in Column 5, is moved to five different frequencies spanning from 123.5 Hz (B_2) to the microtone 133.2 Hz above 130.8 (C_3) utilizing 3 microtones produced by three added frets. Each frequency was produced in graph form and measured singly starting with the highest frequency 246.9 (B_3) down to the lowest frequency (right to left when referring to Table 4.5 and Table 4.6). Then each of the frequencies in Column 2 of each table was measured together with the higher tempered frequency B_3, 246.9 Hz, starting from the highest to the lowest. In Table 4.5, p.79 below, the whole-number ratios for each pitch interval are shown and also the equivalent measurements in cents.

It was found that the microtone 133.2 Hz combined with the tempered frequency 246.9 Hz (B_3) in column six gives a lower ratio (13/7) than the tempered major 7^{th} pitch interval (17/9) produced by the frequencies 130.8 Hz (C_3) and 246.9 Hz (B_3) which indicates that the tempered pitch interval is more dissonant than the pitch interval containing a microtone which produces the ratio 13/7 as discussed above. This is significant because it shows that some pitch intervals involving microtones can be less dissonant then some pitch intervals produced by two tempered notes. The ratio 13/7 is 1068.5 cents – 31.5 cents less than a tempered minor 7^{th} pitch interval and is therefore not near a tempered interval but it is still more consonant. The remaining pitch intervals in Columns 3 and 4 are more dissonant and produce ratios, 25/23 and 63/32 respectively. The frequencies in the Column 2 in both Table 4.5 and Table 4.6 produce an octave-ratio 2/1 (1200 cents).

Table 4.4 below[27] illustrates ratios that are recognized consonants involving the first eight harmonic partials of the overtone series. Outside the special relationships stated here, pitch relationships become dissonant. Pitch intervals further up the overtone series have closer harmonics and thus produce roughness and are therefore more dissonant.

Table 4.4 Special Relationships in Consonant order

Special Relationships
2:1 (1200cents)
3:2 (702 cents),
4:3 (498 cents),
5:3 (884.4 cents)
5:4 (386.3 cents)
6:5 (315.6 cents),
7:4 (968.8 cents)
7:5 (582.5 cents)
8:5 (813.7 cents)
7:6 (266.9 cents)
11:5 (1365 cents)
11:4 (1751.3 cents)
11:3 (2249.4 cents)
11:2 (2951.3 cents)
11:1 (4151.3 cents)
13:6 (1338.5 cents)
13:5 (1652.4 cents)
13:4 (2040.5 cents)
13:3 (2538.6 cents)
13:2 (3240.5 cents)
13:1 (4440.5 cents)

Apart from the first three ratios, 2:1, 3:2 and 4:3 in Table 4.4 above all the other special relationships deviate dramatically from the tempered system, for example, 11:4 and 11:3 are equivalent to tempered quarter tones (50 cent divisions). The notes of the equal temperament system do not fit into the scheme of consonance and dissonance within the overtone series. The tempering of notes dramatically altered the consonant quality of the pitch intervals that are derived from the overtone series. When pitch intervals are formed using the numbers 17 and 19 the results show that some intervals are close to tempered pitches. If pitch intervals formed high up in the overtone series are close to a special relationship then the ear will hear the interval as a 'slightly off' special relationship because of the nature of the aural mechanism.

Table 4.5 Measured frequencies on an electric guitar with added frets (see Figure 3.1,p.54)

Higher Frequency which forms each interval (open 2^{nd} string)	**246.9Hz (B_3)**				
Frequencies which form pitch intervals with B_3. Whole-number ratios for each pitch interval are also shown and equivalent cents (± 2cent).	**123.5 Hz (B_2)** **Ratio 2/1 (octave), 1200 cent**	**125 Hz** **Ratio 63/32,** **1174.5 cent**	**129.3 Hz** **Ratio 25/13** **1119.9 cent**	**130.8 Hz (C_3)** **Ratio 17/9 (Major 7^{th}), 1100 cent**	**133.2 Hz** **Ratio 13/7** **1068.5 cent**
String and fret where each note is found	5^{th} string, normal 2^{nd} fret	5^{th} string, added fret above 2^{nd} fret	5^{th} string, added fret below 3^{rd} fret	5^{th} string, normal 3^{rd} fret	5^{th} string, added fret above 3^{rd} fret
Type of frequency	tempered	microtone	microtone	tempered	microtone

The frequencies in Table 4.5 above were measured singly starting with the highest frequency in Column 6. See Figure 4.2 - Figure 4.7 and each frequency in row 2 was measured in combination with the tempered frequency 246.9Hz (B_3) in row 1. See Figure 4.8 - Figure 4.12.

The following Table 4.6 illustrates a similar experiment taken on an acoustic nylon string guitar. Single measurements (highest to lowest) are shown by Figure 4.13 - Figure 4.18, and in combination with 246.9Hz (B_3), see Figure 4.19 –Figure 4.23 Graph produced by tempered frequencies 246.9Hz (B_3, open 2^{nd} string) and 123.5Hz (B_2, 5^{th} string, 3^{rd} fret), played simultaneously on a nylon string acoustic guitar.

Table 4.6 Measured frequencies on acoustic guitar with added pick-ups under the bridge and nut (see Table 2.1, p.46)

Frequency which forms each interval (open 2nd string)	**246.9Hz (B_3)**				
Frequencies which form pitch intervals with B_3. Whole-number ratios for each pitch interval are also shown and equivalent cents (± 2cent).	**123.5 Hz (B_2)** **Ratio 2/1 (octave), 1200 cent**	**125 Hz** **Ratio 63/32,** **1174.5 cent**	**128.3 Hz** **Ratio 25/13** **1133.3 cent**	**130.8 Hz (C_3)** **Ratio 17/9 (Major 7th), 1100 cent**	**132.6 Hz** **Ratio 41/22** **1076.1 cent**
String and fret where each note is found	5th string, 2nd fret	6th string 19th fret,	6th string, 18th fret	5th string, 3rd fret	6th string, 17th fret
Type of frequency	tempered	microtone	microtone	tempered	microtone

Because there are no added frets on the acoustic guitar, the microtones were found by plucking the stopped string on the opposite side and will produce slightly different pitches. The guitar was amplified.

In the experiment in Table 4.6 above it is found that the three pitch intervals containing a microtone produced higher ratios indicating that they form more dissonant pitch intervals than the tempered dissonant major 7th pitch interval (ratio 17/9). The graphs were measured on a Larson•Davis Meter. See Larson•Davis measurement information before each set of graphs. The first seven graphs, Figure 4.1 - Figure 4.7, show the following information as measured on an electric guitar. The first Graph, Figure 4.1, shows the background noise at the time of measurement.

Figure 4.1 = background noise at time of measurements

Figure 4.2= tempered frequency 246.9Hz (B_3)

Figure 4.3 = microtone 133.2Hz (above C_3)

Figure 4.4 = tempered frequency 130.8Hz (C_3)

Figure 4.5 = microtone 129.3Hz (below C_3)

Figure 4.6 = microtone 125Hz (above B_3)

Figure 4.7 = tempered frequency 123.5Hz (B_2)

In the following five graphs two notes are played together as explained in Table 4.5, p. 79 above.

Figure 4.8 = interval produced by the frequencies 133.2Hz and 246.9Hz (B_3)

Figure 4.9 = interval produced by the frequencies 130.8Hz (C_3) and 246.9Hz (B_3)

Figure 4.10 = interval produced by the frequencies 129.3Hz and 246.9Hz (B_3)

Figure 4.11 = interval produced by the frequencies 125Hz and 246.9Hz (B_3)

Figure 4.12 = interval produced by the frequencies 123.4Hz (B_2) and 246.9Hz (B_3). This pitch interval is a tempered octave.

In the following eleven graphs the measurements are similar to the measurements on electric guitar with added frets except that they were taken on an acoustic guitar:

Figure 4.13 = tempered frequency 246.9Hz (B_3)

Figure 4.14 = microtone 132.6Hz (above C_3)

Figure 4.15 = tempered frequency 130.8Hz (C_3)

Figure 4.16 = microtone 128.3Hz (below C_3)

Figure 4.17 = microtone 125Hz (above B_3)

Figure 4.18 = tempered frequency 123.5Hz (B_3)

In the following five graphs two notes played together as explained in Table 4.6, p. 80 above.

Figure 4.19 = interval produced by the frequencies 132.6Hz and 246.9Hz (B_3).

Figure 4.20 = interval produced by the frequencies 130.8Hz (C_3) and 246.9Hz (B_3).

Fig. 4.21 = interval produced by the frequencies 128.3Hz and 246.9Hz (B_3).

Fig. 4.22= interval produced by the frequencies 125Hz and 246.9Hz (B_3).

Fig. 4.23= interval produced by the frequencies 123.4Hz (B_2) and 246.9Hz (B_3). This pitch interval is a tempered octave.

Figure 4.1 Recorded background noise during the following measurements

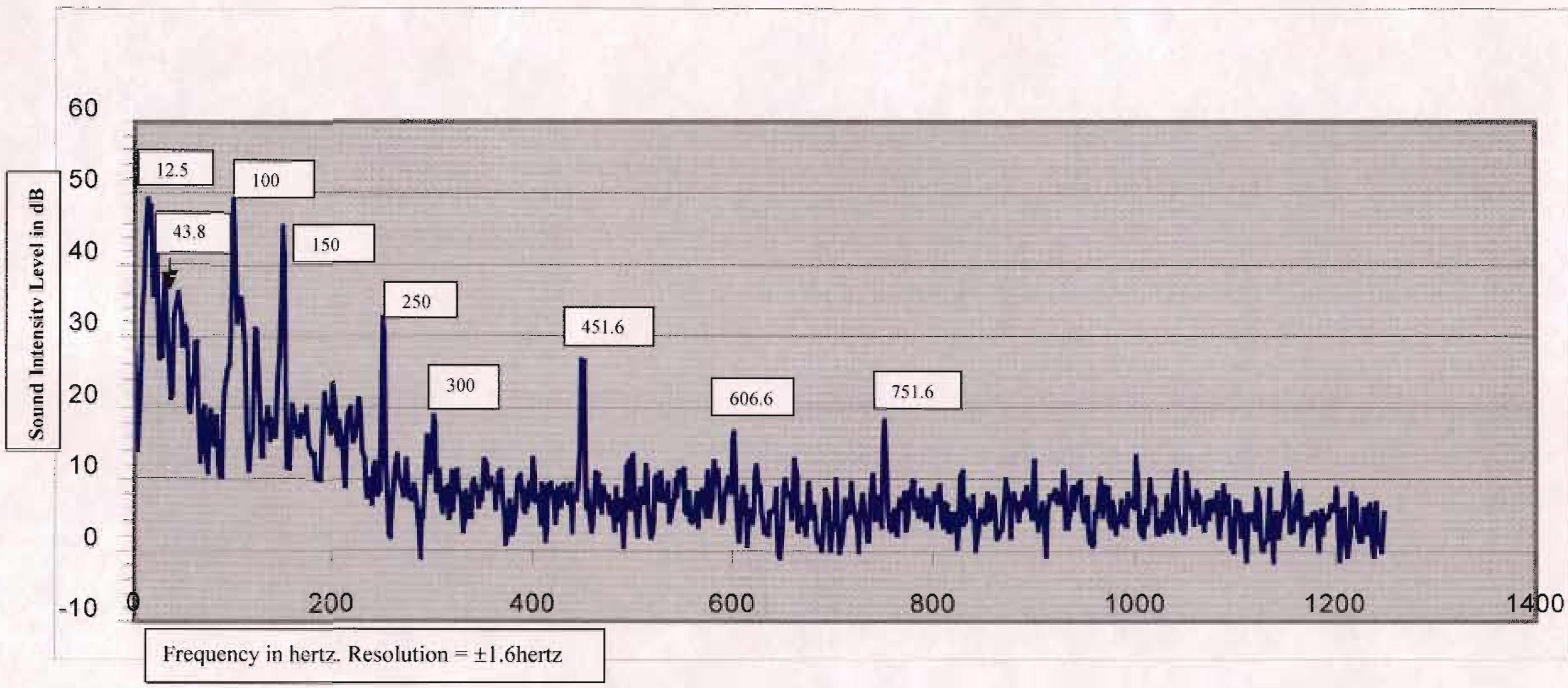


Figure 4.2 Tempered frequency B_3 (246.9Hz, open 2nd string) as produced by an electric guitar

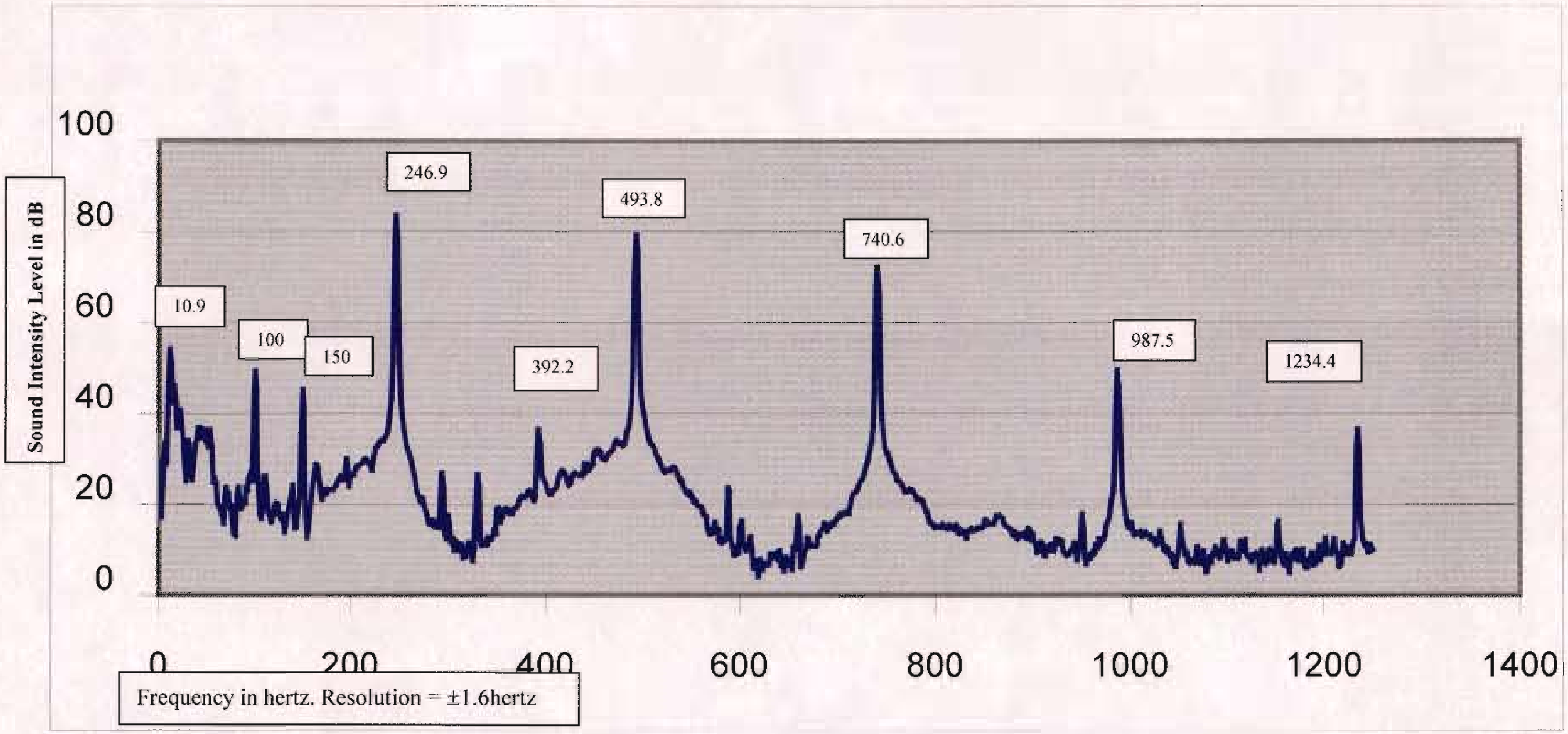


Figure 4.3 Microtone frequency 133.2Hz as produced by electric guitar (added fret above normal 3rd fret, 5th string)

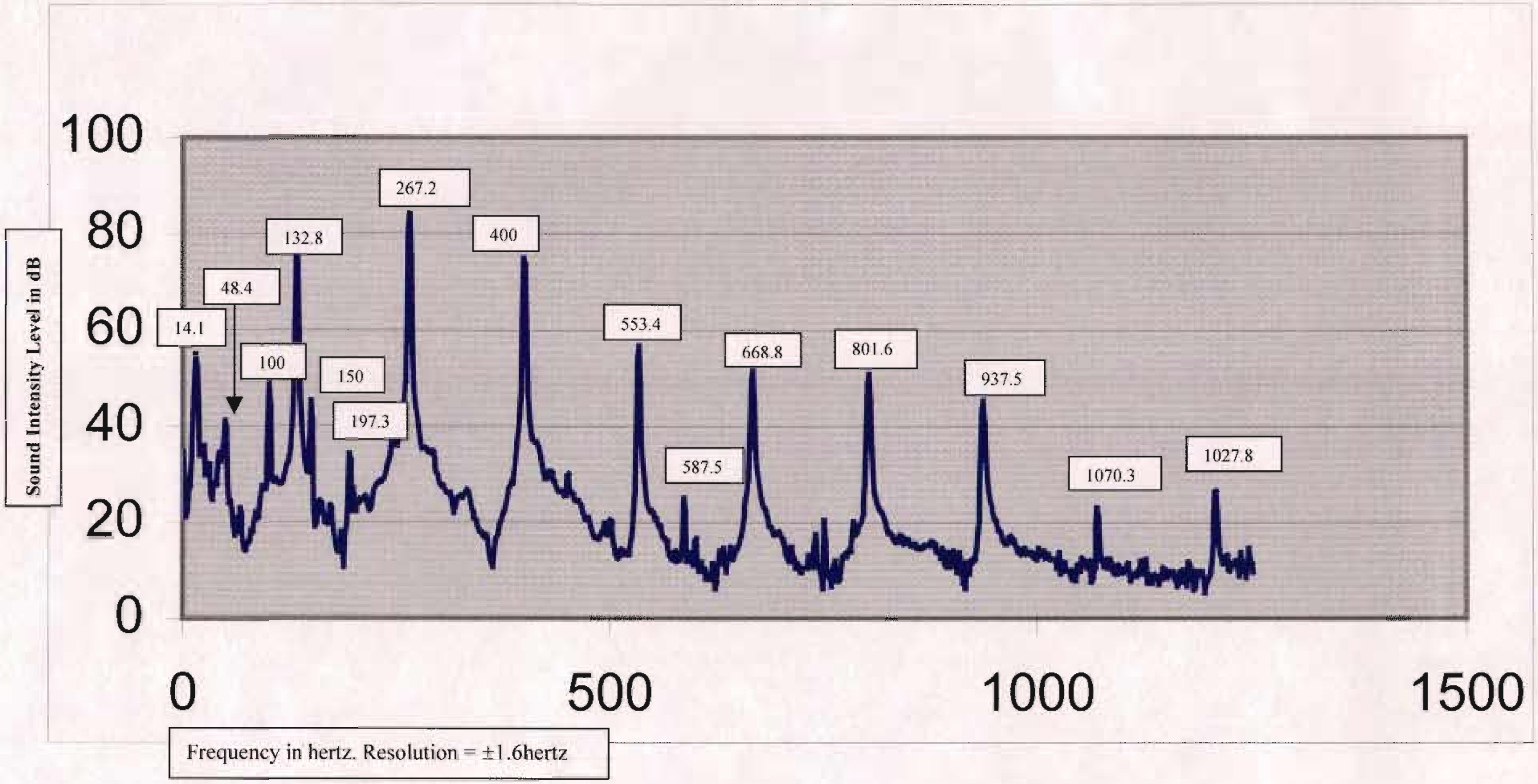


Figure 4.4 Tempered frequency 130.8Hz (C_3) as produced by an electric guitar (normal 3rd fret, 5th string)

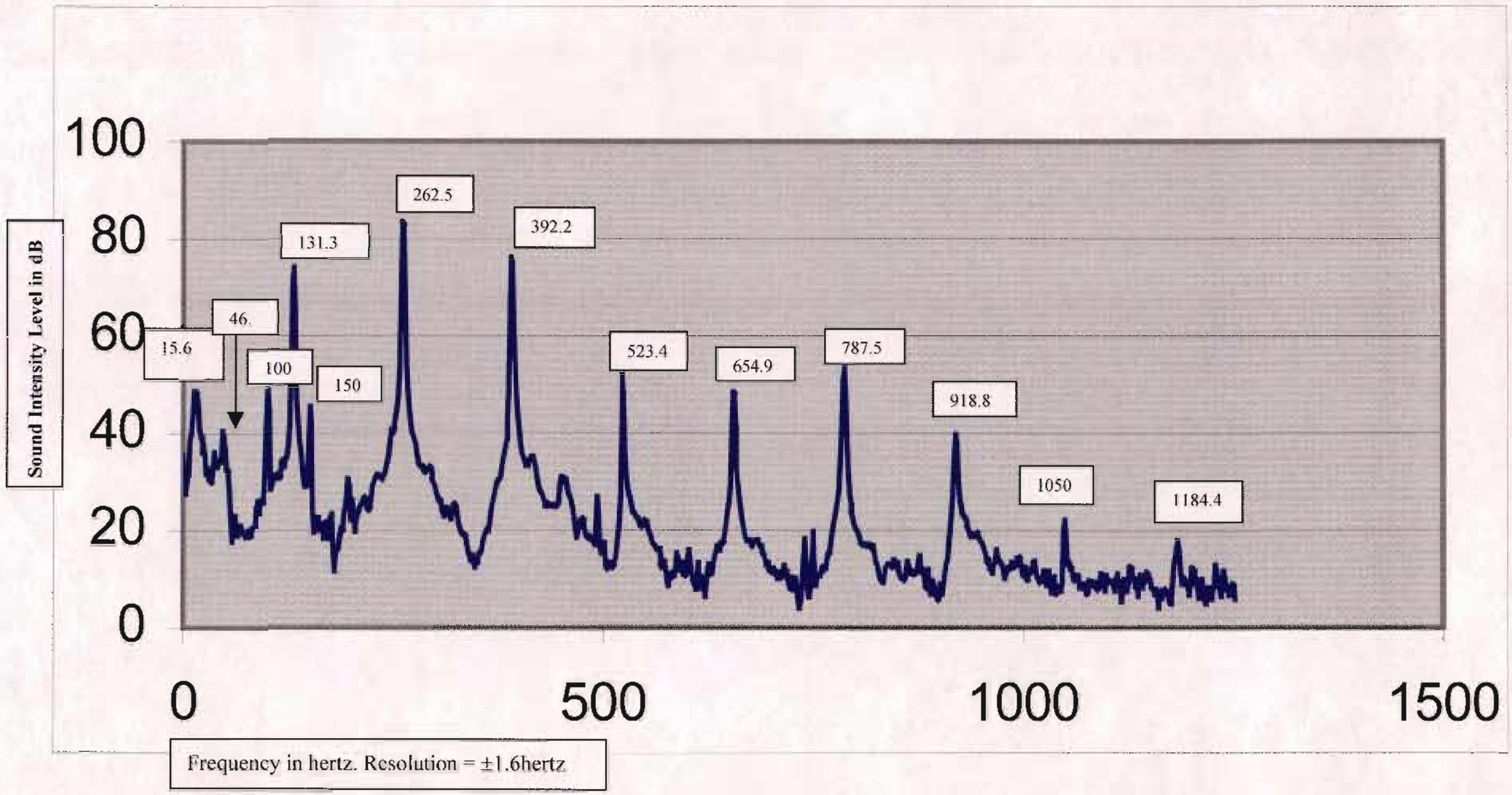


Figure 4.5 Microtone frequency 129.3Hz as produced by electric guitar (added fret below normal 3rd fret, 5th string)

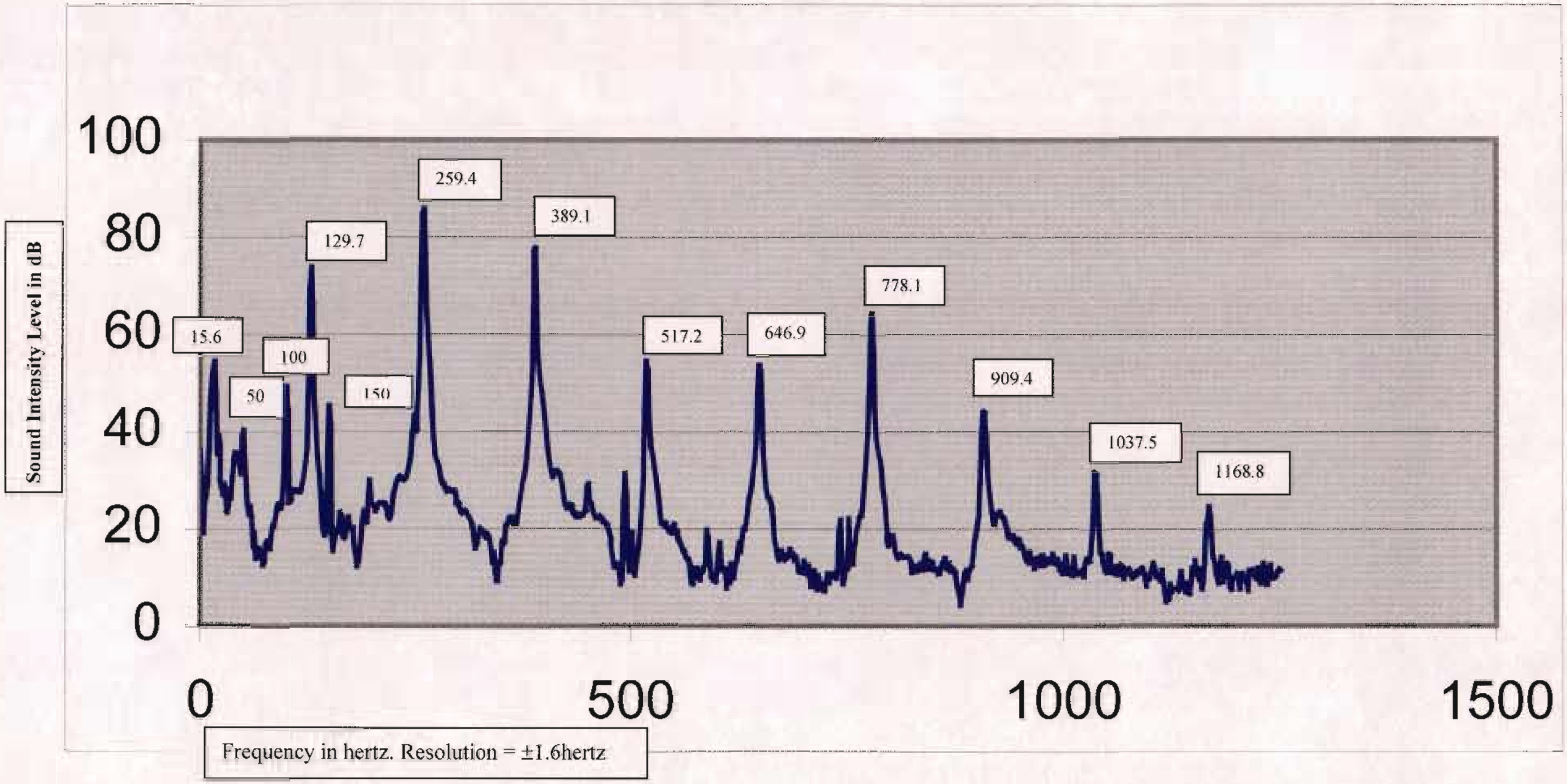


Figure 4.6 Microtone frequency 125Hz as produced by electric guitar (added fret above normal 2nd fret, 5th string)

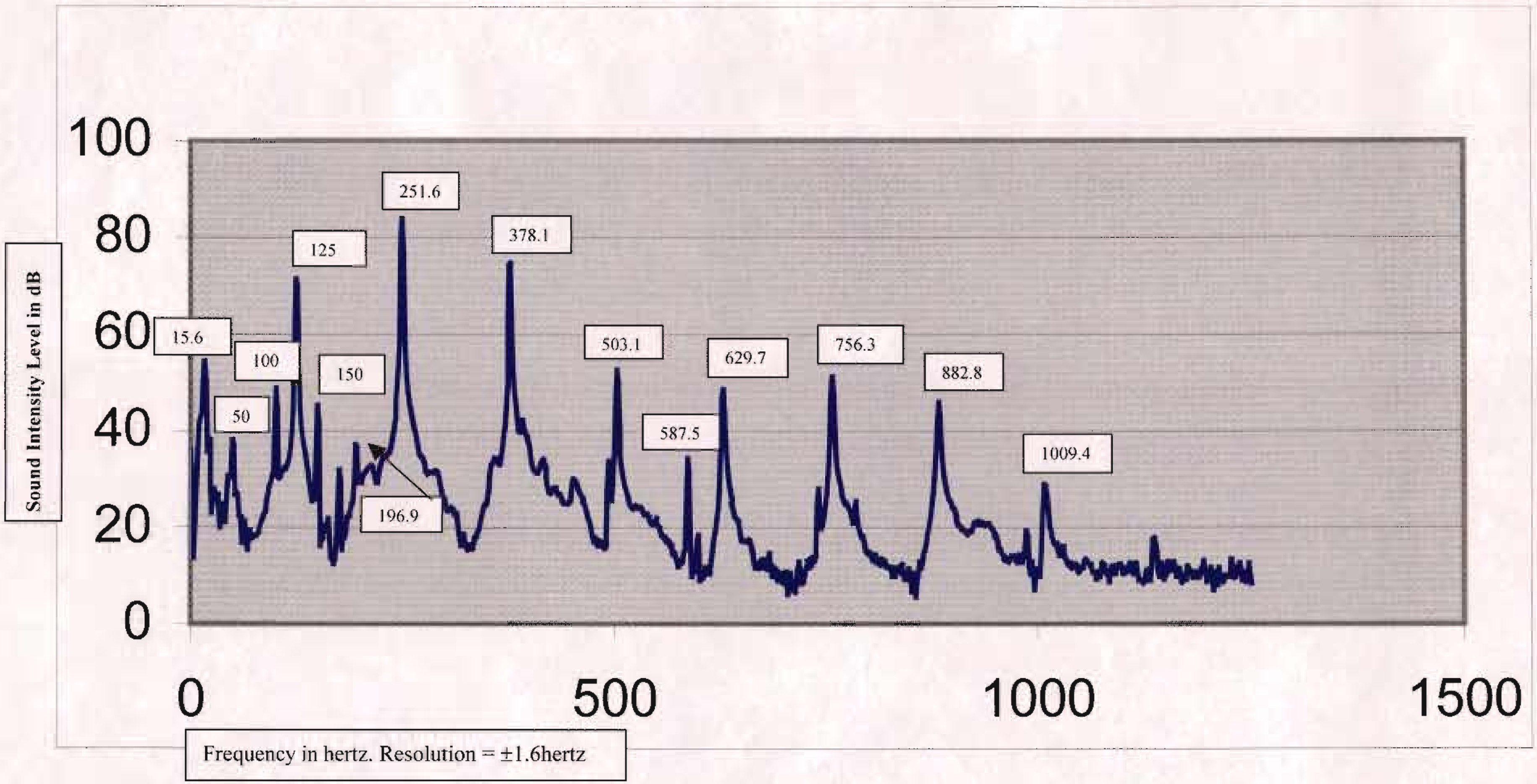
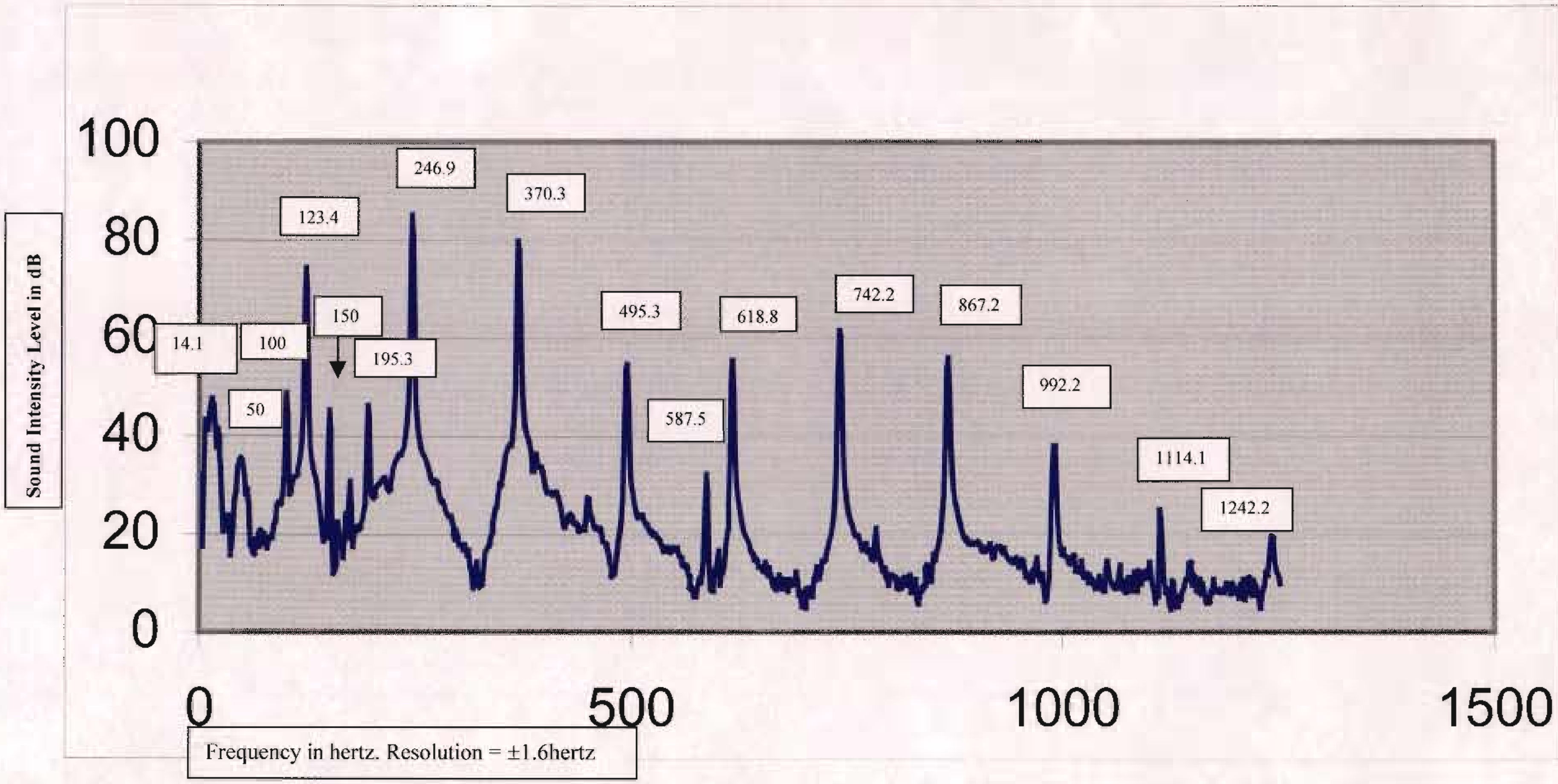


Figure 4.7 Tempered frequency 123.5Hz (B_2) as produced by an electric guitar (normal 2nd fret, 5th string)



The previous graphs show clearly the presence of background noise. This is illustrated separately in Figure 4.1. For example, 43.8Hz or 48.4Hz or 46.9Hz, 100Hz and 150Hz are often prominent in addition to the overtones corresponding to each frequency measurement.

The following five graphs, Figure 4.8-Figure 4.12 contain the measurements of two notes played together as explained in Table 4.5, p.79, above.

Figure 4.8 = interval produced by the frequencies 133.2Hz and 246.9Hz (B_3).

Figure 4.9 = interval produced by the frequencies 130.8Hz (C_3) and 246.9Hz (B_3).

Figure 4.10 = interval produced by the frequencies 129.3Hz and 246.9Hz (B_3).

Figure 4.11 = interval produced by the frequencies 125Hz and 246.9Hz (B_3).

Figure 4.12 = interval produced by the frequencies 123.4Hz (B_2) and 246.9Hz (B_3). This pitch interval is a tempered octave.

Figure 4.8 Graph produced by frequencies 246.9Hz (B_3, open 2nd string) and microtone 133.2Hz (produced by added fret above normal 3rd fret, 5th string), played simultaneously on an electric guitar (136.8 cents between 1st harmonic and B_3)

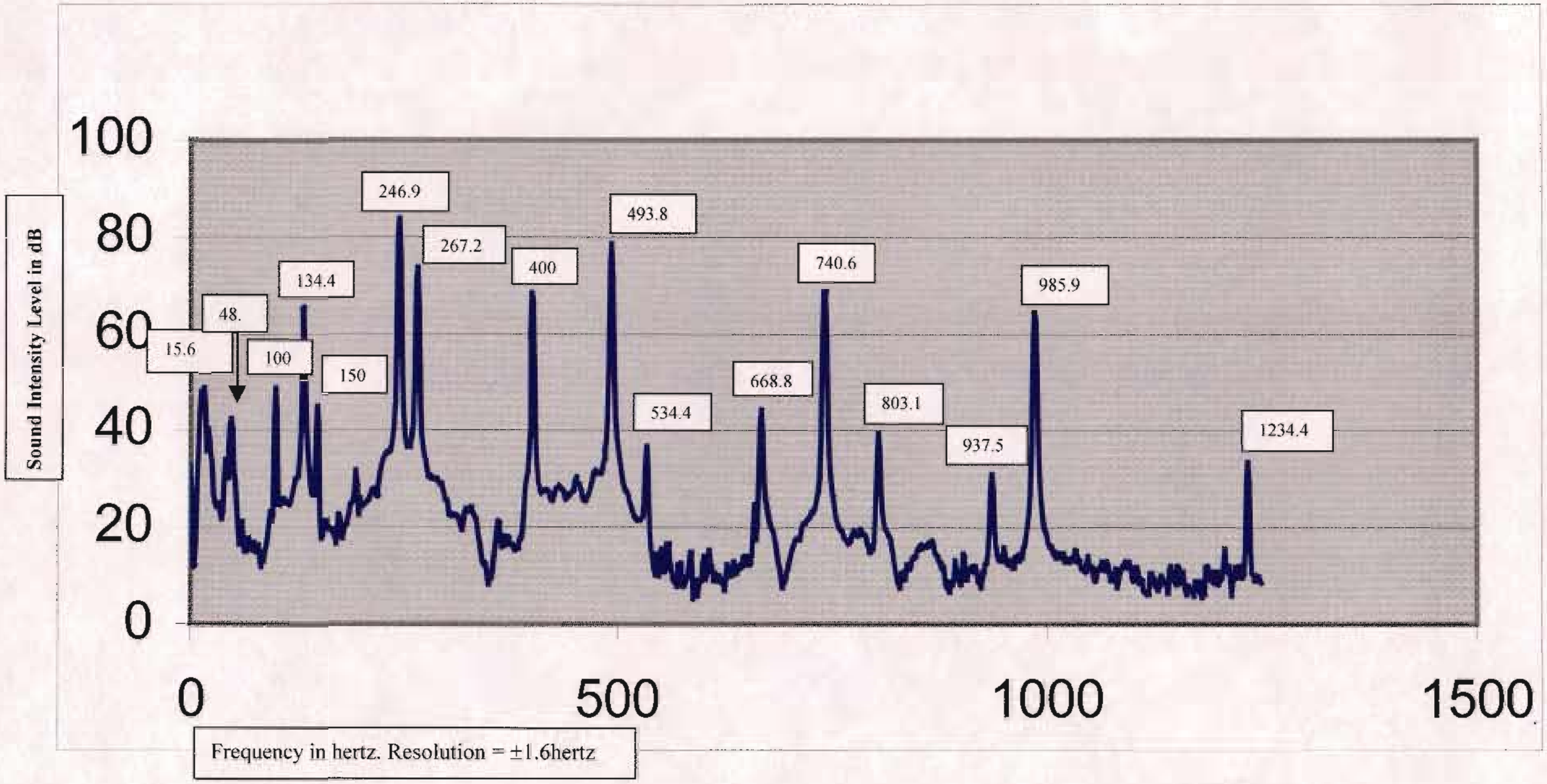


Figure 4.9 Graph produced by tempered frequencies 246.9Hz (B_3, open 2nd string) and 130.8Hz (C_3, normal 3rd fret, 5th string), played simultaneously on an electric guitar (106.1 cents between 1st harmonic and B_3)

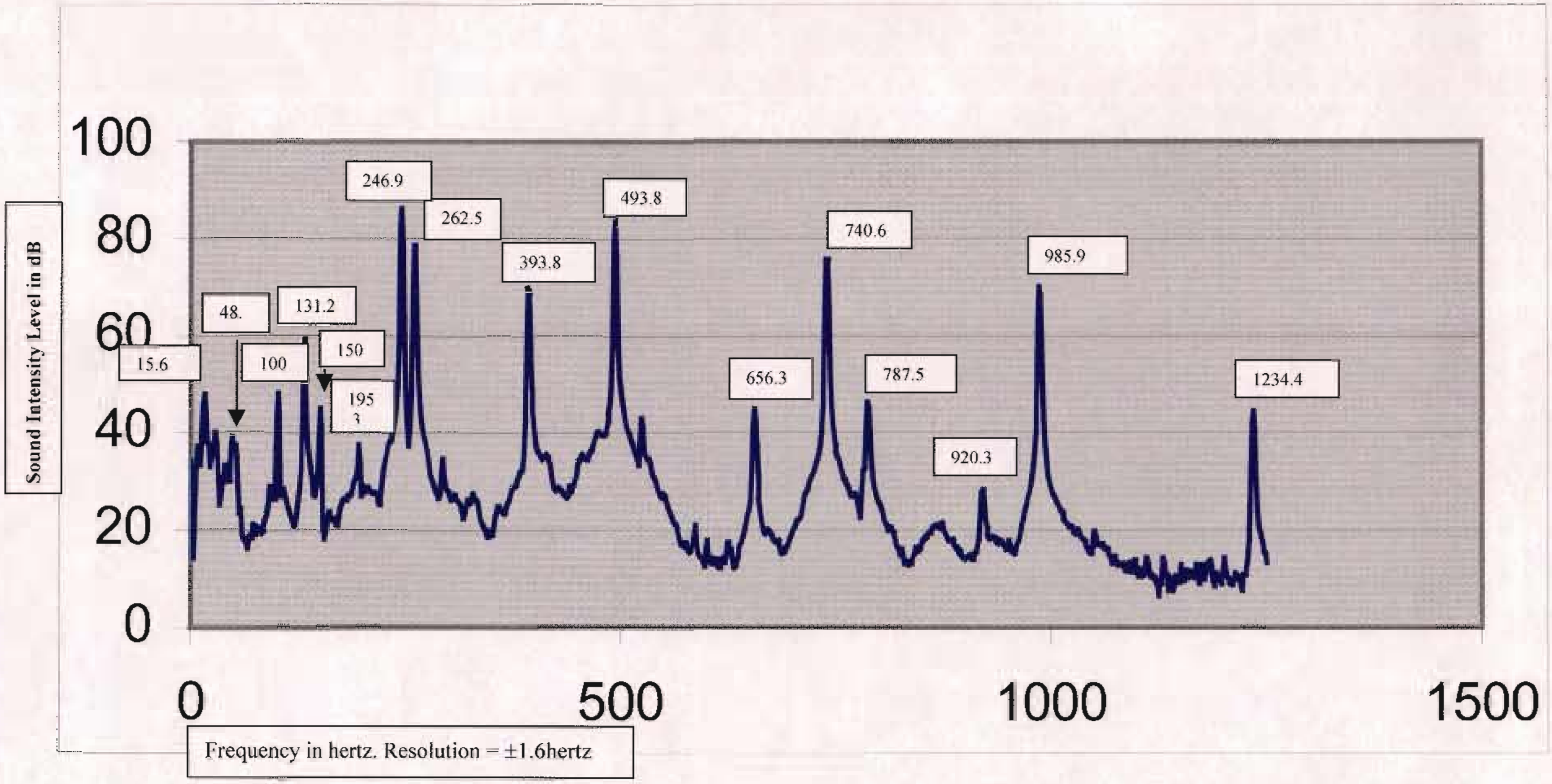


Figure 4.10 Graph produced by frequencies 246.9Hz (B_3, open 2nd string) and microtone 129.3Hz (produced by added fret below normal 3rd fret, 5th string), played simultaneously on an electric guitar. (74.8 cents between 1st harmonic and B_3)

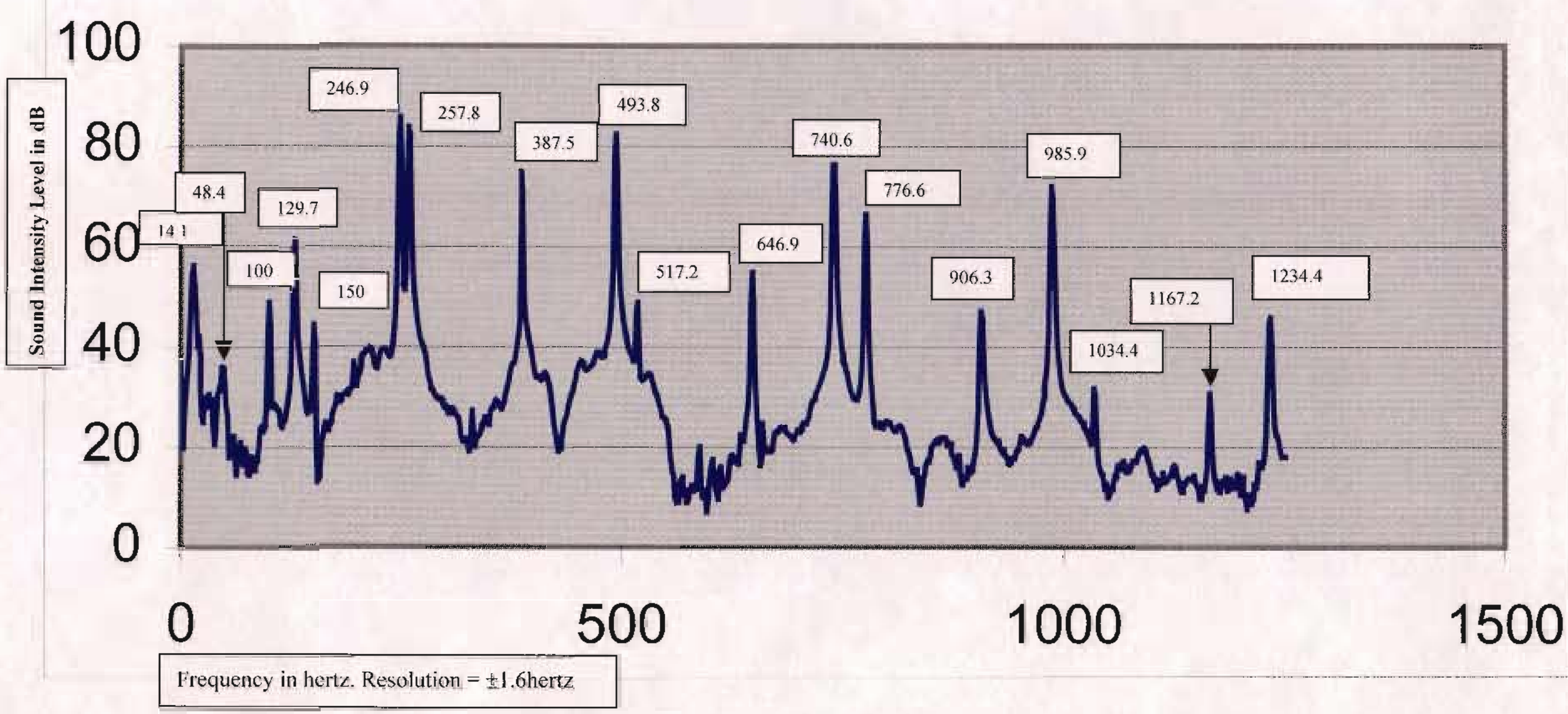


Figure 4.11 Graph produced by frequencies 246.9Hz (B_3, open 2nd string) and microtone 125hz (produced by added fret above normal 2nd fret, 5th string), played simultaneously on an electric guitar. (21.6 cents between 1st harmonic and B_3)

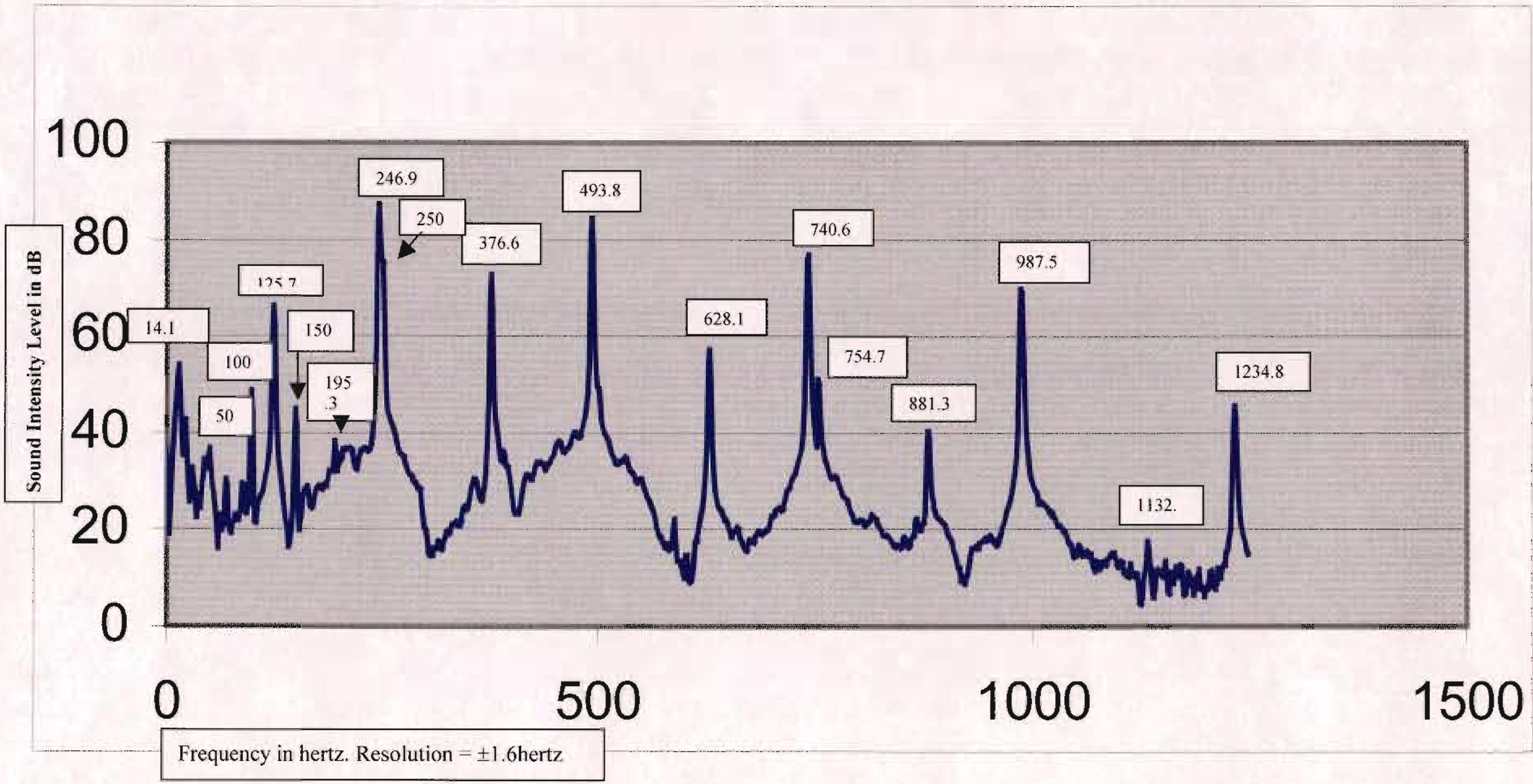
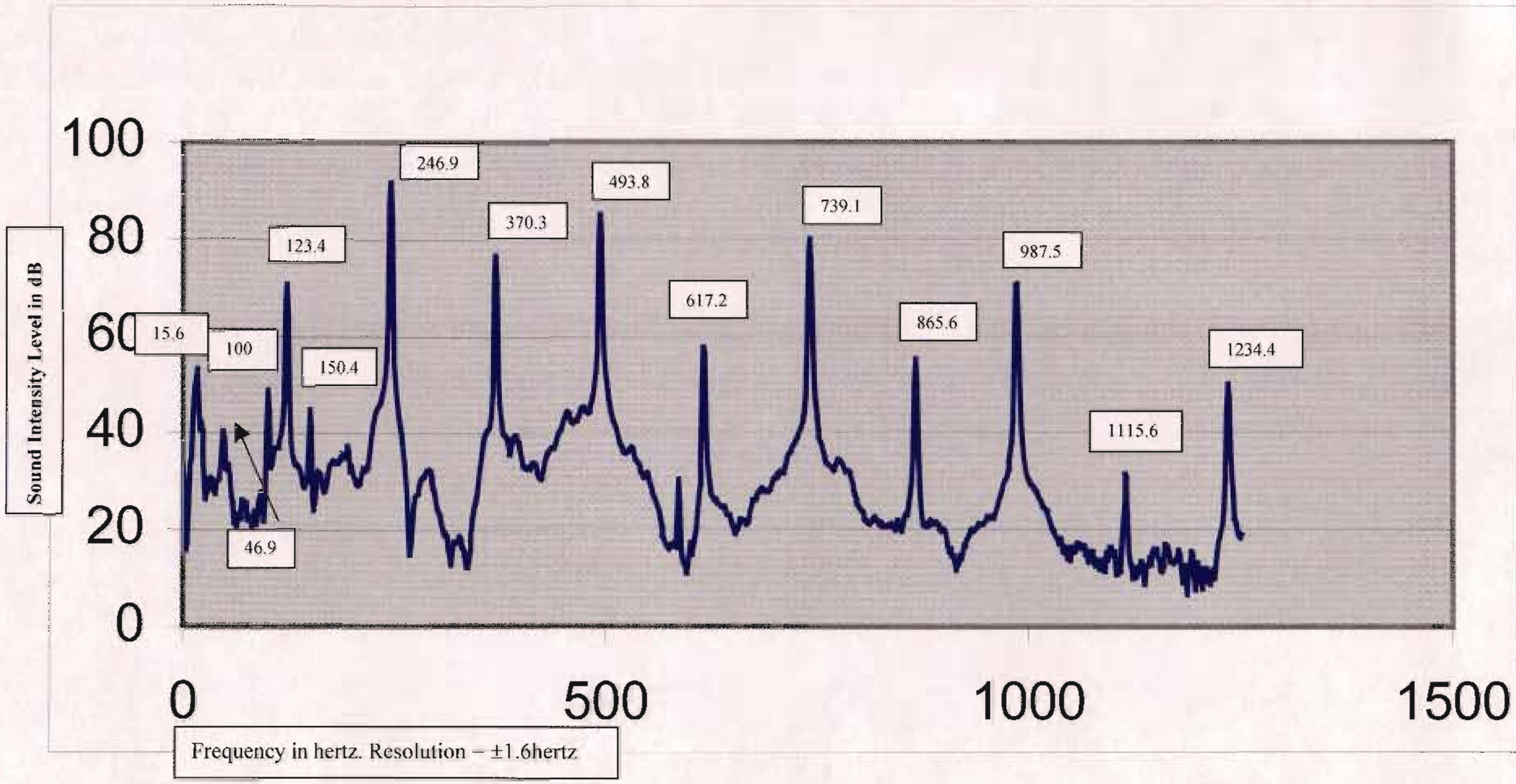


Figure 4.12 Graph produced by tempered frequencies 246.9Hz (B_3, open 2nd string) and 123.5Hz (B_2, normal 2nd fret,5th string). Octave, played simultaneously on an electric guitar (1st, 3rd, 5th, 7th and 9th harmonics are more prominent).



In the following eleven graphs the measurements are similar to Figure 4.2-Figure 4.12, except that they were taken on an acoustic guitar:

Figure 4.13 = tempered frequency 246.9Hz (B_3).

Figure 4.14 = microtone 132.6Hz (above C_3).

Figure 4.15= tempered frequency 130.8Hz (C_3).

Figure 4.16 = microtone 128.3Hz (below C_3).

Figure 4.17 = microtone 125Hz (above B_3).

Figure 4.18 = tempered frequency 123.5Hz (B_3).

Figure 4.13 Tempered interval 246.9Hz (B_3) produced by the open second string of a nylon string acoustic guitar

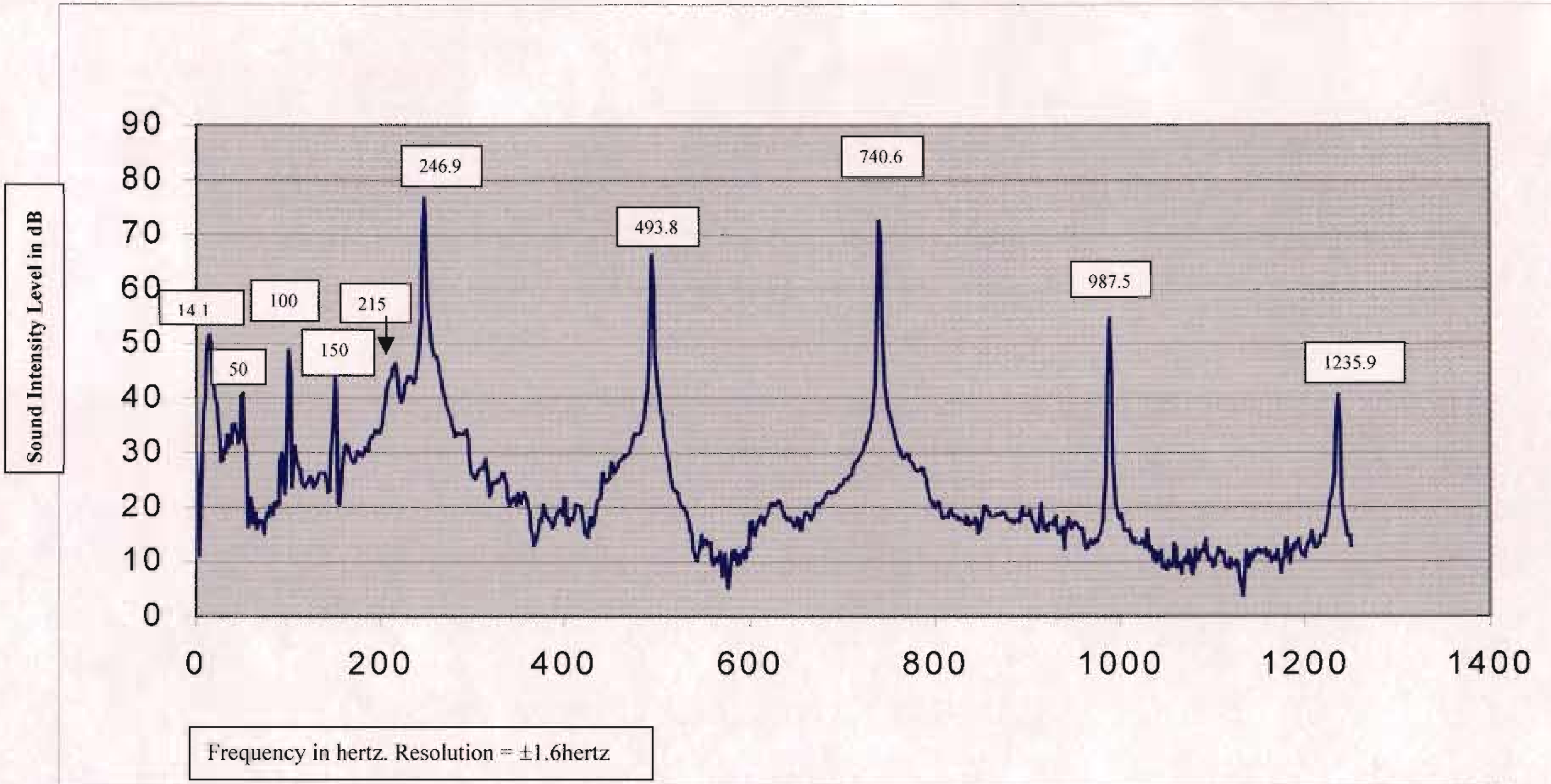


Figure 4.14 Microtone frequency 132.6Hz as produced by plucking a nylon string acoustic guitar on the nut-side of a stopped string (6th string, 17th fret)

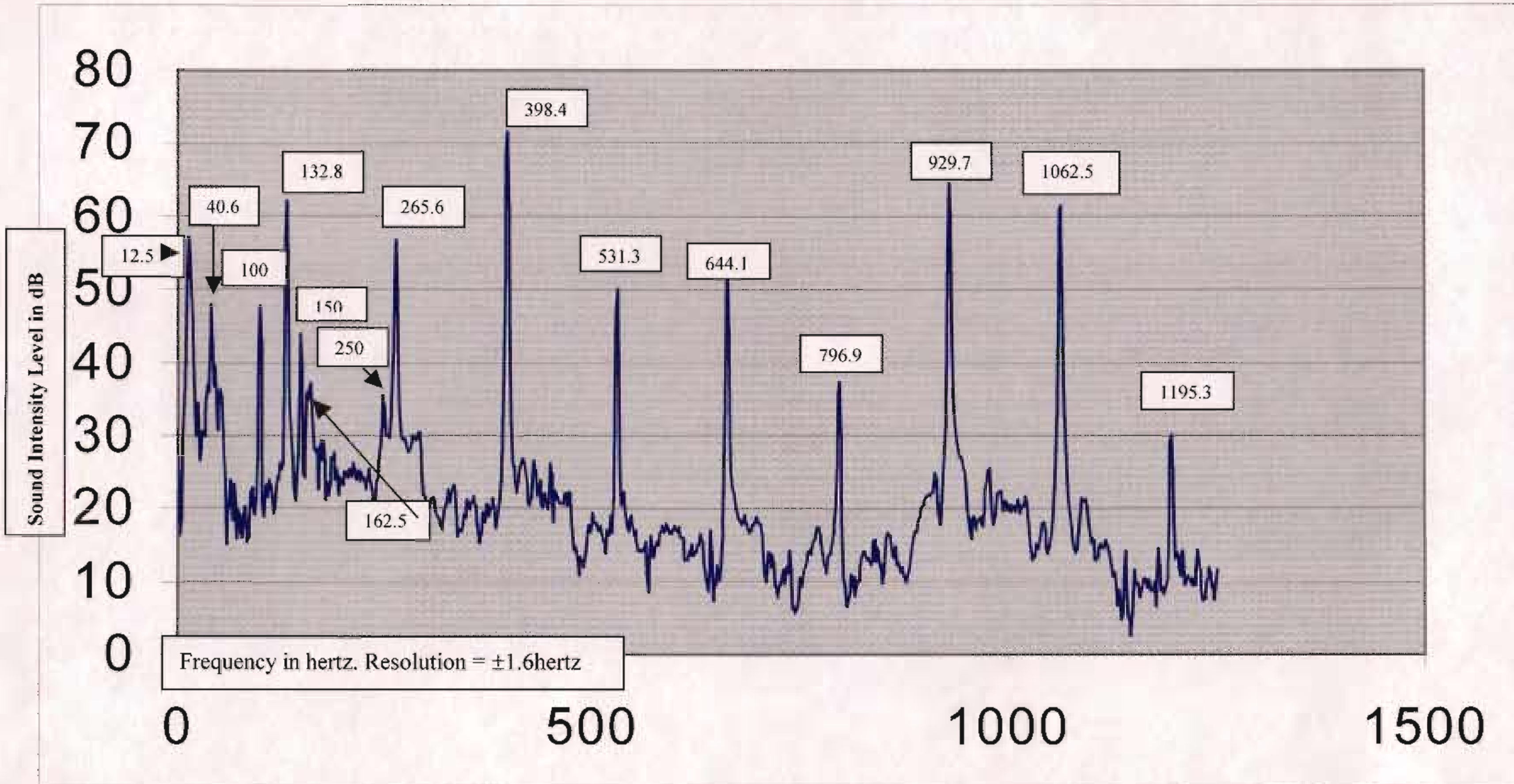


Figure 4.15 Tempered frequency 130hz (C_3) as produced by a nylon string acoustic guitar (6th string, 3rd fret)

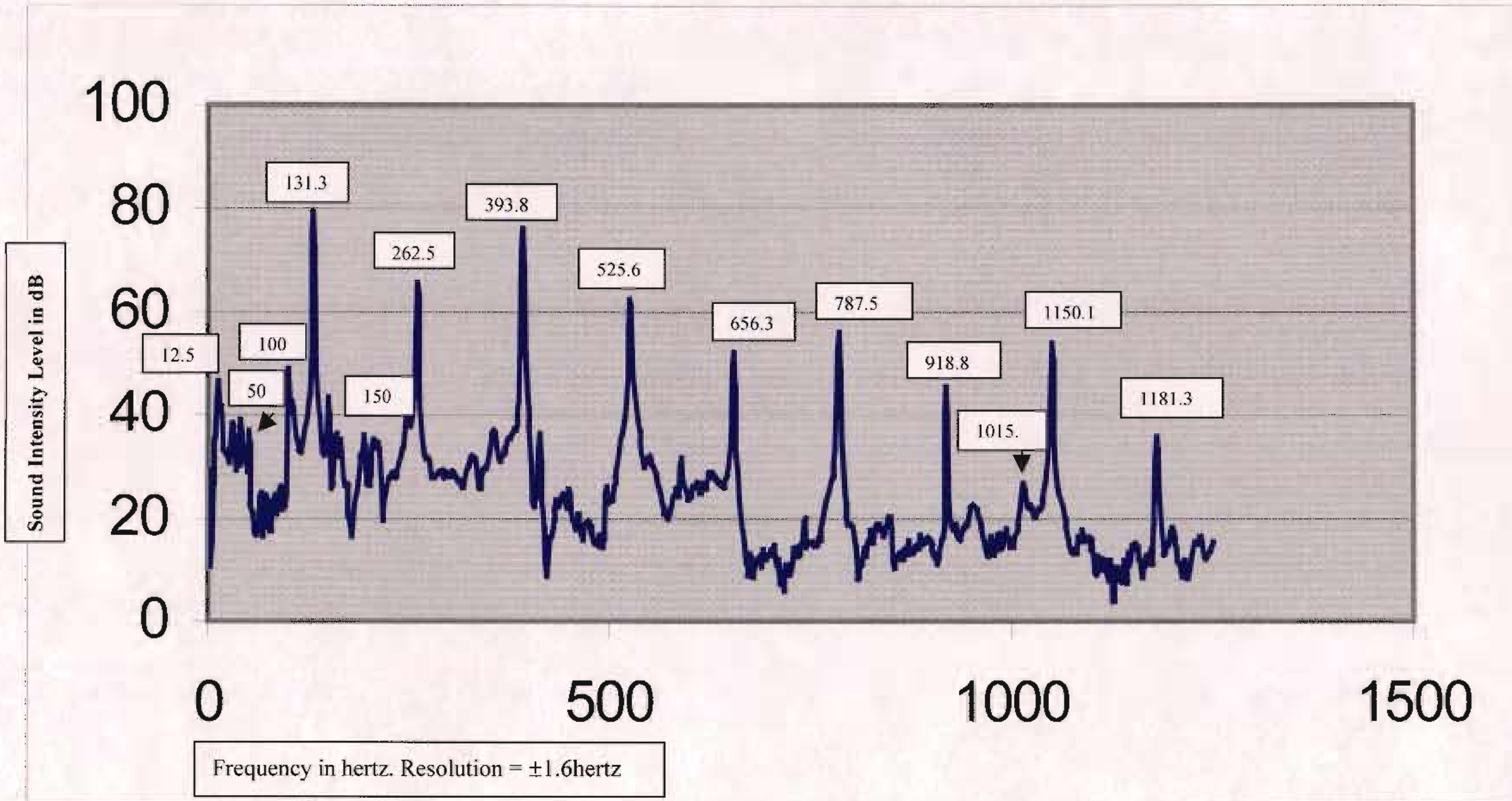


Figure 4.16 Microtone frequency 128.3Hz as produced by a nylon string acoustic guitar (6th string, 18th fret)

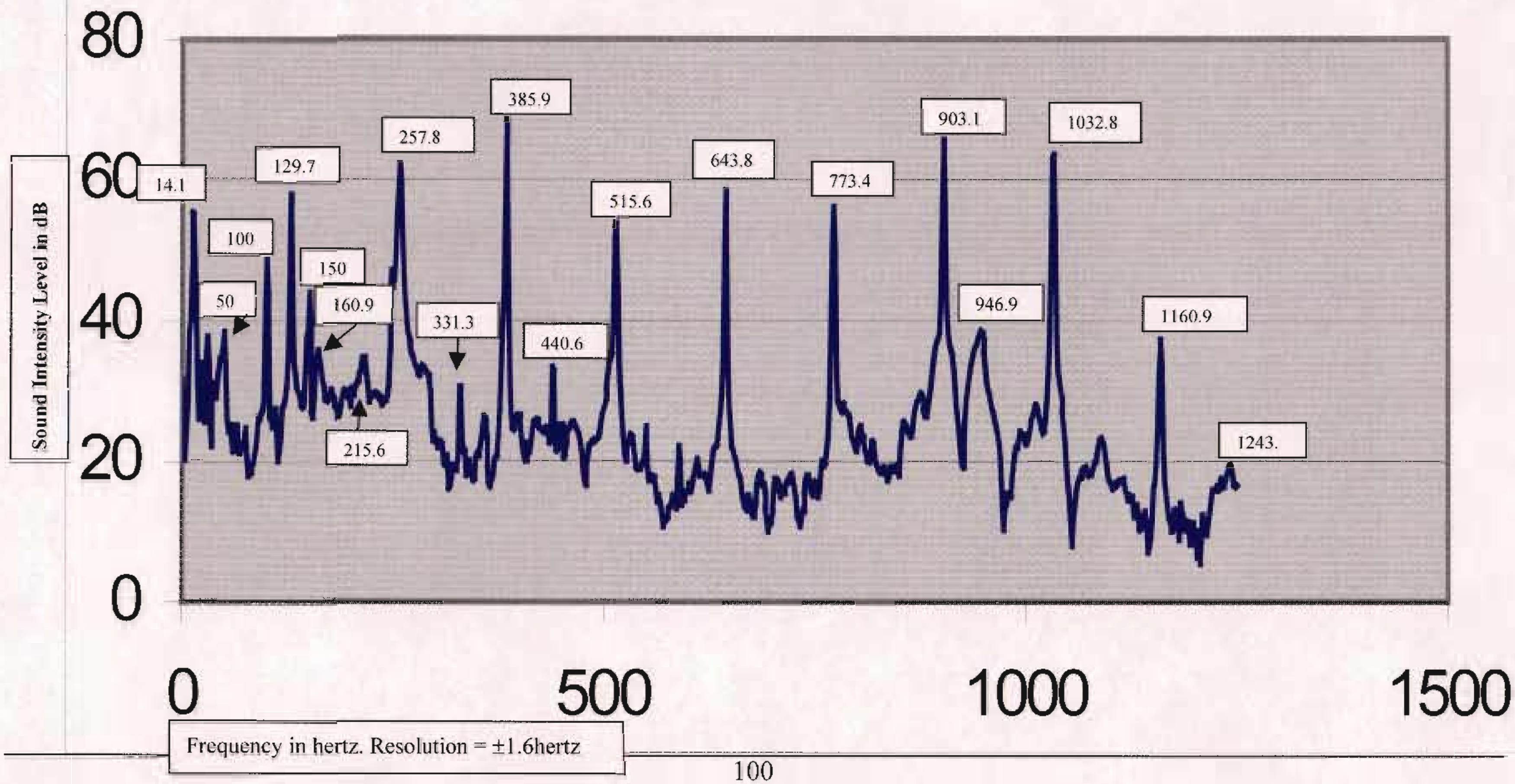


Figure 4.17 Microtone frequency 125Hz as produced on nylon string acoustic guitar (6th string, 19th fret)

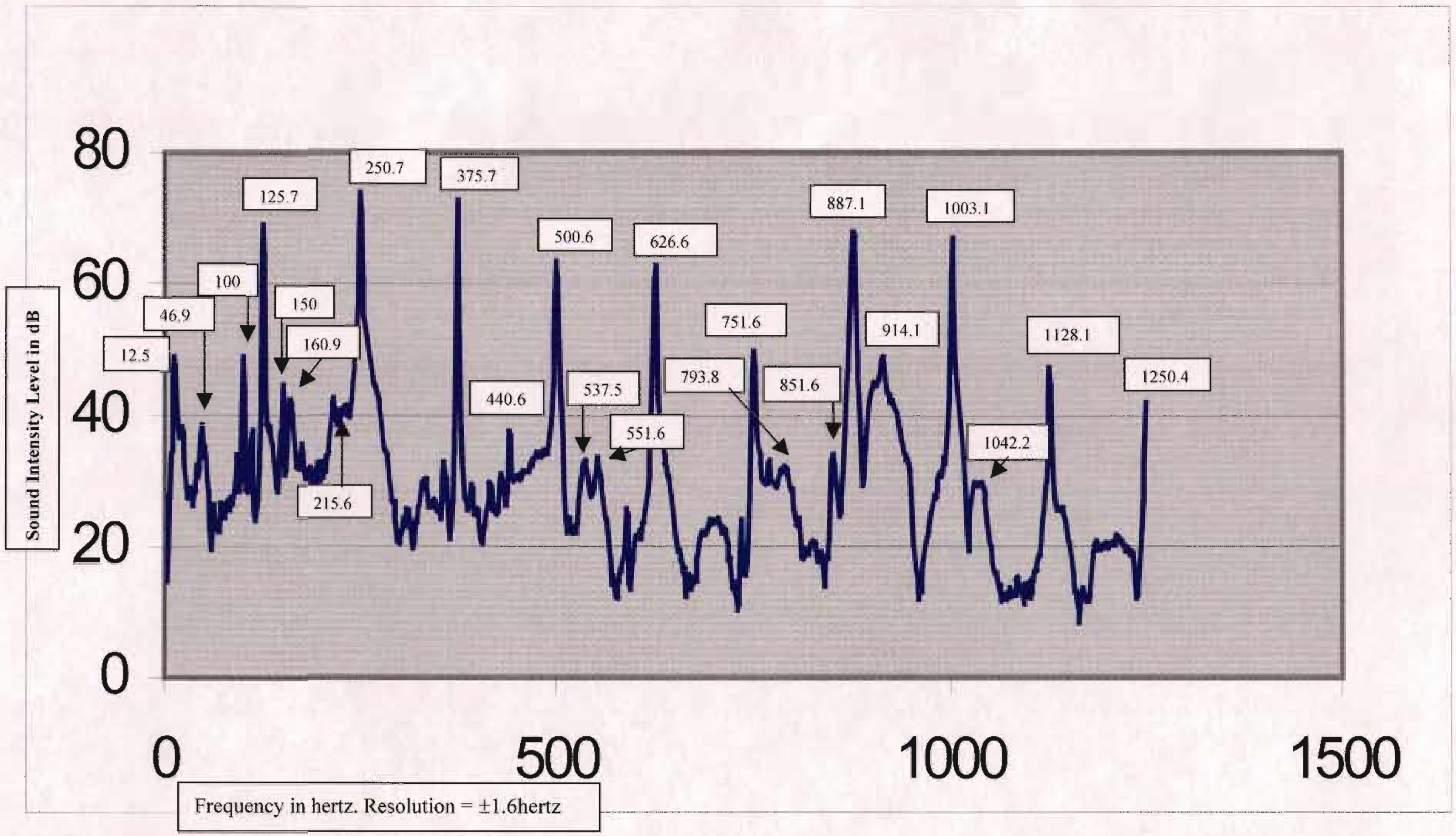
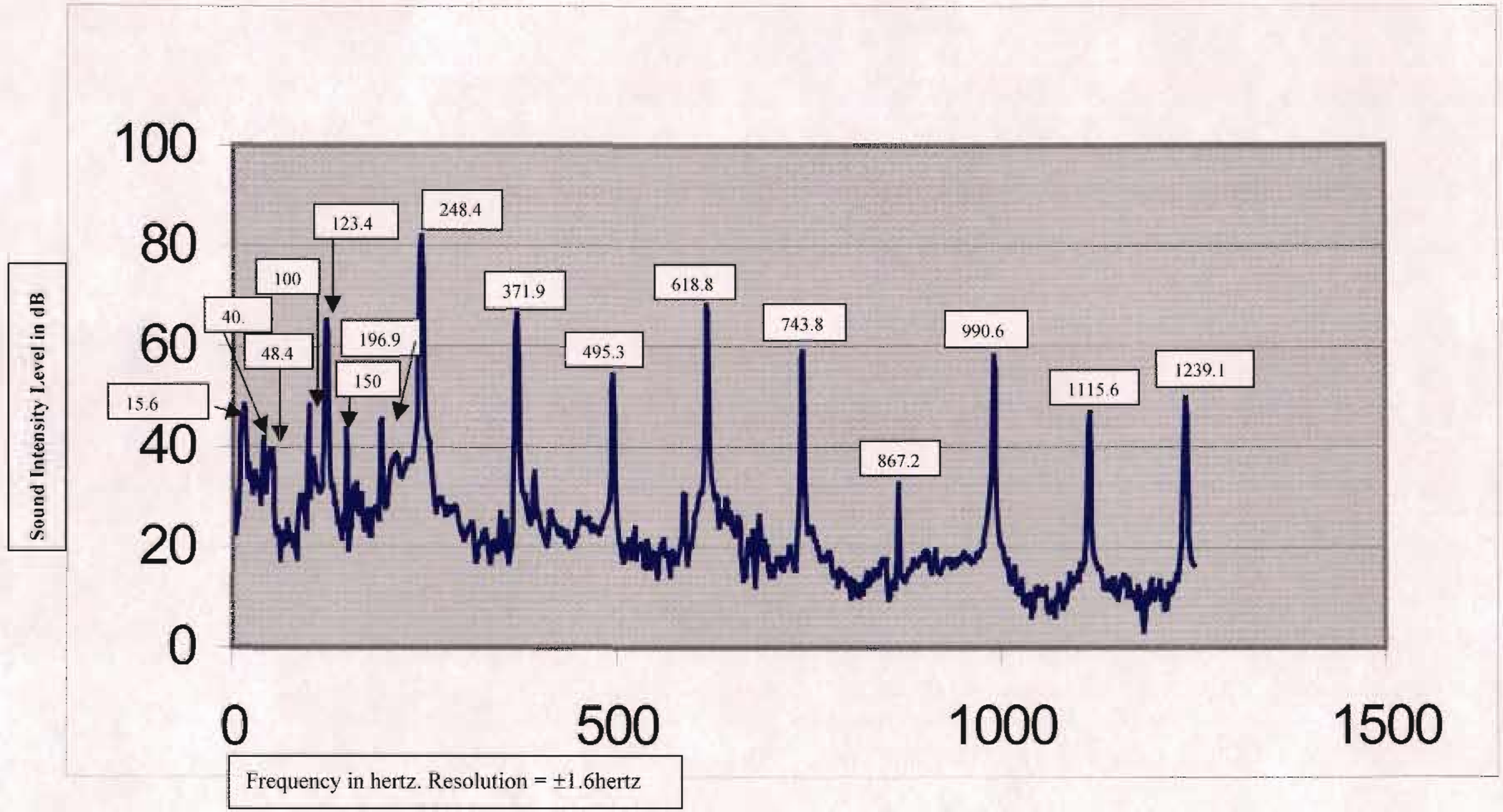


Figure 4.18 Tempered frequency 123.5Hz (B_2) as produced on a nylon string acoustic guitar (5^{th} string, 2^{nd} fret)



In the following five graphs Figure 4.19 – Figure 4.23 two notes were played together as explained in Table 4.6, p.80 above.

Figure 4.19 = interval produced by the frequencies 132.6Hz and 246.9Hz (B_3).

Figure 4.20 = interval produced by the frequencies 130.8Hz (C_3) and 246.9Hz (B_3).

Figure 4.21= interval produced by the frequencies 128.3Hz and 246.9Hz (B_3).

Figure 4.22= interval produced by the frequencies 125Hz and 246.9Hz (B_3).

Figure 4.23= interval produced by the frequencies 123.4Hz (B_2) and 246.9Hz (B_3). This pitch interval is a tempered octave.

In the following graph Figure 4.19 the simplest possible heterodyne components are mapped out in italics; at these points peaks would occur and add to the overall beating of partials. Since the machine measures in a linear mode, it cannot register the heterodyne components that are produced by the non-linear auditory system of the ear when two overlapping frequencies are presented to it.

They can be arranged together with their ancestors in a tabular schema as shown in Table .4.1, p.70.

Figure 4.19 Graph produced by frequencies 246.9Hz (B_3, open 2nd string) and microtone 132.6Hz (6th string, 17th fret), played simultaneously on a nylon string acoustic guitar.

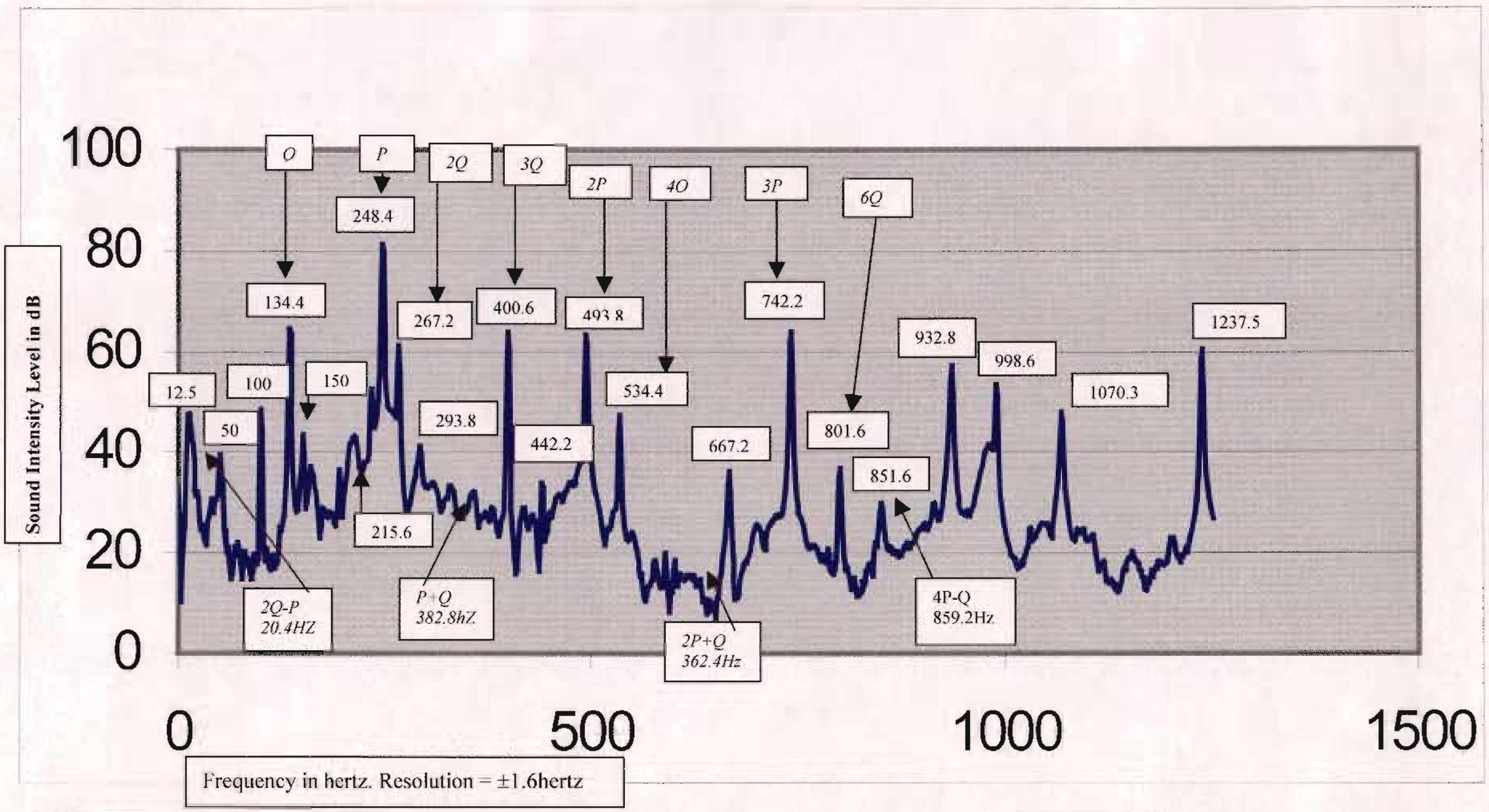


Figure 4.20 Graph produced by tempered frequencies 246.9Hz (B_3, open 2nd string) and 130.8Hz (C_3, 5th string , 3rd fret), played simultaneously on a nylon string acoustic guitar

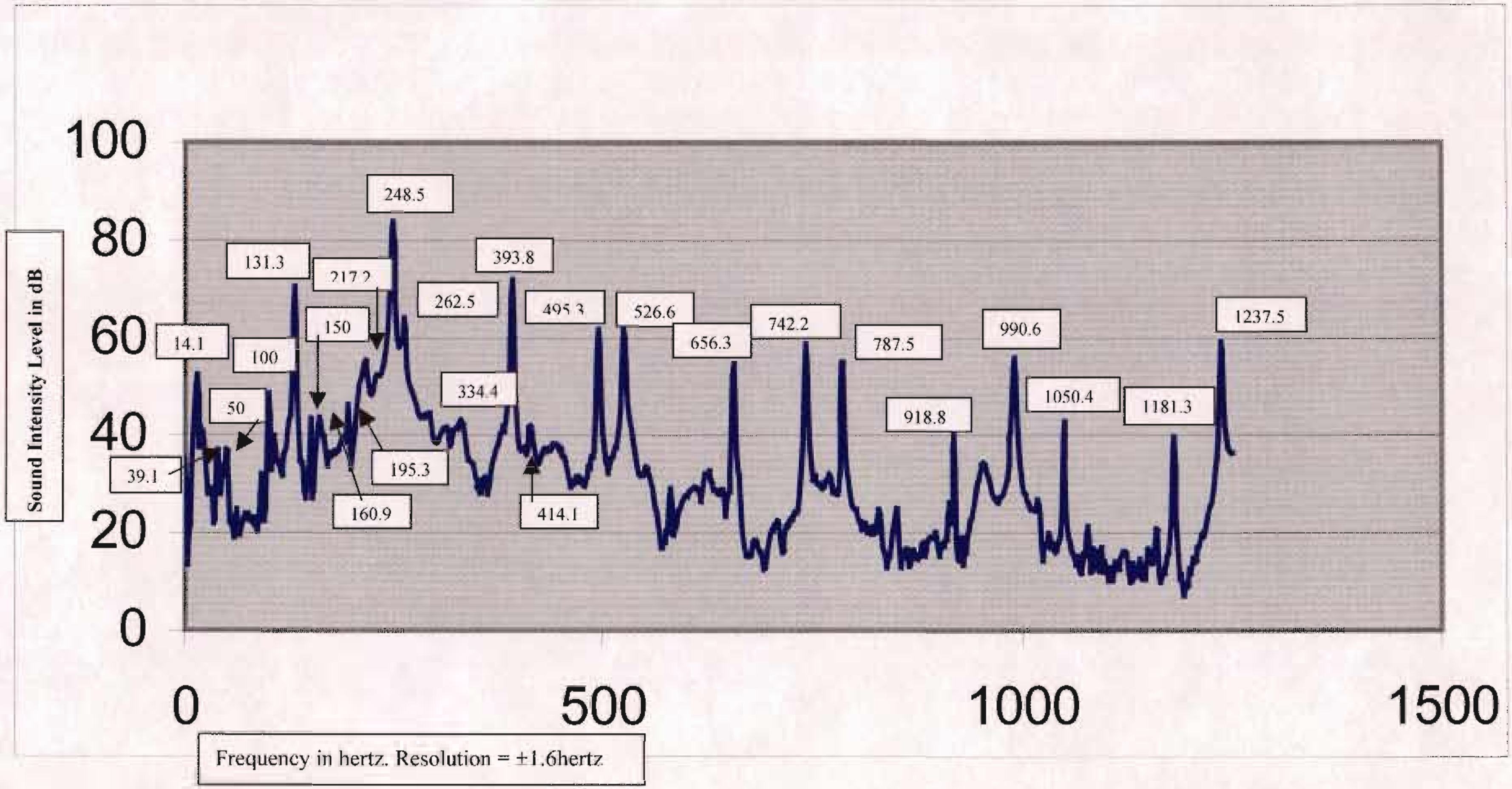


Figure 4.21 Graph produced by frequencies 246.9Hz (B_3, open 2nd string) and microtone 128.3Hz(6th string, 18th fret), played simultaneously on a nylon string acoustic guitar

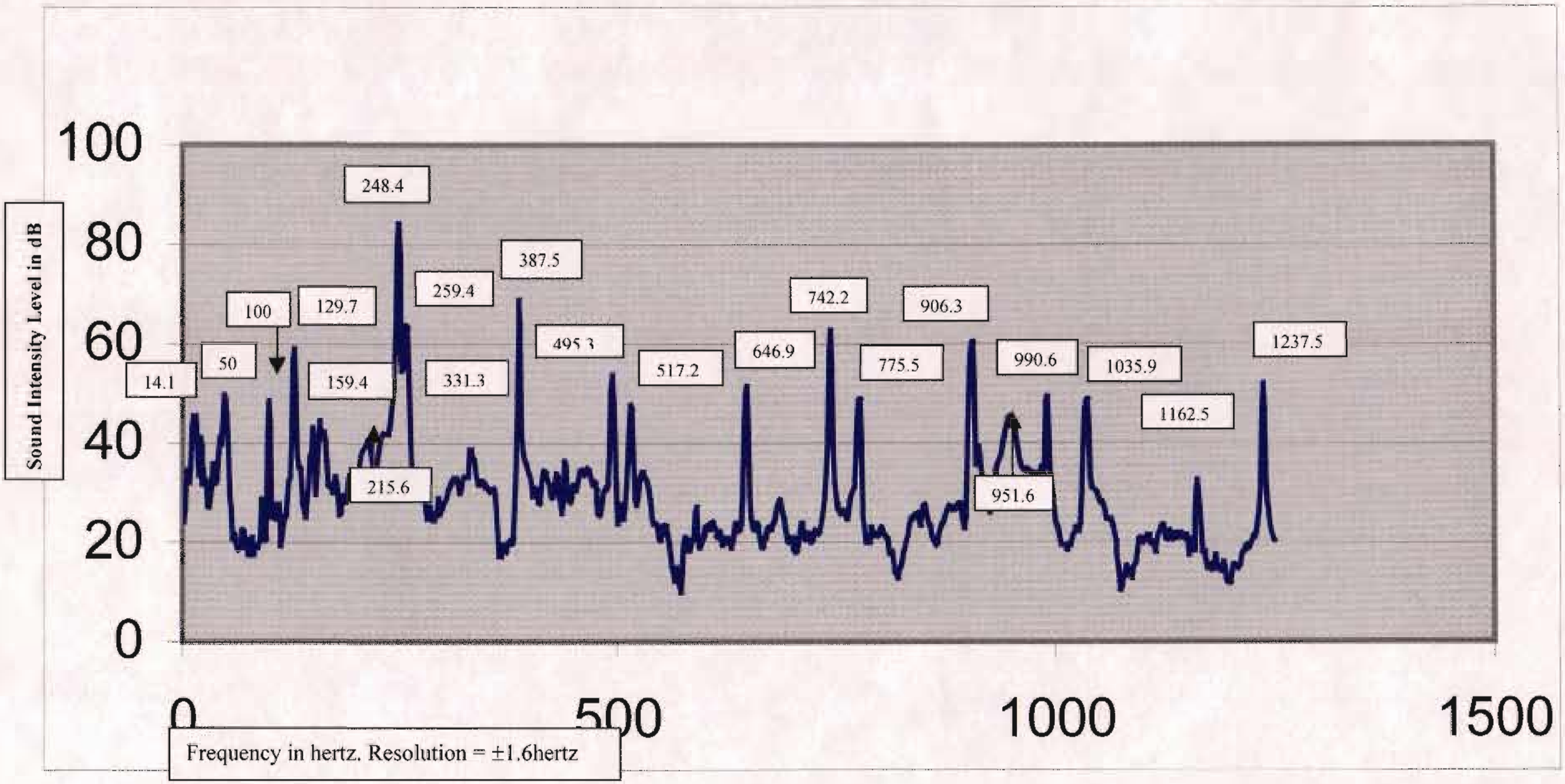


Figure 4.22 Graph produced by frequencies 246.9Hz (B_3, open 2nd string) and microtone 125Hz (6th string, 19th fret), played simultaneously on a nylon string acoustic guitar.

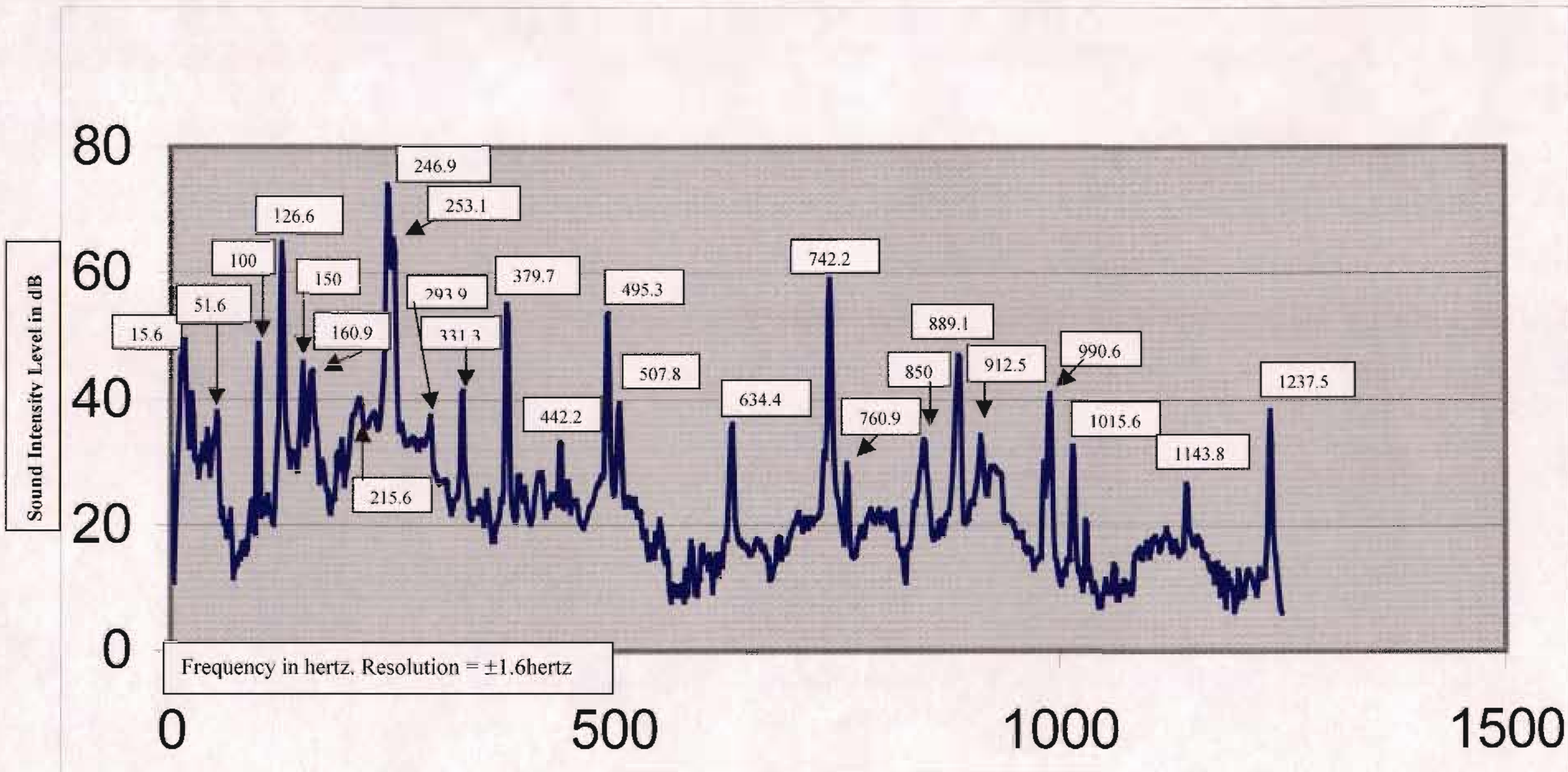
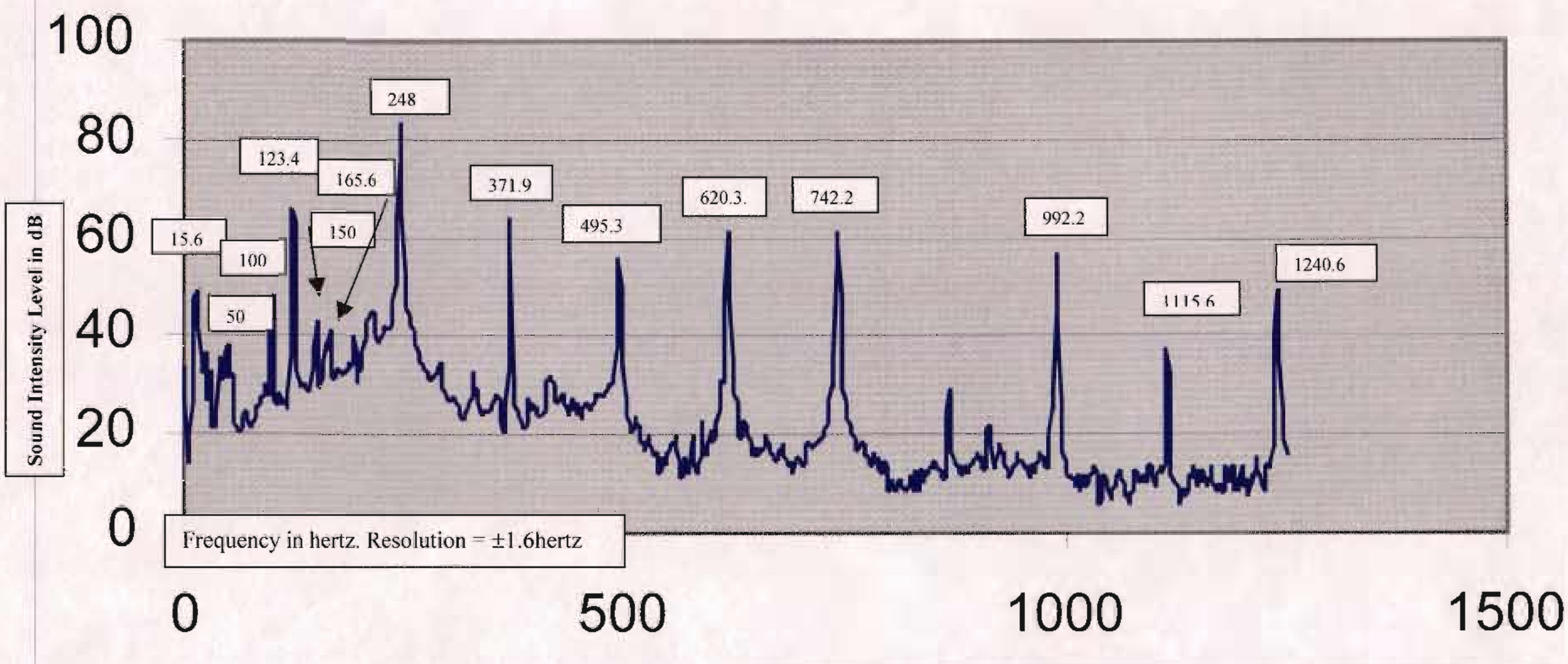


Figure 4.23 Graph produced by tempered frequencies 246.9Hz (B_3, open 2nd string) and 123.5Hz (B_2, 5th string, 3rd fret), played simultaneously on a nylon string acoustic guitar.



4.6 Dissonant and Consonant Qualities of Combined Microtonal Frequencies

Figure 4.24 and Figure 4.25 are examples of a minor 7th pitch interval (1000 cents) distance on a guitar produced by microtones (133.2 Hz-238.3) on electric guitar and (132.6Hz-235.9Hz) on acoustic guitar. The experiment was taken to see if two microtones from the system of microtones used in this thesis are more consonant than a dissonant equally-tempered interval, for example, the tempered major 7th (In the tempered system a minor 7th interval is more consonant than a major 7th interval), see Figure 4.9, and Figure 4.20, See also information supplied in Table 4.5, p.79 and Table 4.6, p.80 about the graphs. The experiment shows that a microtonal pitch interval produced on an acoustic guitar was found to be more consonant than all of the tempered pitch intervals except the main ratios 2:1, 3:2 and 4:3.

The fact that both notes are outside the tempered system has no bearing on the dissonant quality of the pitch interval but the whole-number ratio they produce. The just intonation whole-number ratio 7:4 is an example of a consonant minor 7th pitch interval spanning 968.8cents which is 31.1 cents (see Table 4.4, p.78) less than the 1000 cent equivalent tempered minor 7th. The tempered minor 7th is deemed a mild dissonant[28] pitch interval within the tempered system but a more dissonant pitch interval (41/23) within the overtone series.

In the experiment the microtone pitch interval-133.2 Hz-238.3 (Figure 4.24, p.111), produced on electric guitar was measured, and a whole-number ratio was calculated as 23/13. A similar microtone pitch interval -132.6Hz-235.9Hz (Figure 4.25), produced on acoustic was measured, and a whole-number ratio was calculated as 16/9. The ratio 23/13 is dissonant while the ratio 16/9 is lower in the overtone series than all of the ratios calculated for the equally-tempered system (Table 3.6, p.61), except for the main consonant whole-number ratios, 2/1, 3/2 and 4/3 as stated above.

Ratios based on the numbers 17, 19 are shown to be very close to tempered frequencies. Generally speaking the ratio 8/7 is on the border of dissonance and consonance. When composing within a tempered system using microtones, it is important to realize that microtones are not part of the system, therefore all microtones are equal within the system unless a microtone is close to a tempered frequency, then it will have the effect of a resolving note. If it is within two cents then it will be difficult to distinguish between it and the tempered frequency. The closeness of fundamentals and upper partials is important when considering harmony within microtonal composition. When improvising in a small group without a harmonic instrument the process of improvising with microtones becomes less restricted.

Figure 4.24 Graph produced by microtone frequencies 238.3Hz, (added fret above normal 3rd fret, 3th string)and 133.2Hz (added fret above normal 3rd fret, 5th string), played simultaneously on an electric guitar with added frets (physical guitar shape of a minor seventh).

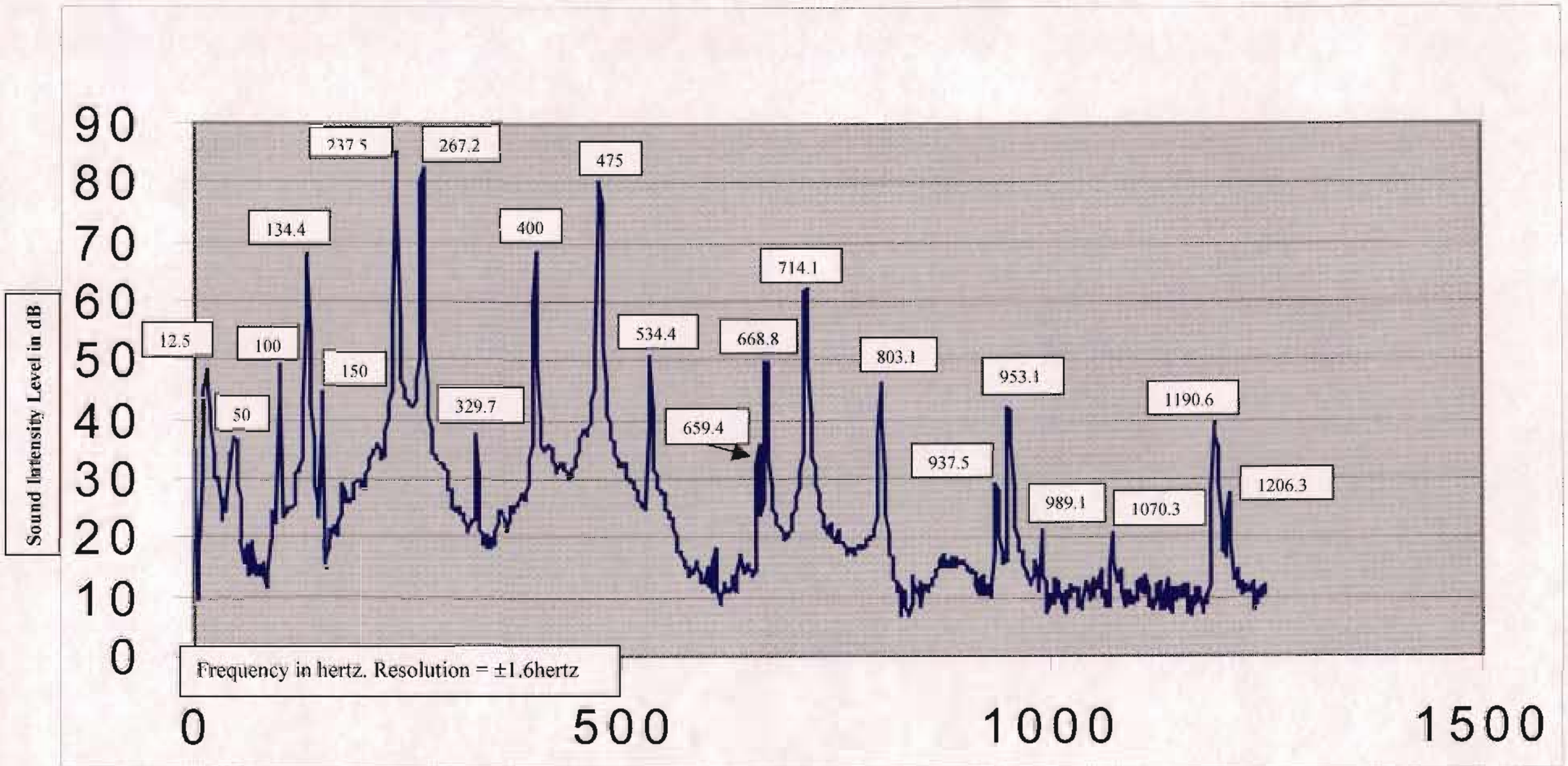
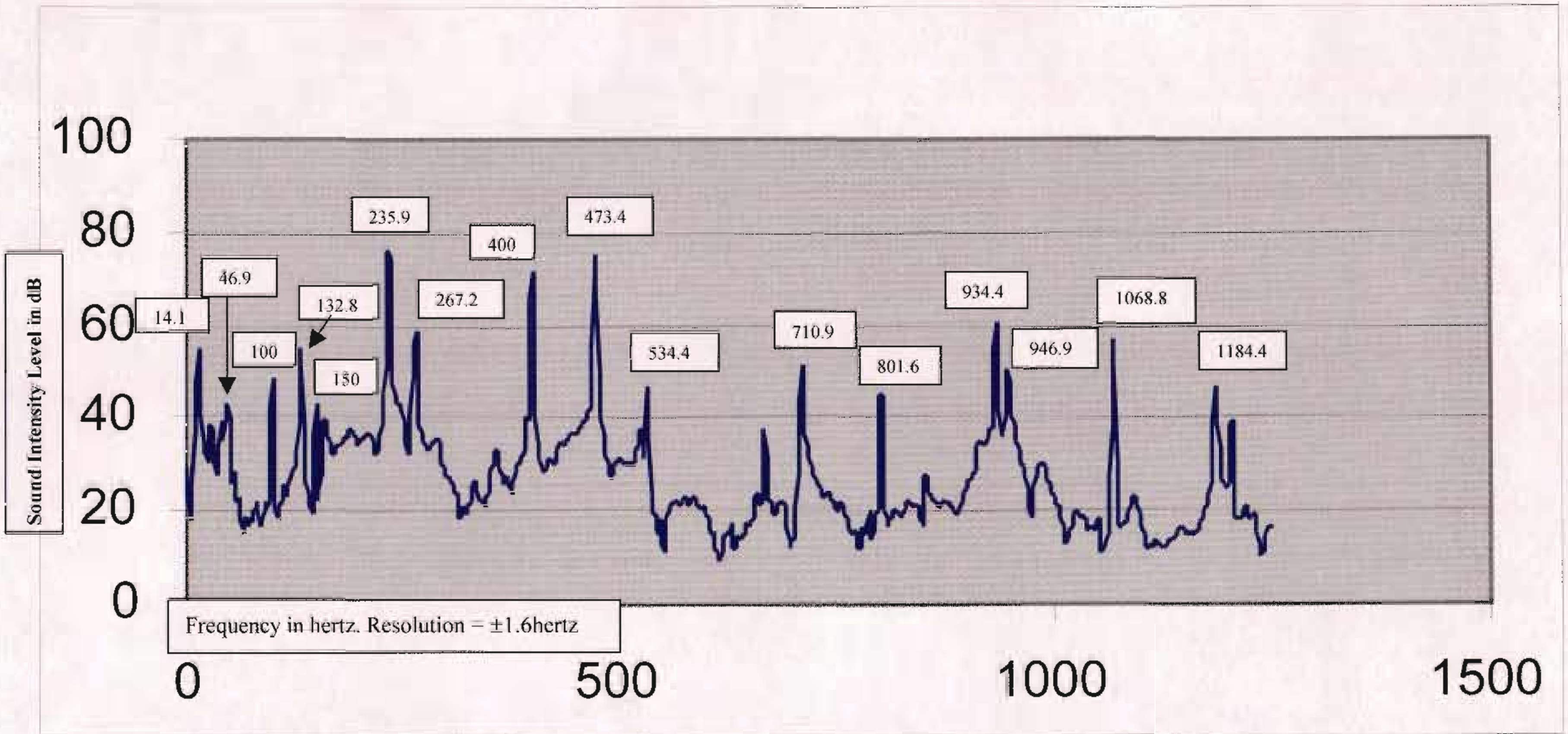


Figure 4.25 Graph produced by microtone frequencies 132.6Hz (18th fret, 6th string) and 235.9Hz (18th fret, 4th string), as played simultaneously on a nylon string acoustic guitar (physical guitar shape of a minor 7th)



4.7 *Guitar Opus 1:* Dissonance and Consonance Involving Microtones

Microtones are utilized in bar 17 of *Guitar Opus 1*. See, Appendix A, p.155. Whole-number ratios were found for pitch intervals involving the three microtones. The pitch interval on the last quarter beat of bar 16 is a tempered major 2nd and is a mild dissonant pitch interval with a ratio of 37:33. The following Table shows the three pitch intervals involving the microtones.

Table 4.7 Ratios and Cents of the Pitch Intervals involving Microtones in Bar 17, *Guitar Opus 1*

	Frequencies	Pitch intervals in cents	Pitch interval ratio
Pitch Interval 1	Microtone **403.1 hertz**	**2249.4**	**11:3**
	Tempered **A_2 - 110 hertz**		
Pitch Interval 2	Microtone **537 hertz**	**1907.5**	**3:1 (+5.5 cents)**
	Tempered **D_3 – 146.8 hertz**		
Pitch Interval 3	Microtone **441.8 hertz**	**1806.8**	**54:19**
	Tempered **Eb_3 – 155.6 hertz**		

The first pitch interval in the above Table is actually a "special relationship" interval with a ratio 11:3 (see Table 4.4, p.78). The second pitch interval is even more consonant producing a ratio 3:1 (octave of 3:2). Even though the pitch interval is larger than 3:1 by 5.5 cents the ear will hear the interval as a special relationship because of the nature of the aural mechanism. The information in the above Table again shows that more consonant pitch intervals can be achieved by a combining a microtone and a tempered frequency. Pitch interval number three is dissonant with a ratio 54:19.

4.8 Conclusion

The microtonal system of frequencies used in this project was measured accurately within two cents and compared to the equally-tempered system and just intonation system Microtones were used along with tempered frequencies in four musical compositions as set out below in Section 5.1, p.118. Whole-number ratios were found for the equally-tempered scale. Whole-number ratios were calculated for the microtonal system in relation to the open string frequencies where they were found. The ratios were compared to "special relationships" (consonant pitch intervals) within the overtone series and their consonant and dissonant quality discussed. It was found in experiments taken in Section 4.5, p.75 and illustrated in Table 4.5, p.79, that a pitch interval between a microtone and tempered note was more consonant than the recognized dissonant pitch intervals within the tempered system. In Section 4.6, p.109 a pitch interval containing two microtones was found to be more consonant than all the tempered pitch intervals of the tempered system except for the main consonant ratios 2:1, 3:2, and, 4:3. In Section 4.7, p.113, whole-number ratios were found for the three pitch intervals containing a microtone in bar 17 of *Guitar Opus 1* (set out in Table 5.2, p.123). Two of the three pitch intervals were "special relationship" intervals (see Table 4.4, p.78).

It is important to realize that apart from the harmonic partials of two tones sounded together, other components also have a large contributing factor to the overall sound produced. Heterodyne components including difference tones, summation tones, et cetera, as discussed in Section 4.3, p.69 affect the outcome of the resultant sound reaching the brain through the non-linear mechanism of the auditory system. It is a complex issue that is the subject of continuing research. Harmonic partials and heterodyne components of similar frequencies are clumped into groups by the ear when two tones are sounded together. Frequencies which are in a special relationship (see Table 4.2, p.72, and Table 4.3, p.73) for example, the unison (1/1), octave (2/1) or fifth (3/2), will contain clumps with larger numbers of heterodyne components than those which are not in a "special relationship". Frequencies in "special relationship" can be fine tuned to produce zero beating and are therefore more consonant. The smaller the number of components in each clump; the less fine-tuning can be achieved. Frequencies that produce whole-number ratios high up in the overtone series are not in "special relationship" (see Table 4.2, p.72), and are therefore more dissonant.

When measuring the consonance/dissonance of two frequencies sounded together it is not important whether the frequencies of a pitch interval are outside the tempered system. What is important is the ratio between the frequencies. This implies that if a series of measurements of similar pitch intervals, for example, if 100 measurements were taken of similar pitch intervals at distances of one cent (spanning a semitone), all of the intervals will have similar ratios therefore similar consonant/dissonant quality. Aurally and physically the pitch intervals are the same when sounded singly.

Finding whole-number ratios for the equally-tempered system is important because they show that the limiting twelve pitch intervals are dissonant except for 2:1, 3:2 and 4:3 (octave, perfect fifth and perfect fourth respectively). Tempered pitches sounded together with microtones can form more consonant pitch intervals than tempered pitch intervals. Also, two microtones forming a pitch interval can be more consonant than some pitches found within the equally-tempered system.

Musically the overtone series gives an infinite number of frequencies to choose from with the most consonant intervals found at the beginning of the series. Any frequency combination can be extracted from the overtone series by finding a whole-number ratio that corresponds to those frequencies.

Through the experience of performance using an electric guitar with added frets (see Figure 3.1, p.54), it is realized that because the microtones produced by the added frets are not part of the tempered system they are musically equal when sounded together with any tempered frequency or harmony. None of the microtonal frequencies are related to the tempered system. When a microtone is very close to a tempered frequency, then it will have the effect of a resolving note and will aurally want to resolve to the nearest tempered note. This is especially true when a composition is more tempered than microtonal.

Because of musical conditioning to the tempered system a person is most likely to be guided-within the framework of a mainly tempered composition with added microtones-towards tempered pitch resolution. To become familiar with the sounds of microtones in composition it is important to have constant exposure to these unfamiliar sounds. The physical analysis of the microtone system here and the tempered system is essential to the analytical process of the compositions produced in this thesis. The new system found and measured physically in this thesis gives composers for guitar new melodic and harmonic material to work from and new ideas in re-fretting a guitar, using unique guitar based microtones and measurements.

Chapter 5 Musical Results and Commentary

5.1 Introduction

A good understanding of dissonance and consonance is essential for composers. In *Guitar Opus 1* and *Opus 2* the intervallic structure is very important. The use of microtones in certain sections of *Guitar Opus 1* and *Guitar Opus 2* suggests that these passages are more dissonant than some passages using only tempered notes. In fact this is not the case because all the intervals of the twelve-note/equal temperament scale have been altered with the exception of the octave from the natural scale. Table 3.6, p.61 shows that when the pitch intervals of the tempered system are converted to whole-number ratios they are considerably higher than the recognized consonant frequencies of the overtone series (see Table 1.11, p.35). The natural scale contains more consonance and more striking dissonances than the twelve-note/equal temperament scale.

In *Guitar Opus 1* and *Guitar Opus 2* microtones are extracted using a nut-side pluck from a normal classical guitar without any modifications or without the use of amplification techniques. The microtones are measured and compared to the nearest tempered notes. The bars that contain the microtones are isolated and commented on, and physical information is shown in tabular format. This particular approach to microtonal composition, using microtones readily available on a normally built and normally tuned classical guitar, does not appear to have been previously explored. *Guitar Opus 1* and *Guitar Opus 2* were composed in order to blend microtones into the dissonant and consonant quality of a twelve-note/equal temperament piece, and to develop sections that would be more microtonal than tempered in content (see Section B of *Guitar Opus 1* and Section D of *Guitar Opus 2*). The microtones in the score are illustrated by the use of square-headed notes.

5.2 Guitar Opus 1

This piece consists of four sections:

Section A	Section B	Section C	Section D
Bars 1-19	Bars 20-37	Bars 38-83	Bars 84-102

Section A is based around the tonality of E. The use of harmonics in Section A, especially in bar 19, establishes the texture which follows in Section B. Section A contains microtones in bar 10 and bar 17. The dissonant and consonant quality of the piece is carefully constructed to enable the microtones to blend in with the overall texture of the piece. In Section B a microtonal melody is used in the lower voice, accompanied by a tempered repeated motif of artificial harmonics in 9/8 time. The notes shown in brackets in the first bar form a chordal shape, which descends in semitone steps. The microtonal melody is in a polymetre of 2:9. The melody changes to a polymetre of 3:9 in bar 26 line 4. At bar 28 line five the metre is in 7/8 changing to 3/4 at bar 30 line six with the melody staggered by an eighth note

Section C is the longest section of the piece. It begins in 3/4 time building up from bar 38 to a *fortissimo* climax in 11/8 at bar 56. A repeated eighth note pattern is used, with a regular accent on the second eighth note of beat two; this suggests a polymetre of 3:2. Bars 60 and 61 are in 5/4 and 11/8 respectively and set up the next part of Section C, which alternates from 5/4 to 11/8 and 5/4 to 9/8. Bars 72 and 73 break the previous pattern which resumes in bar 74. The change to 13/4 at bar 82 signals the end of section.

The first four bars of Section D (bars 84-87) serve the function of an interlude leading back to a reprise of Section A starting in the fourth bar. Harmonics are again utilized leading to the final chord, which is the same as the opening chord of the piece. The texture of the composition is influenced by the use of harmonics, which blend well with the microtones because both are similar in volume, and by the use of strong polyrhythmic motifs and sudden changes of measures.

Microtonal Examples: ***Guitar Opus 1***

The following example utilizes plucked microtones, indicated by square-headed notes from the nut-side of a stopped string. The motif has a descending tempered line starting on the note E_3 (164.8Hz). When this line is plucked on the nut-side of the stopped string it creates an ascending line of microtones. Table 5.1 below shows the microtones in cents on the top row. The second row shows the nearest tempered note (and its octave designations) to each microtone. The third row shows the difference in cents between the microtones and the nearest tempered notes. The fourth line shows the descending tempered line in cents and equivalent octave designations. The microtones in the following tables are shown with fret and string numbers. The frets are numbered in accordance with normal fret numbering. (see Table 3.2, p.53)

Figure 5.1 Section A Bar 10

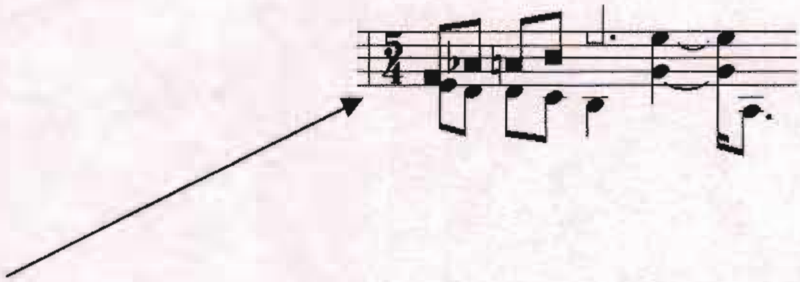


Table 5.1 Section A Bar 10

Microtonal Melody	176.6 (6^{th} string, 12^{th} fret)	205.2 (6^{th} string, 10^{th} fret)	224 (6^{th} string, 9^{th} fret)	248.8 (6^{th} string, 8^{th} fret)	331.6 (6^{th} string, 6^{th} fret)
Nearest temp notes	174.6 (F_3)	207.7 ($G\#_3$)	220 (A_3)	246.9 (B_3)	329.6 (E_4)
Difference in cents	19.7 cents	-21 cents	-31.2 cents	13.3 cents	10.5 cents
Descending temp line of music bar	164.8 (E_3)	146.8 (D_3)	138.6 (Db_3)	130.8 (C_3)	116.5 (Bb_3)

The next three microtones used in *Guitar Opus 1*, illustrated by square-headed notes, G, C, and A, above the stave, are shown in Table 5.2 below. The top row gives the microtones in cents, the second row shows the nearest tempered note (and its octave designations) to each microtone. The third row shows the difference in cents between the microtones and the nearest tempered notes. The fourth line shows the tempered line of the bar in cents and equivalent octave designations.

Figure 5.2 Section A Bar 17

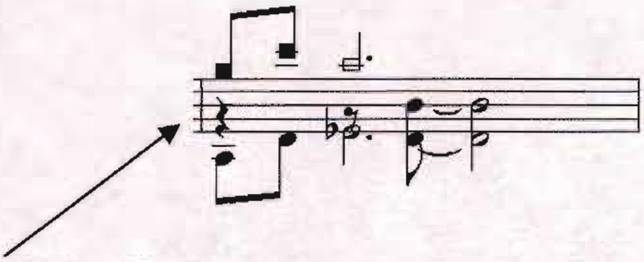


Table 5.2 Section A Bar 17

Microtonal Melody	403.1 (6th string, 5th fret)	537 (5th string, 5th fret)	441.7 (5th string, 6th fret)
Nearest temp notes	392 (G_4)	523.3 (C_5)	440 (A_4)
Difference in cents	48.3 cents	44.7 cents	6.7 cents
Descending temp line of music bar	110(A_2)	146.8 (D_3)	155.6 (Eb_3)

In Section B below a pattern of artificial harmonics in a repeated descending figure [bars 22-35] is combined with a microtonal melody from bar 22 in a polymetre ratio of 2:9. In bars 26-29 the microtonal melody is in a polymetre ratio of 3:9. The metre changes to 7/8 in bars 28 and 29 and to 3/4 at bar 26. There are artificial harmonics on the downbeat and a microtonal melody, which is staggered by an eighth note. The last two bars do not contain any microtones.

Figure 5.3 The following musical passage is Section B in total.

Tables 5.3 – 5.7 below illustrate the notes from bars 22- 35 converted to cents. Row 1 illustrates the artificial harmonics in cents, Row 2 shows the microtonal notes of the melody, Row 3 shows the nearest tempered notes to the microtones and Row 4 shows the difference between the two.

Table 5.3 Section B Bars 22 and 23

Artificial Harmonics	370 ($F\#_4$)	525.3 (C_5)	659.3 (E_5)
Microtonal Melody (two notes)	281.8 (7th fret, 6th string)	333.2 (8th fret, 5th string)	
Nearest temp. note	277.2 ($C\#_4$)	329.6 (E_4)	
Difference in cents	28	18.8	

Table 5.4 Section B Bars 24 and 25

Artificial Harmonics	293.7 (D_4)	493.9 (B_4)	622.3 ($D\#_5$)
Microtonal Melody (two notes)	331.6 (6th fret, 6th string)	377.7 (7th fret, 5th string)	
Nearest temp. note	329.6 (E_4)	370 ($F\#_4$)	
Difference in cents	10.5	35.6	

Table 5.5 Section B Bars 26 and 27

Artificial Harmonics	329.6 (E_4)	466.2 (Bb_4)	587.3 (D_5)
Microtonal Melody (three notes)	403.1 (5th fret, 6th string)	441.8 (6th fret, 5th string)	718 (5th fret, 4th string)
Nearest temp. note	392 (G_4)	440 (A_4)	698.5 (F_5)
Difference in cents	48	7	47

Table 5.6 Section B Bars 28 and 29

Artificial Harmonics	311.1(Eb_4)	440 (A_4)	554.4 (Db_5)
Microtonal Melody (two notes)	521 (4th fret, 6th string)	537 (5th fret, 5th string)	930.5 (4th fret, 4th string
Nearest temp. note	523.3 (C_5)	523.3 (C_5)	932.3(A_5)
Difference in cents	-7.6	44.7	-3.3

Table 5.7 Section B Bars 30 -35

Artificial Harmonics	293.7(D_4)	415.3 (Ab_4)	523.3 (C_5)
Microtonal Melody (two notes)	771.1. (3rd fret, 6th string)	695 (4th fret, 5th string)	
Nearest temp. note	784 (G_5)	698.5 (F_5)	
Difference in cents	-28.7	-8.7	

There are no microtones in Section C. Table 5.8 below illustrates bar 86 of Section D.

Figure 5.4 Section D bar 86

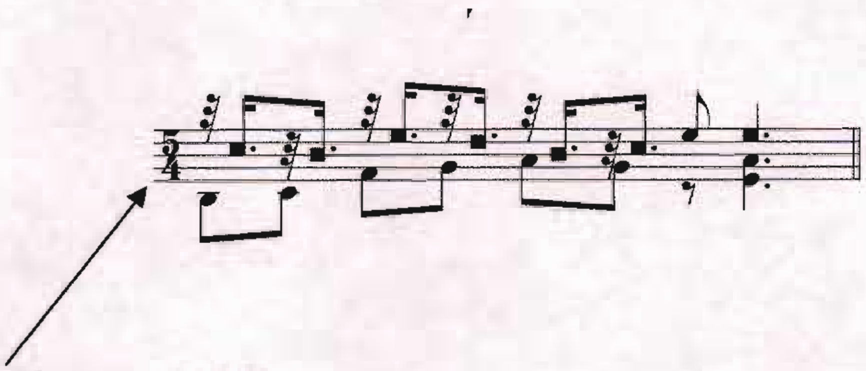


Table 5.8 Section D bar 86

Temp. notes	123.5 (B_2)	130.8 (C_3)	174.6 (F_3)	185 ($F\#_3$)	207.7 ($G\#_3$)	196 (G_3)
Micro. Notes	331.6 7^{th}fr. 6^{th}st.	248.8 8^{th}fr. 6^{th}st.	333.2 8^{th}fr. 5^{th}st.	299.2 9^{th}fr. 5^{th}st.	253.1 11^{th}fr. 5^{th}st.	273.4 10^{th}fr. 5^{th}st.
Nearest temp.	329.6 (E_4)	246.9 (B_3)	329.6 (E_4)	293.7 (D_4)	246.9 (B_3)	277.2 ($C\#_4$)
Difference in cents	10.5	13.3	18.8	32	42.9	-23.9’

5.3 Guitar Opus 2

Guitar Opus 2 continues the microtonal idea used in *Guitar Opus 1* and should be performed after it. The piece is in five sections and utilizes microtones in the melody at the end of Section C and throughout Section D.

The form is broken into five sections:

Section A	Section A_2	Section B	Section C	Section D
Bars 1-8	Bars 1-4, 9-12	Bars 13-23	Bars 24-30	Bars 31-40

Sections A and A_2 are based around the tonality of e minor. There is an e minor/major tonality throughout Sections A and B. Sections A and A_2 use repeated open strings in the treble with a bass melody. The harmonic movement is tonic to dominant in e minor. The first cadence at bar 7 ends in a minor while the second cadence at bar12 ends in e minor.

At Section B bar 13 triplet chords are based on a minor ninth interval. A rapid repeated 5-bar passage from bars 18-22 leads to a dissonant chord based on clustered seconds for increased dissonance.

Section C acts as an interlude between Section B and the final microtonal Section D. Harmonics are introduced to blend in with the texture and amplitude of the microtones. Microtones are found in bars 29 and 30 leading to Section D.

The microtones in Section D are utilized throughout the melody. The tempered harmonic choice is consonant so as to feature a microtonal melody within a consonant tempered harmonic structure. At bar 40 there is a brief reference to the opening motif from Section A

In composing *Guitar Opus 2* it was intended to explore dissonance/consonance through the harmonic structure and texture of the five sections. The aim was to balance the dissonant/consonant quality of Sections A, A_2, and B, which do not include microtones, and the use of microtones in Sections C and D. Section D in terms of dissonant quality is intended to be a complement to Section A. The dissonant quality of the dissonant tempered chords were musically matched by using more consonant tempered chords to underpin the microtonal melody. The changes in metre throughout the piece were not preconceived in the creative process.

Microtonal Examples: *Guitar Opus 2*

Section C bar 29:

This microtone is found by stopping the fourth string at the eighth fret and plucking on the opposite side (pluck the opposite side of the tempered Bb_2 note below).

Figure 5.5

Table 5.9 Section C bar 29

Microtonal Melody	444.35 (8^{th} fret, 4^{th} string)	505.9 (7^{th} fret, 4^{th} string)
.Nearest temp. note	440 (A_4)	493.9(B_4)
Difference in cents	17 cents	41.6

The following bar has one microtone that is found when the opposite side of the first string is plucked and stopped at the eighth fret (tempered C_4)

Figure 5.6

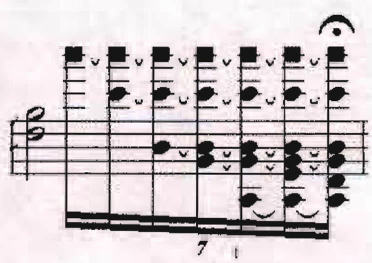
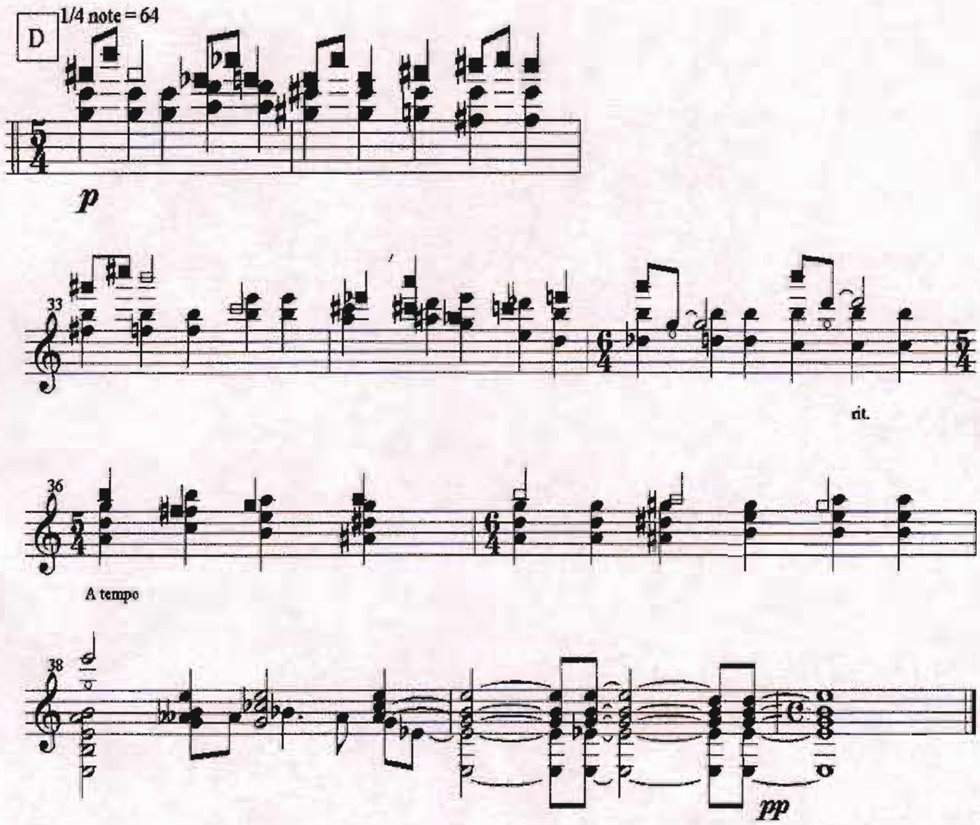


Table 5.10 Section C Bar 30:

Microtonal Melody **(two notes)**	999 (8^{th} fret, 1^{st} string)
Nearest temp. note	987.8 (B_5)
Difference in cents	19.5 cents

The next microtonal example is in Section D.



Section D bars 31-37:

Table 5.11 Section D bar 31:

Microtonal Melody (five notes)	750.8 (8th fret, 2nd string)	999 (8th fret, 1st string)	613.7 (10th fret, 2nd string)	818.8 (10th fret, 1st string)	675 (9th fret, 2nd string)
Nearest temp. note	740 (F#$_5$)	987.8 (B$_5$)	622.3 (Eb$_5$)	830.6 (G$_5$)	659.3(E$_5$)
Difference in Cents	25.1	19.5	-24.1	-24.8	40.7

Table 5.12 Section D bar 32:

Microtonal Melody (two notes)	675 (9th fret, 2nd string)	897.3 (9th fret, 1st string)	750.8 (8th fret, 2nd string)	853.5 (7th fret, 2nd string)
Nearest temp. note	659.3 (E_5)	880(A_5)	740 ($F\#_5$)	830.6 (Ab_5)
Difference in Cents	40.7	33.7	25.1	47.1

Table 5.13 Section D bar 33

Microtonal Melody (three notes)	853.5 (7th fret, 2nd string)	1135.9 (7th fret, 1st string)	995 (6th fret, 2nd string)	529.3 (12th fret, 2nd string)
Nearest temp. note	830.6 (Ab_5)	1108.7 ($C\#_6$)	987.8 (B_5)	523.3 (C_5)
Difference in Cents	47.1	42	12.6	19.7

Table 5.14 Section D bar 34

Microtonal Melody	613.7 (10th fret, 2nd string)	569.5 (11th fret, 2nd string)	818.8 (10th fret, 1st string)	423.8 (12th fret, 3rd string)	538.3 (9th fret, 3rd string)	684 (7th fret, 3rd string)
Nearest temp. note	622.3 (Eb_5)	554.4 ($C\#_5$)	830.6 (G_5)	415.3 ($G\#_4$)	523.3 (C_5)	698.5 (F_5)
Difference in Cents	-24.1	46.5	-24.8	35.1	48.9	-36.3

Notes in Columns 3 and 4 above are sounded together.

Table 5.15 Section D bar 35

Microtonal Melody (two notes)	801.2 (6th fret, 3rd string)	977 (5th fret, 3rd string)
Nearest temp. note	784 (G_5)	987.8 (B_5)
Difference in Cents	37.5	-19

Table 5.16 Section D bar 36

Microtonal Melody (two notes)	505.9 (7th fret, 4th string)	364.3 (10th fret, 4th string)	400.4 (9th fret, 4th string)	444.5 (8th fret, 4th string)
Nearest temp. note	493.9 (B_4)	370 (Gb_4)	392 (G_4)	440 (A_4)
Difference in Cents	41.6	-26.9	36.7	17.6

Table 5.17 Section D bar 37

Microtonal Melody (two notes)	505.9 (7th fret, 4th string)	444.5 (8th fret, 4th string)	400.39 (9th fret, 4th string)
Nearest temp. note	493.9 (B_4)	440 (A_4)	392 (G_4)
Difference in Cents	41.6	17.6	36.7

Etude for Electric Guitar with Added Frets

See section 2.3, p.48.

Etude for electric guitar with added frets

Mike Nielsen

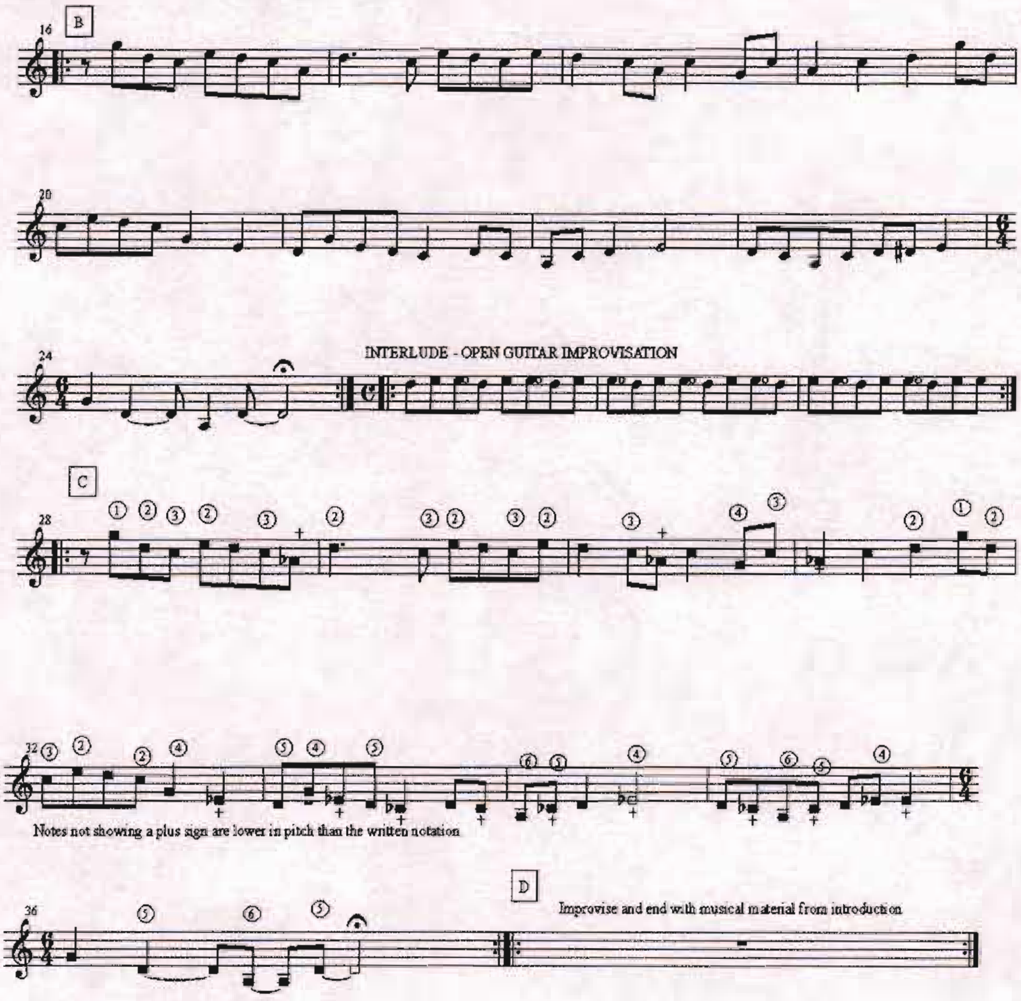


Table 5.18 below shows the measured microtonal frequencies and normal tempered frequencies from the composition, *Etude for Electric Guitar with Added Frets*. Highlighted coloured figures illustrate the different melodic sections of the composition.

Table 5.18 Etude for electric guitar with added frets

Strings ► / Frets ▼	6th	5th	4th	3rd	2nd	1st
	Frequency /Hz					
	±0.4Hz	±0.4Hz	±0.8Hz	±0.8Hz	±0.8Hz	±1.6
Open String Frequencies ►	*82.4 (E_2)*	*110 (A_2)*	*146.8 (D_3)*	*196 (G_3)*	*246.9 (B_3)*	*329.6 (E_4)*
1 (Between 0&1)	84 (E_2 +33 cents)	112.1	150	200.8	251.6	334.4
2 (Between 0&1)	85.6 (F_2 −39.1 cents)	114.1	153.1 (47)	203.9	257	340.6
Normal 1st fret	**87.5 (F_2)**	**116.5 (Bb_2)**	*155.6 (Eb_3)* (47)	**207.7 *(Ab_3)***	**261.6 *(C_4)***	**349.2 *(F_4)***
3 (Between 1&2)	89.1 (F_2 +30.6 cents)	119.1	159.4 (47) 159.38 (30,33,41,48)	213.3 (7,16,20)	268.8	356.3
Normal 2nd fret	**92.5 ($F\#_2$)**	**123.5 (B_2)**	*164.8 (E_3)* (30,33,41,48)	*220 (A_3)* (7,16,20)	**277.2*($C\#_4$)***	**370 *($F\#_4$)***
4 (Between 2&3)	94.2 ($F\#_2$ +30.6 cents)	125	168 (30,33,41,48)	224.2 (7,16,20)	282.8	375
5 (Between 2&3)	96.9 (G_2 -20 cents)	129.3 (35,37,39,43,45)	172.7	230.5	290.6 (2,5,8,11,14,22,24,27,)	385.9 (1,23)
Normal 3rd fret	**98 (G_2)**	*130.8 (C_3)* (35,37,39,43,45)	**174.6 *(F_3)***	**233.1 *(Bb_3)***	*293.7 (D_4)* (2,5,8,11,14,22,24,27,)	*392 (G_4)* (1,23)
6 (Between 3&4)	100.4 (G_2 +47.7 cents)	133.2 (35,37,39,43,45)	178.1	238.3	300 (2,5,8,11,14,22,24,27,)	398.4 (1,23)
Normal 4th fret	**103.8 (Ab_2)**	**138.6 (Db_3)**	**185 *(Gb_3)***	**246.9 *(B_3)***	**311.1 *(Eb_4)***	**415.3 *(Ab_4)***
7 (Between 4&5)	107.8 (38,44,51) (A_2 −34.8 cents)	143.4 (31,34,36,40,42,46,50,52)	192.4 (18,29,32,49)	256.3 (3,6,9,12,15,17,19,21,25,28)	322.7 (4,10,13,26)	428.1
Normal 5th fret	*110 (A_2)* (38,44,51)	*146.8 (D_3)* (31,34,36,40,42,46,50,52)	*196 (G_3)* (18,29,32,49)	*261.6 (C_4)* (3,6,9,12,15,17,19,21,25,28)	*329.6 (E_4)* (4,10,13,26)	**440 *(A_4)***
8 (Between 5&6)	112.5 (38,44,51) (A_2 +38.9 cents)	149.2 (31,34,36,40,42,46,50,52)	200 (18,29,32,49)	267.2 (3,6,9,12,15,17,19,21,25,28)	336.7 (4,10,13,26)	446.875
Normal 6th fret	**116.5 (Bb_2)**	**155.6 *(Eb_3)***	**207.7 *(Ab_3)***	**277.2*(Db_4)***	**349.2 *(F_4)***	**466.2 *(Bb_4)***
Normal 7th fret	**123.5 (B_2)**	**164.8 *(E_3)***	**220 *(A_3)***	**293.7 *(D_4)***	**370 *($F\#_4$)***	**493.9 *(B_4)***

The frequencies supplied in Table 5.18 above were taken from Table 3.4, p.57. The highlighted figures illustrate the three different melody sections of the composition, *Etude for electric guitar with added frets.* Section B is in red (tempered frequencies), Section A is in green (microtonal frequencies) and Section C is in yellow (microtonal frequencies). The exact physical and melodic form of the melody is moved up a fret (green) from the tempered frequencies (red) and down a fret (yellow) from the tempered frequencies (red).

The melody of *Etude for electric guitar with added frets,* was initially conceived as a tempered melody (red). The composition begins with the melody in microtonal form in Section A and is shown as green and is up a fret from the tempered melody, red. The melody is then played as a tempered melody (red) in Section B; Section C (yellow) is another microtonal form of the melody - down a fret from the tempered melody. The total melody contains fifty-two notes. The figures in brackets underneath each melody note show at what point each note is used within the melody.

For example, the frequencies 392 Hz [red], 398.4 Hz [green] or 385.9 Hz [yellow] are the first melody note of Sections B, A and C respectively, and occur again as the 23rd melody note of each melodic form. Frequency 159.4 Hz is the only shared note between the three different forms of the melody and is shared by the green and yellow microtonal versions. The shared note is melody note 47 in the green version The same melody note functions as the melody notes, 30,33,41,48 in the yellow version. The equivalent tempered melody frequency is Eb_3 or 155.6Hz. The introduction and interludes of *Etude for electric guitar with added frets* use a melodic figure of two tempered notes, 293.7 (D_4) and 329.6 (E_4) and one microtonal note (322.7 Hz) which is approx. Eb_4 + 35cents. This melodic figure is the initial springboard for improvisation at these points, smoothly connecting each melody form. Section D is an improvised ending.

5.4 Etude For Amplified Classical Guitar

This piece was composed using an amplified classical guitar with two specially designed pick-ups one placed under the saddle and one under the nut. A 'hammer-on' technique is used throughout which demonstrates how both sides of stopped strings of equal loudness can be utilized in composition. This technique prompted the initial idea for the system of microtones used in this project. The string when playing, is more prominent than normal this produces more string noise because of the two added pick-ups but this can be used to advantage as the string noise can produce similar sound to drums.

Etude for Amplified Classical Guitar consists of and introduction and eight sections:

Introduction –Improvised drum sounds using string noise

Section A is comprised of eight bars in 5/8 time and uses a left hand hammer-on technique. The figure is composed of a repeated five-note figure in eighth notes. Table 5.19, p.144 below illustrates the five-note figure and shows the nut-side microtonal notes and tempered notes that are produced simultaneously by the hammer-on technique. The tempered notes of the figure are only shown in the composition. The microtonal notes of the figure can sound because the palm of the right hand dampens bridge-side. The right hand palm is then rolled back towards the bridge until both sides of stopped strings are sounded. This process is then repeated.

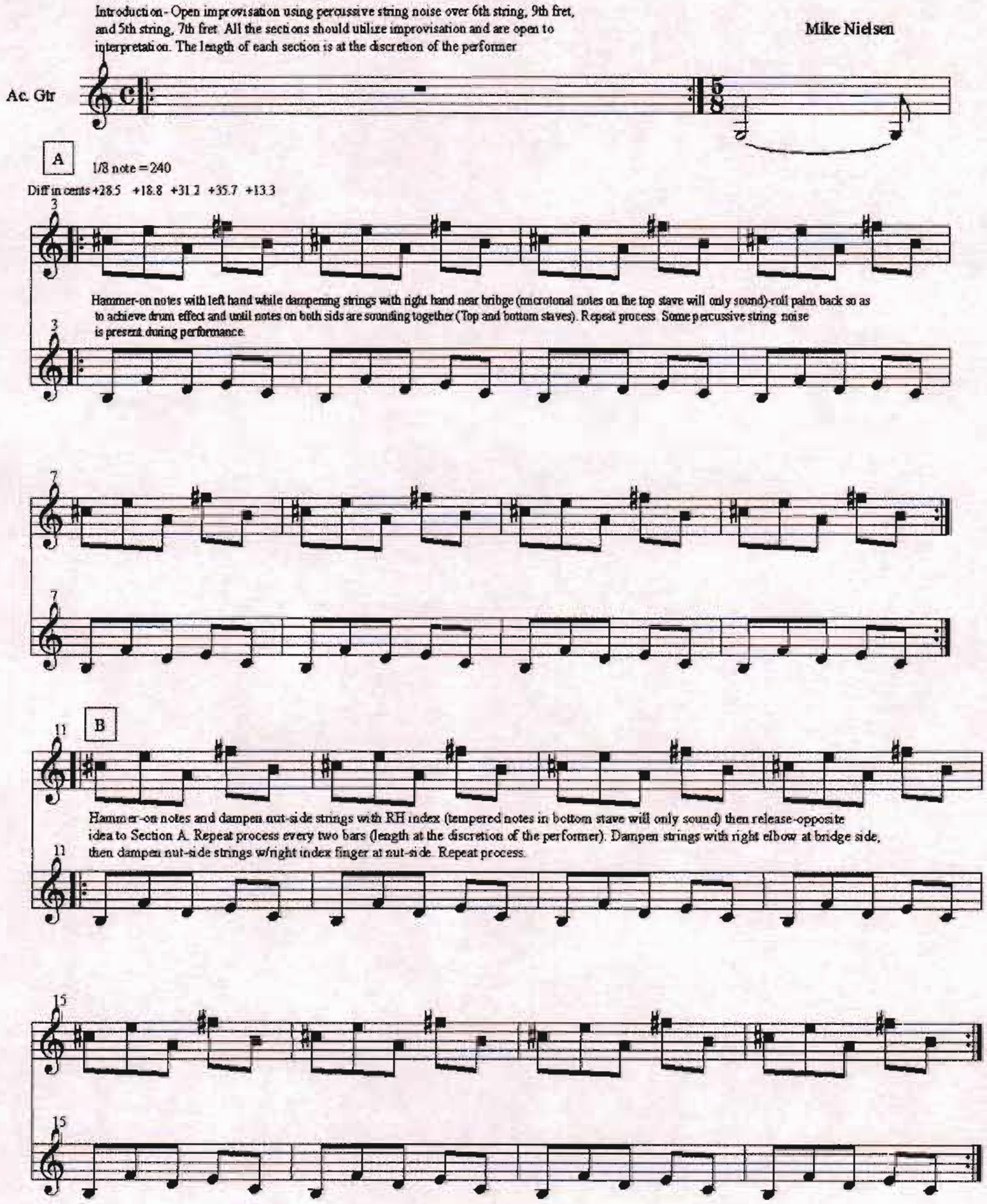
Section B continues with the same figure as Section A. The same figure is used throughout the composition. The nut-side notes are dampened allowing the tempered bridge-side notes to sound. Then the strings are released allowing both sides of stopped strings to sound. The process is then repeated. Section C continues with the same figure as Sections A and B except that it is notated on the lower stave. The figure is repeated while a chord is tapped on to the 4th fret on the nut-side. The tempered bridge-side notes sound throughout. The chord is shown as whole notes but the rhythm of the chord is improvised.

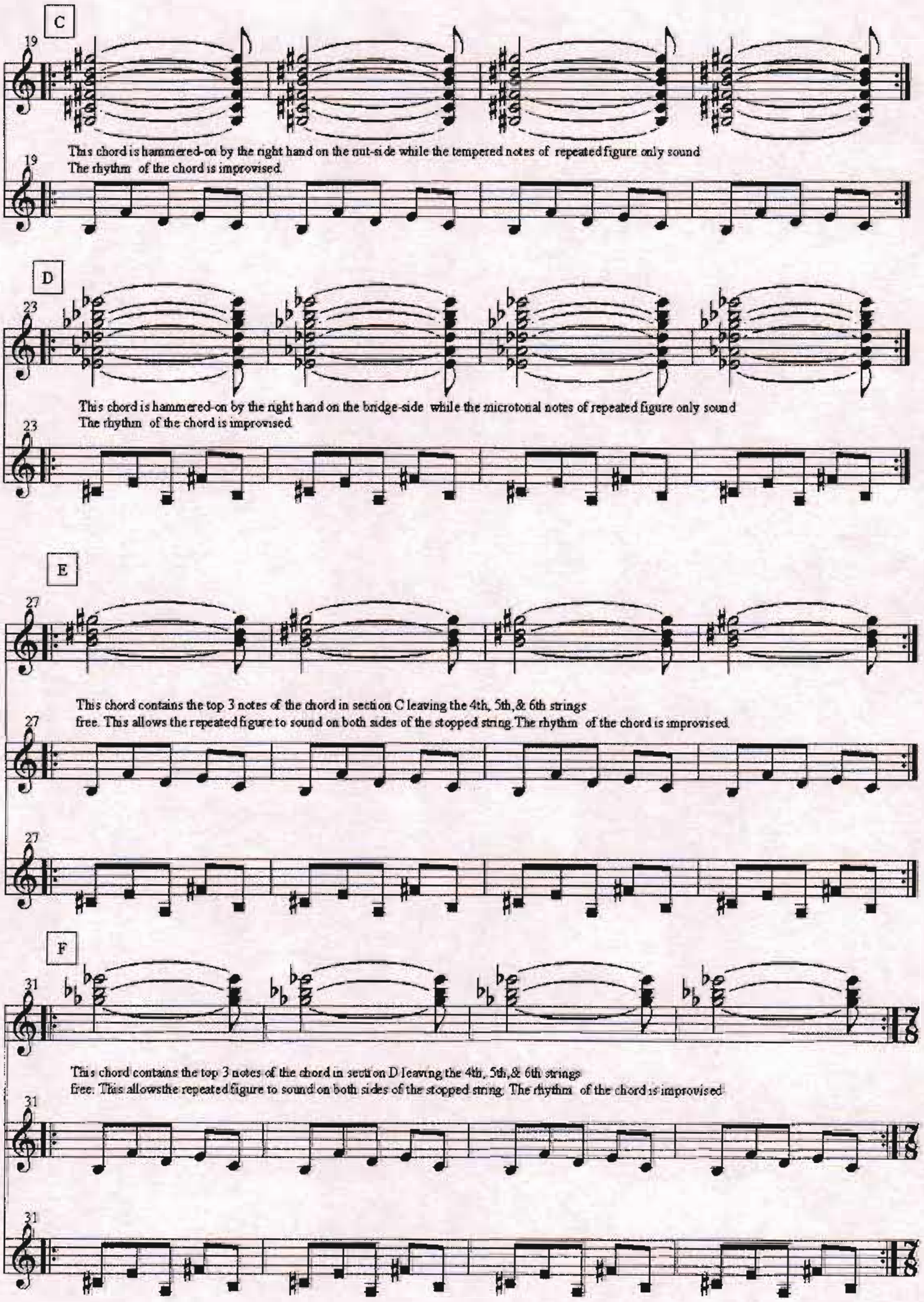
Section D continues with the same figure as Section A and Section B on the lower stave. The figure is repeated while a chord is tapped on to the 11th fret. The rhythm of the chord is improvised as in the previous chord. The nut-side notes of the repeated figure sound throughout. Section E continues with the same repeated figure as the previous sections. The figure is repeated while a three-note chord is tapped on to the 4^{th} fret. This chord is similar to the upper three notes of the chord in Section C. Both sides of stopped strings of the repeated figure sound throughout.

Section F continues with the same figure as in the previous sections. The figure repeats while a three-note chord is tapped on to the 11^{th} fret. This chord is similar to the upper three notes of the chord in Section D. Both sides of the stopped strings of the figure sound throughout. Section G contains a new microtone, 441.8 hertz which is 7cents higher that the tempered A note 440hertz. After Section G is played the performer repeats back to any section and improvises on the written musical material. After the improvisation the performer plays Section G again and finishes with Section H. Section H starts with an improvisation in 5/8 on the given material and finishes with the percussive string noise as in the Introduction.
The nine sections are used as a basis for improvisation. Variations and instant composition of new material based on this composition should be improvised. The sections can also be repeated in any order and be of any length.

ETUDE
For amplified classical guitar

Mike Nielsen




C
19
This chord is hammered-on by the right hand on the nut-side while the tempered notes of repeated figure only sound
The rhythm of the chord is improvised.
19
D
23
This chord is hammered-on by the right hand on the bridge-side while the microtonal notes of repeated figure only sound
The rhythm of the chord is improvised.
23
E
27
This chord contains the top 3 notes of the chord in section C leaving the 4th, 5th,& 6th strings
free. This allows the repeated figure to sound on both sides of the stopped string. The rhythm of the chord is improvised.
27
27
F
31
This chord contains the top 3 notes of the chord in section D leaving the 4th, 5th,& 6th strings
free. This allowsthe repeated figure to sound on both sides of the stopped string. The rhythm of the chord is improvised.
31
31

After section G, repeat to any section and improvise using the written musical material.
To finish play G again and then H section which ends with percussive string noise like introduction..

Table 5.19 Microtonal and tempered notes sounded by the repeated figure

	Microtonal Notes in hertz	Tempered Note in hertz
1	281.8 = C#$_4$ +28.5 cents (7th fret, 6th string)	123.5 (B$_2$)
2	333.2 = E$_4$ + 18.8 cents (8th fret, 5th string)	174.6 (F$_3$)
3	224 = A$_3$ + 31.2 cents (9th fret, 6th string)	138.6 (C#$_3$)
4	377.7 = F#$_4$ + 35.7 cents (7th fret, 5th string)	164.8 (E$_3$)
5	248.8 = B$_3$ + 13.3 cents (8th fret, 6th string)	130.8 (C$_3$)

The five notes in the repeated figure in *Etude for Amplified Classical Guitar* are contained in Table 5.19 above.

Conclusions

In the course of this research project the theories of sound, pitch, loudness, quality/timbre were studied. The vibrational frequencies of strings, and physics of musical pitch and the evolution of pitch measurement, musical scales and tuning systems in western music were discussed. The search by composers for alternative scale systems was studied in the historical context.

The microtonal system used in composition in this project was measured on a high precision sound lever meter and frequency analyzer and the results were illustrated. Whole-number ratios were also calculated in relation to the open strings from which they originated. Inverse proportion was used to calculate the predicted microtones used here and cent (1/100 of a semitone) measurements were defined and used when calculating the pitch intervals between two frequencies to within an accuracy of two cents.

Microtones are musical notes that are higher or lower in frequency than the notes of the twelve-note/equal temperament scale. 'Tempering' is when a musical interval is lessened or enlarged away from the 'natural' scale (that deducible by physical law). From a physical point of view, the previous two statements suggest that the twelve-note/equal temperament scale is in fact a microtonal scale, relative to the natural scale from which it deviates. Equal temperament was developed to enable composers to modulate to different keys using fewer notes; it also simplified the construction of musical instruments and playing technique.

The microtonal system used for this research project was extracted from a normal classical guitar and then compared to the overtone series. The consonant/dissonant quality of the tempered system was found by converting the pitch intervals to whole-number ratios. The results were compared to the "special relationship" pitch intervals of the overtone series. The microtonal system was compared to the "special relationship" pitch intervals of the overtone series and the tempered system.

It was found that some pitch intervals containing a tempered note and a microtone and some pitch intervals comprised solely of microtones were less dissonant than some of the dissonant intervals of the tempered system. It is shown that bar 17 of *Guitar Opus 1* contains two pitch intervals comprised of tempered and microtonal notes that are "special relationship" whole-number ratios and therefore more consonant than any of the tempered interval except the octave-2:1, fifth-3:2 and fourth-4:3.

During the medieval period the foot rule, used throughout Europe, had an important influence on organ building and pitch levels. A foot was a different measurement in different countries and even within a country the measurement could vary. As a result, different pitched organs were built depending on the foot size. The pitch A_4 varied throughout Europe changing between 0 and 7.4 semitones (0 and 740cents) until the recognized international standard pitch for A_4 = 440 hertz was agreed in 1939.

On fixed pitch instruments like the guitar and piano these enharmonic pitches are played at the same pitch, but on non-fixed pitch instruments such as the violin enharmonic notes, are played at different pitches, for example an F# note is played sharper than a Gb note. When a violinist performs with a pianist the violinist must alter the pitch aurally to suit the equally-tempered pitch of the piano. Tuning a piano is a process of tempering the natural intervals in order to have equidistant tempered intervals that will make the pitch intervals more dissonant.

If brass or woodwind instruments based on a conical tube are blown to produce their natural notes, the frequencies produced are those of the harmonic series. Performers of such instruments need to adjust their embouchure throughout a performance in order to produce equally-tempered pitches. It is important to note that if a similar frequency is played on different instruments the frequency will have different degrees of dissonance because of the distribution of the upper partials. Different instruments sounding together will affect the sound produced, for example, an oboe and a clarinet or saxophone and clarinet playing two different frequencies simultaneously will produce a different quality of sound because of the distribution of the upper partials of each instrument. The combined sound of each pairing will be different because of the upper partial phenomenon.

Changes of spacing, timbre and register will affect the consonant and dissonant response also. According to theorists ‘perfect consonance’ would be total silence. So total dissonance must be noise - the inability to distinguish a single pitch - the loudness and quality of the noise would also be a factor in determining the dissonant quality of a noise. With the natural scale as the basis of musical pitch, a fixed pitch tuning system like the equally-tempered system can never be set in stone because it is only one system of tuning formulated from the overtone series. A composer could formulate an infinite number of microtonal systems using frequencies solely from the overtone series or using a mixture of microtonal and tempered notes.

Conditioning within the equally-tempered system will initially affect the ability of listeners exposed to microtonal music to come to terms with such music. The calculation of whole-number ratios for the equally-tempered intervals shows that the pitch intervals are quite high up in the overtone series showing the tempered system to be dissonant. The whole-number ratios found for the pitch intervals involving microtones in bar 17 of *Guitar Opus 1* showed that two of the pitch intervals were very consonant as they were 'special relationship' whole-number ratios. This is an important discovery and further work in analyzing all the pitch intervals and harmony containing microtones in the compositions produced here would further explain the consonant/dissonant quality of the compositions.

In this project, microtonal notes, which can be extracted from a normal classical guitar, are used in composition. It was the initial intention to introduce microtones to tempered composition and gradually become more microtonal. This was realized in the compositions *Guitar Opus 1* and *Guitar Opus 2* (CD tracks 1 and 2). The increased aural perception developed during the project is evident in the later compositions: *Etude for Amplified Classical Guitar* (CD track 3) is more microtonal and *Etude for Electric Guitar with Added Frets* (CD track 4) contains two different microtonal versions of the tempered melody.

The microtonal notes are found by plucking the opposite side of a stopped string and when a 'hammer-on' technique is used, both sides of the string sound. Since the overtone series is infinite, the frequencies found by plucking the opposite side of the stopped string are obviously part of this system and calculations show them to be mostly dissonant but in combination with tempered notes the produced pitch intervals can be more consonant than the tempered system. The first task was to find whole-number ratios for the pitch intervals of the equally-tempered system within the overtone series using a resolution of two cents. Apart from the octave, perfect fifth and perfect fourth, the other calculated ratios are dissonant and quite high up in the overtone series. The calculation of whole-number ratios for the equally-tempered pitch intervals was very important in understanding the dissonant quality of the equally-tempered system. Tempering has lessened the consonant quality of the frequencies of the equally tempered scale.

In this project equally-tempered notes along with the microtonal notes are used as a source for composition. The idea is to allow the listener to gradually become used to microtonal notes that are outside the familiar sound of equal temperament. They are already audible to guitarists as component frequencies of some sounded notes on the guitar. A survey of the consonance and dissonance quality within the tempered scale shows that pitch intervals containing microtones can be less dissonant than pitch intervals containing tempered notes. Using the information from the calculated whole-number ratios for both the equally-tempered system and the microtonal system shows clearly that they are not within the recognized consonant whole-number ratios of the overtone series. Some of the whole-number ratios calculated for the measured microtones are lower down the overtone series than some of the ratios of the tempered system, making them more consonant.

Experimentation in microtonal music provides composers today with new directions in composition. The outcome of this research project confirms that the guitar is particularly suited to further exploration in microtonality. As shown by the compositions *Guitar Opus 1* and *Guitar Opus 2* modification is not necessary because microtones can be extracted from a normal classical guitar. The system of microtones that is present on a guitar, does not fit into any other microtonal system used by composers. As highlighted in *Etude for Amplified Classical Guitar* these microtonal notes are heard together with a tempered note when a finger is hammered on to an open string - a common guitar technique. The application of this research project provides an exciting challenge for composers of guitar music who wish to explore further the uncharted realms of microtonal composition.

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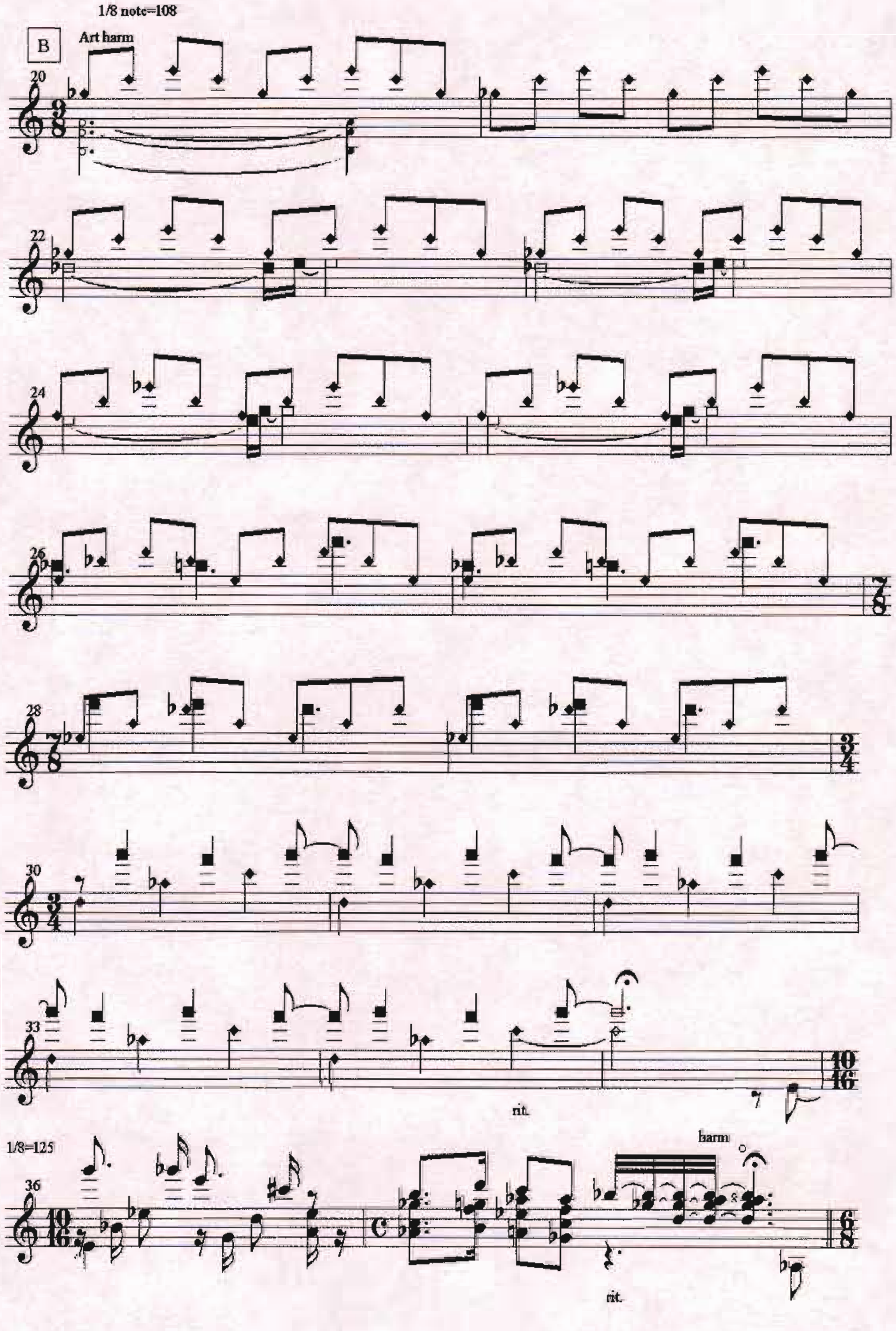
Appendix A Composition

Opus 1

Mike Nielsen


1/8 note=108
B
Art harm
rit.
1/8=125
harm
rit.


C
♩= 150-202
38
accel.
mp
42
46
50
54
ff
con brio
57
60
mp
62
mf


63
f
64
mf
mp
66
mf
f
68
mf
mp
70
mf
73
75
76


78
79
80
mp
82
f
rit.
83
pp
D
♩= 50
84
Rubato
86
3
88
3


90
92
94
97
Ad lib
accel.
99
101
Molto rit

Opus 2

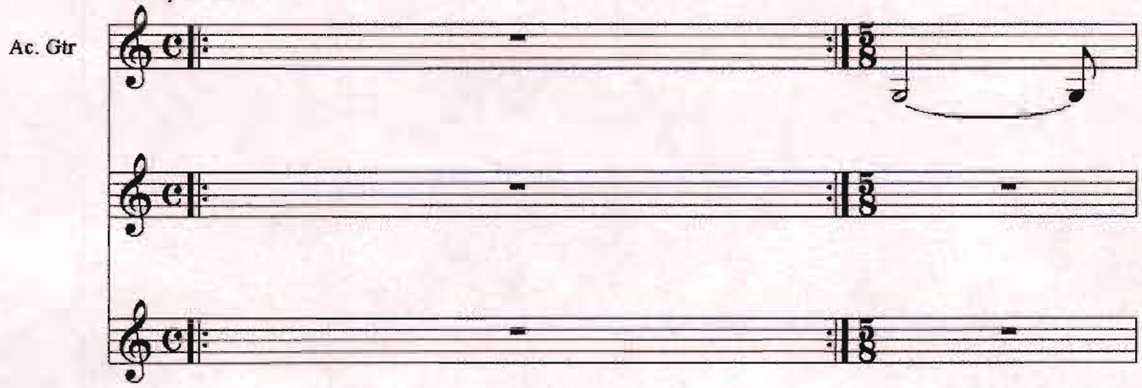
Mike Nielsen

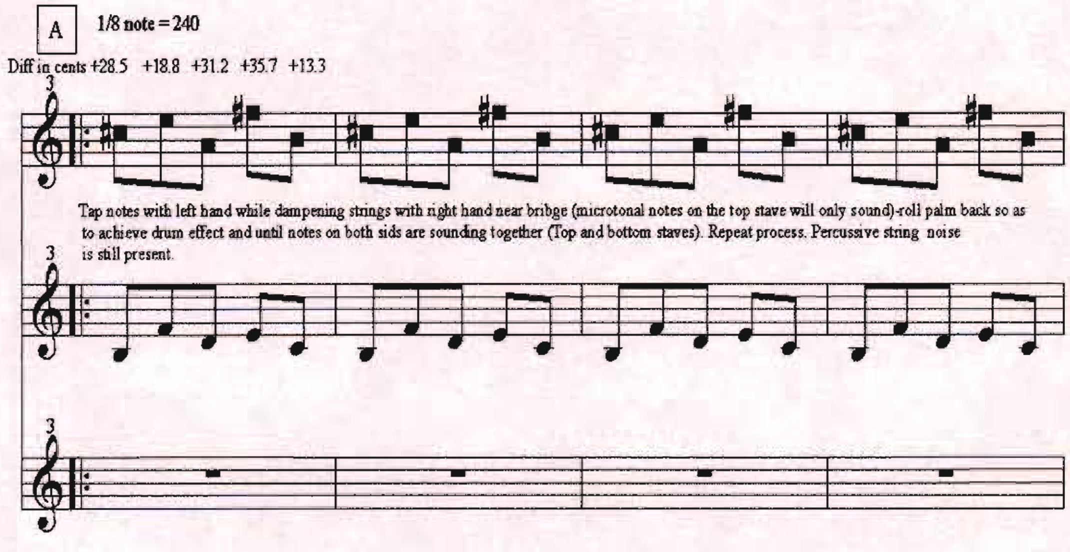

25
29
D
1/4 note = 64
p
7
33
rit.
36
A tempo
38
pp

ETUDE
For amplified classical guitar

Introduction- Open improvisation using percussive string noise over 6th string, 9th fret, and 5th string, 7th fret. All the sections should utlize improvisation and are not played exactly as written.

MIKE NIELSEN

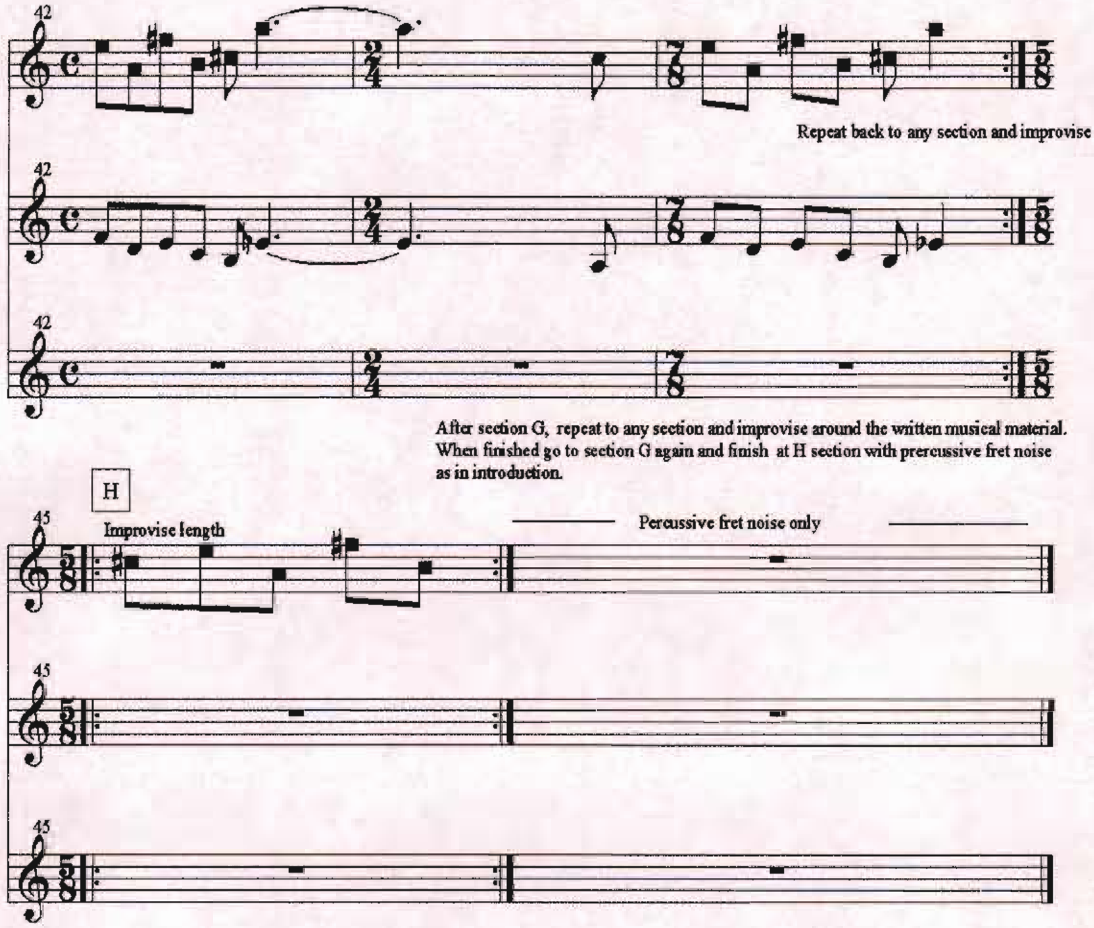





7
7
7
B
10
Dampen nut side strings with RH index (tempered notes in bottom stave will sound then release-opposite idea to Section A.
Repeat process every two bars. Dampen strings with right elbow at bridge side, then dampen nut-side strings w/right
index finger at nut-side. Repeat process
10
10
14
14
14

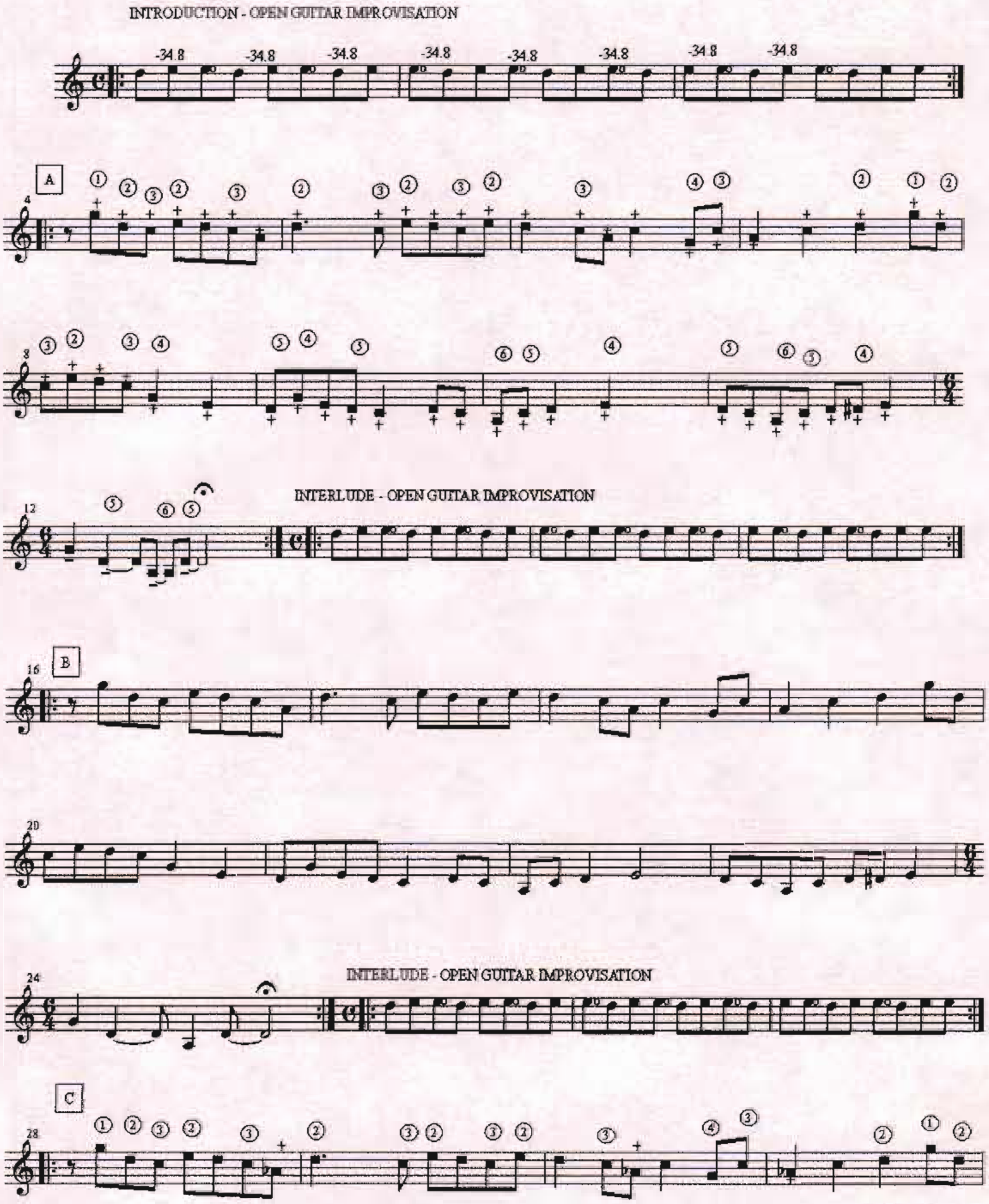

C
18
This chord is tapped by the right hand on the nut-side while the tempered notes of repeated figure only sound
The rhythm of the chord should be improvised.
18
18
D
22
This chord is tapped by the right hand on the bridge-side while the microtonal notes of repeated figure only sound
The rhythm of the chord should be improvised.
22
22
E
26
This chord contains the top 3 notes of the chord in section C leaving the 4th, 5th,& 6th strings
free. This allowsthe repeated figure to sound on both sides of the stopped string.
26
26

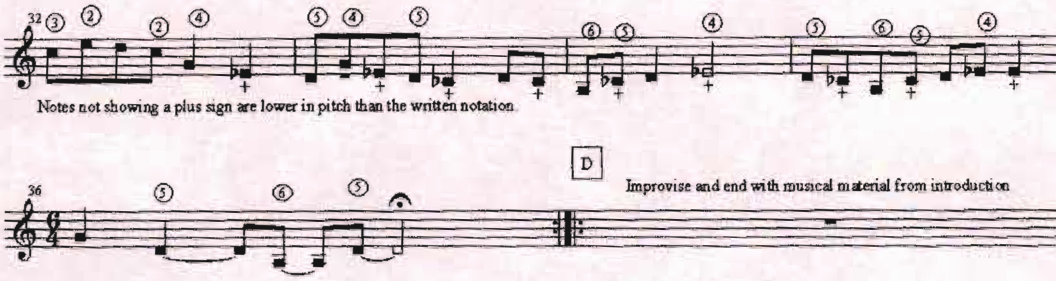

F
This chord contains the top 3 notes of the chord in section D leaving the 4th, 5th,& 6th strings free. This allowsthe repeated figure to sound on both sides of the stopped string.
G


42
Repeat back to any section and improvise
42
42
After section G, repeat to any section and improvise around the written musical material.
When finished go to section G again and finish at H section with precussive fret noise
as in introduction.
H
45
Improvise length
Percussive fret noise only
45
45

Etude for electric guitar with added frets

Mike Nielsen




32
Notes not showing a plus sign are lower in pitch than the written notation.
36
D
Improvise and end with musical material from introduction

Appendix B Development of Stringed Instruments

Development of stringed instruments	Country	Period	Scale structures and tuning
Lyre, lower sound-chest harp, upper sound-chest harp.	Ancient Mesopotamia *(Royal tombs of Ur)*	c.2500B.C.	Ancient Chaldaean scale (Most probably a scale of 7 notes)
Harp (lower sound-chest). c. Pyramids of Gizeh. An older version than Royal tombs of Ur. Closer to its ancestor-the hunters bow. Ten and twelve stringed-six foot tall harp. Three stringed lute and lyre introduced	Ancient Egypt	c.2600B.C. c.1890B.C. c. 1250B.C.	5 and 7-note scales
Sitar, fiddle, lute, sarod, vina, tambura (long necked fretless lute for creating drone)	India	c.1500B.C.	Seven notes (*svaras*) per octave-further divided into 22 microtonal steps (*srutis*) of nearly equal size
Zithers including; the long-necked three-stringed guitar and lute (*p'i-p'a*) and seven-stringed zither (*ch'in*). Two stringed fiddle.	China	c.1050B.C.	5-note scales
Six-stringed zither (*yamato-goto*), long-necked three-stringed guitar (*samisen*), koto, lute (*biwa*)	Japan	c.1000B.C. c.1560A.D. *(Samisen, koto)*	5 and 7-note scales. Melodic microtonal graces used in koto (tuned to a 5-note scale) playing.
Lute (*pi-wan, pi-ban*), 4-stringed fiddle	Tibet	Early Chinese influence	Religious music (7-note scale), folk music (5-note scale)
Spike fiddle-*rabab* of Arab origin and zithers of foreign origin	Indonesia	c.1300A.D.	*Pelog* (5 notes), *slendro* (divides octave into approx 5 equal notes-20 cents per note.
Lyre, Bouzouki	Greece	c.800B.C.	Quarter-tones (early Greek music). Various 7 note modes which are used in western music today (modified by Temperament). A theoretical 21-pitch scale contained all diatonic, chromatic and enharmonic notes used by the Greeks. Parallel seconds were used to accompany a melody. A Pythagorean scale based on whole-number ratios was used.
Lute (*'ud*), Lyre *(mi'zaf)*, upper-chested harp	Other Arab countries	c.224A.D.	A Pre-Islamic scale was arrived at by, dividing the string or fingerboard into 40 parts. Similar scale to a Pythagorean scale was used (intervals though read ascending and not descending as in Greek music). Musical treatise included; *The Book about Music* and, *The Book of Musical Modes*. 14th Century Arabic music used Pythagorean scale. Also, the introduction of modern quarter-tone (*rub'*) scale.
Laud (of Arabic *al-'ud*)	Spain	c.713A.D	The Laud (fretted) was tuned to the Pythagorean scale.
Rebec (pre-Viol) ,Cittern	Europe	c.1000A.D.	Pythagorean intonation. Rebec was descendent of Arab *rebab*