2006

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Recommended Citation

Caprani, C., O'Brien, E., Gonzalez, A., Li, Y., Rattigan, P., Harris, N.: Bridge dynamics and loading. Keynote paper, Third National Symposium on Bridge and Infrastructure Engineering in Ireland, eds N.A. Ni Nuallain, A. O'Connor and K. Gavin, Trinity College Dublin and University College Dublin, pp.347-358. 2006.

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2006-01-01

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Bridge Dynamics and Loading

Eugene OBrien, Arturo González, Colin Caprani, Yingyan Li, Paraic Rattigan & Niall Harris

Abstract

Recent UCD research on dynamic bridge-truck interaction and bridge traffic loading in general is reported. The accuracy of bridge traffic load assessment has been significantly improved by identifying the differences in the sources of traffic loading on short- to medium-span bridges and treating them separately. It is shown that results can be significantly improved by treating 1-, 2-, 3- and 4-truck loading events separately. Gaps between trucks are also identified as a critical issue that strongly influences the frequency of key 3- and 4-truck events. Trends in the dynamics of traffic loading are identified using simple force models on simply supported beams. It is shown that an accurate assessment of the influence of dynamics requires a probabilistic approach. This is illustrated through an assessment of the dynamic factor for a bridge in Slovenia.

Keywords: Bridge, traffic, load, static, gap, interaction, dynamic, DAF, bivariate.

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BACKGROUND

As bridge stocks age across Europe and the world, maintenance costs represent an increasing proportion of total road infrastructure expenditure. Due to our past history of low investment in transport infrastructure, Ireland is at an earlier stage than most of the developed world. Nevertheless, it is now time to prepare for the long term maintenance, repair and renewal of our rapidly growing bridge stock. A recent study [1] of the 15 EU member states prior to May 2004, estimates expenditure on the repair, rehabilitation and maintenance of bridge structures to be €4–6 bn annually. In the 25 states of the enlarged EU, the annual spend on bridge maintenance is likely to be at least €6 bn.

The significant cost attributable to the maintenance of highway bridge infrastructure has driven much research in this area over the past number of years. If it is can be proven that a bridge is safe without strengthening, the useful life of a bridge can be extended. In addition to the cost savings, there are considerable environmental benefits through reduced use of non-renewable materials. Hence, there is a drive to improve the accuracy of methods of bridge assessment.

Whilst the accuracy of assessments of bridge capacity is improving, the assessment of traffic loading on bridges is at an early stage of development. Some countries (e.g., UK) have notional load models for assessment [2]. While such models can allow for some variation in traffic conditions between sites [3], they are necessarily conservative as they are deemed to represent a wide range of situations. Bridges can often be shown to be safe for the traffic loading to which they are subject, even if they do not have the capacity to resist the notional assessment load. It is reasonable to assume that the notional load model in use in the South East of England is inappropriate for use in Ireland where loading patterns are considerably different.

In this paper, the concept of site-specific bridge load assessment is presented. This involves the measurement of the weights and frequencies of heavy trucks at the site of the bridge. The bridge is then assessed for the level of loading that has an acceptably low probability of being exceeded.

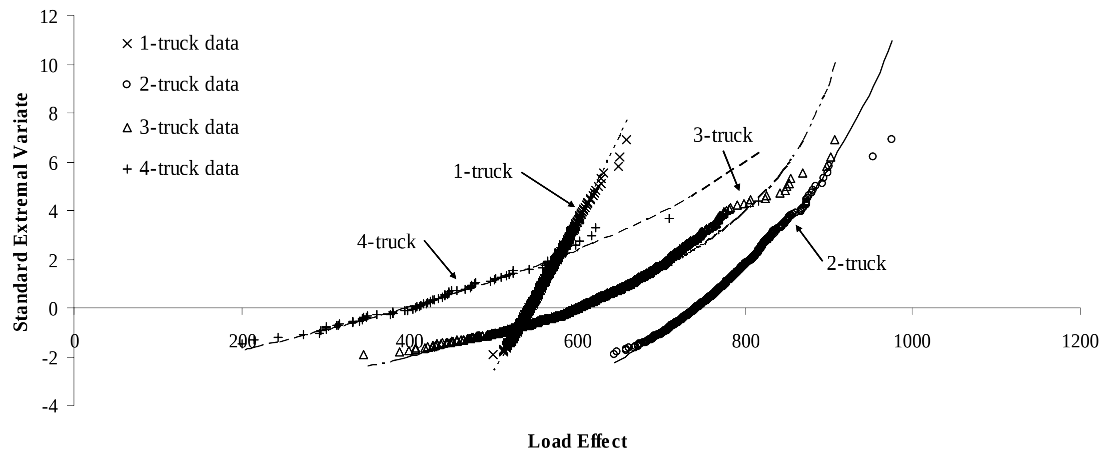
STATISTICAL EXTREMES OF STATIC LOADING

A load effect on a bridge is any effect, such as bending moment or strain, which results from any form of loading event. This study addresses load effects in short- to medium-span (20 to 50 m), bidirectional, two-lane bridges. For these bridge lengths, when an allowance for dynamic effects is made, free-flowing traffic generally governs [4]. Many authors approach the problem by identifying the maximum load effect recorded during a loading event [5-7] or in a reference period such as a day [8-10, 3] or a week [11], and fitting these maxima to an extreme value statistical distribution. This approach is based on the assumption that individual loading events are independent and identically distributed (iid) [12-14]. However, load effects can be the result of any of a number of quite different loading events, involving different numbers of trucks. A single truck crossing event is relatively simple with variables of truck weight, axle configuration and distribution of weight between axles. Events involving two trucks are more complex involving distributions for weight and geometric variables for both trucks and new statistical variables such as the location of the second truck relative to the first. It is therefore clear that, in general, a load effect due to the passage of a single vehicle has a different statistical distribution to the same load effect due to an occurrence of multiple vehicles. To mix load effects from such different loading events violates the iid assumption used in extreme value analysis.

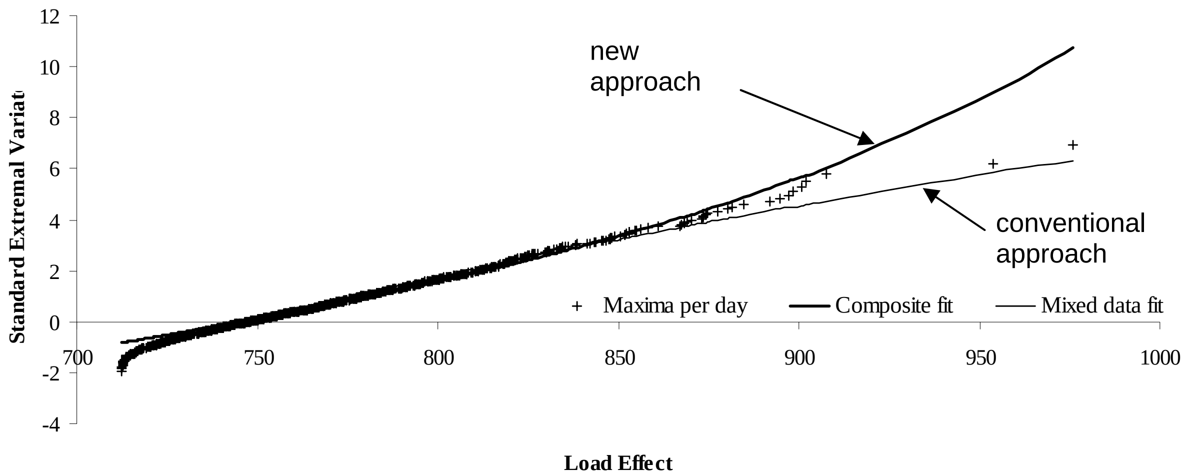
Fig. 1 illustrates the phenomenon of mixing statistical distributions for central support bending moment in a 2-span 30m (2×15m) bridge. Bending moment is calculated using numerical simulations for typical truck crossing and meeting events. In total, 1000 days of traffic is simulated and the daily maximum recorded for each event type – using the number of trucks involved to define type.

Data of each type is ranked – largest point, followed by 2nd largest, etc., but the rankings are scaled on the vertical axis so that the points can be more easily seen. The scale used here corresponds to the inverse Gumbel statistical distribution but any scale could be used. The daily maxima by event type are plotted in Fig. 1(a) with the individual best fits. Within the 1000 day simulation period, 2- and 3-truck events result in most of the daily maxima. However, different trends are evident in each of the data sets. It is of particular interest that the 4-truck events, while very rare and only occasionally featuring as the maximum in the day, shows a trend that indicates that it will feature strongly in a 1000 year return period. This is the level of safety recommended in the Eurocode [30] and corresponds to about 12.5 on the vertical axis of Fig. 1. The quality of the fit to the 2- and 3-truck data is imperfect in the tail but this is the result of a small number of data points and is not representative of the general trend. In any case, the characteristic value is much more strongly influenced by 4-truck than 2- or 3-truck events.

The conventional approach [3, 5-10] is to identify the maximum load effect, regardless of event type, and to fit this mixed maximum-per-day data to a statistical distribution. The mixed maximum-per-day data is illustrated in Fig. 1(b) with the conventional approach and the fit by event type. The mixed data appears to fit better to a GEV statistical distribution than the new approach. However, this is in fact an over-fitting to a mixture of non-iid data and is not representative of the four underlying trends shown in Fig. 1(a). The conventional approach is in fact following a trend supported by the two most extreme data points which are not critical as they are the result of 2-truck events. There are pronounced differences in the characteristic values when the 1000 days of data is used to predict the 1000 year extreme. The conventional approach predicts a characteristic load effect of 1333 kNm, 34% in excess of 995 kNm, the corresponding value predicted by the new approach.



(a) **Daily maxima by event type**



(b) **Mixed daily maxima**

Fig. 1 Results for Central Support Moment on 30 m long bridge

The fact that 3- and 4-truck events are important for short- to medium-span bridges is a new finding and raises new issues. The gap between trucks now becomes much more important – the frequency of very short gaps is key to determining the frequency of extreme 3- and 4-truck events. Researchers in the past [4, 15-26] have made various assumptions about the gaps between vehicles. In this study, statistical distributions are fitted to *measured* headways to accurately represent actual site conditions. Five days of Weigh-In-Motion (WIM) data was processed from the two outermost (slow) lanes of the 4-lane A6 motorway near Auxerre in France.

For headways of less than 1.5 seconds, the correlation between hourly flow and headway is weak as can be seen from Fig. 2. Hence, it is reasonable to assume a distribution of headway that is independent of flow. This approach is supported by the theory that, for small headways, driver perception of safe distance rather than traffic flow determines the minimum headways [27, 28]. Combining all available headway data less than 1.5 seconds, for both directions, gives the cumulative distribution function which is fitted with two quadratic equations, one for less than 1 second and another between 1 and 1.5 seconds. For headways between 1.5 and 4 seconds, there is a correlation between headway and flow.

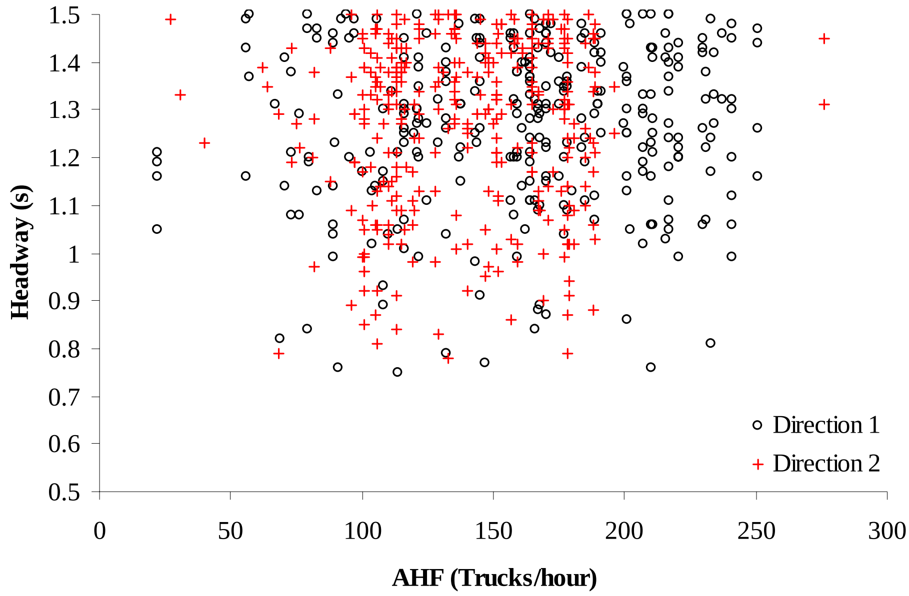


Fig. 2 Occurrences of headways less than 1.5 seconds and corresponding average hourly flow (AHF)

Statistics for the numbers of trucks involved in the daily maximum load effects are shown in Fig. 3 for 250 simulated days of traffic. There are no single truck loading events that feature in the daily maxima but 2-truck events are prominent, particularly for shorter bridge lengths. Surprisingly and contrary to assumptions made in some past studies, 3- and 4-truck events feature significantly for 40 m and 50 m bridges. Similar graphs for conventional gap assumptions [29] show that these also feature significant frequencies of 4-truck and even 5-truck events which is consistent with the finding (Fig 1) that they must be considered.

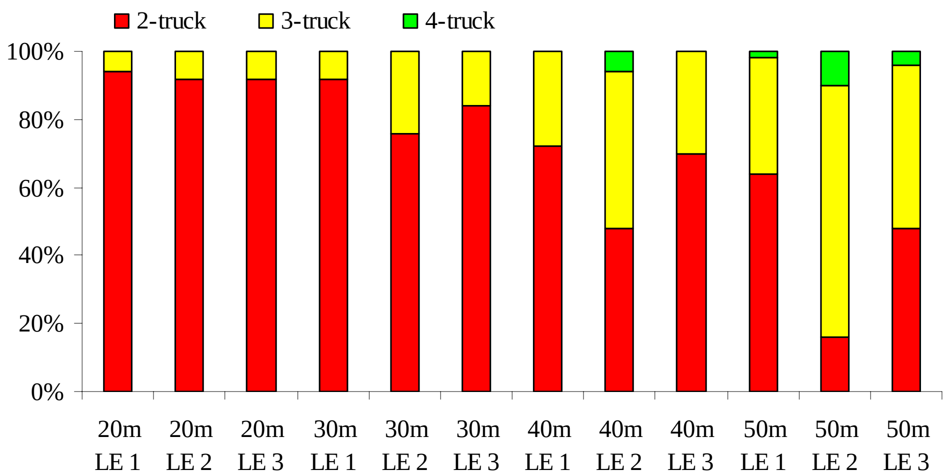
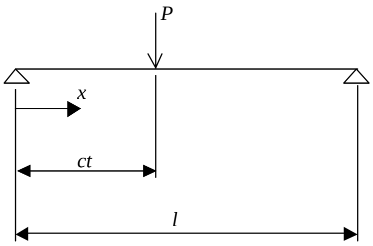


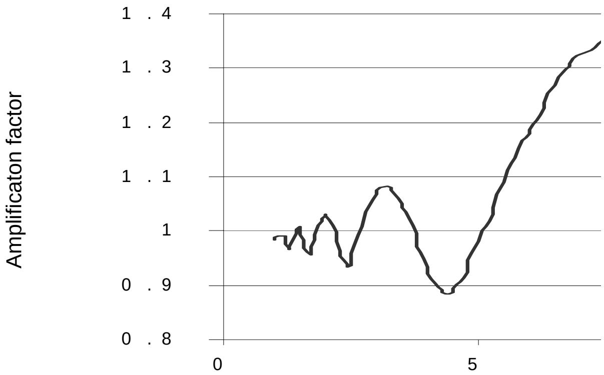
Fig. 3 Number of trucks involved in maximum-per-day load effects (LE i = Load Effect i; LE1 & LE2 are Mid-span Moment and End Shear respectively in simply supported bridge; LE3 is Central Supported Moment in 2-span Continuous Bridge)

DYNAMIC AMPLIFICATION OF BRIDGE LOAD EFFECT

When trucks pass over a bridge, they bounce and rock, the bridge vibrates and there is dynamic interaction between them. The total load effect on the bridge is usually greater than the corresponding static load effect. The Eurocode [30] traffic load model has been developed from the simulation of static loading events [4]. Dynamic amplification factors of 20% to 70% are applied to the 1000-year characteristic static load effects to give the load model used for design. These dynamic amplification factors are based on field trials where single trucks have been driven over bridges. However the critical loading combination involves a number of trucks meeting on the bridge for which the dynamic effect is considerably less.

One of the simplest representations of truck-bridge dynamics is a single point force on a simply supported beam (Fig. 4(a)). Using equations first developed by Fryba [31], it can be shown that there are a number of critical speeds (load circular frequencies) at which the dynamic amplification reaches a local maximum. This is the result of a matching of truck speed with bridge first natural frequency.



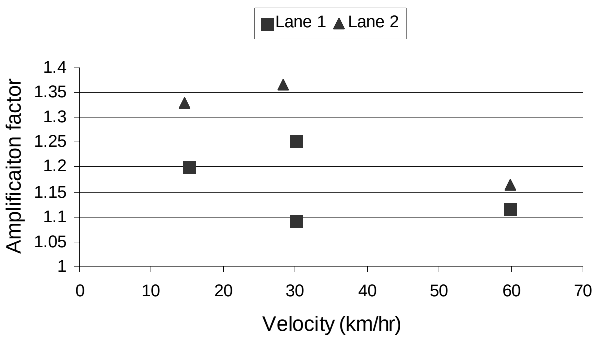


(a) Schematic of beam and load model

(b) Dynamic Amplification as function of load circular frequency (or speed)

Fig. 4 Dynamic amplification factor versus load circular frequency for 25m bridge with 3% damping

This is a very simple model but field trials – Fig. 5 – have confirmed that dynamic amplification is quite sensitive to vehicle speed. However, the problem is complicated by the vehicle mass, the suspension of the vehicle and the road surface roughness.

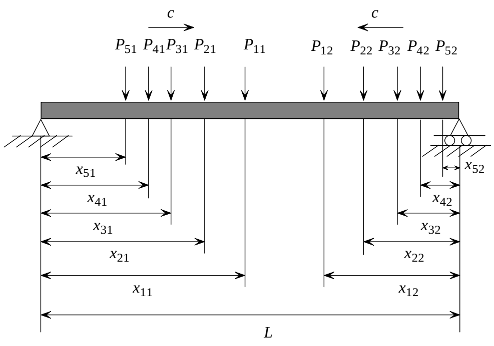


(a) – Measured Amplifications

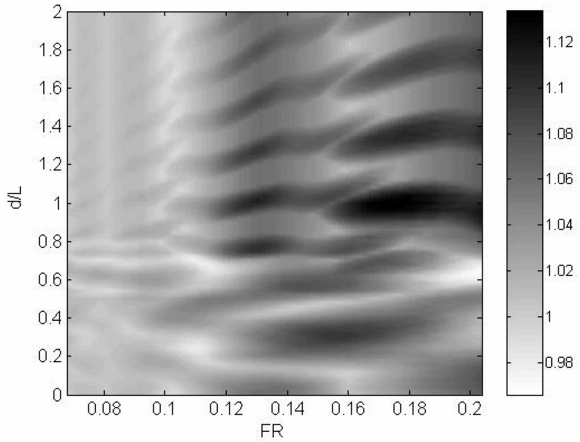
(b) – The vehicle

Fig. 5 Measurement of Dynamic Amplification Factor at Range of Speeds

Slightly more elaborate theoretical models can represent typical 5-axle trucks with five point forces. In such a case, the relative meeting point, represented in Fig. 6 as d/L, as well as the Frequency Ratio, the ratio of the truck speed to the bridge natural frequency, are important.



(a) Representation of two 5-axle trucks



(b) Contours of Dyanamic Amplification Factor against Frequency Ratio (FR) and Relative Position of trucks (d/L)

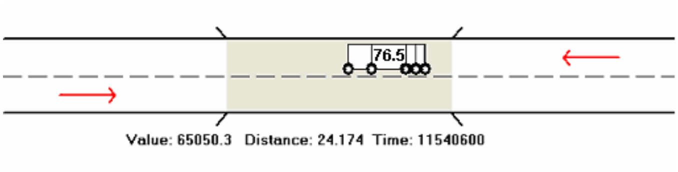
Fig. 6 Dynamic amplification factor for two 5P-load trucks on 25 m bridge with 5% damping

While these models are simple, they are useful in identifying the nature of dynamic amplification. When trucks are more than half the bridge span apart – d/L ≥ 0.5, there are a series of peaks in dynamic amplification. These correspond to critical combinations of speed and meeting point on the bridge. However, the more interesting issue is when trucks are closer together as it is this that constitutes the critical load case. Here there is one major peak – around FR = 0.16 in the figure. For most short to medium span bridges, this maximum amplification corresponds to highly improbable truck speeds – about 300 kph. It would be inappropriate to use this dynamic amplification in an assessment. It becomes clear that the appropriate dynamic amplification factor to be used for bridge assessment can only be found through a probabilistic analysis.

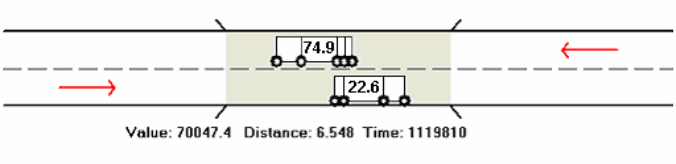
For the 32 m long Mura River test bridge in Slovenia, the dynamic amplification factor has been calculated using a probabilistic approach. Using measured statistics of truck weights and frequencies, Monte Carlo simulation is used to generate 10 years of typical bi-directional, free-flowing traffic data and this traffic is passed over the influence line for an edge beam to determine the load effects that result. Each year of simulation is broken into ‘months’ of 25 working days each and there are thus 10 such months in each year of simulation (allowing for weekends and about 10 national holidays). As a basis for further analysis, the events corresponding to monthly-maximum static load effect are retained. This is done to minimize the number of events that are to be dynamically analysed, as well as providing a shorter ‘extrapolation distance’.

Of the 100 monthly-maximum events, 20 were found to be 1-truck events, 77 to be 2-truck events and 3 were 3-truck events. Fig. 7 illustrates some examples of the monthly-maximum events; heavier trucks are in Lane 1 (top lane) as the edge beam considered is under this lane.

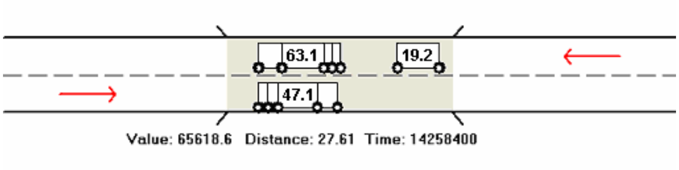
The 100 monthly-maximum loading events obtained from the simulations are analysed using 3-Dimensional Finite Element bridge-truck interaction models developed by González [32] and Rattigan et al [33] – Fig. 8 – and validated experimentally [34]. The truck models used rigid bodies supported by suspension and tyre systems. The trailer and tractor masses in the trucks are modelled as point loads distributed throughout the frame by rigid elements. The suspensions and tyres are modelled as spring dashpot systems.



(a) Loading Event 1



(b) Loading Event 2



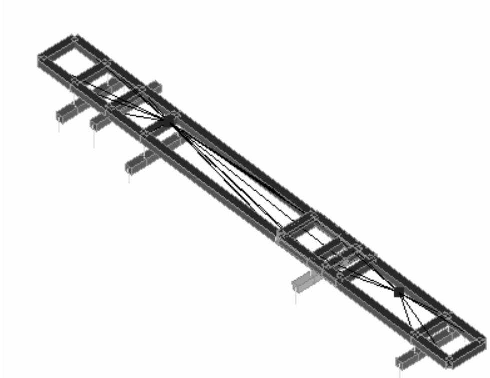
(c) Loading Event 3

Fig. 7 Monthly Maximum Loading Events Found by Simulation

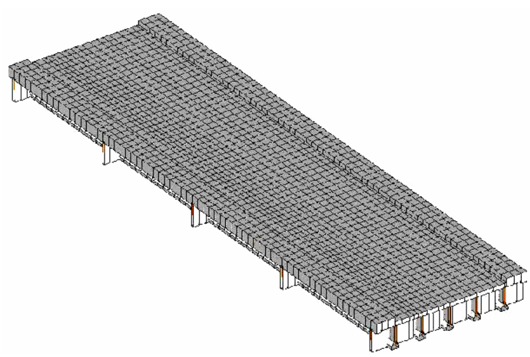
The mechanical characteristics (suspension and tyre properties) of the rigid 2-axle and 3-axle configuration truck models are based on parameters given by [35] and [36]. The mechanical properties of the articulated truck models are based on values proposed by [37] and are kept constant throughout.

The end result of these bridge-truck interaction simulations is a population of 100 monthly-extreme loading events for which both static and total (static + dynamic) load effects are known. It is acknowledged that only the static load effects are maximum-per-month and it is possible that other events that are below the maximum statically could result in a greater total load effect. It is also acknowledged that variations in truck mechanical properties will influence the total load effect.

The static and total load effect data is fitted using the Gumbel logistic bivariate extreme value statistical distribution. Fig. 9 shows a contour plot of the bivariate probability density function which best fits the simulated data.



(a) Dynamic Model of 5-Axle Articulated Truck



(b) Finite Element Model of Bridge Deck

Fig. 8 Model of Bridge-Truck Dynamic Interaction on Mura River Bridge

It can be seen that the static and total stresses are strongly correlated, as would be expected – high static increases the probability of high total and vice versa. The general trend in the data is to the right of the 45^o line. This tells us that total stress is generally (though not always) greater than the static stress. In other words, dynamic amplification is generally greater than unity. What is critical in the graph however, is not the general trend but the curvature in the contours – the amount by which total stress exceeds static, i.e., the dynamic amplification, tends to get less as stress increases. This is critically important in a statistical extrapolation to determine dynamic amplification appropriate to extreme stresses.

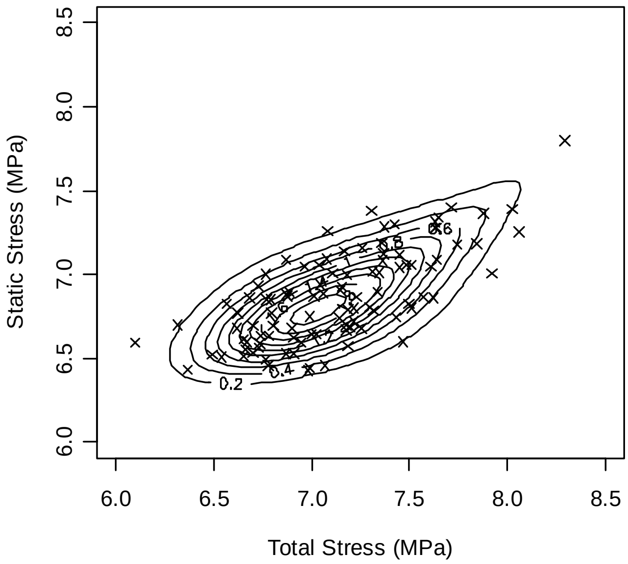


Fig. 9 Contours of Probability Density Function which Best Fits Results

To estimate the distribution of the 100-year lifetime load effect, a parametric bootstrapping approach is used [38]. The 100-year lifetime of the bridge is simulated from the fitted bivariate statistical distribution of monthly maxima. To do this, 1000 synthetic monthly-maximum events (100 years with ten 25-day months per year) are simulated from the fitted model illustrated above. The monthly maxima illustrated above are reproduced in Fig. 10 together with the corresponding 100-year lifetime maxima. The curvature in the contours of Fig. 9 results in a reduction of the dynamic effect as the trend is extrapolated. It is clear that there is dependence between the static and total maximum-in-lifetime load effect values – even though they are not related through individual loading events which is a result of the dependence in the parent distributions.

The characteristic loading is defined in the Eurocode [30] as being that value which is expected to be exceeded with a probability of 10% in the 100 year lifetime. This corresponds to the top 10% of the maximum-in-lifetime points in Fig. 10. The top 10% total stress level divided by the top 10% static stress level gives the dynamic amplification that should be used for bridge assessment. For this particular study, the ratio was found to be 5.8%.

CONCLUSIONS

This paper reviews recent work completed in University College Dublin on bridge traffic loading and the dynamic interaction between moving traffic and a bridge. It is shown that loading event type is an important feature of traffic loading. For example, 4-truck loading events, although very rare, have the potential to cause very high load effects in medium-span bridges and can govern a bridge load assessment. It follows that gaps between vehicles are important as the frequency of small inter-truck gaps has a strong influence on the frequency of 3- and 4-truck loading events. When taken together, an accurate allowance for statistical measurements of gap and a separation of loading events according to the number of trucks involved, makes a most significant difference to the calculated characteristic load effect.

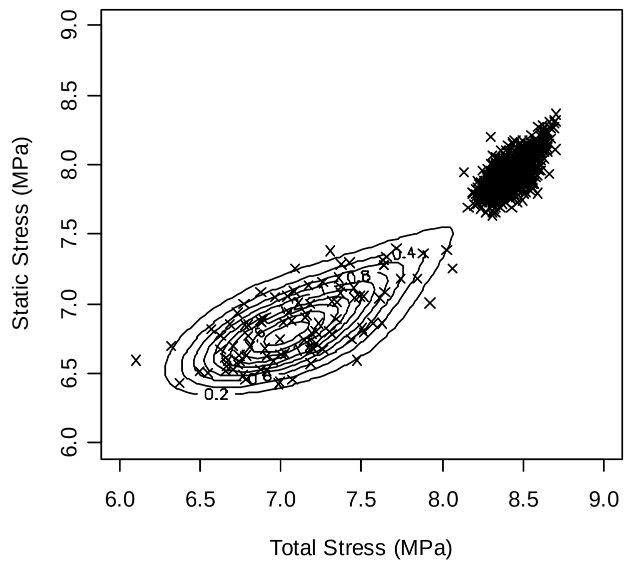


Fig. 10 Monthly Maximum Stresses (lower left) and Corresponding 100-year Lifetime Maxima (upper right)

These factors must be taken into account in any site-specific assessment of bridge traffic loading.

Bridge and vehicle dynamics are also an important issue for load effect assessment in short to medium span bridges. This is a major source of conservatism in current assessments with allowances in excess of 40% being typical for 2-span bridges. Studies are reported which highly the influence of truck speed and relative location on the bridge. A case study of a bridge in Slovenia is used to demonstrate how a complete probabilistic analysis can be used to calculate a dynamic allowance. For the example considered, this is significantly less than 40%.

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