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Thermodynamic Optimisation of the Otto / Stirling Combined Cycle

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Abstract: Combined cycle systems are an established method for increasing primary energy efficiency of power generation systems. Some ongoing research is concerned with investigating the novel combined cycle system involving the Otto and Stirling thermodynamic cycles. The Otto cycle is to act as the topping cycle, with the Stirling cycle acting to recover heat from the exhaust for the purpose of additional power generation.

The present work investigates the thermodynamic optimisation of the combined cycle system for the case of the engines operating under two imposed parametric constraints: 1) imposed heat addition to the Otto cycle, and 2) imposed maximum cycle temperature of the Otto cycle. These conditions are analogous to a specified fuel consumption of the engine and the metallurgical limit of the operating components respectively. The optimum work output for each scenario is analysed with respect to the particular physical constraints of the Stirling cycle heat exchangers – effectiveness, NTU, heat transfer coefficient and heat transfer area. Only interactions between the engine and the external source and sink are considered in this treatment. Regeneration within the cycle, as would typically be used within the practical engine, is considered as perfect.

The existence of an optimum power output for the combined system is proven analytically. A numerical study is then presented to further investigate the performance for each of the parameters named above.

Keywords: Combined Cycles, Otto Cycle, Stirling Cycle, Thermodynamic Optimisation

1. Introduction

Combined cycle power generation systems are an established method for increasing primary generation efficiency. An established technology that have benefitted from several decades of development, they are more typically associated with large scale centralised generation systems, and almost exclusively involve turbine plant operating on Brayton / Rankine combinations. Current thinking, however, favours development towards Distributed Generation (DG) networks in an effort to increase energy efficiency for both environmental and security reasons [1-5].

Arising from this is an interest in small-scale combined cycle systems involving reciprocating engines such as gas fired Otto cycle and Diesel cycle engines. These engines traditionally dominate this smaller scale (<5MW) power systems market as Combined Heat and Power (CHP) generators, renewable gas prime movers and standby generators [6, 7]. Some recent work has focused on use of the Rankine cycle as a heat recovery device on a reciprocating internal combustion engine. Gambarotta and Vaja [8] investigated the Organic Rankine Cycle (ORC) as a bottoming cycle on an Otto Cycle spark ignition engine. Badami et al [9] investigated the standard Rankine cycle

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Figure 1: The Combined Cycle represented on the T-S plane

Endo et al [10] have investigated the exergy gains possible from utilising a Rankine cycle as a bottoming cycle on an automotive internal combustion engine. Similar work has been conducted by Chammas and Clodic [11].

Use of the Stirling cycle as a bottoming cycle has been studied also by several different parties, for example [12-15]. Use of the Stirling cycle as a bottoming cycle is of interest due to the high theoretical output and efficiency that could be attained, in addition to potentially smaller plant footprint and quiet operation.

Some ongoing work [16-18] has been investigating this alternative combined cycle utilising a Stirling cycle engine as a bottoming cycle on an Otto cycle engine. Whereas these previous works have dealt with the modelling and simulation of the combined cycle for use in instances such as stationary and vehicular power generation, the present work seeks to contribute to the general model by developing an optimised thermodynamic model of the system considering parameters concerning the Stirling engine source and sink heat exchangers: effectiveness (ε) and NTU. This further implies the opportunity to study the optimal heat exchanger area (A) and overall heat transfer coefficient (U). The optimisation procedure can be considered as being in terms of Finite Dimension Thermodynamics (FDT) [19], as the performance of the cycles is considered with regard to the physical characteristics of the heat exchangers.

2. The Combined Cycle

In the combined cycle analysis, the thermodynamic cycles are considered with the Stirling cycle as the bottoming cycle on the Otto cycle. The combined system is depicted on the T-S plane in Fig. 1. Of interest is the optimisation of the combined power output and efficiency under two operating conditions: 1) fixed heat input to the Otto cycle and 2) fixed maximum cycle temperature in the Otto engine. These conditions are analogous to specified fixed fuel consumption of the engine and a metallurgical limit on the system components respectively. The coupling of the two relies on
The correct specification of the heat exchanger inventory between both the Stirling cycle source – the Otto cycle exhaust stream, and sink – a water cooling circuit.

The following section details the basic relationships of the mathematical model. Section 4.0 analyses the optima in terms of a fixed heat addition. Section 5.0 provides a similar analysis for the case of imposed maximum cycle temperature in the Otto cycle.

### 3. Basic Mathematical Model

Depicted in Fig. 1 is the air standard Otto/Stirling combined cycle. The power output of the air standard Otto cycle is readily expressed as:

$$-\dot{W}_{\text{otto}} = |\dot{Q}_{\text{in}} + \dot{Q}_{\text{out}}|$$  \(1\)

$$-\dot{W}_{\text{otto}} = \dot{C}_o[(T_3 - T_2) - (T_4 - T_1)] - \frac{K_L(T_3 - T_{\text{amb}})}{T_C}$$  \(2\)

Where \(K_L\) is a generalised heat conductance term to account for global losses from the cycle that can not otherwise be accommodated in the analysis. Similarly, the power output of the Stirling cycle can be expressed as:

$$-\dot{W}_{\text{stirling}} = \varepsilon_H\dot{C}_o(T_4 - T_H) + \varepsilon_C\dot{C}_s(T_1 - T_C)$$  \(3\)

Where \(\dot{C}_o\) and \(\dot{C}_s\) are the minimum heat capacitance rates of the Stirling cycle source and sink heat exchangers respectively:

$$\dot{C}_o = m_o C_{v,o}$$  \(4\)

$$\dot{C}_s = m_s C_{p,s}$$  \(5\)

The entropy balance of the Otto engine, with inclusion for the heat transfer to the Stirling cycle, can be expressed as:

$$\oint \frac{\delta Q_{\text{combustion}}}{T} + \frac{\delta Q_{\text{heat transfer}}}{T} + \frac{\delta Q_{\text{exhaust}}}{T} = 0$$  \(6\)

This can be reduced to:

$$\int_{T_2}^{T_3} \frac{dT}{T} + \int_{T_4}^{T_5} \frac{dT}{T} + \int_{T_5}^{T_4} \frac{dT}{T} = -\frac{\dot{S}}{\varepsilon_0}$$  \(7\)

giving the entropy generation constraint for the Otto cycle:

$$K_o = e^{-\frac{\dot{S}}{\varepsilon_0}} = \frac{T_1T_2}{K_o T_3 T_4}$$  \(8\)

This entropy constraint would permit analysis with inclusion for internal irreversibilities, with \(0 < K_o \leq 1\). Under the endoreversible conditions, \(K_o = 1\). A similar condition exists for the endoreversible Stirling cycle:

$$\frac{T_{\text{amb}}}{T_C} = K_S - \frac{\varepsilon_H\dot{C}_o T_4}{\varepsilon_C C_{C_S} T_H}$$  \(9\)

Where

$$K_S = 1 + \frac{\varepsilon_H\dot{C}_o}{\varepsilon_C C_{C_S}} - \frac{\dot{S}_{1,s}}{\varepsilon_C C_{C_S}}$$  \(10\)

It can be seen from equation (10) that for the endoreversible case, \(K_S > 1\). Study of the cycles in the present work is limited to the endoreversible case; however the inclusion of both \(K_o\) and \(K_S\) in the analysis permits investigation with inclusion for irreversibilities.

#### 3.1 Case of Imposed Heat Addition

The objective function of the combined cycle is ultimately the work output. We can achieve an optimum scenario by considering cycle temperatures. The optimum combined work output may most simply be expressed as:

$$-\dot{W}_{\text{TOTAL}} = -\dot{W}_{\text{otto}} - \dot{W}_{\text{stirling}}$$  \(11\)

In order to complete the optimisation, we must specify the constraints to be imposed. In this first scenario, the heat addition to the Otto cycle – and therefore the combined cycle as a whole, is to be fixed. The heat added to the Otto cycle is:

$$\dot{q}_o = \dot{C}_o(T_3 - T_2) - K_L(T_3 - T_{\text{amb}})$$  \(12\)

This serves as the first constraint. This equation expresses the cycle heat addition as that admitted to the engine less a cooling loss term between the engine and the surroundings. This term accounts for non-adiabatic operation of the engine. The other constraints to be considered relate to the endoreversibility criteria. For the Otto cycle:

$$\frac{T_{\text{amb}}}{K_o}T_3 - T_2 T_4 = 0$$  \(13\)

For the Stirling cycle, equation (6) acts as the entropy generation constraint.

With these constraints, the objective function therefore becomes:

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\[ -W_{TOTAL} = \dot{q}_0 - K_L T_3 + K_L T_{amb} + \dot{C}_o T_{amb} - \dot{C}_o T_4 + \epsilon_H \dot{C}_o T_4 - \epsilon_H \dot{C}_o T_H + \epsilon_C \dot{C}_S T_{amb} - \epsilon_C \dot{C}_S T_{amb} - \frac{\epsilon_H \dot{C}_o T_4}{K_S \epsilon_C \dot{C}_S T_H} \]

(14)

This function represents the combined work output of the combined cycle plant and can be optimised to determine expressions for the optimal cycle temperatures that will maximise the work output. For the condition of imposed heat addition, maximum work output corresponds to maximum efficiency also.

Performing the optimisation provides the optimum temperature relationships detailed in the following.

The temperature at the end of expansion and immediately before exhaust, \( T_4^* \) can be determined by numerical solution of the polynomial:

\[ \frac{(\dot{q}_0 + K_L T_{amb}) K_o T_{amb} K_o}{(C_o + K_L)(K_o T_4 - C_o T_{amb})^2} = \frac{\epsilon_H \dot{C}_o}{K_S \epsilon_C \dot{C}_S} + \sqrt{\frac{T_{amb}}{T_4}} \]

(15)

The temperature after initial compression of the working gas, \( T_2^* \) is computed from:

\[ T_2^* = \frac{\dot{q}_0 T_{amb} - K_L T_{amb}^2}{K_o T_o C_o + K_o T_o C_o T_2 - C_o T_{amb}} \]

(16)

And the optimum temperature after combustion \( T_3^* \) is determined from:

\[ T_3^* = \frac{\dot{c}_o T_2^* - T_{amb} K_L + \dot{q}_0}{\epsilon_C + K_L} \]

(17)

The optimum temperatures of the Stirling source can be described by:

\[ T_H^* = \frac{1}{K_S} \left[ \frac{\epsilon_H \dot{C}_o}{\epsilon_C \dot{C}_S} + \sqrt{T_{amb} T_4} \right] \]

(18)

The corresponding sink temperature can then be derived from equation (10). Power output of the combined cycle can therefore be calculated by substituting the preceding optimal temperatures into equation (11):

\[ -W_{TOTAL,q}^* = \dot{q}_0 - K_L T_3^* + K_L T_{amb} + \dot{C}_o T_{amb} - \dot{C}_o T_4^* + \epsilon_H \dot{C}_o T_4^* - \epsilon_H \dot{C}_o T_H^* + \epsilon_C \dot{C}_S T_{amb} - \epsilon_C \dot{C}_S T_{amb} - \frac{\epsilon_H \dot{C}_o T_4^*}{K_S \epsilon_C \dot{C}_S T_H} \]

(19)

And efficiency can be calculated in the usual manner:

\[ \eta_T^* = \frac{-W_{TOTAL,q}^*}{\dot{q}_0} \]

(20)

The preceding analysis offers expressions for power output and efficiency for the combined cycle when the heat supplied to the total system is fixed as an imposed parameter. It is also useful to examine the optimisation of the combined system with a maximum cycle temperature in the Otto cycle used as an imposed constraint. This is given in the next section.

### 3.2 Case of Imposed Maximum Cycle Temperature

In this scenario, it is assumed that a maximum cycle temperature is to be imposed. This is a common optimisation constraint for thermal power plant and allows consideration of the engine in terms of real metallurgical limits.

In this case, the first imposed constraint becomes:

\[ T_3 = T_{max} \]

(21)

And the Otto cycle entropy constraint becomes:

\[ \frac{T_{amb} T_{max}}{K_o} - T_2 T_4 = 0 \]

(22)

Temperature \( T_2^* \) is computed from the entropy constraint given in equation (22):

\[ T_2^* = \frac{T_{max} T_{amb}}{K_o T_2^*} \]

(23)

The optimum heat requirement to the cycle is free to vary subject to the maximum temperature constraint:

\[ \dot{q}_{SH}^* = \dot{C}_o (T_{max} - T_2^*) + K_L (T_{max} - T_{amb}) \]

(24)

\( T_2^* \) is a function of \( T_4^* \), the optimum temperature after expansion in the Otto cycle. To determine this temperature, the objective function can again be optimised for maximum power and efficiency under the given constraints. Temperature \( T_4^* \) can therefore once again be calculated by solution of a polynomial:
The effectiveness-NTU relationship used in the heat exchanger is given by:

\[ \frac{\hat{C}_o T_{amb} T_{max}}{K_o T_4^2} = \hat{C}_o (1 - \epsilon_h) \]

\[ + \frac{\epsilon_h \hat{C}_o}{K_s} \left[ \frac{T_{amb}}{T_4} \right] \]

The optimized NTU is computed by solving the following equation:

\[ \frac{\hat{C}_o T_{amb} T_{max}}{K_o T_4^2} = \hat{C}_o (1 - \epsilon_h) \]

\[ + \frac{\epsilon_h \hat{C}_o}{K_s} \left[ \frac{T_{amb}}{T_4} \right] \]

The optimum temperatures for the Stirling cycle are computed as before. This is possible as the source temperature, \( T_{H}^* \), is a function of the optimum \( T_4^* \) for the given constraints. The power output of the combined cycle under the given constraints can therefore be expressed in terms of the optimal temperature values as:

\[ -\dot{W}_{TOTAL} = \hat{C}_o \left[ T_{max} - \frac{T_{amb} T_{max}}{K_o T_4^*} + T_{amb} - \frac{T_{4}^*}{T_4^*} \right] + \epsilon_h \hat{C}_o (T_4^* - T_{H}^*) + \epsilon_c \hat{C}_s T_{amb} \left[ 1 - \frac{1}{(K_s - \epsilon_h \hat{C}_o T_4^*) / (\epsilon_c \hat{C}_s T_{H})} \right] \]

And the efficiency under these constraints becomes:

\[ \eta_T^* = \frac{-\dot{W}_{TOTAL}^*}{\hat{Q}_o^*} \]

### 3.3 Study of Heat Exchanger Parameters: NTU, Heat Transfer Coefficient, Heat Transfer Area

The model equations presented in the foregoing sections offers expressions for the optimal work output as a function of the heat exchanger effectiveness, \( \epsilon \). More direct analysis in terms of physical characteristics of the system can be completed by analysis in terms of the heat exchanger Number of Transfer Units (NTU). The effectiveness-NTU relationship used in the present work is [20]:

\[ \epsilon = 1 - \exp (-NTU) \]

The NTU value is a dimensionless quantity that includes for the heat exchange area, \( A \), the overall heat transfer coefficient, \( U \) and the heat capacitance rate, \( C \):

\[ NTU = \frac{UA}{C} \]

The total NTU for the Stirling cycle engine is considered as the sum of the source and sink heat exchanger NTU values:

\[ NTU_T = NTU_H + NTU_C \]

This permits an expression for the heat transfer area distribution for the engine:

\[ \frac{A_H}{A_T} = \left( \frac{U_C}{U_H} \right) \left( \frac{C_C}{C_T} \right) \left[ 1 - \frac{NTU_C}{NTU_T} \right] \]

This equation allows study of the optimal heat exchanger area distribution in terms of the ratio of the overall heat transfer coefficients, \( \frac{U_C}{U_H} \) the ratio of the heat capacitance rates, \( \frac{C_C}{C_T} \) and the NTU ratio \( \frac{NTU_C}{NTU_T} \).

### 4. Results

The optimised system is represented in the following figures for the two cases of imposed heat addition and maximum cycle temperature constraint. The two cases of imposed heat addition and imposed maximum cycle temperature are analyzed in section 3.1 and 3.2 respectively. Within each scenario, performance is analysed against heat exchanger effectiveness and NTU separately.

#### 4.1 Imposed Heat Addition

In this scenario, the following parameters were imposed: \( Q_{in} = 27kW \), \( \hat{C}_o = 10W/K \), \( \hat{C}_s = 15W/K \), \( T_1 = 300K \), \( K_d = 0.5W/K \)

#### 4.1.1 Heat Exchanger Effectiveness Analysis

![Figure 2. Total Power Vs Heat Exchanger \( \epsilon \)](http://www.ecos2010.ch)
It is possible also to express the model results against the heat exchanger NTU. This offers the advantage of allowing analysis in terms of the physical inventory of the heat exchangers. It can be seen that an expression in terms of NTU yields a clearer optimum point for Power output and Efficiency, as may be seen in Fig. 6 and Fig. 7.

4.2 Maximum Cycle Temperature

In this scenario, it was desired to investigate the system optimisation considering a maximum cycle temperature of the Otto cycle to be an imposed constraint. The constrained temperature parameter imposed was: $T_3 = T_{\text{max}} = 3000K,$
Figures 2 to 11 present the results of the model presented in section 3. Figures 2 to 5 show the results of the analysis for the condition of imposed heat addition to the combined cycle. It is evident from Fig. 2 that maximum power is achieved for the case of maximum heat exchanger performance. Similarly, efficiency increases to a maximum for perfect heat exchange. It is notable though that in order to maximise the total combined output for this constraint, the Otto cycle compression ratio is reduced considerably below typical expected values. This has the net effect of increasing the temperature available at the Stirling source and thereby increasing the power output and efficiency of the Stirling bottoming cycle. These trends are visible in Fig. 4 and Fig. 5. In Fig. 5 it is notable that the Stirling cycle dominates the total output, accounting for over 80% of the total output. Figures 6 and 7 show the total combined power output and efficiency as a function of the NTU of each of the Stirling cycle heat exchangers. This method has the advantage of demonstrating a clearer optimum than the case of the heat exchanger effectiveness values. It also permits a clearer analysis in terms of the physical characteristics of the heat exchangers.

Results for the case of imposed maximum temperature in the Otto cycle are presented in Fig. 8 to 11. It is seen that maximum power is achieved at maximum heat exchanger effectiveness, whilst efficiency is minimised at these conditions. As in the case of imposed heat input, it is seen in Fig. 10 that for maximum power conditions, the Otto cycle compression ratio is reduced, although in this instance it remains in the range of that expected for a real engine. This is a consequence of the temperature limitation placed on the cycle. Figure 11 demonstrates that, at most, the Stirling bottoming cycle produces approximately 15% of the total power output.

It is clear from the results presented in section 4 that clear optima exist for each of the analysed constraints. As regards thermodynamic design of a power plant, it is evident that for the case of imposed heat, maximum power and maximum efficiency coincide. To achieve this, the effectiveness of the Stirling heat exchangers must be maximised. For the case of maximum imposed temperature, maximum power and efficiency are not coincident. Maximum efficiency occurs for the case of the Otto cycle acting alone. In this instance, the efficiency gain from the increased compression ratio of the Otto cycle appears to overcome that from the addition of the bottoming cycle, as to increase the power and efficiency of the Stirling cycle the compression ratio is reduced to raise the
temperature of the exhaust, which acts as the thermal source for the Stirling cycle.

5. Conclusions
A thermodynamic model of the combined Otto / Stirling cycle is presented and optimised for maximum total power output, with the Otto exhaust acting as the thermal source to the Stirling cycle. The optimisation is performed for two different imposed criteria. The first imposed constraint is a fixed heat input to the Otto cycle. The second scenario investigated was the case of an imposed maximum cycle temperature for the Otto cycle, $T_3$. In both scenarios the optimum temperatures in each cycle are presented. Calculation of the optimum temperatures allows calculation of the optimum work in each case.

The novel hybrid system described here, that of the Otto/Stirling combined cycle, is of interest to the power generation industry. Attention is currently being given to the use of Organic Rankine cycle (ORC) systems as bottoming cycles on natural gas fired Otto engines. The Stirling cycle is pertinent as an alternative bottoming cycle due to its high theoretical efficiency, its quiet operation, its compact construction and modest footprint.

References


