Comparison of Finite Difference and Finite Element (Telemac) Models of Dublin Bay

Garrett Keane  
*Technological University Dublin*, garrett.keane@tudublin.ie

Zeinab Bedri  
*University College Dublin*

John O'Sullivan  
*University College Dublin*

Follow this and additional works at: https://arrow.tudublin.ie/engineducpres

Part of the Bioresource and Agricultural Engineering Commons

**Recommended Citation**


This Other is brought to you for free and open access by the Engineering: Education and Innovation at ARROW@TU Dublin. It has been accepted for inclusion in Presentations by an authorized administrator of ARROW@TU Dublin. For more information, please contact yvonne.desmond@tudublin.ie, arrow.admin@tudublin.ie, brian.widdis@tudublin.ie.

This work is licensed under a Creative Commons Attribution-Noncommercial-Share Alike 3.0 License
Comparison of Finite Difference (SW2D) and Finite Element (Telemac) Models of Dublin Bay

Garrett Keane\textsuperscript{1,2}, Zeinab Bedri\textsuperscript{1}, John O’Sullivan\textsuperscript{1}, Michael Bruen\textsuperscript{1}

\textsuperscript{1}School of Civil, Structural and Environmental Engineering, UCD
\textsuperscript{2}School of Civil and Structural Engineering, DIT
Context of Study

- **CWRR, UCD** (Centre for Water Resources Research)
- **Dublin Bay** – Hydrodynamic/Water Quality Models
- **Qiang (93)** - Dublin Bay Water Quality Management Plan
- **Hussey (96)** – SW2D - Extended Area
  - Eulerian-Langrangian 2D Finite Difference Model
  - Dublin Bay Water Quality Management Plan Study
  - Howth Outfall Study
- **Bedri (07)**
  - 3D Hydrodynamic/Water Quality Telemac Model of Inner Bay
SW2D Model Domain

Model Domain
-6° 15’ to -5° 50’ E-W
53° 10’ to 53° 30’ N-S
over 72000 grid boxes

Bathymetry
Admiralty Charts 1447 & 1468
Surveys for Various Studies
- Irish Hydrodata Ltd - Howth
- BKS – Tolka Mudflats
- ESB International – Bull Island

Boundary Conditions
North and South - Elevations for Spring and Neap Tides
East – “Glass Wall”
Objectives of Study

TELEMAC vs SW2D

- Accuracy
- Stability
- Computational Time
- Ease of use - man hour costs.

Schedule of Simulations

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>SW2D</td>
<td>The original finite difference Extended Dublin Bay model using a grid size of 100.79m by 92.75m.</td>
</tr>
<tr>
<td>T0</td>
<td>The Baseline TELEMAC finite element model with a uniform mesh with a resolution of 104m between the nodes.</td>
</tr>
<tr>
<td>T1</td>
<td>The four meshes used in the Telemac Convergence Study to determine the optimum mesh.</td>
</tr>
<tr>
<td>T1</td>
<td>12985 nodes</td>
</tr>
<tr>
<td>T2</td>
<td>22611 nodes</td>
</tr>
<tr>
<td>T3</td>
<td>31653 nodes</td>
</tr>
<tr>
<td>T4</td>
<td>49381 nodes</td>
</tr>
</tbody>
</table>
Field Measurements - Tides

Tidal Gauges

October 1998

Tidal Constituents

- North Wall Lighthouse
- North Bank Lighthouse
- Kish Bank Lighthouse
- Howth Harbour
- Dun Laoghaire
Field Measurements - Currents

• Spring and Neap Tides

• Locations 1-4
  Environmental Study of Howth
  - Irish Hydrodata (98)

• Locations 5-8
  Environmental Study of Liffey Estuary and Dublin Bay
  - Crisp (76)

Location of Current Meters
Depth Averaged Fluid Equations

Continuity Equation

\[
\frac{\partial \xi}{\partial t} + \frac{\partial HU}{\partial x} + \frac{\partial HV}{\partial x} = 0
\]

Momentum Equation in \(x\)-Direction

\[
\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} - fV + g \frac{\partial \xi}{\partial x} + \gamma U - \frac{\theta \rho_a W^2 \sin \psi}{\rho H} - k \left( \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial x^2} \right) = 0
\]

Momentum Equation in \(y\)-Direction

\[
\frac{\partial V}{\partial t} + U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} + fU + g \frac{\partial \xi}{\partial y} + \gamma V - \frac{\theta \rho_a W^2 \cos \psi}{\rho H} - k \left( \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} \right) = 0
\]
The SW2D - Momentum Equation is given by:

$$\frac{DU}{Dt} = fV - g \frac{\partial \xi}{\partial x} - \gamma U + \frac{\theta \rho_a W^2 \sin \psi}{\rho H} + k \left( \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial x^2} \right)$$

The numerical solution for the $(i+\frac{1}{2}, j)$th grid point is:

$$\frac{U^{n+1}_{i+\frac{1}{2}, j} - U^n_{i+\frac{1}{2}, j}}{\Delta t} = fV^n_{i+\frac{1}{2}, j} - \frac{g \alpha_1}{\Delta x} \left( \frac{\xi_{n+1} - \xi_{n+1}^{i+1, j}}{\xi_{i, j}} \right) - \frac{g(1-\alpha_1)}{\Delta x} \left( \frac{\xi_{n} - \xi_{n}^{i+1, j}}{\xi_{i, j}} \right) +$$

$$k \left( \frac{U^n_{i-\frac{1}{2}, j} - 2U^n_{i, j} + U^n_{i+\frac{1}{2}, j}}{\Delta x^2} + \frac{U^n_{i+\frac{1}{2}, j-1} - 2U^n_{i+\frac{1}{2}, j} + U^n_{i+\frac{1}{2}, j+1}}{\Delta y^2} \right) -$$

$$\frac{\alpha_2 \gamma^n_{i+\frac{1}{2}, j}}{H^n_{i+\frac{1}{2}, j}} U^{n+1}_{i+\frac{1}{2}, j} - \frac{(1-\alpha_2) \gamma^n_{i+\frac{1}{2}, j}}{H^n_{i+\frac{1}{2}, j}} U^n_{i+\frac{1}{2}, j} + \left( \frac{\theta \rho_a W^2 \sin \psi}{\rho H} \right)^n_{i+\frac{1}{2}, j}$$

$$U^{n+1}_{i, j+\frac{1}{2}} = \left( 1 + \frac{\alpha_2 \Delta t \gamma^n_{i+\frac{1}{2}, j}}{H^n_{i+\frac{1}{2}, j}} \right)^{-1} \left[ F U^n_{i+\frac{1}{2}, j} - \frac{g \Delta t \alpha_1}{\Delta x} \left( \frac{\xi_{n+1} - \xi_{n+1}^{i+1, j}}{\xi_{i, j}} \right) \right]$$
SW2D - Continuity Equation

\[
\left( 1 + AAU_{i+\frac{1}{2},j}^n + AAU_{i-\frac{1}{2},j}^n \right. \\
+ AAV_{i,j+\frac{1}{2}}^n + AAV_{i,j-\frac{1}{2}}^n \left. \right) \xi_{i,j}^{n+1} \\
- AAU_{i+\frac{1}{2},j}^n \xi_{i+\frac{1}{2},j}^{n+1} \\
- AAU_{i-\frac{1}{2},j}^n \xi_{i-\frac{1}{2},j}^{n+1} \\
- AAV_{i,j+\frac{1}{2}}^n \xi_{i,j+\frac{1}{2}}^{n+1} \\
- AV_{i,j-\frac{1}{2}}^n \xi_{i,j-\frac{1}{2}}^{n+1} = B_{i,j}^n \xi_{i,j}^n
\]
SW2D - Reverse Particle Tracking

- Euler Method

\[ U^n_{i+\frac{1}{2}-a,j-b} \]

\[ x^{n+\frac{1}{10}} = x^n + U\Delta t_s \]

\[ y^{n+\frac{1}{10}} = y^n + V\Delta t_s \]
Bottom Friction Parameter

\[ \gamma = \frac{\sqrt{U^2 + V^2}}{C_z^2} \]

where \( C_z = \frac{1}{H^6} \cdot \frac{1}{n} \)
Surfer Grid & SW2D Pre-Processor

Bathymetry - Surfer Grid

Excel VBA – Preprocessor
SW2D – Vector Plots

Low Water

Mid-Flood
SW2D – Vector Plots

High Water

Mid-Ebb
## SW2D- Tabulated Results

<table>
<thead>
<tr>
<th></th>
<th>North Wall</th>
<th>North Bank</th>
<th>Dun Laoghaire</th>
<th>Kish Lighthouse</th>
<th>Howth Harbour</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Low Tide (m)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>model</td>
<td>-1.607</td>
<td>-1.575</td>
<td>-1.597</td>
<td>-1.542</td>
<td>-1.750</td>
</tr>
<tr>
<td>measured</td>
<td>-1.720</td>
<td>-1.560</td>
<td>-1.665</td>
<td>-1.530</td>
<td>-1.730</td>
</tr>
<tr>
<td>% diff</td>
<td>6.6%</td>
<td>-1.0%</td>
<td>4.1%</td>
<td>-0.8%</td>
<td>-1.2%</td>
</tr>
<tr>
<td><strong>High Tide (m)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>model</td>
<td>1.890</td>
<td>1.880</td>
<td>1.870</td>
<td>1.800</td>
<td>2.025</td>
</tr>
<tr>
<td>measured</td>
<td>2.060</td>
<td>1.900</td>
<td>1.930</td>
<td>1.900</td>
<td>2.080</td>
</tr>
<tr>
<td>% diff</td>
<td>8.3%</td>
<td>1.1%</td>
<td>3.1%</td>
<td>5.3%</td>
<td>2.6%</td>
</tr>
<tr>
<td><strong>Tidal Range (m)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>model</td>
<td>3.497</td>
<td>3.455</td>
<td>3.467</td>
<td>3.342</td>
<td>3.775</td>
</tr>
<tr>
<td>measured</td>
<td>3.780</td>
<td>3.460</td>
<td>3.595</td>
<td>3.430</td>
<td>3.810</td>
</tr>
<tr>
<td>% diff</td>
<td>7.5%</td>
<td>0.1%</td>
<td>3.6%</td>
<td>2.6%</td>
<td>0.9%</td>
</tr>
<tr>
<td><strong>Time of Low Tide</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>model</td>
<td>09:11</td>
<td>09:20</td>
<td>09:11</td>
<td>09:06</td>
<td>09:05</td>
</tr>
<tr>
<td>measured</td>
<td>09:07</td>
<td>09:06</td>
<td>09:01</td>
<td>09:11</td>
<td>09:45</td>
</tr>
<tr>
<td>diff (mins)</td>
<td>+4</td>
<td>+14</td>
<td>+10</td>
<td>+5</td>
<td>-40</td>
</tr>
<tr>
<td><strong>Time of High Tide</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>measured</td>
<td>15:20</td>
<td>15:16</td>
<td>15:23</td>
<td>15:11</td>
<td>15:56</td>
</tr>
<tr>
<td>diff (mins)</td>
<td>-4</td>
<td>-2</td>
<td>-5</td>
<td>+3</td>
<td>-43</td>
</tr>
</tbody>
</table>
Telemac – Structured Mesh

Prepared with Blue Kenue
Canadian Hydraulics Centre of the National Research Council Canada
A finite element solution is generally considered to be unique if the "entropy" condition is satisfied (Hervouet, 2007). In the Saint-Venant equations, the entropy is equal to the total energy of a column of water written as:

$$E = \frac{hu^2}{2} + g \frac{h^2}{2} + ghZ_f$$

The entropy condition is given by:

$$\frac{\partial E}{\partial t} + div \left[ u \left( E + g \frac{h^2}{2} \right) \right] \leq 0$$

The energy flux into the model domain increases as the mesh is refined at an open boundary. The entropy condition is not satisfied if the energy flux becomes too large, resulting in the possibility of an infinite number of solutions of the Saint-Venant equations and instability.
Telemac – Unstructured Mesh
Tidal Elevations
Measured, SW2D & Telemac

North Wall
North Bank
Howth
Kish Bank
Dun Laoghaire

27th October, 1989
27th October, 1989
27th October, 1989
27th October, 1989
Location 1 – Spring Tide

- **Magnitude of Velocity**
  - Graph showing velocity (m/s) over time relative to high water (hr).
  - Data points and trend lines for SW2D, T3, and average experiment (avg expt).

- **Direction of Velocity**
  - Graph showing direction wrt N (deg) over time relative to high water (hr).
  - Data points and trend lines for SW2D, T3, and average experiment (avg expt).
Location 2 – Spring Tide

Magnitude of Velocity

- Velocity (m/s) vs. Time Relative to High Water (hr)
- SW2D, T3, avg expt

Direction of Velocity

- Direction wrt N (deg) vs. Time Relative to High Water (hr)
- SW2D, T3, avg expt
Location 3 – Spring Tide

Magnitude of Velocity

Direction of Velocity
Location 4 – Spring Tide

**Magnitude of Velocity**

- SW2D
- T3
- avg expt

**Direction of Velocity**

- SW2D
- T3
- avg expt
Location 5 – Spring Tide

**Magnitude of Velocity**

- Time Relative to High Water (hr)
- Velocity (m/s)

**Direction of Velocity**

- Time Relative to High Water (hr)
- Direction wrt N (deg)

Legend:
- SW2D
- T3
- avg expt
Location 6 – Spring Tide

![Graph of Magnitude of Velocity](chart1.png)

![Graph of Direction of Velocity](chart2.png)
Location 7 – Spring Tide

Magnitude of Velocity

- SW2D
- T3
- avg expt

Direction of Velocity

- SW2D
- T3
- avg expt
Location 8 – Spring Tide

**Magnitude of Velocity**

- **SW2D**
- **T3**
- **avg expt**

**Direction of Velocity**

- **SW2D**
- **T3**
- **avg expt**
Simulation Times

- Dell OptiPlex 780 – Intel Pentium CPU G840 @ 2.80GHz chip.
- Equivalent Simulation Time – 5 cycles.

<table>
<thead>
<tr>
<th>Model</th>
<th>Nodes (1000)</th>
<th>Timestep (s)</th>
<th>CPU time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SW2D</td>
<td>72</td>
<td>30</td>
<td>121,000</td>
</tr>
<tr>
<td>T5</td>
<td>75</td>
<td>5</td>
<td>5428</td>
</tr>
<tr>
<td>T5</td>
<td>75</td>
<td>15</td>
<td>2007</td>
</tr>
<tr>
<td>T3</td>
<td>25</td>
<td>5</td>
<td>1645</td>
</tr>
<tr>
<td>T3</td>
<td>25</td>
<td>15</td>
<td>601</td>
</tr>
<tr>
<td>T3</td>
<td>25</td>
<td>30</td>
<td>341</td>
</tr>
</tbody>
</table>