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# Relativistic particle acceleration in tangled magnetic fields

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**Abstract.**
We present simulations of the transport of fast particles through three-dimensional turbulent magnetic field configurations. A time dependency is imposed on the plane wave modes used in constructing these fields such than acceleration via the second-order Fermi process is possible. We consider simulations of models with low and high turbulence levels for non-relativistic waves. The predictions of quasi-linear theory are discussed with respect to the simulation data. We conclude that for pure stochastic acceleration via Alfvén waves to be plausible as the generator of UHECR in Cen A, the baryon number density would need to be several orders of magnitude below currently held upper-limits.

## 1. Introduction

The origin of ultra high energy cosmic rays (UHECR) remains one of the long-standing problems in high energy astrophysics. The gyroradii of these particles are so large that it is difficult to contain them in our galaxy, and they are almost certainly extra-galactic in origin. Recent observations by the Pierre Auger Observatory (Auger) have indicated that UHECR with energies $> 10^{19.7}$ eV, appear to have statistically significant (at the $3\sigma$ level) correlations with maps of the local ($< 75$ Mpc) AGN population (Abraham et al. 2008). In particular, Cen A has been associated with a number of these events.

Although the acceleration mechanism is uncertain, the magnetic fields and sizes of these objects make them potential sites for the confinement and acceleration of UHECR. Within the AGN class of objects, several different acceleration sites present themselves as candidate locations for the acceleration of UHECR. Among these are the central AGN, the jet driving the lobe, and the large radio lobes themselves (for a review see Begelman et al. 1994).

## 2. Quasi-linear theory

The evolution of the energetic particle distribution can be described by the momentum diffusion equation Tverskoi (1967)

$$\frac{\partial f(p,t)}{\partial t} = \frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2 D(p)\frac{\partial f(p,t)}{\partial p}\right]. \tag{1}$$

which is valid for scattering times-scales much shorter than the crossing time of the physical system.

Neglecting numerical factors, and considering a one-dimensional spectrum $W(k) \propto k^{-q}$, where $\delta B^2/8\pi = \int_{k_{\rm min}}^{k_{\rm max}} W(k){\rm d}k$, with $k = 2\pi/\lambda$, the momentum diffusion coefficient may be shown to take the form (Schlickeiser 1989)

$$D(p) \approx \beta_{\rm A}^2 \frac{\delta B^2}{B_0^2}\left(\frac{r_{\rm g}}{\lambda_{\rm max}}\right)^{q-1}\frac{p^2c^2}{r_{\rm g}c} \propto p^q. \tag{2}$$

where $r_g = pc/eB_0$ is the gyroradius of a particle, and $\beta_{\rm A}$ is the Alfvén velocity normalized to the speed of light. This result is valid for particles with gyroradii smaller than the correlation length of the field. The diffusion of particles with larger gyroradii are essentially independent of momentum since such particles interact with the entire spectrum (Tsytovich 1966). This results in an increase in acceleration times, a fact which we demonstrate numerically in Section 3.4.. The associated mean free path, $L$, is

$$L \approx \frac{B_0^2}{\delta B^2}\left(\frac{r_g}{\lambda_{\rm max}}\right)^{1-q} r_g. \tag{3}$$

Equation 1 contains a systematic acceleration timescale given by

$$t_{\rm acc} = \frac{p^2}{D(p)} \approx \beta_{\rm A}^{-2}\frac{L}{c} \propto p^{2-q}. \tag{4}$$

## 3. Model

While in this work we focus on isotropic turbulence, we note that numerical investigations on the effects of anisotropy in the turbulent field have been reported by Michałek et al. (1999), by considering waves drawn randomly from within cones of differing opening angle about the mean field. They find that the diffusion coefficients vary by less than an order of magnitude.

It has also been argued by Yan & Lazarian (2004) that Alfvénic turbulence is anisotropic, with eddies elongated along the (local) mean magnetic field (ie. $k_\perp > k_\parallel$), resulting in inefficient gyroresonant acceleration due to the reduction in the total power in parallel modes. The importance of this effect is unclear in strongly turbulent fields as may be prevalent in radio lobes.

For instance, there is observational evidence from polarization maps for chaotic fields in the southern lobe of Cen A (Junkes et al. 1993). Furthermore, lobes such as Cen A with morphologies characterized by extended filamentary structures (Carilli et al. 1989) are known to be weakly magnetized (Clarke 1996) in the sense that $\delta B > B_0$. Consequentially, hydrodynamic turbulence, as might arise from high velocity injection of plasma into the lobes, will result in a strongly chaotic magnetic field component.

### 3.1. Turbulent field in three dimensions

We now describe the construction of a magnetic field from modes with wavevectors directed isotropically in three-dimensions and a corresponding electric field such that $\delta\mathbf{E}\cdot\mathbf{B}=0$.

Appealing to Ohm's law for an ideal plasma with infinite conductivity, we may write

$$\delta\mathbf{E} = -\delta\mathbf{u}\times\mathbf{B}, \tag{5}$$

where $\delta\mathbf{E}$, $\delta\mathbf{u}$, $\mathbf{B}$ are the total electric, bulk plasma velocity in units of $c$, and magnetic fields respectively in the observer frame.

Linearizing Faraday's law, and using the fact that linear Alfvén waves are non-compressional (ie. $\mathbf{k}_j\cdot\delta\mathbf{u}_j=0$), we have

$$\delta\mathbf{u}_j = -\mathrm{sgn}(\cos\theta)\frac{\beta_A}{B_0}\delta\mathbf{B}_j \tag{6}$$

for the contribution to the velocity field due to each Alfvén mode. Hence

$$\delta\mathbf{u} = -\frac{\beta_A}{B_0}(\mathbf{B}^+ - \mathbf{B}^-), \tag{7}$$

where $\mathbf{B}^+(\mathbf{B}^-)$ is the sum of all forward(backward) moving modes with respect to the mean field. From equation 5 then

$$\delta\mathbf{E} = -\delta\mathbf{u}\times\mathbf{B}_0 + 2\frac{\beta_A}{B_0}(\mathbf{B}^+\times\mathbf{B}^-). \tag{8}$$

Therefore, retaining the second order correction to $\delta\mathbf{E}$ given by the second term on the right-hand side of equation 8 provides a total electric field satisfying $\delta\mathbf{E}\cdot\mathbf{B}=0$. In the absence of this term, only $\delta\mathbf{E}\cdot\mathbf{B}_0=0$ and the electric field can still have a parallel component to the perturbing magnetic field, leading to rapid electrical acceleration of a charged particle propagating in the total B-field, $\mathbf{B}$.

### 3.2. Particle equations of motion

Given the time dependent fields $\boldsymbol{B}(\mathbf{r})$ and $\delta\boldsymbol{E}(\mathbf{r})$, the relativistic equations of motion for a particle of charge number $Z$, velocity $\beta\equiv\mathbf{v}/c$, and rest mass $m$ are

$$\frac{\mathrm{d}\gamma\boldsymbol{\beta}}{\mathrm{d}t} = \frac{Ze}{mc}\left(\delta\boldsymbol{E}+\boldsymbol{\beta}\times\boldsymbol{B}\right), \tag{9}$$

$$\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t} = \boldsymbol{\beta}c, \tag{10}$$

where $e$ is the electronic charge. In a magnetic field of mean magnitude $B_0$, the characteristic relativistic Larmor angular frequency is $\omega_{\mathrm{g}}\equiv ZeB_0/\gamma mc$ where $\gamma\equiv(1-\beta^2)^{-1/2}$. The corresponding maximal Larmor radius is $r_{\mathrm{g}}\equiv\omega_{\mathrm{g}}^{-1}\beta c$.

### 3.3. Simulation parameters

We select Cen A as the reference object for the physical parameters of the models considered here. To test the results of quasi-linear theorypresented in Section 2., we consider a homogeneous region of infinite extent with a mean field of $B_0 = 3\,\mu$G onto which is superimposed an isotropic turbulent component $\delta B$. The turbulence level of the field is determined via the parameter $(\sqrt{\langle \delta B^2 \rangle}/B_0)^2 \equiv (\delta B/B_0)^2$. The dynamic range of the turbulent field extends from $\lambda_{\rm min} = 10^{-8}\,$kpc to $\lambda_{\rm max} = 1\,$kpc and is resolved via 513 modes evenly spaced logarithmically. We have confirmed that our results are adequately converged at this mode density and dynamic range.

The Alfvén speed in the simulations is determined by the proton density according to

$$\beta_{\rm A} = \frac{1}{c}\frac{B_0}{\sqrt{4\pi m_{\rm p} n_{\rm p}}} \tag{11}$$

where $m_{\rm p}$ and $n_{\rm p}$ are the proton mass and number density respectively.

We consider two models in this work: model **A** describes a low turbulence level regime, $(\delta B/B_0)^2 = 0.1$, appropriate to representing the conditions required by quasi-linear theory; for model **B** , the turbulence level is chosen to be high with $(\delta B/B_0)^2 = 1.0$. For both models, we have chosen the canonical value $n_{\rm p} = 10^{-4}\,{\rm cm}^{-3}$ for the proton number density. Non-relativistic three-dimensional Alfvén turbulence is assumed with $\beta_{\rm A} = 0.002$. (Mildly relativistic turbulence models with $\beta_{\rm A} = 0.2$ have been considered by O'Sullivan et al. (2009).) Numerically, the turbulence is constructed by means of a summation of plane wave Alfvén modes similarly to the procedure outlined by Giacalone & Jokipii (1999).

With these configurations, the acceleration timescales are too long to capture the evolution of an injected particle distribution over a significant energy range with sufficient accuracy to resolve any statistical signal from numerical error. Therefore, we rely on snapshots of the acceleration rate at energies $\log_{10}\gamma = \{5, 6, 7, 8, 9\}$. Furthermore, we find that integrations over $T = \{64, 32, 16, 8, 4\} \times 10^3 \omega_{0\,{\rm g}}^{-1}$ respectively, where $\omega_{0\,{\rm g}}(\gamma_0)$ is the relativistic Larmor frequency *at the corresponding injection energy* $\gamma_0$, are sufficient for the momentum distributions to complete the initial ballistic transport phase and become diffusive. While $t \ll \tau_{\rm acc}$, the momentum distributions may be approximated via Gaussian profiles.

For each of the snapshot energies, 1500 particles are injected with random position and velocity orientation in a randomly generated field. No more than three particles are released in any given realization in an effort to reduce numerical noise.

### 3.4. Results

As previously remarked, for non-relativistic turbulence with $\beta_{\rm A} = 0.002$, integration over a large energy range is not feasible computationally. Instead, ensembles of particles are injected at discrete energies over the range of interest and instantaneous acceleration times are inferred. Once the the injected delta function particle distribution has been evolved for a sufficient time for it to relax

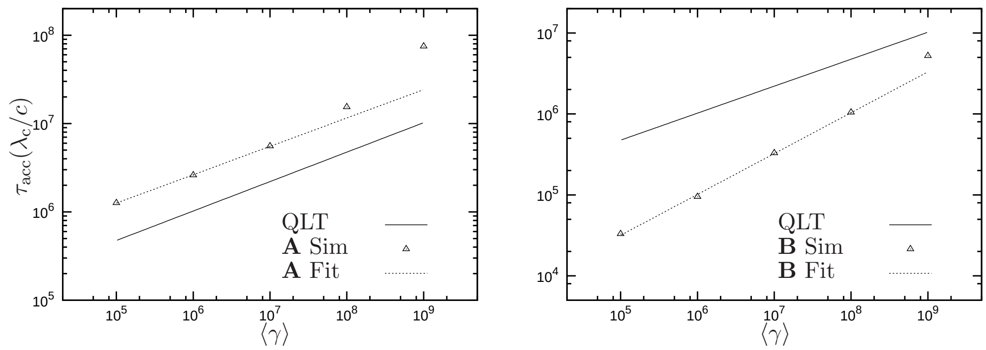


Figure 1. Acceleration $\tau_{\rm acc}$ as a function of energy. $\tau_{\rm acc}$ is derived from best fits of Gaussian profiles to the standard deviation of the momentum distributions via $\gamma_0^2/D_p$ where $D_p \equiv \langle\Delta\gamma^2\rangle/2\Delta t$. Note that for $\rho \equiv r_{\rm g}/\lambda_{\rm c} \gtrsim 1$ the efficacy of the acceleration process is damped. Left panel: Model **A** . The expected value from quasi-linear theory, $10224\gamma^{1/3}$, is represented by the solid line. The best fit to the data for $\rho < 1$ ($\langle\gamma\rangle < 3.6\times10^8$) is given by $19138\gamma^{0.36}$ (dotted line). Right panel: Model **B** . The expected value from quasi-linear theory, $1022\gamma^{1/3}$, is represented by the solid line. The best fit to the data for $\rho < 1$ ($\langle\gamma\rangle < 3.6\times10^8$) is given by $95\gamma^{0.50}$ (dotted line).

to a Gaussian distribution, the variance of the profile is obtained by a best fit. The momentum diffusion coefficient is then approximated by

$$D_p = \frac{\langle\Delta\gamma^2\rangle}{2\Delta t} \tag{12}$$

where we have used the fact that the mean drift in the distribution is negligible over the integrated times ie. $\Delta\gamma = \gamma - \langle\gamma\rangle \approx \gamma - \gamma_0$. From this expression, the acceleration time, $\tau_{\rm acc}$, is then obtained using

$$\tau_{\rm acc} = \frac{\gamma_0^2}{D_p}. \tag{13}$$

The results for model **A** are plotted in the left panel of Figure 1. An index of 0.36 is found for $\tau_{\rm acc}$ using a best fit containing all points excluding that for the $10^9$ ensemble. This is in close agreement with the theoretical value of 1/3 dictated by quasi-linear theory, although the index does appear to approach this value asymptotically for lower rigidities. Notably the three-dimensional field presents acceleration times that are a factor of 2 larger than the one-dimensional quasi-linear theory. It is also observed that for $\rho \gtrsim 1$ ($\gamma \gtrsim 3.6\times10^8$) the acceleration time strongly diverges from the theoretical result as particles' gyromotions exceed the scale at which resonant waves are present in the field.

Regarding the acceleration of UHECR, this emphasizes the fact that the maximum energy is quite typically governed by the correlation length of the field.

For model **B** , where the turbulence level is high, the agreement with quasi-linear theory is less compelling, as illustrated in the right panel of Figure 1. Here, the index derived is 0.50, and the experimentally determined acceleration time is an order of magnitude faster than predicted in quasi-linear theory.

The divergence from quasi-linear theory values with increasing turbulence level is not surprising, as formally quasi-linear theory is valid in the limit of weak perturbations to the mean field.

## 4. Conclusions

For non-relativistic turbulence, we find that agreement with quasi-linear theory is good at low turbulence levels. The three-dimensional acceleration timescales are found to be in excess of one-dimensional quasi-linear theory by a factor of $\sim 2$. At higher turbulence levels, this agreement is less pronounced with the power law dependence of the acceleration time on the energy becoming steeper.

Notably, above the critical energy (corresponding to resonance with the largest wave in the system, $\rho = 1$) the particles interact with the the entire Alfvén spectrum such that the diffusion becomes almost independent of energy. This dramatically reduces the efficacy of the acceleration process.

Given that the results of the numerical simulations are in reasonable agreement with the results of quasi-linear theory, if ultra-high energy protons are stochastically accelerated in radio lobes, the Alfvén speed must be considerably larger than currently held estimates suggest. In particular, lower density and/or higher magnetic fields are required. In support of this conclusion, recent work by Feain et al. (2009) indicates density limits are lower by a factor of 2 - 3 than earlier accepted values.

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