

2007-01-01

## Thermal control of an occupied room: investigating the use of carbon dioxide concentration as an auxiliary control variable

Kevin O'Connell

*Technological University Dublin, Kevin.OConnell@tudublin.ie*

Aidan O'Dwyer

*Technological University Dublin, aidan.odwyer@tudublin.ie*

Follow this and additional works at: <https://arrow.tudublin.ie/engscheleart>



Part of the [Energy Systems Commons](#)

---

### Recommended Citation

O'Connell, Kevin and O'Dwyer, Aidan : Thermal control of an occupied room: investigating the use of carbon dioxide concentration as an auxiliary control variable. Proceedings of IMC-24; the 24th International Manufacturing Conference, pp. 77-85, Waterford Institute of Technology, August,2007. doi:10.21427/25nb-1h98

This Conference Paper is brought to you for free and open access by the School of Electrical and Electronic Engineering at ARROW@TU Dublin. It has been accepted for inclusion in Conference papers by an authorized administrator of ARROW@TU Dublin. For more information, please contact [arrow.admin@tudublin.ie](mailto:arrow.admin@tudublin.ie), [aisling.coyne@tudublin.ie](mailto:aisling.coyne@tudublin.ie).



This work is licensed under a [Creative Commons Attribution-NonCommercial-Share Alike 4.0 License](#)

## **THERMAL CONTROL OF AN OCCUPIED ROOM: INVESTIGATING THE USE OF CARBON DIOXIDE CONCENTRATION AS AN AUXILIARY CONTROL VARIABLE**

K. O'Connell and A. O'Dwyer \*,  
School of Control Systems and Electrical Engineering,  
Dublin Institute of Technology, Kevin St., Dublin 8.  
\* Principal author of paper. E-mail: aidan.odwyer@dit.ie

### **ABSTRACT**

This paper details the theoretical and experimental work done in the examination of the relationship between human generated carbon dioxide (CO<sub>2</sub>) concentration and temperature rise in an occupied room. Generation of CO<sub>2</sub> by humans is linked to metabolic rate and heat generation. Therefore, CO<sub>2</sub> concentration may be used as an indicator of occupancy in a room. Typically, a heating control system will react slowly to changes in room occupancy if temperature is used as the process variable. The aim of the work was to establish if CO<sub>2</sub> concentration in the space could be used as an auxiliary control variable so as to allow a more rapid temperature control system response. Models of the relationship between room air temperature and occupancy, and CO<sub>2</sub> concentration and occupancy are mathematically developed and experimentally validated. The paper concludes that CO<sub>2</sub> concentration is an appropriate auxiliary control variable for temperature control when the room, having been occupied, is vacated.

**KEYWORDS:** Temperature control, buildings, auxiliary control variable.

### **1. INTRODUCTION**

A room that is suddenly occupied or vacated will experience a sudden change in sensible heat gain. The sensible gain from occupancy can vary from 65 to 220 watts per person (for an adult male) depending upon metabolic rate [1]. Temperature control in such occupied spaces is usually implemented by a control system that uses a temperature sensor located in the space. The controller compares the signal from the temperature sensor with the set point and implements corrective action. In the case of an all-air system, the temperature or volume of the supply air is modulated to change the desired space temperature. In situations where the thermal load is subject to rapid change, the system lags can cause a considerable delay before corrective action is taken by the control system.

This paper reports on the investigation of the use of CO<sub>2</sub> concentration as an auxiliary control variable. Section 2 details the mathematical relationships between room temperature and occupancy, and carbon dioxide concentration and occupancy, for a small lecture theatre with a number of sedentary occupants. Both models are first order in nature. Subsequently, in Section 3, the models are validated. The lecture theatre is fitted with both a temperature and carbon dioxide sensor and data is collected over a three-week period. Detailed attention is given to measurement issues. Data is adjusted to take account of external factors such as heat loss due to the room fabric. Conclusions are reached in Section 4.

## 2. MATHEMATICAL RELATIONSHIPS

### 2.1 The room

Room K43 at DIT Kevin St. was chosen to explore the dynamic relationship between heat gain and CO<sub>2</sub> concentration. This room is located on the ground floor of a 4-story over basement building. The room has a capacity for 54 persons, though normal occupancy is 20 persons. Room furnishings consist of laminate wood finish on the desk units and cloth upholstery covering the seating. The room contains twelve 58-watt fluorescent lights. Allowing a factor of 1.15 for ballast losses, the heat gain to the room from this source is 800 W. The space heating for the room is supplied by hot air from a fan coil unit located in the adjoining room. An on/off thermostat located in the return air duct thermostatically controls the fan coil unit output. The dimensions and layout of the room are shown in Figure 1.

The external walls of the room are composed of two-leaf 100-mm solid concrete blocks forming an 80-mm cavity. The wall internal surface is finished with 20-mm insulating plaster and the external surface is finished with 20-mm concrete plaster. The wall cavity contains a thickness of 50-mm expanded polystyrene insulation. The internal wall is composed of single-leaf 150-mm solid concrete blocks plastered on both sides with finishing plaster. The floor consists of 50-mm heavy duty PVC covering laid on a prefabricated reinforced concrete slab. A 500-mm suspended ceiling is fitted with 150-mm fibreglass insulation on top of proprietary acoustic tiling. The structural ceiling is pre-cast reinforced concrete; however, it is thermally insulated from the room air. There is aluminium frame 6-mm single glazing. Each glazing section contains two large openings. The total glazed area = 20 m<sup>2</sup>.

The room construction can be characterised as 'heavyweight', with the furnishing and ceilings being 'lightweight' [2]. When a change in heat is suddenly input to the room, it is expected that the temperature of the heavyweight surfaces will remain unchanged in the short term. It is expected that the temperature of the lightweight surfaces will rise, and these surfaces will not be expected to accept a significant amount of heat from the room air because of their small thermal mass.

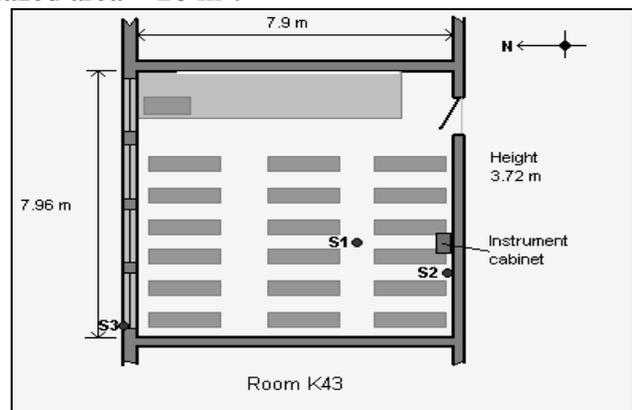


Figure 1: Dimensions and layout of the room

### 2.2 Relationship between room temperature and occupancy

A mathematical relationship between room temperature and occupancy was developed for room K43, which is now outlined; this is based on the work of Underwood [3].

Firstly, the air change rate in the room is measured by introducing a tracer gas and recording its rate of decay. CO<sub>2</sub> was used as a tracer gas, introduced using a fire extinguisher. The decay rate is measured relative to the ambient CO<sub>2</sub> levels. To ensure complete dispersion of the tracer gas in the room, the gas was slowly introduced and agitated over a period of 5 minutes. From the resulting decay curve, the air change rate is measured as approximately 1.15 air changes per hour = 0.0191 air changes per minute = 0.00032 air changes per second (or 0.074 m<sup>3</sup>/s).

Now, from Underwood [3], the rate of change of energy stored in the room air = (Heat gain) minus (Transmission loss to Room) minus (Infiltration loss to room). In differential equation format:

$$V_r \rho_a C_{pa} \frac{d\theta_r}{dt} = q_{plant} + q_{gain} + q_{occ} - A \bar{U}_f (\theta_r - \bar{\theta}_f) - 0.33 h_r V_r (\theta_r - \theta_o) \quad (1),$$

where  $V_r$  = room volume ( $m^3$ ),  $\rho_a$  = density of the air ( $kg/m^3$ ),  $C_{pa}$  = specific heat capacity of the air ( $J/kgK$ ),  $\theta_r$  = room air temperature ( $^{\circ}C$ ),  $q_{plant}$  = energy input from heater (W),  $q_{gain}$  = heat gain from lighting and other equipment (W),  $q_{occ}$  = room occupancy heat gain (W),  $A$  = area of the room fabric ( $m^2$ ),  $\bar{U}_f$  = weighted mean figure for the fabric heat transfer coefficient ( $W/m^2K$ ),  $\bar{\theta}_f$  = mean surface temperature of the room fabric ( $^{\circ}C$ ),  $h_r$  = air change rate ( $m^3/s$ ),  $V_r$  = room volume ( $m^3$ ) and  $\theta_o$  = external temperature ( $^{\circ}C$ ). In the application,  $q_{plant}$  and  $q_{gain}$  are considered fixed. The windows are north facing and therefore any solar gains may be neglected.

Expressing equation (1) in terms of deviation variables rather than in terms of absolute value variables, setting all initial conditions to zero, and translating into the Laplace domain gives, after some development:

$$\frac{\theta_r(s)}{q_{occ}(s)} = \frac{K_{ml}}{1 + T_{ml}s}, \text{ with } K_{ml} = \frac{1}{A \bar{U}_f + 0.33 h_r V_r} \text{ and } T_{ml} = \frac{V_r \rho_a C_{pa}}{A \bar{U}_f + 0.33 h_r V_r} \quad (2)$$

It is assumed that the room is occupied by  $n$  sedentary persons, each with a metabolic rate of 140 W (90 W sensible heat gain, 50 W latent heat gain); the sensible heat gain contributes to a temperature rise in the space, with the latent heat gain contributing to a rise in air moisture content [1]. Then,  $q_{occ} = 90n$  W. Detailed work reveals that  $V_r = 234 m^3$ ,  $A = 224 m^2$  and  $\bar{U}_f = 8.0 W/m^2K$ . Then, from equation (2),  $K_{ml} = 5.56 \cdot 10^{-4} \text{ }^{\circ}C/W$  and  $T_{ml} = 159s = 2.65$  minutes, assuming  $\rho_a = 1.2 kg/m^3$  and  $C_{pa} = 1020 J/kgK$ . Thus, the maximum predicted temperature rise, due to room occupancy, from this model =  $90n(5.56 \cdot 10^{-4}) = 1^{\circ}C$ , if  $n = 20$ .

Space does not permit the development of further theoretical work showing the relationship between room air temperature and outside ambient temperature, and room air temperature and room wall temperature. This work will be discussed in detail in the presentation at the conference. An outline of the results is provided. A first order model is developed for the relationship between room air temperature and outside ambient temperature; the time constant of the model is 2.5 minutes (i.e. it is similar to  $T_{ml}$  above). For static conditions, the change in room air temperature, for a sudden change  $\Delta T$  in the outside ambient temperature, is theoretically determined to be  $0.1(\Delta T) \text{ }^{\circ}C$ . A first order model is also developed for the relationship between room air temperature and room wall temperature, for the case when the wall temperature is greater than the room temperature. The time constant of this model is 180 minutes; such a large time constant corresponds to the view of Underwood [3] that the room fabric does not influence the dynamic thermal responses for periods shorter than one hour. In conclusion, the theoretical development shows that room air temperature is influenced by occupancy rate and outside air temperature, in steady state, according to the ratio  $0.05n:0.1(\Delta T)$ , respectively. Thus, each change of  $1^{\circ}C$  in the ambient temperature changes the room temperature by an amount equal to that contributed by 2 extra persons in the room.

### 2.3 Relationship between carbon dioxide concentration and occupancy

Carbon dioxide (CO<sub>2</sub>) is a by-product of metabolism. The normal urban atmospheric concentration of carbon dioxide varies between 375 and 450 parts per million (p.p.m.) [4]. CO<sub>2</sub> rapidly diffuses in the atmosphere and has a seasonal variation of  $\pm 25$  p.p.m. The normal ambient concentration of CO<sub>2</sub> is taken as 400 p.p.m. From a ventilation perspective, 2000 p.p.m. is considered the upper limit to preserve comfort conditions because the presence of this level of CO<sub>2</sub> also indicates the presence of undesirable odours generated by human bioeffluents. In human metabolism, carbon and hydrogen in foods are oxidised to CO<sub>2</sub> and water, which are eliminated by the body as waste products. The lungs extract oxygen from inspired air and this oxygen is absorbed by the blood, which carry it to muscle cells; the cells release CO<sub>2</sub>, which is carried back to the lungs and expired. Inspired fresh air contains approximately 0.035% CO<sub>2</sub>, while expired air contains approximately 4.0% CO<sub>2</sub> [5].

The rate of generation of CO<sub>2</sub> is a function of metabolic rate. The rate of CO<sub>2</sub> generation is defined by [4] to be 0.00004M l/s (M = metabolic rate in watts) i.e. if M = 140 W for adult males in sedentary occupations, at an ambient temperature of 22 °C [1], the rate of CO<sub>2</sub> generation is 0.0056 l/s. Separately, the rate of CO<sub>2</sub> generation is defined by Smith [6] to be  $2.3376 \cdot 10^{-2} \text{ m}^3/\text{hr} = 0.0065 \text{ l/s}$ , for adults in classroom conditions. In this work, the rate of CO<sub>2</sub> generation is assumed to be 0.0061 l/s (the average of 0.0056 l/s and 0.0065 l/s).

The CO<sub>2</sub> concentration balance for the room air is given by the following linear first order differential equation:

$$Xdt = V_r dL + Lh_r V_r dt,$$

where CO<sub>2</sub> gain = X (m<sup>3</sup>s<sup>-1</sup>), CO<sub>2</sub> concentration = L (m<sup>3</sup>/m<sup>3</sup>), with h<sub>r</sub> = 0.00032 air changes per second. Expressing this equation in terms of deviation variables, setting all initial values to zero and transforming to the Laplace domain yields, after some development:

$$\frac{L(s)}{X(s)} = \frac{K_{m2}}{1 + T_{m2}s}, \text{ with } K_{m2} = \frac{1}{V_r h_r} \text{ and } T_{m2} = \frac{1}{h_r} \quad (3)$$

It is assumed that the room is occupied by  $n$  sedentary persons (as before). The rate of CO<sub>2</sub> generation for  $n$  persons is thus 0.0061 $n$  l/s or 0.0000061 $n$  m<sup>3</sup>/sec. Now, V<sub>r</sub> = 234 m<sup>3</sup> and h<sub>r</sub> = 0.00032s<sup>-1</sup>. Then, from (3), K<sub>m2</sub> = 13.36 and T<sub>m2</sub> = 3125s  $\approx$  52 minutes. Thus, maximum predicted CO<sub>2</sub> concentration rise from this model = 0.0000061 $n$ (13.36) = 0.00163 = 1630 p.p.m., if  $n = 20$ .

## 3. MODEL VALIDATION

### 3.1 Introduction

In preliminary validation work, a hand-held instrument was used to take measurements of CO<sub>2</sub> concentration, which showed that, initially, CO<sub>2</sub> concentration was localised about individual persons and appeared to follow convection air currents. Within a period of 15 minutes, the increased levels of CO<sub>2</sub> concentration could be measured in every corner of the room, with uniformity being observed. Thus, it was decided that the CO<sub>2</sub> sensor should be mounted approximately 2 m from the floor, in a ventilated instrument cabinet (Figure 1). A temperature sensor was mounted 2 m from the floor (labelled S2 in Figure 1). Two other air temperature

sensors were used, one suspended 0.6 m from the ceiling (labelled S1 in Figure 1); this sensor allowed measurement of the room air temperature independent of the room wall thermal dynamics. The final temperature sensor was mounted external to the building on a windowsill (labelled S3 in Figure 1); this sensor was shielded from direct sunlight.

The data acquisition equipment used in the work was capable of providing simultaneous readings of CO<sub>2</sub> concentration and temperature from a number of air temperature sensors. The data collected was automatically logged to facilitate subsequent analysis. Data logging, recording and viewing occurred at a remote site to the data acquisition location, so that the equipment is secure and the experiment does not intrude on the live classroom situation. Precision thermistor temperature sensors and a non-dispersive infrared CO<sub>2</sub> concentration sensor were used. Further details of the equipment used will be provided at the conference.

### 3.2 Validating the room thermal model

Models were developed from eleven separate sets of temperature data, recorded on three separate days. Data was recorded for different numbers of people entering the room and leaving the room. For cross-referencing, data was also gathered when 2kW of heater power was switched on in the room, and when it was subsequently switched off. Empirical fitting of the data with a first order model revealed that the average value of gain,  $K_{m1}$ , was  $1.2 \cdot 10^{-3} \text{ } ^\circ\text{C/W}$ , with a range of  $1.0 \cdot 10^{-3} \text{ } ^\circ\text{C/W}$  to  $1.4 \cdot 10^{-3} \text{ } ^\circ\text{C/W}$ . The average value of the time constant,  $T_{m1}$ , was 22 minutes, with a range of 17 to 32 minutes. Sample results are provided in Figures 2 and 3. For Figure 2,  $K_{m1} = 1.06 \cdot 10^{-3} \text{ } ^\circ\text{C/W}$ ,  $T_{m1} = 30$  minutes; for Figure 3,  $K_{m1} = 0.99 \cdot 10^{-3} \text{ } ^\circ\text{C/W}$ ,  $T_{m1} = 22$  minutes.

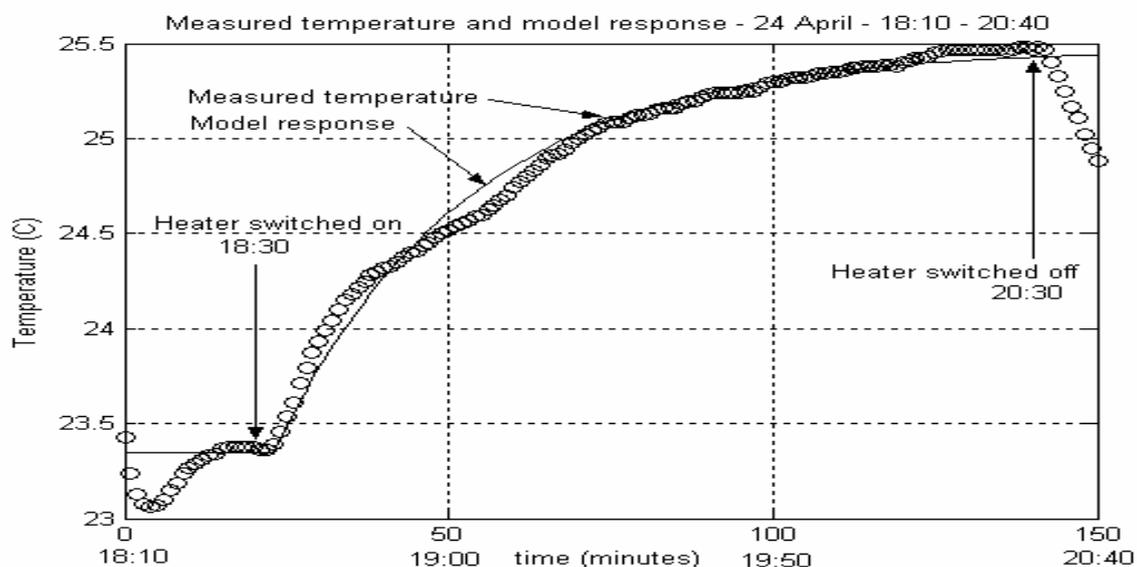


Figure 2: 24 April 2002, 18:10–20:40. A step input of 2 kW in heater power was introduced.

The average values of  $K_{m1}$  and  $T_{m1}$  were both greater than that predicted from theory in Section 2.2; the wall surface temperature recorded increased from ambient during all the experiments. Going back to equation (2),  $\bar{U}_f$  was back-calculated to be  $3.7 \text{ W/m}^2\text{K}$  from the average value of  $K_{m1}$  determined empirically; this is approximately a halving in the weighted mean figure for the fabric heat transfer coefficient. This new value of  $\bar{U}_f$  incorporates the effect of the increase

in the wall surface temperature recorded. Similarly,  $C_{pa}$  can be back-calculated to be 3900 J/kgK from the average value of  $T_{m1}$  determined, which is an increase of a factor of approximately 3.8 in the thermal capacity compared to that used in the theoretical modelling work; this increase reflects the effect of the thermal capacity of the fabric, which was not incorporated in the theoretical model.

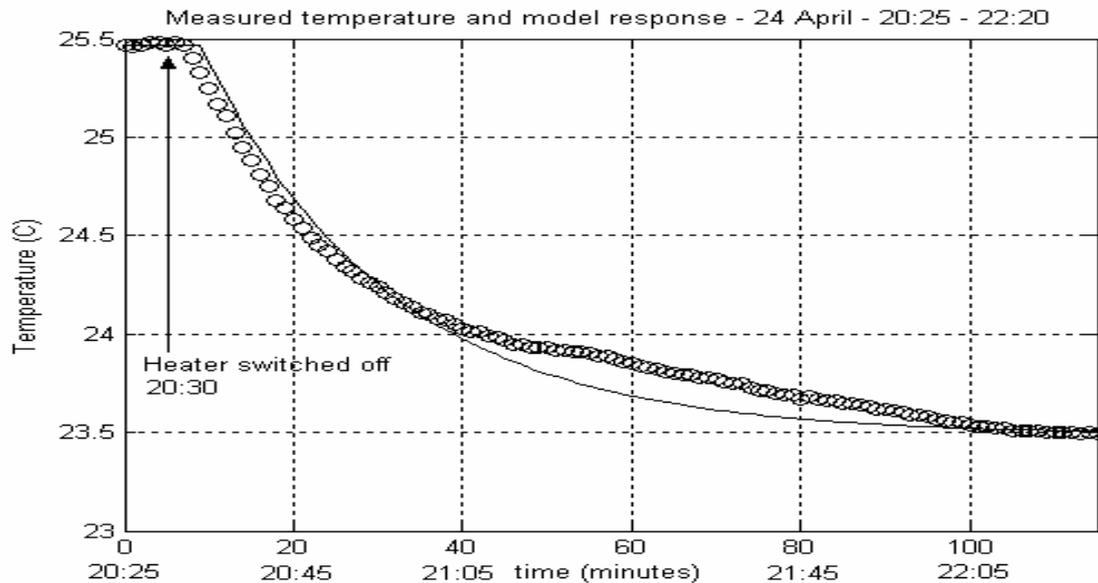


Figure 3: 24 April 2002, 20:25–22:20. The 2 kW heater was switched off.

The updating of  $\bar{U}_f$  and  $C_{pa}$  also affects the transfer function relationship between room air temperature and outside ambient temperature. Calculations show that the revised time constant is 20 minutes (i.e. it is similar to the occupancy model time constant). The revised steady state change in the room air temperature, for a change  $\Delta T$  in the outside ambient temperature, is now  $0.15(\Delta T)$  °C; with these revised figures, each change of 1°C in the ambient temperature changes the room temperature by an amount equal to that contributed by the equivalent of 1.4 extra persons in the room.

### 3.3 Validating the room CO<sub>2</sub> model

Models were developed from thirteen separate sets of CO<sub>2</sub> concentration data, recorded on three separate days. Data was recorded for different numbers of people entering the room and leaving the room. Interestingly, a model for carbon dioxide concentration response may be obtained under a wider variety of conditions than can a model for air temperature response. This was particularly noticeable in experimental situations where it is desired to obtain an appropriate model when people leave a room. Empirical fitting of the data with a first order model revealed that the average value of gain,  $K_{m2}$ , was 9.5 [concentration (ppm.10<sup>-6</sup>)/CO<sub>2</sub> generation (m<sup>3</sup>s<sup>-1</sup>)], with a numerical range of 6.1 to 14.0. The average value of the time constant,  $T_{m2}$ , when people enter the room was 29 minutes; the time constant varied from 19 to 38 minutes. The average value of the time constant for people leaving the room was 7.7 minutes, with a variation of 1.6 minutes to 12.5 minutes. Sample results are provided in Figures 4 and 5. For Figure 4,

$K_{m_2} = 8.6$  [concentration (ppm. $10^{-6}$ )/CO<sub>2</sub> generation (m<sup>3</sup>s<sup>-1</sup>)],  $T_{m_2} = 19.5$  minutes; for Figure 5,  $K_{m_2} = 6.66$  [concentration (ppm. $10^{-6}$ )/CO<sub>2</sub> generation (m<sup>3</sup>s<sup>-1</sup>)],  $T_{m_2} = 8.3$  minutes.

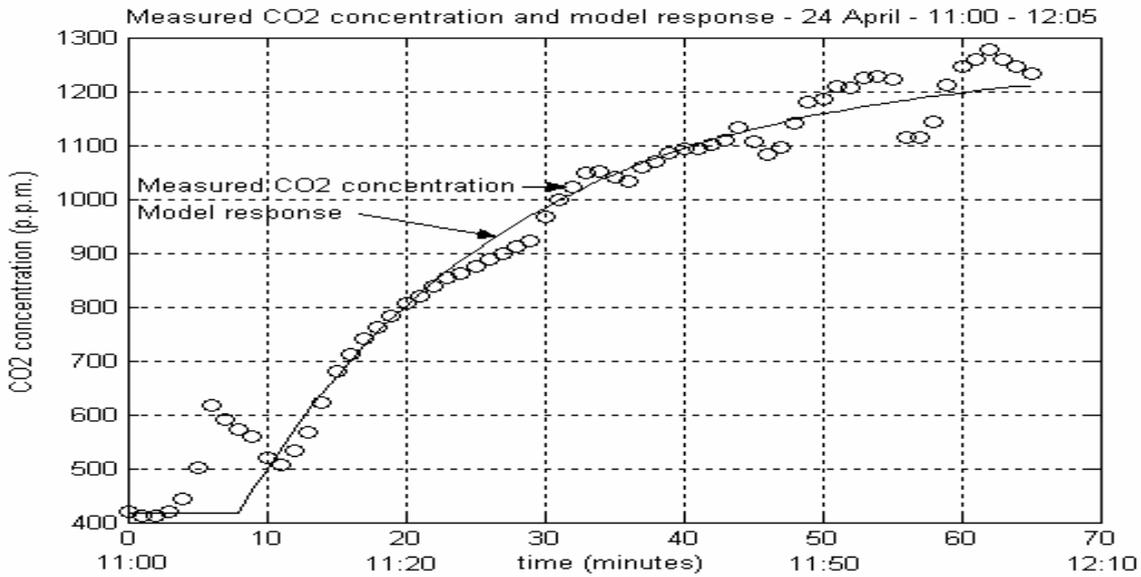


Figure 4: CO<sub>2</sub> concentration, 24 April 2002, 11:00-12:05. Occupancy: 16 people enter the room at approximately 11:03 and leave the room at approximately 12:04.

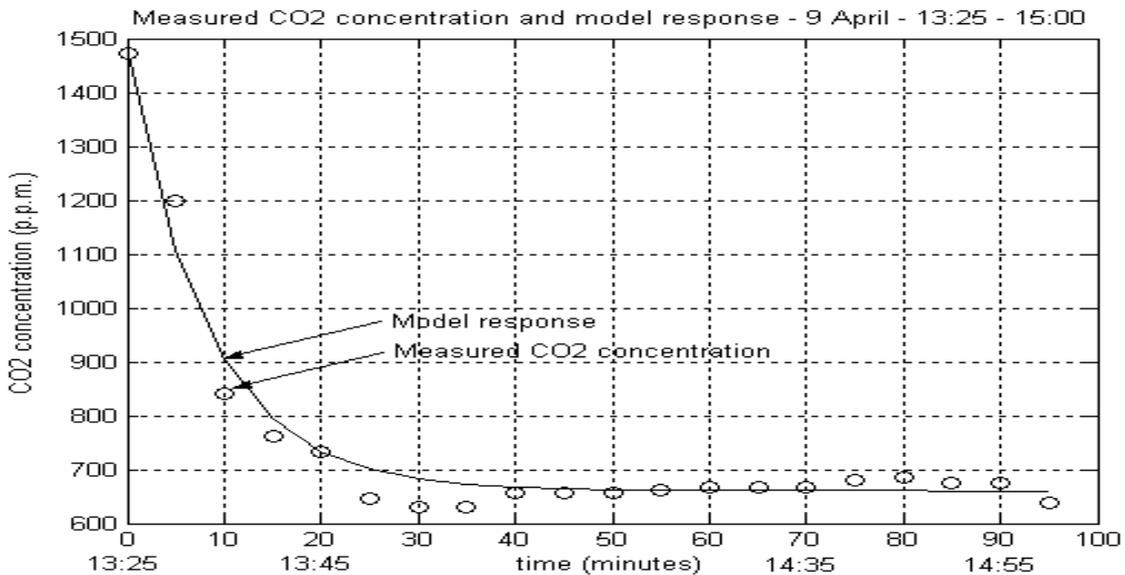


Figure 5: CO<sub>2</sub> concentration, 9 April 2002, 13:25-15:00. Occupancy: 20 people leave the room at 13:25 (approximately).

The average values of  $K_{m_2}$  and  $T_{m_2}$  were both greater than that predicted from theory in Section 2.3. Going back to equation (3),  $K_{m_2}$  may be empirically determined to be  $\alpha/V_r h_r$ ,  $\alpha = 0.7$  on average, with  $T_{m_2}$  empirically determined to be  $\beta/h_r$ ,  $\beta = 0.55$  on average (for persons entering the room), with  $\beta = 0.15$  on average (for persons leaving the room).

Overall, the time constant is significantly smaller when people leave the room, compared to when people enter the room. This is because when people enter the room, the door to the corridor is closed, and the CO<sub>2</sub> concentration builds gradually; when people leave the room, the door to the corridor is left open, and thus the CO<sub>2</sub> concentration falls quickly. The time constant of the air temperature response is, on average, 22 minutes, and does not vary unduly whether people enter or leave the room. Detailed analysis of the results reveals that the time constant of the CO<sub>2</sub> concentration response is always greater than the time constant of the air temperature response, when people enter the room. Thus, CO<sub>2</sub> concentration should not be used as an auxiliary variable in these circumstances. On the other hand, further detailed analysis reveals that the time constant of the CO<sub>2</sub> concentration response when people leave the room is between 3.9 and 11.9 times faster than the time constant of the air temperature response. Thus, CO<sub>2</sub> concentration can be used as an auxiliary variable to detect when people leave the room.

#### 4. CONCLUSIONS

The aim of the work was to examine the relationship between human CO<sub>2</sub> generation rate and temperature rise, in an enclosed space, and to establish if the CO<sub>2</sub> concentration in the space can be used as an auxiliary control variable to give an early indication of a change in temperature. It is concluded that CO<sub>2</sub> concentration can be used as an auxiliary control variable for temperature control when the room, having been occupied, is vacated. The paper also developed models for the relationship between room air temperature and occupancy, and room CO<sub>2</sub> concentration and occupancy; such models are useful in controller design and should allow the specification of improved closed loop control systems. The linear models developed are first order in nature; the parameters of the room CO<sub>2</sub> concentration model, in particular, vary significantly, suggesting that further work should concentrate on the development of an appropriate nonlinear model. In addition, some of the experimental results do show higher order effects, which could be incorporated in the theoretical model [3]. Interestingly, the data from the wall mounted temperature sensor clearly indicates a significant change in the heavyweight fabric temperatures over the test periods, which was unexpected.

Finally, measured CO<sub>2</sub> concentration is independent of factors which influence room temperature, such as changes in the ambient air temperature; thus, variations in measured CO<sub>2</sub> concentration could be used in a simple manner to detect the presence or absence of persons in an enclosed space.

#### REFERENCES

- [1] Environmental design - CIBSE Guide A (1999). Sixth Edition, Table 6.1, p. 6-2.
- [2] Environmental design - CIBSE Guide A (1999). Sixth Edition, p. 5-13.
- [3] C.P. Underwood, *HVAC Control Systems: Modelling Analysis and Design*, E.& F.N. Spon (1999), pp. 78-116.
- [4] M. Schell and D. Int-hout, Demand Control Ventilation using CO<sub>2</sub>, *ASHRAE Journal* (February 2001), 18-29.
- [5] British Standard 5925:1991. *Ventilation principles and designing for natural ventilation*.
- [6] P.N. Smith, Determination of Ventilation Rates in Occupied Buildings from Metabolic CO<sub>2</sub> Concentrations and Production Rates, *Building and Environment*, 23(2), (1988), 95-102.