2005-07-01

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Recommended Citation

Fitzmaurice, Ronan, Burke, Ted, De Paor, Annraoi: Electromagnetic measurement of spinal curvature. Proceedings of IEEE EMBSS UKRI PGBIOMED'05, University of Reading, UK, July 18-20, 2005, pp.13-14.

ELECTROMAGNETIC MEASUREMENT OF SPINAL CURVATURE

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Abstract - In this paper, we describe an original mathematical technique for calculating the position and orientation of an electromagnetic coil from a minimum of four collinear magnetic field measurements. This problem arose in the development of a system for evaluating the efficacy of inflatable back rafts designed to mitigate complications that arise from the immobilisation of patients with suspected spinal injuries on spinal boards during transport to hospital. Electromagnetic markers are attached to points of interest on the back of an immobilised test subject. Spinal curvature is then measured by passing a magnetometer underneath the board.

I. INTRODUCTION

It is common practice for a patient with a suspected spinal injury to be immobilised on a spinal board during transport to hospital. This minimises the risk of further spinal cord damage occurring. Typically, a rigid plastic board is used, to which the patient is tightly strapped. Prolonged immobilisation can cause serious side effects, such as pressure sores and flattening of the spine. The inflatable *back raft* is a cushioning device designed to alleviate these problems. In evaluating the efficacy of the raft, we designed a system to measure spinal curvature during immobilisation.

Electromagnetic marker coils are attached to points of interest on the back of a subject who is immobilised on a spinal board, beneath which a magnetometer is passed. The position of each marker is calculated from the magnetic field readings. The coils are pulsed with current, one at a time, under computer control.

Here, we describe the mathematical theory used to calculate the position and orientation of each coil from the magnetometer readings.

II. THEORY

Consider a circular coil of radius r_{coil}, with N turns, centred on a point, **c,** and carrying a current, I. Point **p** is relatively distant from **c**.

$$\mathbf{r} = \mathbf{p} - \mathbf{c}$$

$$|\,\mathbf{r}\,| >> r_{coil}$$

The magnetic flux density at **p,** due to the coil, may be approximated as that due to an equivalent magnetic dipole [1].

$$\mathbf{B}(\mathbf{p}) = \frac{\mu_0}{4\pi}\left[\frac{3(\mathbf{m}.\mathbf{r})\mathbf{r}}{|\mathbf{r}|^5} - \frac{\mathbf{m}}{|\mathbf{r}|^3}\right]$$

where **B** is magnetic flux density, μ_0 is the permeability of free space and **m** is the magnetic moment vector of the equivalent dipole. The direction of **m** is perpendicular to the plane in which the coil lies and its magnitude is defined as follows [1],

$$|\mathbf{m}| = NI\pi\, r_{coil}{}^2$$

The dipole axis is the line perpendicular to the plane in which the coil lies, passing through the point **c**. All lines tangential to the field intersect the dipole axis at some point, except those that are parallel to it.

Consider a line defined by a point $\mathbf{p_i} = (x_i, y_i, z_i)$ and a directional vector $\mathbf{h_i} = (l_i, m_i, n_i)$. We define the following associated values,

$$u_i = m_i z_i - n_i y_i$$

$$v_i = n_i x_i - l_i z_i$$

$$w_i = l_i y_i - m_i x_i$$

Two such lines intersect if and only if they satisfy the following condition [2].

$$u_1 l_2 + v_1 m_2 + w_1 n_2 + u_2 l_1 + v_2 m_1 + w_2 n_1 = 0$$

At each of four points on the x-axis, the magnetic flux density, $\mathbf{B}(\mathbf{p_i}) = (b_{ix}, b_{iy}, b_{iz})$, is measured and a line, L_i, tangential to the field at that point is defined. Since, in general, the four tangent lines, L_1, L_2, L_3 and L_4 are skew, there exist only two lines that intersect with all four. One is the observation line (the x-axis) and the other is the dipole axis. Using the intersection condition stated above, the following four simultaneous equations are generated from the tangent lines.

$$b_{1x}u + b_{1y}v + b_{1z}w + b_{1z}x_1 m - b_{1y}x_1 n = 0$$

$$b_{2x}u + b_{2y}v + b_{2z}w + b_{2z}x_2 m - b_{2y}x_2 n = 0$$

$$b_{3x}u + b_{3y}v + b_{3z}w + b_{3z}x_3 m - b_{3y}x_3 n = 0$$

$$b_{4x}u + b_{4y}v + b_{4z}w + b_{4z}x_4 m - b_{4y}x_4 n = 0$$

One of the two solution lines of this set of equations is the observation line and the other is the dipole axis, from which $\hat{\mathbf{m}}$, the unit vector in the direction of the dipole moment, can readily be obtained.

Now, using any one of the four tangent lines, the point **c** can be calculated. Here, we choose L_1. First, let us rewrite the expression for the magnetic flux density at a point **p** as follows.

$$\mathbf{B}(\mathbf{p}) = \frac{\mu_0|\mathbf{m}|}{4\pi|\mathbf{r}|^3}\left[3(\hat{\mathbf{m}}.\hat{\mathbf{r}})\hat{\mathbf{r}} - \hat{\mathbf{m}}\right],$$

where $\hat{\mathbf{r}}$ is a unit vector in the direction of **r**. Note that the vector part of this expression (contained in the square brackets) depends only on the directions of **m** and **r** (and not on their magnitudes), so lines tangential to the magnetic field at every point on any straight line passing through **c** are parallel. Hence, knowing the angle between the field tangent at a point and the dipole axis, we can calculate the angle between the vector **r** and the dipole axis. This in turn reveals the position of **c**.

Since $\hat{\mathbf{m}}$ and $\hat{\mathbf{r}}$ are unit vectors, their dot product is simply the cosine of θ, the angle between them. So, the expression for the magnetic flux density may once again be rewritten:

$$\mathbf{B}(\mathbf{p}) = k\left[3\cos\theta\,\hat{\mathbf{r}} - \hat{\mathbf{m}}\right],$$

where k is simply the scalar multiple from the previous form of the expression. Consider a circle lying in the plane containing both L_1 and the dipole axis, with a diameter of 3 and passing through **c**, as shown in Figure 1. The point **d** is defined by

$$\mathbf{d} = \mathbf{c} + \hat{\mathbf{m}}$$

The line L_1' is the line parallel to L_1 that passes through **d**. **b** is the point of intersection of L_1' with the circle. Since α is the angle between the vector $\hat{\mathbf{m}}$ and the field vector at $\mathbf{p_1}$, it can be calculated from their dot product as follows,

$$\alpha = \cos^{-1}\left(\frac{\mathbf{B}(\mathbf{p}_1).\hat{\mathbf{m}}}{\left|\mathbf{B}(\mathbf{p}_1)\right|}\right)$$

Applying the sine rule to triangle **abd** gives

$$\frac{\sin\alpha}{1.5} = \frac{\sin\beta}{0.5},$$

from which we obtain the following.

$$\beta = \sin^{-1}\left(\frac{\sin\alpha}{3}\right)$$

And since triangles **abc** and **abd** share a common vertex angle,

$$\theta = \frac{\alpha+\beta}{2}$$

The distance from **b** to **d** is

$$\left|\mathbf{b}-\mathbf{d}\right| = 0.5\cos\alpha + 1.5\cos\beta,$$

The unit vector, $\hat{\mathbf{r}}$, may now be expressed as follows,

$$\hat{\mathbf{r}} = \frac{1}{3\cos\theta}\left(\hat{\mathbf{m}} + \frac{\left|\mathbf{b}-\mathbf{d}\right|\mathbf{B}(\mathbf{p_1})}{\left|\mathbf{B}(\mathbf{p_1})\right|}\right)$$

The perpendicular distance between the point $\mathbf{p_1}$ and the dipole axis may be expressed as $h = \left|\hat{\mathbf{m}} \times (\mathbf{e} - \mathbf{p_1})\right|$, where **e** is any point on the axis [3]. Using this, we can calculate the magnitude of **r,**

$$\left|\mathbf{r}\right| = \frac{h}{\cos\theta}$$

Finally, since the magnitude of **r** is now known in addition to its unit vector, the point **c** at which the coil is located may be calculated.

$$\mathbf{c} = \mathbf{p}_1 - \mathbf{r}$$

III. CONCLUSION

We emphasise that the validity of the method is not specific to the magnetometer used. Given *exact* magnetic flux density readings, this technique delivers the *exact* coil position. This was double-checked using simulated readings generated from the dipole field equation. The expected error in the coil position estimate depends on the precision of the specific magnetometer used as well as on the position of each of the four points relative to the coil. To ensure coil position estimates within the specification required in this application (5mm), many readings were taken for each coil, from which a large number of estimates were calculated. Outlying estimates were discarded and the average of those that remained was taken as the final estimate. The final estimates were shown, theoretically and experimentally, to satisfy the specification.

The original technique described here for measuring magnetic coil position using four collinear field measurements undoubtedly has numerous potential uses in a wide variety of applications. It provides a simple and efficient means of orientation and position sensing of objects in otherwise difficult circumstances.

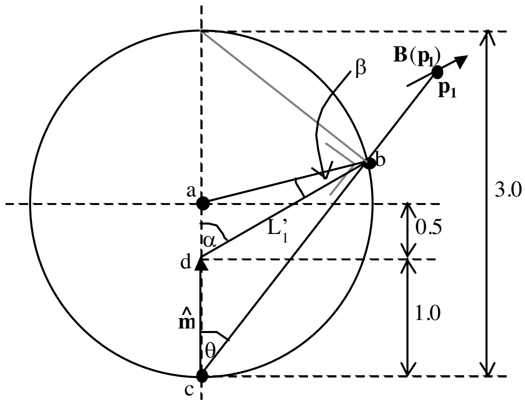

Fig. 1: $\left|\mathbf{b}-\mathbf{c}\right| = 3\cos\theta \Rightarrow \left|\mathbf{b}-\mathbf{d}\right| = 3\cos\theta - \hat{\mathbf{m}}$

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