

2014-12-07

Applied Energy Systems - Rudimentary Fluid Mechanics

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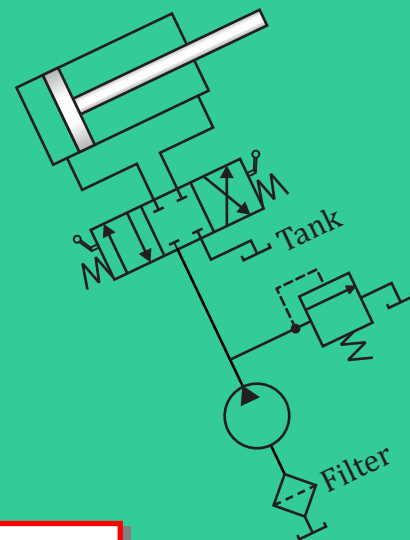
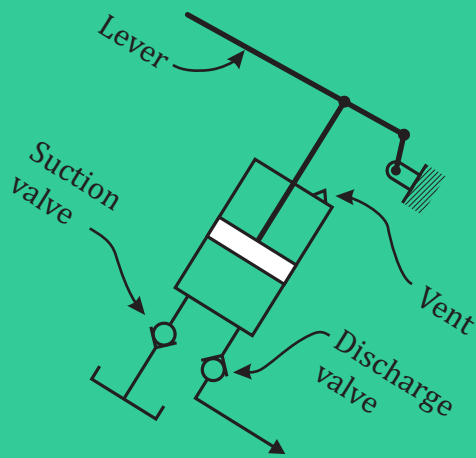
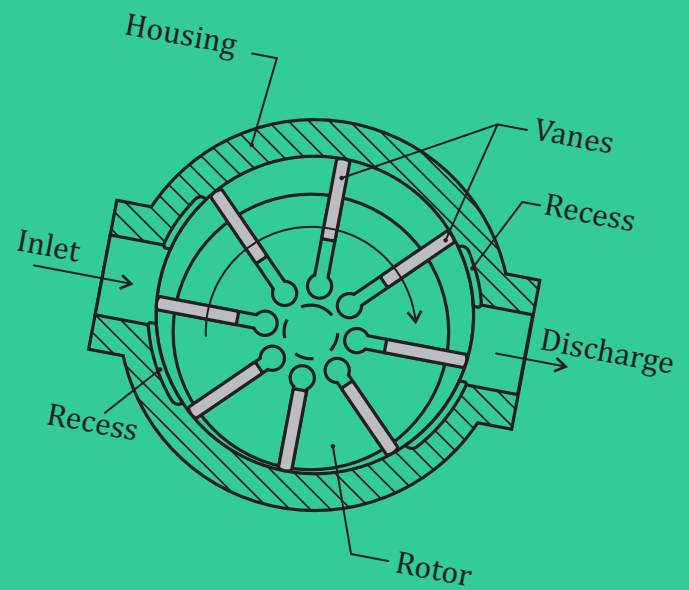
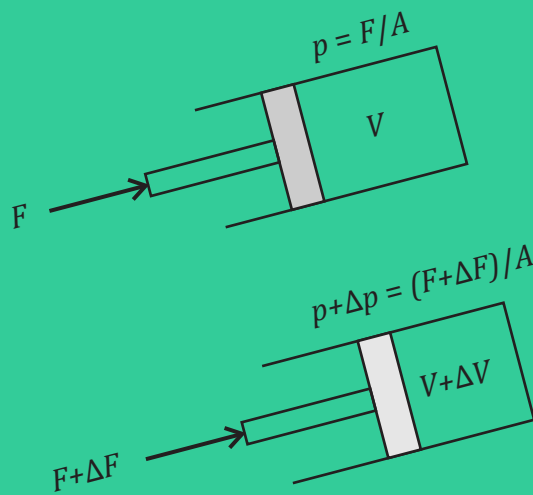
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Applied Energy Systems — Rudimentary Fluid Mechanics

Jim McGovern



SI Units 2014 Edition

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Jim McGovern

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Table of Contents

Nomenclature.....	iv
Preface.....	vii
Chapter 1. Units and Dimensions.....	1
Problems.....	5
Chapter 2. Fluids, Pressure and Compressibility.....	6
What is a Fluid?.....	6
Normal Stress and Shear Stress.....	6
Pressure.....	8
Compressibility and Bulk Modulus.....	10
Problems.....	12
Chapter 3. Hydrostatic Work and Power Transmission.....	13
Pascal's Law.....	13
Continuity.....	14
Mass and Volume Flow Rates.....	15
Work, Flow Work and Power.....	16
Design and Operating Principles of a Hydraulic Jack.....	18
Problems.....	19
Chapter 4. Positive Displacement Pumps.....	21
Simple Reciprocating Pump.....	21
Swash Plate Pump.....	22
Rotary Positive Displacement Pumps.....	23
Ideal Flow Rate of a Positive Displacement Pump.....	25
Efficiency of Hydraulic Pumps.....	26
Problems.....	28
Chapter 5. Hydraulic System Components.....	30
Check Valve and Pressure Relief Valve.....	30
Hydraulic Cylinders or Rams.....	31
Cylinder Force Calculations.....	32
Directional Control Valves.....	34
The Hydraulic Circuit Diagram.....	36
Problems.....	36
Chapter 6. Design of Hydraulic Systems.....	38
Flow Control.....	38

Hydraulic System for a Lift.....	40
Hydraulic Press	41
Problems	44
Chapter 7. Viscosity.....	46
Newton’s Viscosity Equation.....	47
Kinematic Viscosity	50
More about Viscosity Units and Values	51
Problems	53
Chapter 8. Pressure Losses in Pipes and Fittings	55
Laminar and Turbulent Flow	55
Reynolds Number.....	57
Head Loss Due to Friction.....	59
Flow through Valves and Fittings	62
Flow Around Elbows and Bends.....	63
Equivalent Pipe Length.....	63
Pressure Loss and Pumping Power.....	64
Summary of Key Formulae for Pipe Flow.....	65
Problems	69
Chapter 9. Valves and Heat Exchanger Flow Paths	71
Valves.....	71
Head Loss Coefficients for Open Valves	74
Heat Exchangers	75
Pressure Drops or Head Losses in Heat Exchangers.....	79
Problems	80
Appendix A Formulae.....	81
Newton’s Viscosity Equation.....	81
Kinematic viscosity.....	81
Newton’s Second Law Applied to S.I. Units.....	82
Pressure.....	82
Pressure and Head.....	82
Compressibility and Bulk Modulus.....	83
Force Produced by a Hydraulic Cylinder	83
Displacement of a Hydraulic Piston Pump.....	84
Ideal Flow Rate of a Positive Displacement Pump.....	84

Efficiency of a Hydraulic Pump	84
Area of a Circle.....	85
Mass Flow Rate.....	85
Volume Flow Rate	85
Reynolds Number.....	86
The Darcy-Weisbach Equation.....	86
Friction Factor for Laminar Flow.....	87
Head Loss Coefficient for Fittings and Features.....	87
Appendix B Glossary	88

Nomenclature

Symbol	Description	SI units
A	area	m^2
A_{vc}	area at vena contracta	m^2
A_o	area of orifice	m^2
a	acceleration	ms^{-2}
a	constant	
b	constant	
d	distance or diameter	m
d_{eq}	equivalent diameter	m
F	force	N
f	friction factor	
g	acceleration due to gravity	ms^{-2}
h	head, height or depth	m
h_f	head loss due to friction	m
K	bulk modulus	Pa
K	head loss coefficient	
k	surface roughness	m
L	length dimension	
L	length	m
M	mass dimension	
m	mass	kg

Symbol	Description	SI units	
\dot{m}	mass flow rate	kg s^{-1}	
n_{cyl}	number of cylinders		
P	Power	W	
p	pressure	Nm^{-2} or Pa	
Q	volume flow rate	m^3s^{-1}	
Re	Reynolds number		
SI	Système International, International System		
s	distance	m	
T	time dimension		
t	time	s	
V	volume	m^3	
v	velocity	m s^{-1}	
v_n	velocity at position n	m s^{-1}	
W	work	J	
x	distance (usually horizontal)	m	
y	distance (usually vertical)	m	
Greek letters			
β	compressibility	Pa^{-1}	(beta)
Δ	indicates an increment of the following variable e.g. Δy is a difference between two values of y		(uppercase delta)

Symbol	Description	SI units	
Δp_{hx}	pressure drop across a heat exchanger	Pa	
Δs	displacement (change of position)	m	
Θ	temperature dimension		(uppercase theta)
μ	absolute viscosity	Pas or Nm^{-2}s or $\text{kgm}^{-1}\text{s}^{-1}$	(mu)
ν	kinematic viscosity	m^2s^{-1}	(nu)
ρ	density	kg/m^3	(rho)
σ	normal stress	Nm^{-2}	(sigma)
τ	shear stress	Nm^{-2}	(tau)

Preface

Applied Energy Systems modules have developed and evolved at the Dublin Institute of Technology (DIT) within the framework of the Bachelor of Engineering Technology degree and the programmes that preceded it. The present work is a textbook for such a module addressing rudimentary fluid mechanics. Specifically, it is an introduction to fluid mechanics for incompressible fluids. To allow for learners of diverse backgrounds, the level of mathematics is kept simple. The book does not attempt to cover the principles of fluid mechanics comprehensively. Rather, it provides a practical but theoretically sound foundation, linked to some key areas.

‘Energy’ is often very topical. There is a widespread appreciation that energy must be used efficiently and a general willingness to make use of all forms of energy that are renewable and sustainable. Likewise there is a general desire to avoid, where possible, making use of forms of energy that are finite (i.e. already stored in fixed quantities on the planet Earth) or that damage the environment when they are used. The underlying principles and scientific knowledge that relate to energy utilization are common, whether energy is being used well or ineffectively and irrespective of the source of the energy. This present work fits into that general picture.

The use of fluids, e.g. gases like air or natural gas or liquids like water or oil, forms a major sub-category of energy systems generally. This text focuses on two practical areas in the use of fluids: hydrostatic power transmission systems and the flow of fluids through pipes and fittings. This selection allows the two areas to be presented in some detail. Through this approach learners can put the principles of fluid mechanics into context, while developing their knowledge and understanding. Thus, they are prepared for applying the same and similar principles to a much broader range of practical applications in the future.

Hydrostatic power transmission systems, often described less precisely as hydraulic machines, are employed in a vast range of

heavy and light machinery. Hydraulic machines are perhaps most visible in the construction industry, e.g. excavators, loaders, concrete pumps and dumpers. In manufacturing industry, hydraulic power systems are common, e.g. for the operation of presses that can apply huge forces to form workpieces or to allow moulding dies to be held firmly together and subsequently separated quickly. Hydrostatic power transmission systems are employed in vehicles such as cars, trains, ships and aircraft. Furthermore, many of the principles of hydraulic power transmission also apply in situations where pressures act but there is not necessarily transmission of power. For instance, a submarine is subjected to hydrostatic pressure.

The flow of fluid in pipes is such a common phenomenon that we take it completely for granted. Air flows into and out of our lungs through our bronchial tubes and our blood flows through pipes that are usually called veins and arteries. Natural gas, oil and water are just three fluids that commonly flow through pipes. It is a simple fact that energy is dissipated, or transformed to a different form, whenever a fluid flows. Mankind needs to use energy to cause fluids to flow. While, in the present work, the application area of fluid flow relates to pipes and fittings, the very same principles are common to other application areas such as wind turbines, aerodynamics and biomedical devices such as artificial hearts.

Learners who have ambition to work with the most exciting and innovative ways of harnessing energy will find that what they learn through taking an Applied Energy Systems module in Rudimentary Fluid Mechanics will serve them well. Of course, not all applied technologists work at the forefront of technical development and innovation. The everyday work of society must go on. If that work involves fluids in any way, the learner will find that their learning within the scope of such a module will have been relevant and worthwhile. However, the relevance of a module of this type is significantly broader yet. In common with other modules on an engineering programme, it involves engineering units (specifically the SI system) and involves important and generic engineering approaches such as experimentation, measurement, representation of practical problems in mathematical form, teamwork and

communication. As in any area so centrally important to society, there are ethical and environmental considerations to be aware of too.

The present work is intentionally compact and concise. It requires further explanation or elaboration. Learners are encouraged to make use of the Internet and library services for additional and supplementary information.

Chapter 1. Units and Dimensions

A physical quantity such as temperature is quantified as a pure number such as 301.5 multiplied by a unit such as the kelvin. Hence, a temperature might be written as 301.5 K. If we divide the temperature by its unit we are left with just the pure number:

$$\frac{301.5 \text{ [K]}}{\text{[K]}} = 301.5.$$

Table 1-1 lists SI and some non-SI units that are used in the present work. Of all the physical quantities that are encountered, the units for these quantities are made up of four fundamental types of units, or dimensions. Symbols can be used to represent these *dimensions* as follows: mass M, length L, time T and temperature Θ . Acceleration would have the fundamental units LT^{-2} .

Table 1-1 Units

Physical quantity	SI base unit	Other units
mass	kg (kilogram)	1 g = 10^{-3} kg 1 tonne = 10^3 kg
length	m (metre)	1 mm = 10^{-3} m 1 km = 10^3 m 1 cm = 10^{-2} m
time	s (second)	1 min = 60 s 1 h = 3600 s
force	N (newton)	1 kN = 10^3 N 1 MN = 10^6 N
area	m ²	1 cm ² = 10^{-4} m ² 1 mm ² = 10^{-6} m ²

Physical quantity	SI base unit	Other units
volume	m^3	$1 \text{ cm}^3 = 10^{-6} \text{ m}^3$ $1 \text{ mm}^3 = 10^{-9} \text{ m}^3$ liter: $1 \text{ L} = 10^{-3} \text{ m}^3$ milliliter: $1 \text{ mL} = 10^{-6} \text{ m}^3$
pressure	Pa (pascal)	$1 \text{ Pa} = 1 \text{ Nm}^{-2}$ $1 \text{ kPa} = 10^3 \text{ Pa}$ $1 \text{ MPa} = 10^6 \text{ Pa}$ $1 \text{ GPa} = 10^9 \text{ Pa}$ $1 \text{ hPa} = 10^2 \text{ Pa}$ $1 \text{ bar} = 10^5 \text{ Pa}$ atmosphere: $1 \text{ atm} = 101325 \text{ Pa}$
stress	Pa (pascal)	$1 \text{ Pa} = 1 \text{ Nm}^{-2}$
absolute or dynamic viscosity	Pa s or $\text{N m}^{-2}\text{s}$, or $\text{kg m}^{-1}\text{s}^{-1}$	$1 \text{ poise} = 10^{-1} \text{ Pa s}$ $1 \text{ centipoise} = 10^{-2} \text{ poise} = 10^{-3} \text{ Pa s}$
kinematic viscosity	m^2s^{-1}	$1 \text{ stoke} = 10^{-4} \text{ m}^2\text{s}^{-1}$ $1 \text{ centistoke} = 10^{-2} \text{ stoke} = 10^{-6} \text{ m}^2\text{s}^{-1}$
velocity	m s^{-1}	
acceleration	m s^{-2}	

Physical quantity	SI base unit	Other units
density	kg m^{-3}	
energy, work, heat	J	$1 \text{ J} = 1 \text{ Nm}$ $1 \text{ kJ} = 10^3 \text{ J}$ $1 \text{ MJ} = 10^6 \text{ J}$
power	W	$1 \text{ W} = 1 \text{ J s}^{-1}$ $1 \text{ kW} = 10^3 \text{ W}$
bulk modulus of elasticity	Pa	$1 \text{ Pa} = 1 \text{ Nm}^{-2}$
compressibility	Pa^{-1}	
conventional temperature	$^{\circ}\text{C}$ (degree Celsius)	
thermodynamic or absolute temperature	K (kelvin)	
torque (moment)	Nm	
angle	rad	$2\pi \text{ rad} = 360^{\circ}$
angular velocity	rad s^{-1}	revolutions per second: $1 \text{ r.p.s.} = 2\pi \text{ rad s}^{-1}$ revolutions per minute: $1 \text{ r.p.m.} = \frac{2\pi}{60} \text{ rad s}^{-1}$
angular acceleration	rad s^{-2}	
volume flow rate	$\text{m}^3 \text{ s}^{-1}$	$1 \text{ L s}^{-1} = 10^{-3} \text{ m}^3 \text{ s}^{-1}$ 1 L min^{-1} $= \frac{10^{-3}}{60} \text{ m}^3 \text{ s}^{-1}$

Newton's 2nd Law and the SI Units for Force

Newton's second law has a central role in mechanics, including fluid mechanics. It is mentioned here in order to explain and define the relationship between the units for force, mass, distance and time.

Newton's second law describes the force that must be applied to a mass to cause it to accelerate. It can be summarized in words as:

Force equals mass times acceleration.

This can be expressed mathematically as

$$F = ma \quad (1-1)$$

where

$$\begin{aligned} F &= \text{force} && \text{N} \\ m &= \text{mass} && \text{kg} \\ a &= \text{acceleration} && \text{m s}^{-2} \end{aligned}$$

As with all equations that represent physical quantities, the units of the left hand side of the equation must be equivalent to the units of the right hand side of the equation. Newton's second law, Equation 1-1, therefore defines the relationship between the newton, the kilogram, the metre and the second. Of these four units, any one can be expressed in terms of the other three. For example

$$1 \text{ N} = 1 \text{ kg m s}^{-2}$$

$$1 \text{ kg} = 1 \text{ N m}^{-1} \text{ s}^2.$$

Hence, wherever the unit the newton is used it can be replaced by the combination of units kg m s^{-2} . Also, it can be deduced that force has the dimensions MLT^{-2} . Pressure would therefore have the dimensions $\text{MLT}^{-2}\text{L}^{-2}$, which is equivalent to $\text{ML}^{-1} \text{T}^{-2}$.

Problems

1-1 Write the fundamental SI units (symbol only) for the quantities in the table below. Also write the fundamental dimensions of each quantity using only M for mass, L for distance, T for time and Θ for temperature.

Quantity	Fundamental SI Units	Fundamental Dimensions
Mass		
Length		
Time		
Temperature		
Angle		
Area		
Volume		
Velocity		
Angular velocity		
Acceleration		
Angular acceleration		
Volume flow rate		
Kinematic viscosity		$L^2 T^{-1}$
Force		
Energy		
Moment or torque		
Power		
Density		
Pressure		
Shear stress		
Bulk modulus		

Chapter 2. Fluids, Pressure and Compressibility

What is a Fluid?

A fluid is a substance that can flow. Generally it would be a liquid or a gas. A solid is not a fluid. So, what distinguishes a fluid from a non-fluid? The following is a suitable definition:

A fluid is a substance that cannot sustain a shear stress without undergoing relative movement.

The definition of a fluid involves the term 'shear stress,' which itself requires some explanation, as provided below.

Normal Stress and Shear Stress

A stress is a force per unit area (S.I. units newton/m²). Two main categories of stresses are normal stresses and shear stresses. If a force is normal to the area over which it acts it is said to be a normal force and the stress is therefore a normal stress. A tensile force applied to a bar, as shown in Figure 2-1, would be an example of a normal force. The normal stress exists because the bar is being pulled from both ends; as the bar is in tension this normal stress can be described as a tensile stress.

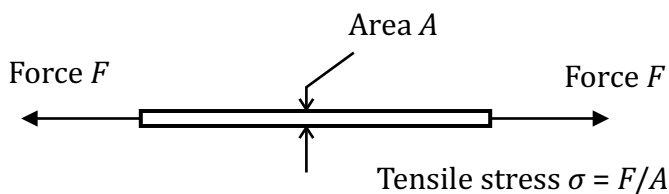


Figure 2-1 Normal force and stress for a bar in tension

For a bar in tension, the tensile stress is given by

$$\sigma = \frac{F}{A} \quad (2-1)$$

where

σ = tensile stress Nm^{-2}

F = force N
 A = area m^2

If a force is in the same plane as the area over which it acts it is called a shear force and the corresponding stress is therefore a shear stress. Figure 2-2 illustrates shear force and shear stress. The shear stress exists because the bar is subjected to forces by blades of a guillotine or shears that are tending to shear the bar in the plane in which the forces act, the shear plane.

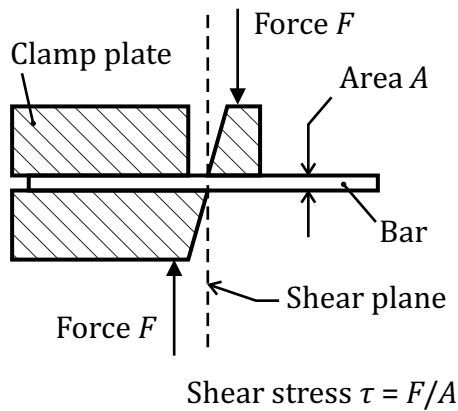


Figure 2-2 Shear force and stress for a bar in shear

The shear stress in the bar is given by

$$\tau = \frac{F}{A} \tag{2-2}$$

where

σ = shear stress Nm^{-2}

Solid materials can withstand high shear forces without any relative movement occurring within the material. However, when a fluid is subjected to a shear stress, relative movement always occurs within the fluid, no matter how small the shear stress. Because of this characteristic, a liquid will always flow under the influence of gravity unless it is prevented from flowing further by a container wall below the liquid. The surface of a liquid at equilibrium within a container will be horizontal, because if it were

not horizontal, shear forces capable of causing relative movement would exist within the liquid.

Pressure

Pressure is force exerted per unit area in a direction that is normal to and towards the area. It is expressed as

$$p = \frac{F}{A} \quad (2-3)$$

where

p	=	pressure	Pa or Nm^{-2}
F	=	force	N
A	=	area	m^2

One pascal (Pa) is a very low pressure; for instance, atmospheric pressure is about 10^5 Pa.

Absolute Pressure

Absolute pressure is true pressure, which is the force per unit area exerted on a surface. The entire force normal to the area is included. The lowest possible value of absolute pressure is zero (in any pressure units).

Atmospheric Pressure

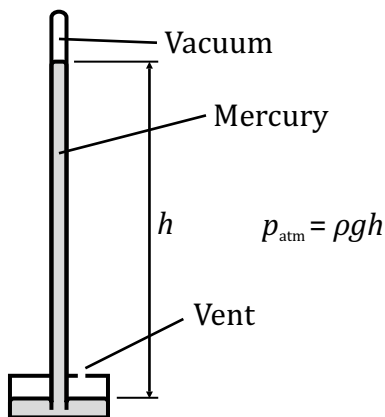


Figure 2-3 Mercury barometer

The pressure of the atmosphere is caused by the weight of the air that constitutes it. At sea level the atmospheric pressure equals the weight of air that is supported by a unit area of horizontal surface. At higher levels, less air is supported so the atmospheric pressure is correspondingly lower.

1 standard atmosphere = 1.01325×10^5 Pa

Atmospheric pressure can be measured by means of a traditional mercury barometer, Figure 2-3, or, for instance, by an aneroid barometer, which contains a sealed cell in the form of a bellows as in Figure 2-4.

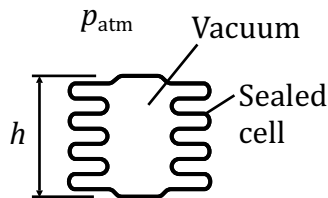


Figure 2-4 Sealed bellows cell of an aneroid barometer

Gauge Pressure

Gauge pressure is a relative pressure: it is the amount by which the pressure exceeds the pressure of the atmosphere. Gauge pressure is commonly used in engineering and many common pressure-measuring devices sense gauge pressure rather than absolute pressure. A 'pressure gauge' usually measures gauge pressure.

Vacuum and Vacuum Pressure

Vacuum pressure is the amount by which a pressure is less than atmospheric pressure. In a perfect vacuum the absolute pressure would be zero.

Pressure Head

A fluid exerts a pressure by virtue of its weight. At a depth h below the free surface of a liquid at rest the pressure due to the weight of the liquid is given by:

$$p = \rho gh \quad (2-4)$$

where

ρ = density kg/m^3

g = acceleration due to gravity ms^{-2}

h = depth below free surface m

It is common to describe a pressure by quoting the corresponding head of water or, sometimes, of other fluids, e.g. mercury (density $13,546 \text{ kg m}^{-3}$).

Compressibility and Bulk Modulus

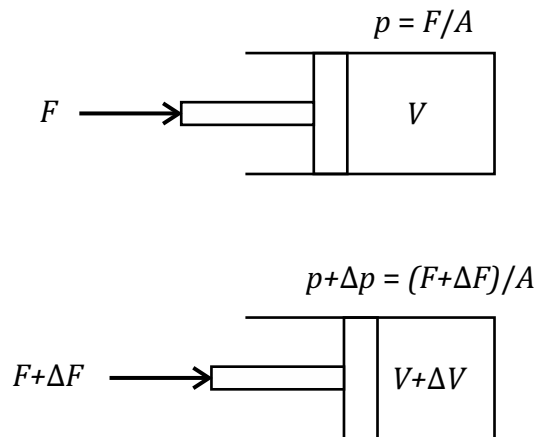


Figure 2-5 Change in volume owing to an increase in pressure

The compressibility of a fluid is a measure of how much its volume changes with pressure. The volumetric change is described as volumetric strain, i.e. the change in volume divided by the original volume.

Suppose a fluid is compressed, as shown in the Figure 2-5, due to the application of a small additional force, ΔF . The compressibility of the fluid is given by Equation (2-5).

$$\text{Compressibility, } \beta = \frac{\left(-\frac{\Delta V}{V}\right)}{\Delta p} = -\frac{1}{V} \frac{\Delta V}{\Delta p} \quad (2-5)$$

where

β	=	compressibility	Pa^{-1}
V	=	volume	m^3
p	=	pressure	Nm^{-2} or Pa
ΔV	=	change (increase) in volume	m^3
Δp	=	change (increase) in pressure	Nm^{-2} or Pa

The term ‘bulk modulus of elasticity’ is used for the inverse of compressibility, Equation (2-6)

$$\text{Bulk modulus, } K = \frac{\Delta p}{\left(-\frac{\Delta V}{V}\right)} = -V \frac{\Delta p}{\Delta V} \quad (2-6)$$

where

K	=	bulk modulus of elasticity	Pa
-----	---	----------------------------	----

Bulk modulus has the same units as pressure, i.e. Pa. The bulk modulus of liquid water is about 2.2 GPa (1 GPa = 10^9 Pa), which indicates how much pressure is required to produce volumetric strain.

All liquids have high values of bulk modulus: for hydraulic fluids it ranges from about 1.4 to 1.8 GPa. Gases have very low bulk modulus—for isothermal compression the bulk modulus is roughly equal to the pressure.

In an oil-hydraulic power system the compressibility can usually be ignored when sizing components. The fluid is treated as ‘incompressible’.

Example 2-1 Bulk Modulus

What increase in pressure would be required to cause a volumetric strain of 1% in water (i.e. to reduce the volume by 1%)?

Solution

$$\Delta p = -K \left(\frac{\Delta V}{V} \right) = 2.2 \times 10^9 \text{ [Pa]} \times 0.01 = 2.2 \times 10^7 \text{ [Pa]}$$

Thus, a pressure of about 220 atmospheres would be necessary to reduce the volume of liquid water by 1%.

Problems

- 2-1 What is a fluid (in 20 words or less)?
- 2-2 A mineral oil is placed in a rigid cylindrical container that has a flat base and a flat leak-tight piston at the top in contact with the mineral oil. A force is applied to the piston, causing the pressure to increase by 15.8 MPa. As a result, the mineral oil undergoes a decrease in its volume and the volumetric strain, $-\Delta V/V$, is 0.00913. Calculate the compressibility and the bulk modulus of the mineral oil.
- 2-3 Hydraulic oil at a pressure of 2 MPa and having a bulk modulus of elasticity of 1.6 GPa fills a rigid cylinder of diameter 100 mm and height 500 mm. The ends of the cylinder are flat and the top end is a frictionless and leak-tight piston. If the force on the piston is increased so that the oil pressure rises to 8 MPa, calculate the distance that the piston would move.

Chapter 3. Hydrostatic Work and Power Transmission

Pascal's Law

It is possible to talk about the pressure at a point in a fluid by considering the force on a very small area centred about that point. Pascal's law¹ can be stated as follows:

Pressure at a point within a fluid at rest acts equally in all directions and at right angles to any surface on which it acts.

Hydraulic power transmission systems take advantage of this law in transmitting pressure from one part of a system to another. While differences in head may exist within a hydraulic power system, the overall height of the system is usually such that pressure differences between different levels within the fluid are negligible in comparison to the operating pressures that are used. The term 'hydrostatic machine' can be used for a machine in which Pascal's law applies to a good approximation within bodies of connected fluid. Figure 3-1 illustrates how forces depend on areas in a hydrostatic system or machine.

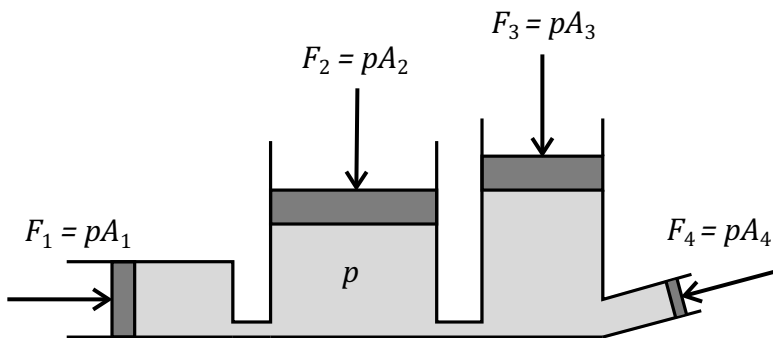


Figure 3-1 Schematic diagram of a general hydrostatic machine

¹ More generally, Pascal's law also states that if a small pressure change is caused at one point, or position, within a fluid at rest then the same pressure change occurs at all points within the fluid.

Continuity

In the system shown in Figure 3-2 an incompressible fluid is contained between two pistons. The piston on the left moves to the right applying a force to the fluid. The fluid applies a force to the piston on the right causing it to move to the right and upwards in the direction of the pipe.

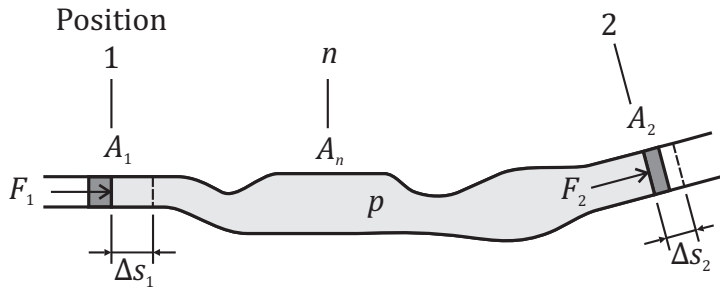


Figure 3-2 An incompressible fluid between pistons

Each piston face moves through the same volume, ΔV , such that

$$\Delta V = A_1 \Delta s_1 = A_2 \Delta s_2.$$

(3-1)

where

Δs = displacement of piston face m

ΔV = volume associated with Δs m³

A = area m²

Let Δt be the time for displacements Δs_1 and Δs_2

$$A_1 \frac{\Delta s_1}{\Delta t} = A_2 \frac{\Delta s_2}{\Delta t}.$$

where

Δt = time interval s

But $\frac{\Delta s_1}{\Delta t} = v_1$, the velocity of piston 1 and $\frac{\Delta s_2}{\Delta t} = v_2$, the velocity of piston 2, so

$$A_1 v_1 = A_2 v_2 = Q \quad (3-2)$$

where

v = velocity m/s

Q = volume flow rate m^3/s

Note that in Equation (3-2) Q is the volume flow rate just ahead of piston 1 or just behind piston 2. Equation (3-2) is known as the continuity equation. It applies equally well if there are no pistons at positions 1 and 2, as shown in Figure 3-3. The fundamental SI units for volume flow rate are m^3/s .

Mass and Volume Flow Rates

From Equation (3-2) the volume flow rate at a given cross section of a pipe is given by the product of the area and the average velocity,

$$Q = Av. \quad (3-3)$$

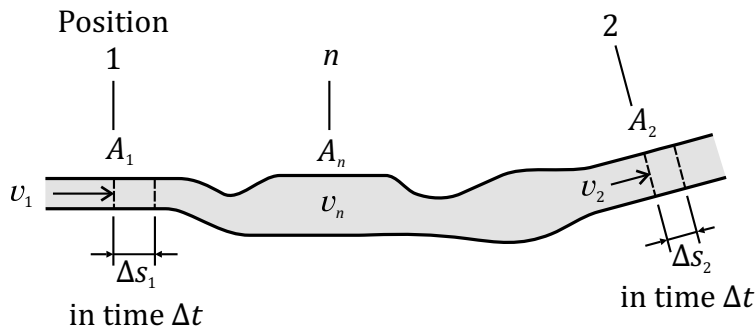


Figure 3-3 Flow of an incompressible fluid

If the density of the fluid ρ is known then the mass flow rate at the cross section is given by

$$\dot{m} = \rho Av. \quad (3-4)$$

where

\dot{m} = mass flow rate kg/s

ρ = density kg/m^3

The fundamental SI units for mass flow rate are kg/s.

Work, Flow Work and Power

Figure 3-2 and Figure 3-3 can help us to grasp the meaning of ‘flow work’. In Figure 3-2 work is done by one piston and on another piston, while in Figure 3-3 equivalent ‘flow work’ or ‘flow power’ occurs without the presence of pistons at the flow cross sections of interest. In a sense, the flow work for a volume of fluid that passes through a cross section is the energy being transferred through that cross section by the hydraulic fluid and the flow work per unit time is the power being transmitted by the hydraulic fluid.

The work done on the fluid by piston 1 in Figure 3-2 is given by

$$\Delta W_1 = F_1 \Delta s_1 = p A_1 \Delta s_1.$$

where

ΔW = increment of work J

F = force N

Similarly, the work done on piston 2 by the fluid is given by

$$\Delta W_2 = F_2 \Delta s_2 = p A_2 \Delta s_2.$$

From Pascal’s law and neglecting friction, the same pressure, p , acts at positions 1 and 2 and throughout the connected fluid. Hence

$$\Delta W_1 = \Delta W_2 = \Delta W = p \Delta V \quad (3-5)$$

where ΔV is the volume through which each piston sweeps. Equation (3-5) describes the work done by the left-hand piston on the fluid in Figure 3-2 and also the work done by the fluid on the right-hand piston. In each case the volume displacement at the piston is ΔV .

If the work at the pistons occurs over a small time interval Δt then the rate at which work is done is given by the work amount divided by the time interval:

$$P = \frac{\Delta W}{\Delta t} = p \frac{\Delta V}{\Delta t} = pQ \quad (3-6)$$

where

P = power W

p = pressure Pa

Q = volume flow rate m^3/s

t = time s

V = volume m^3

W = work J

Example 3-1 Piston Work

- What work is done by a piston of diameter 0.05 m when it exerts a pressure of 200 kPa and moves through a distance of 0.04 m?
- If the movement occurs in 2 s, what is the average rate of work, or power?

Solution

$$\begin{aligned} W &= pA\Delta s = 200 \times 10^3 \text{ [Pa]} \times \frac{\pi 0.05^2}{4} \text{ [m}^2\text{]} \times 0.04 \text{ [m]} \\ &= 15.7 \text{ J} \quad \text{answer a.} \end{aligned}$$

$$P = \frac{W}{\Delta t} = \frac{15.7 \text{ [J]}}{2 \text{ [s]}} = 7.85 \text{ W} \quad \text{answer b.}$$

Design and Operating Principles of a Hydraulic Jack

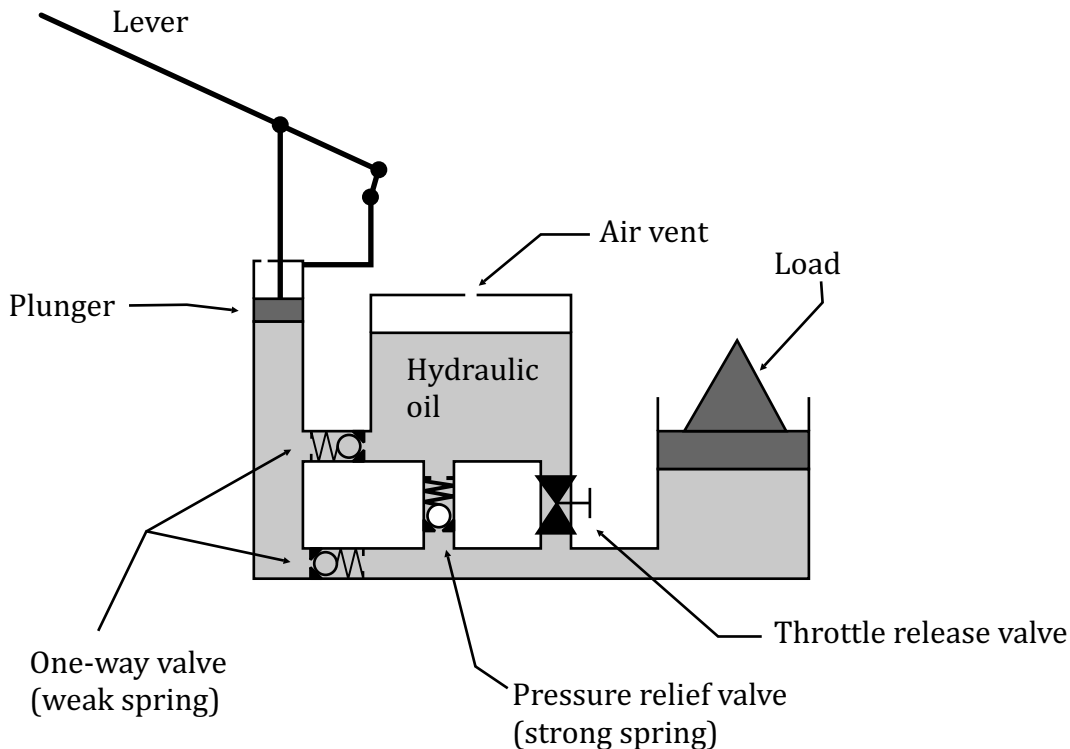


Figure 3-4 Schematic representation of a hydraulic jack

The hydraulic jack shown in Figure 3-4 is in a stationary state, with the throttle release valve closed. The oil in the space below the piston on the right is under pressure, owing to the weight of the piston and the load. If an operator applies downward pressure on the lever, the plunger on the left will move downwards in the cylinder. Oil will flow towards the right through the one-way valve in the bottom pipe and will cause the load to be raised.

The pressure relief valve is similar to a one-way valve, but has a very high spring force applied so that it will only open when the maximum desired system pressure is reached.

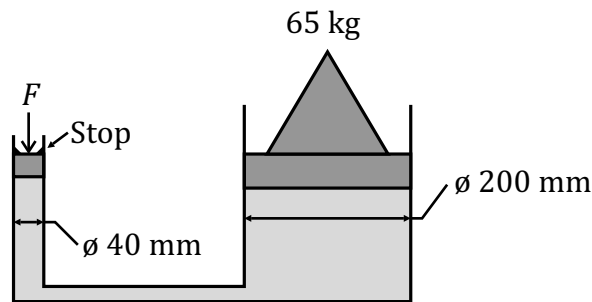
If the throttle release valve is opened, the load on the right will cause the oil to flow through the restriction of the throttle release valve into the hydraulic oil tank. In this way the load can be lowered smoothly.

If the left hand plunger is in its lowest position and is caused to rise by an operator lifting the lever, oil will flow into the left hand cylinder from the hydraulic oil tank (or reservoir).

Therefore the machine (a hand-operated hydraulic jack) allows the load to be raised or lowered in a controlled manner. The pressure relief valve provides protection against over pressure.

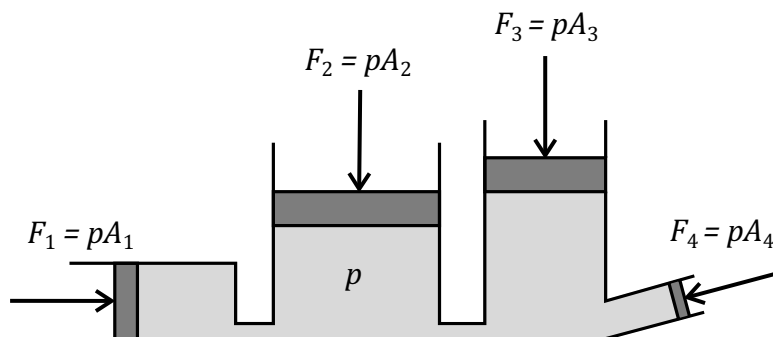
Problems

3-1



In the arrangement shown above, a mass of 65 kg is to be raised through a distance of 30 mm by the application of a force, F , to the small piston on the left. The masses of the two pistons are such that they apply the same pressure to the hydraulic fluid owing to their weight (the two pistons have the same mass or weight per unit area). Calculate the force that must be applied to the left hand piston and the distance through which it must travel.

3-2

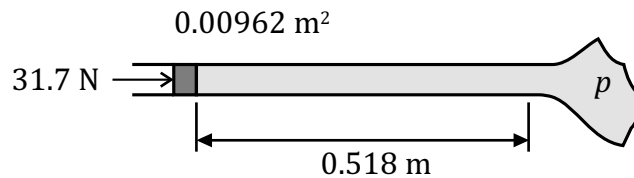


In the figure above, take it that the four pistons shown are massless, frictionless and leak tight and enclose liquid water in the system shown. The pressure within the system is 2.35

MPa gauge and all four pistons are held at rest by the application of the four forces shown. Area A_3 is 0.0303 m^2 and the diameter of piston 1 equals 75% of the diameter of piston 3. The diameter of piston 4 is 64.2 mm and $F_2 = 2.31 F_3$. Calculate the value of each of the four forces shown and the diameter of piston 2.

- 3-3 Water flows in a pipe that has an internal diameter of 50 mm at a mean velocity of 1.6 m/s. Calculate the volume flow rate through the pipe. If the internal diameter of the pipe reduces to 30 mm further downstream, calculate the mean velocity at that position for the same flow rate through the pipe.

3-4



The diagram above shows a piston that applies a constant force of 31.7 N to hydraulic fluid as it moves through a distance of 0.518 m in 1.62 seconds. The area of the piston is 0.00962 m^2 . Calculate the pressure of the hydraulic fluid that resists the motion, the work done and the power required to push the piston.

Chapter 4. Positive Displacement Pumps

A positive displacement pump is a pump that discharges a fixed volume of pressurized fluid for each rotation of the input shaft or for each cycle in the case of a reciprocating pump. The standard hydraulic symbol for a pump is shown in Figure 4-1.

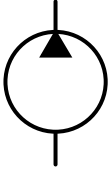


Figure 4-1 Hydraulic pump symbol

Simple Reciprocating Pump

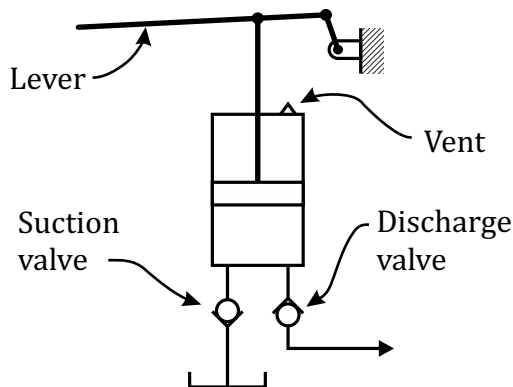


Figure 4-2 Schematic representation of a hydraulic hand pump

A reciprocating positive displacement pump, as shown in Figure 4-2, discharges a fixed amount of pressurized fluid for each cycle of the piston. The cycle is made up of an intake stroke and a discharge stroke. The displacement of the pump per cycle is equal to the product of the cross-sectional area of the cylinder and the stroke, Equation (4-1).

$$V_{\text{displ}} = A_{\text{cyl}} L_{\text{stroke}} \quad (4-1)$$

where

V_{displ} = pump displacement per cycle

A_{cyl} = cross-sectional area of cylinder

L_{stroke} = pump stroke

Example 4-1 Hand Pump Displacement

If the piston of a hand-operated pump has a diameter of 30 mm and a stroke of 20 mm, determine the displacement per cycle.

Solution

$$A = \frac{\pi d^2}{4} = \frac{\pi (0.03)^2 [\text{m}^2]}{4} = 0.7068 \times 10^{-3} \text{ m}^2$$

Hence,

$$V_{\text{displ}} = 0.7068 \times 10^{-3} [\text{m}^2] (0.02 [\text{m}]) = 14.14 \times 10^{-6} \text{ m}^3$$

Swash Plate Pump

The swash plate pump, Figure 4-3, contains a rotating piston block that houses a number of pistons. The piston ends press against a swash plate that causes the pistons to undergo a reciprocating motion. Note the following features:

A swash plate, which is a flat thrust plate at an angle to the axis of rotation of the pump

Check valves for intake and discharge

A fixed valve plate that contains two curved slots: one to distribute the intake fluid and the other to collect the discharge fluid

A piston block, pistons, a shoe plate and piston shoes, all of which rotate with the shaft

A spring that pushes the shoe plate against the swash plate and the piston block against the valve plate

A drain port so that any leakage past the pistons can be returned to the tank of the hydraulic system

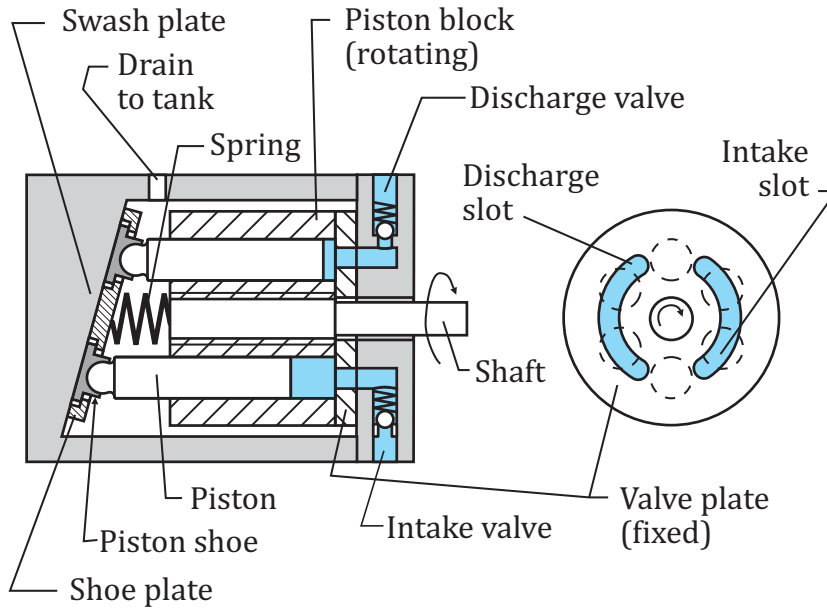


Figure 4-3 Schematic diagram of a swash plate piston pump

The displacement of the swash plate pump per revolution of the pump drive shaft is given by

$$V_{\text{displ}} = n_{\text{cyl}} A_{\text{cyl}} L_{\text{stroke}} \quad (4-2)$$

where

n_{cyl} = number of cylinders

Rotary Positive Displacement Pumps

Rotary positive displacement pumps, e.g. gear pumps or vane pumps, discharge a fixed amount of pressurized fluid for each rotation of the input shaft.

Gear Pumps

In a gear pump, Figure 4-4, the fluid is transported from the intake port to the discharge port. Note carefully the directions of rotation of the gears and the path followed by the fluid. The displacement of the gear pump per revolution of the drive shaft can be calculated from the dimensions and geometry of the gears and the housing.

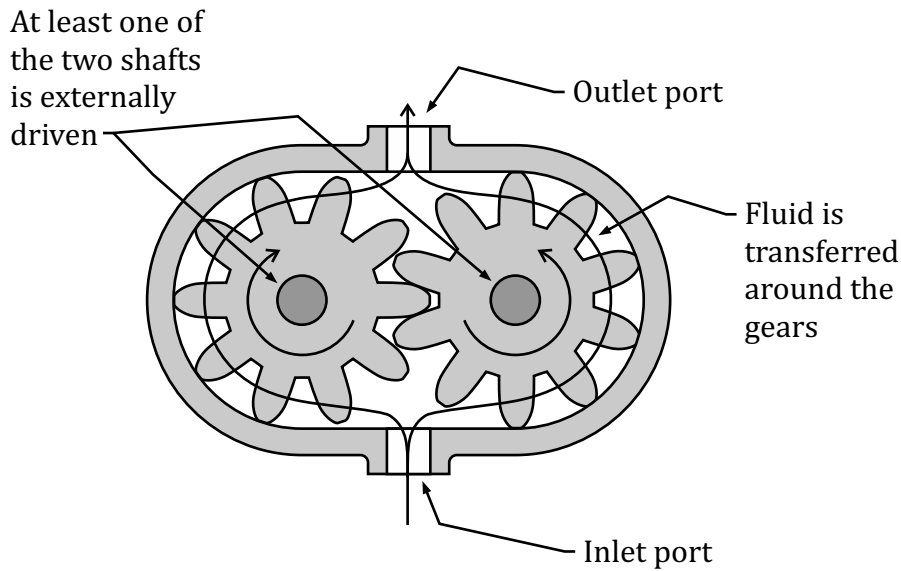


Figure 4-4 Schematic representation of a gear pump

Vane Pump

A basic type of vane pump is shown schematically in Figure 4-5. Volumes of fluid are enclosed on the inlet side of the pump and transported to the discharge side.

As with other rotary positive displacement pumps, the displacement per rotation of the drive shaft can be calculated from the dimensions and geometry of the housing, rotor and vanes. Note that the casing recesses in Figure 4-5 are necessary as the fluid is incompressible and the enclosed volume should not change as it is brought from the inlet to the outlet side of the pump.

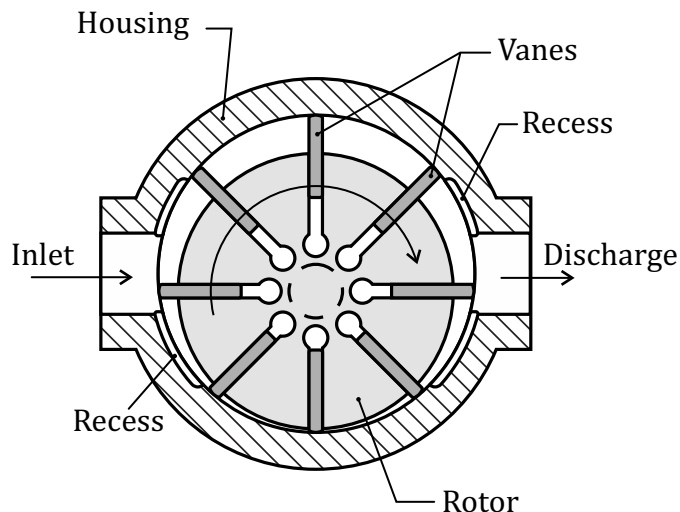


Figure 4-5 Schematic diagram of a vane pump

Ideal Flow Rate of a Positive Displacement Pump

The ideal flow rate of a positive displacement pump can be calculated as the product of the displacement of the pump per rotation (or cycle) and the speed in rotations (or cycles) per unit time,

$$Q_{\text{ideal}} = V_{\text{displ}} N \quad (4-3)$$

where

Q_{ideal}	=	ideal volume flow rate	m^3/s
V_{displ}	=	pump displacement per revolution (or cycle)	$\frac{\text{m}^3}{\text{rev}}$
N	=	speed (rotations or cycles per unit time)	$\frac{\text{rev}}{\text{s}}$

Example 4-2 Ideal Flow Rate of a Vane Pump

A vane pump having a displacement of 25 mL per revolution runs at a speed of 1750 r.p.m. What flow rate would it ideally produce? Express the answer in m^3/s and also in litres per hour.

Solution

$$V_{\text{displ}} = 25 \frac{\text{mL}}{\text{rev}} = 25 \times 10^{-6} \frac{\text{m}^3}{\text{rev}}$$

$$N = 1750 \frac{\text{revs}}{\text{min}} = \frac{1750}{60} \frac{\text{rev}}{\text{s}} = 29.17 \frac{\text{rev}}{\text{s}}$$

$$\begin{aligned} Q_{\text{ideal}} &= V_{\text{displ}} N = 25 \times 10^{-6} \left[\frac{\text{m}^3}{\text{rev}} \right] \times 29.17 \left[\frac{\text{rev}}{\text{s}} \right] \\ &= 729.3 \times 10^{-6} \frac{\text{m}^3}{\text{s}} \end{aligned}$$

or

$$Q_{\text{ideal}} = 729.3 \times 10^{-6} \left[\frac{\text{m}^3}{\text{s}} \right] \times 1000 \left[\frac{\text{L}}{\text{m}^3} \right] \times 3600 \left[\frac{\text{s}}{\text{h}} \right]$$

$$= 2625 \frac{\text{L}}{\text{h}}$$

Efficiency of Hydraulic Pumps

Mechanical Efficiency of a Pump

The mechanical efficiency of a pump is defined as

$$E_{\text{pump, mech}} = \frac{Q_{\text{ideal}} \Delta p}{P_{\text{actual}}} \quad (4-4)$$

where

$E_{\text{pump, mech}}$ = pump mechanical efficiency

Q_{ideal} = ideal volume flow rate m^3/s

Δp = pressure difference across pump Pa

P_{actual} = actual pump input power W

It is important to note that the flow rate used in the definition of the mechanical efficiency, Equation (4-4), is the ideal flow rate, based on the theoretical displacement of the pump.

Volumetric Efficiency of a Pump

An ideal positive displacement pump would have a fixed displacement and would always discharge that amount of fluid for each rotation (or cycle, in the case of a reciprocating pump). In practice, there is usually some leakage backwards through the pump so that the amount of fluid discharged is less than the ideal amount (corresponding to the theoretical displacement of the pump). The leakage within a pump depends on the pressure difference: higher leakage rates occur when the pump is operating against higher pressure differences.

The volumetric efficiency of a pump is defined as

$$E_{\text{pump, vol}} = \frac{Q_{\text{actual}}}{Q_{\text{ideal}}} \quad (4-5)$$

where

$E_{\text{pump, vol}}$ = pump volumetric efficiency

Q_{actual} = actual volume flow rate m^3/s

Overall Efficiency of a Pump

The overall efficiency of a pump, Equation (4-6), is the product of the mechanical and volumetric efficiencies. Some typical values of hydraulic pump efficiencies are shown in Table 4-1.

$$\begin{aligned}
 E_{\text{pump, overall}} &= E_{\text{pump, mech}} \times E_{\text{pump, vol}} \\
 &= \frac{Q_{\text{ideal}} \Delta p}{P_{\text{actual}}} \frac{Q_{\text{actual}}}{Q_{\text{ideal}}} = \frac{Q_{\text{actual}} \Delta p}{P_{\text{actual}}}
 \end{aligned}
 \tag{4-6}$$

where

$E_{\text{pump, overall}}$ = pump overall efficiency

Table 4-1 Typical hydraulic pump efficiencies

Pump Type	Mechanical Effic. /[%]	Volumetric Effic. /[%]	Overall Effic. /[%]
Gear	90–95	80–90	72–86
Vane	90–95	82–92	74–87
Piston	90–95	90–98	81–93

Example 4-3 Gear Pump Efficiency

A gear pump operates at 1,500 r.p.m. with a pressure difference of 15 MPa and has a displacement of 12 mL per revolution. The measured flow rate is 0.948 m^3/h . The mechanical power input to drive the pump is 4.77 kW. Calculate the mechanical efficiency, the volumetric efficiency and the overall efficiency of the pump.

Solution

The ideal flow rate of the pump is given by

$$\begin{aligned}
 Q_{\text{ideal}} &= 12 \times 10^{-6} [\text{m}^3] \times \frac{1500}{60} [\text{s}^{-1}] \\
 &= 300 \times 10^{-6} \frac{\text{m}^3}{\text{s}} \\
 E_{\text{pump, mech}} &= \frac{Q_{\text{ideal}} \Delta p}{P_{\text{actual}}} \\
 &= \frac{300 \times 10^{-6} \left[\frac{\text{m}^3}{\text{s}} \right] \times 15 \times 10^6 \left[\frac{\text{N}}{\text{m}^2} \right]}{4.77 \times 10^3 [\text{W}]} \\
 &= 0.943 = 94.3\% \\
 Q_{\text{actual}} &= \frac{0.984 \left[\frac{\text{m}^3}{\text{s}} \right]}{3600} \\
 &= 273.3 \times 10^{-6} \frac{\text{m}^3}{\text{s}} \\
 E_{\text{pump, vol}} &= \frac{Q_{\text{actual}}}{Q_{\text{ideal}}} = \frac{273.3}{300} = 0.911 = 91.1\% \\
 E_{\text{pump, overall}} &= E_{\text{pump, mech}} \times E_{\text{pump, vol}} \\
 &= 0.943 \times 0.911 = 0.859 = 85.9\%
 \end{aligned}$$

Problems

- 4-1 An ideal hydraulic gear pump has a displacement of 25 mL per revolution and operates at 3,000 r.p.m. Calculate the flow rate produced by the pump.
- 4-2 An ideal swash plate pump operates at 1,500 r.p.m. and has eight pistons. Each piston has a diameter of 1 cm and a stroke of 2 cm. Calculate the volume flow rate of the pump.
- 4-3 Suppose each of the pumps in questions 1 and 2 above is connected to a hydraulic cylinder with a bore of 75 mm and a

stroke of 500 mm: how long would it take for the cylinder to extend fully in each case? The volume of fluid required to extend the cylinder equals the product of the cross-sectional area and the stroke, $V = \frac{\pi d^2}{4} L_{\text{stroke}}$.

- 4-4 The following equations relating to the efficiency of pumps are given:

$$E_{\text{pump, mech}} = \frac{Q_{\text{ideal}} \Delta p}{P_{\text{actual}}}$$

$$E_{\text{pump, vol}} = \frac{Q_{\text{actual}}}{Q_{\text{ideal}}}$$

$$E_{\text{pump, overall}} = E_{\text{pump, mech}} \times E_{\text{pump, vol}}$$

$$= \frac{Q_{\text{actual}} \Delta p}{P_{\text{actual}}}$$

A vane pump has a displacement of 3.61 mL per revolution and operates at 1,450 r.p.m. with a pressure difference of 791 kPa. The measured flow rate is 4.79 L/min. The measured mechanical power input to drive the pump is 77.4 W. Calculate the mechanical efficiency, the volumetric efficiency and the overall efficiency of the pump.

Chapter 5. Hydraulic System Components

Check Valve and Pressure Relief Valve

Figure 5-1 illustrates the construction of a spring loaded one-way valve. Figures 5-2 and 5-3 are the schematic representations of check valves with and without springs respectively.

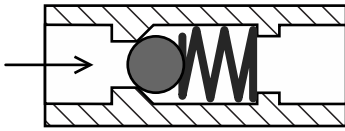


Figure 5-1 Simple one-way or check valve



Figure 5-2 Check valve symbol (with spring)



Figure 5-3 Check valve symbol (without spring)

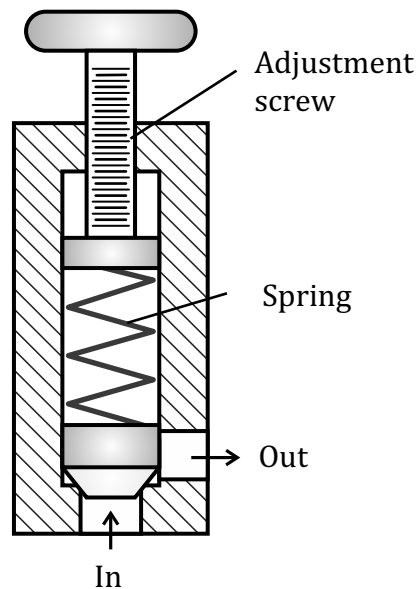


Figure 5-4 Operating principle of a pressure relief valve

Figure 5-4 illustrates the construction principles of a pressure relief valve. The pressure relief valve can be adjusting by turning a knob. The resulting force, exerted by the compressed spring, determines the pressure at which the valve will open. The standard symbol for a pressure relief valve is shown in Figure 5-5.

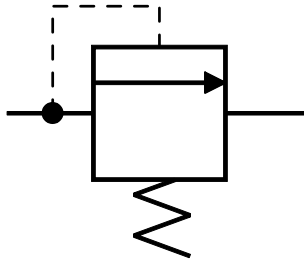


Figure 5-5 Pressure relief valve symbol

Hydraulic Cylinders or Rams

Hydraulic cylinders or rams are linear actuators that can be used to apply high forces and provide pushing or pulling movements. A typical construction for a hydraulic ram is illustrated in Figure 5-6.

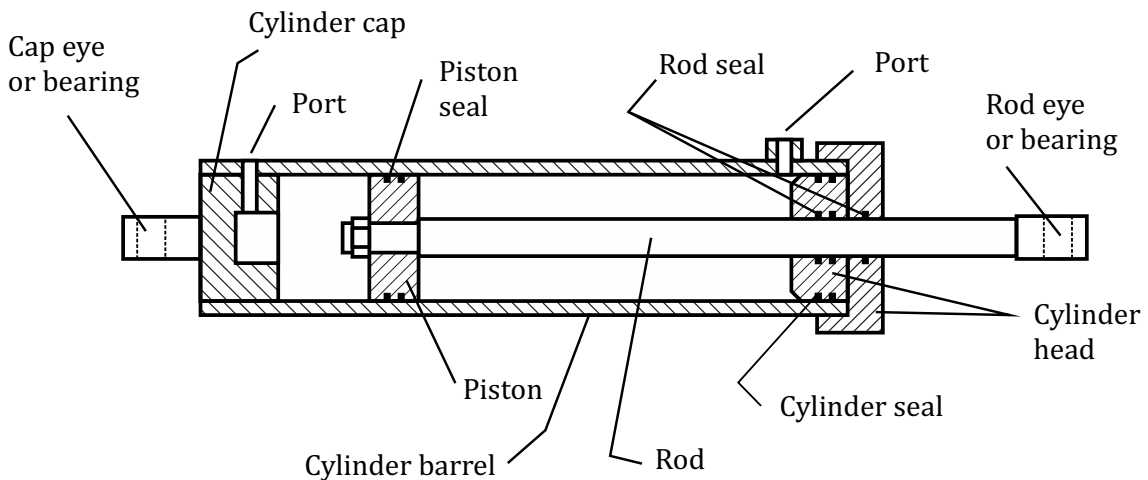


Figure 5-6 Typical construction of a commercial hydraulic cylinder

A ram is double acting if it can apply force while extending and while retracting. If the ram can only apply force in one direction it is said to be single acting—some external means will be required to return the piston to its original position. A hydraulic ram may have a rod that extends from one end only or may have a rod that extends from both ends. These configurations are described as

'single rod' and 'double rod' respectively. Figures 5-7 to 5-9 are the schematic representations of three possible configurations.

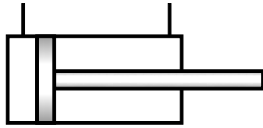


Figure 5-7 Double acting single rod cylinder



Figure 5-8 Single acting cylinder

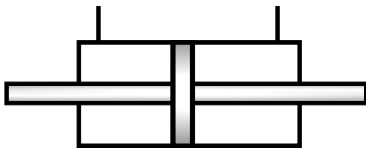


Figure 5-9 Double acting double rod cylinder

Cylinder Force Calculations

The main parameters of a hydraulic cylinder are the bore, the cylinder rod diameter and the stroke of the piston. These parameters determine the volumes of hydraulic fluid required to fill the cylinder. The cylinder bore and the piston rod diameter determine the thrust or tension generated at a given operating pressure.

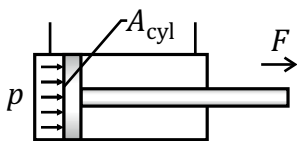


Figure 5-10 Force calculation for the cap end of a cylinder

Figure 5-10 shows the force produced by the pressure acting on the piston at the cap end of a single-acting cylinder. It is calculated as

$$F = pA_{cyl} \quad (5-1)$$

where

F = force produced in the piston rod

p = pressure

A_{cyl} = cylinder cross-sectional area

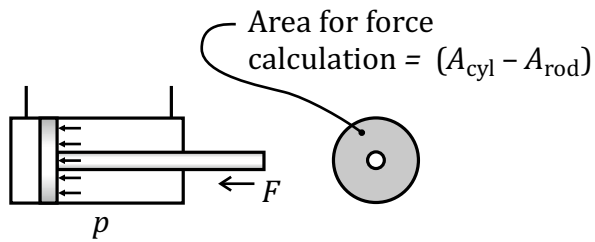


Figure 5-11 Force calculation for the rod end of a cylinder

Figure 5-11 shows the force produced by the pressure acting on the piston at the rod end of a single-acting cylinder. It is a tensile force within the rod and is calculated as

$$F = p(A_{\text{cyl}} - A_{\text{rod}}) \quad (5-2)$$

where

A_{rod} = rod cross-sectional area

Example 5-1 Cylinder Forces

- A single-rod double-acting cylinder with a bore of 160 mm is required to exert a force of 200 kN on the outstroke. Determine the required oil pressure in the cap end of the cylinder.
- If the diameter of the cylinder rod of the same double-acting cylinder is 90 mm, determine the pulling force of the rod on the retract stroke with the same oil pressure, this time in the rod end of the cylinder.

Solution

$$\begin{aligned} A_{\text{cyl}} &= \frac{\pi d^2}{4} = \frac{\pi 0.16^2}{4} [\text{m}^2] \\ &= 0.02011 \text{ m}^2 \end{aligned}$$

$$\begin{aligned}
 p &= \frac{F}{A_{\text{cyl}}} = \frac{200 \times 10^3}{0.02011} \left[\frac{\text{N}}{\text{m}^2} \right] \\
 &= 9.945 \times 10^6 \text{ Pa} \\
 &= 9.95 \text{ MPa}
 \end{aligned}$$

a.

$$\begin{aligned}
 A_{\text{rod}} &= \frac{\pi d^2}{4} = \frac{\pi 0.09^2}{4} [\text{m}^2] \\
 &= 0.006362 \text{ m}^2
 \end{aligned}$$

$$A_{\text{eff}} = A_{\text{cyl}} - A_{\text{rod}}$$

$$= 0.02011 [\text{m}^2] - 0.00636 [\text{m}^2] = 0.01375 \text{ m}^2$$

$$F = pA_{\text{eff}} = 9.945 \times 10^6 [\text{Pa}] \times 0.01375 [\text{m}^2]$$

$$= 136,743 \text{ N} = 136.7 \text{ kN}$$

b.

Directional Control Valves

In order to be able to switch between extending or retracting a hydraulic cylinder, a directional control valve is required. In the valve symbols shown in Figure 5-12 and Figure 5-13 each square box represents a possible configuration of flow paths between the four ports of the valve.

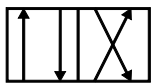


Figure 5-12 Two position directional control valve symbol

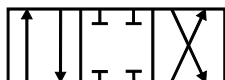


Figure 5-13 Three position direction control valve

Many different methods of actuating a valve are possible, some of which are illustrated by the standard symbols shown in Figure 5-14.

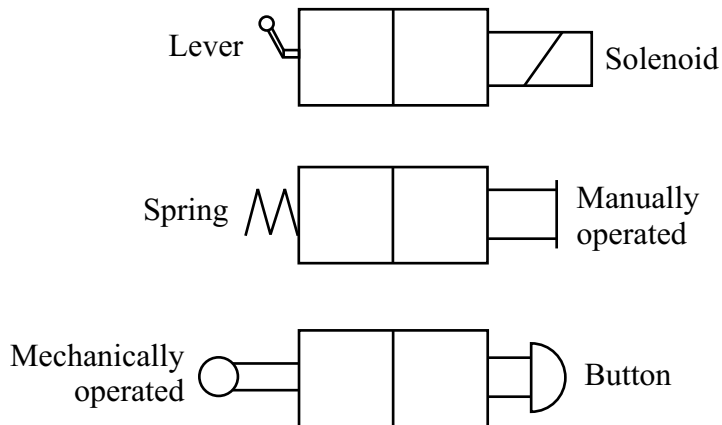


Figure 5-14 Symbolic representations of different valve actuation types

Directional Spool Valve

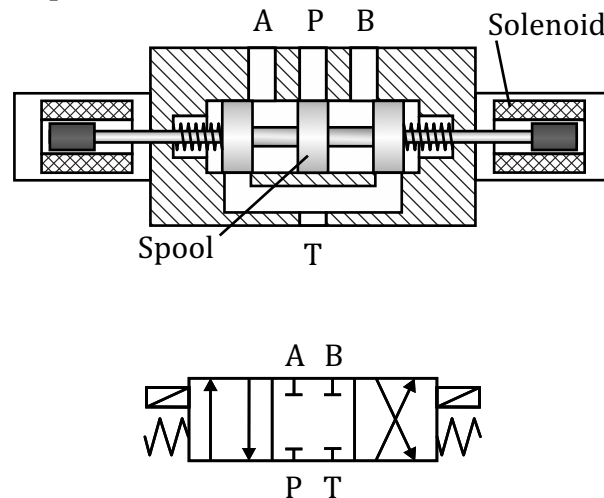


Figure 5-15 Three-position directional spool valve

Various designs for directional control valves are possible, but a common arrangement makes use of a spool, which is a solid object in the form of a series of cylinders of different diameters. Figure 5-15 includes a schematic illustration of the construction of a three position, spring return, solenoid actuated directional control spool valve as well as the standard symbolic representation of such a device. Letters A and B label the connections to the device being controlled, such as a hydraulic ram or a hydraulic motor. P labels

the connection from the pump and T labels the connection to the tank.

The Hydraulic Circuit Diagram

Figure 5-16 is an example of a hydraulic circuit for a double acting cylinder and illustrates how a directional control valve is connected. This circuit is made up entirely of standard symbols for hydraulic components such as the cylinder, the pump, the pressure relief valve, a filter and the hydraulic tank or reservoir. It should be noted that the tank is shown three times in the circuit diagram, although there is only one hydraulic fluid tank. This avoids the clutter that would be caused if all return lines were shown connected to just one symbol for the tank.

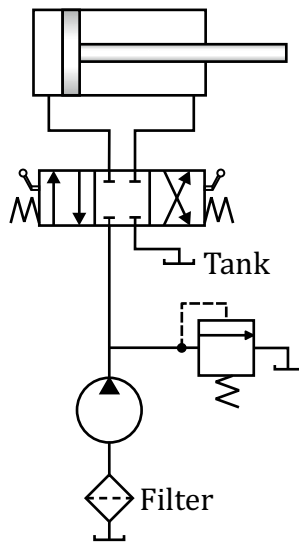


Figure 5-16 Basic hydraulic circuit for a double acting single rod cylinder

Problems

5-1 Size a cylinder to lift 5 tonne through 250 mm. This involves specifying the diameter of the cylinder and specifying its stroke. The force developed is related to the pressure by the expression $p = F/A$. Hence, $A = F/p$. One of the following standard bores should be used: 25, 32, 40, 50, 63, 80, 100, 125, 160, 200, 250, 320 mm.

For our purposes, anything in the range of 70 to 210 bar is acceptable.

- 5-2 Size a pump to drive an 80 mm bore cylinder at 100 mm/s.
- 5-3 What hydraulic power is required for the previous example if the operating pressure is 100 bar?
- 5-4 In a double-acting single-rod hydraulic cylinder the inside diameter of the cylinder is 120 mm and the rod diameter is 40 mm. Calculate the maximum force that can be developed by the cylinder on the outstroke and on the return stroke if the hydraulic pump can provide a maximum pressure of 8.5 MPa. Note: in each case the force is the product of the supply pressure and the effective area of the piston.

Chapter 6. Design of Hydraulic Systems

Control of hydraulic systems is introduced through the topic of flow control. The principles of hydraulic system design are introduced by means of two worked examples, for a lift and a press respectively. It is important to keep in mind that in design there are always choices and judgements to be made.

Flow Control

Flow Restriction and Flow Control

The flow of fluid in a hydraulic circuit can be reduced by means of a flow restrictor, Figure 6-1. This causes a pressure drop in the direction of flow. By means of a variable area valve, the flow rate can be controlled. Generally, the flow rate through a restrictor is proportional to the effective flow area and to the square root of the pressure drop across it.



Figure 6-1 Symbols for a fixed area flow restrictor and a variable area flow restrictor

Power is always dissipated in a flow restrictor. This has the effect of increasing the temperature of the hydraulic fluid.

Hydraulic Ram Speed Control

The speed of a hydraulic ram can be controlled by the use of a flow control valve in conjunction with a pressure relief valve, as illustrated in Figures 6-2 to 6-4. Three possible arrangements are illustrated, named ‘meter-in’, ‘meter out’ and ‘bleed-off’ flow control.

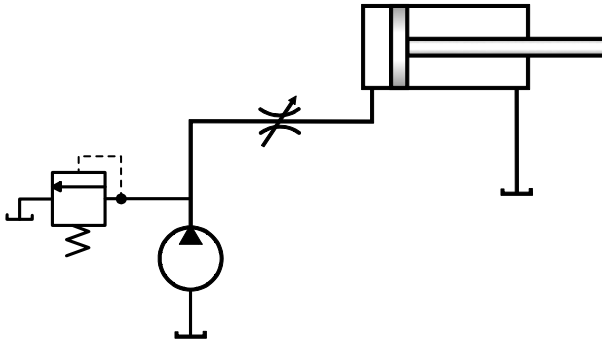


Figure 6-2 Meter-in speed control metering circuit

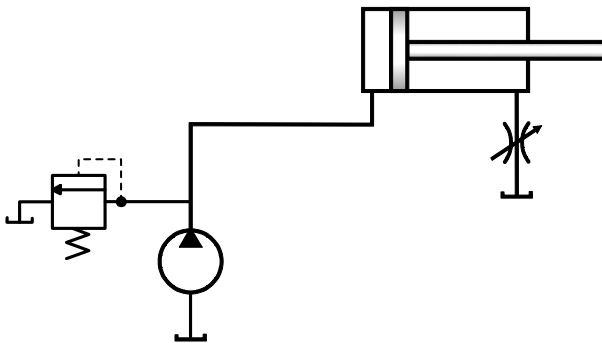


Figure 6-3 Meter-out speed control metering circuit

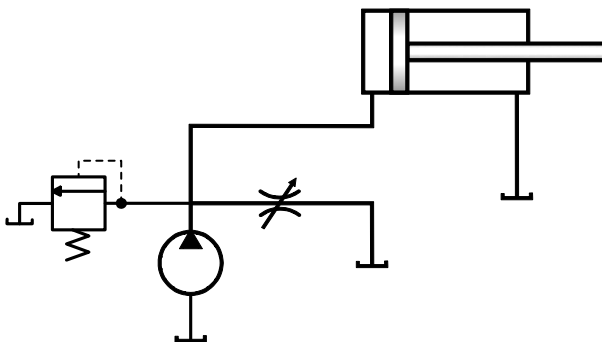


Figure 6-4 Bleed-off speed control metering circuit

With meter-in and meter-out flow control, a proportion of the flow is forced through the pressure relief valve. With meter-in control the force available to drive the load is greater. Meter-out control creates a back pressure on the rod side of the piston which can help prevent lunging if the load decreases rapidly or reverses.

With bleed-off control, fluid is bled-off through the restrictor at a lower pressure than the pressure relief valve setting so there is less power wastage. Bleed-off control can only be used where the load opposes the motion.

Power wastage is inevitable when metering flow control is used. An alternative would be to use a hydraulic pump whose displacement can be varied. Figure 6-5 is the standard symbol for such a pump.

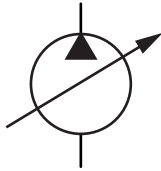


Figure 6-5 Standard hydraulic symbol for a variable displacement pump

Hydraulic System for a Lift

Example 6-1 Component Sizing for a Hydraulic Lift

A hydraulic lift is required to raise loads of up to 20 tonne through a distance of 4 m in 30 s. Select a reasonable operating pressure and use this to size a suitable cylinder and pump. Do not exceed a running pressure of 120 bar. Determine the exact running pressure and specify a relief valve setting. Assume an overall pump efficiency of 60% and estimate the required electric motor power output.

The following standard bores should be used:

25, 32, 40, 50, 63, 80, 100, 125, 160, 200, 250, 320 mm.

Solution

First, to find the required cylinder bore, use $A = F/p$, and a trial pressure of 120 bar.

$$F = 20 \times 1,000 \text{ [kg]} \times 9.81 \text{ [m s}^{-2}\text{]} = 196,200 \text{ N}$$

$$p = 120 \times 10^5 \frac{\text{N}}{\text{m}^2}$$

$$A = \frac{1.962 \times 10^5 \text{ [N]}}{120 \times 10^5 \text{ [N m}^{-2}\text{]}} = 0.0164 \text{ m}^2$$

and

$$D = \sqrt{4A/\pi} = \sqrt{4 \times 0.0164/\pi} \text{ m} = 0.1445 \text{ m} = 145 \text{ mm.}$$

We must choose a standard size, the nearest ones being 125 and 160 mm.

In order to keep the running pressure below 120 bar, we choose the larger (160 mm) bore.

Running pressure

$$p_{\text{run}} = \frac{F}{A} = \frac{1.962 \times 10^5 \text{ [N]}}{\frac{\pi 0.160^2}{4} \text{ [m}^2\text{]}} = 9.758 \times 10^6 \frac{\text{N}}{\text{m}^2} = 97.6 \text{ bar}$$

It would be reasonable to set the pressure relief valve at a pressure that is 10% higher than the pressure required by the maximum load. Hence, the setting would be

$$p_{\text{rv}} = 97.6 \text{ [bar]} \times 110\% = 107 \text{ bar.}$$

Pump size:

$$\begin{aligned} Q &= A \times v = \left(\pi \frac{0.160^2}{4} \right) \text{ [m}^2\text{]} \left(\frac{4}{30} \right) \text{ [m s}^{-1}\text{]} \\ &= 0.00268 \frac{\text{m}^3}{\text{s}} = 2.68 \frac{\text{litre}}{\text{s}} = 161 \frac{\text{litre}}{\text{min}} \end{aligned}$$

Electric motor size:

$$\begin{aligned} \text{Hydraulic power} &= p \times Q = 107 \times 10^5 \times 0.00268 \text{ [W]} \\ &= 28,676 \text{ W} = 28.7 \text{ kW} \end{aligned}$$

Estimated electric motor power

$$= \frac{\text{Hydraulic output power}}{\text{Overall pump efficiency}} = \frac{28.7 \text{ [kW]}}{0.6} = 47.8 \text{ kW}$$

The next standard motor power rating above this would be selected, e.g. 50 kW.

Hydraulic Press

Figure 6-6 is a possible circuit for a hydraulic press. With the variable area flow restrictor fully open only a slight pressure will

be applied to the piston. The pressure can be increased progressively by gradually closing the restrictor.

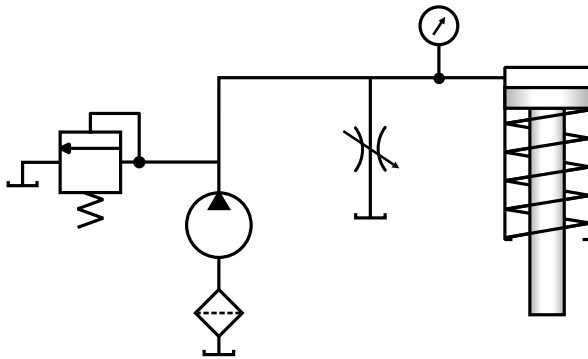


Figure 6-6 Circuit for a hydraulic press

Figure 6-7 shows how a hydraulic cylinder might be incorporated into a load frame of a press.

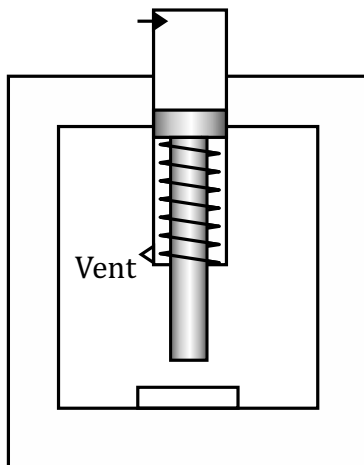


Figure 6-7 Load frame of a hydraulic press

Example 6-2 Design of a Simple Hydraulic Press

A 25 tonne hydraulic press based on a single-acting spring-return cylinder is required to stroke through 300 mm in 6 seconds. If the maximum allowable pressure rating is 150 bar

- (a) Specify a suitable standard cylinder bore and calculate the required operating pressure.
- (b) Size a suitable pump.

The following standard bores should be considered:

25, 32, 40, 50, 63, 80, 100, 125, 160, 200, 250 & 320 mm.

Solution

A suitable design for the press might be as follows:

(a) Specify a suitable standard bore for the cylinder

The force exerted by the press is 25 tonne weight (strictly, the weight of a mass of 25 tonne). This must be converted to newtons.

1 tonne weight = $1 \times 1,000 \times 9.81$ N, where 9.81 m/s^2 is the acceleration due to gravity. Hence

$$F = 25 \times 1,000 \times 9.81 = 245,250 \text{ N.}$$

The maximum pressure of 150 bar must be converted to N/m^2 . Hence

$$p = 150 \times 10^5 \text{ Nm}^{-2}.$$

The force is also equal to the product of the pressure by the area of the piston. Hence in terms of the piston diameter or cylinder bore, d ,

$$\frac{\pi d^2}{4} p = F$$

$$\text{or } d = \sqrt{\frac{4F}{\pi p}}.$$

Hence

$$d = \sqrt{\frac{4 \times 245,250}{\pi \times 150 \times 10^5}} \sqrt{\frac{[\text{N}]}{[\text{Nm}^{-2}]}} = 0.1443 \text{ m.}$$

Therefore a standard bore of 0.160 m could be used. The required operating pressure would therefore be

$$p_{\text{op}} = 150 [\text{bar}] \times \left(\frac{0.1443}{0.16}\right)^2 = 122.01 \text{ bar}$$

The pressure relief valve would be set at perhaps 10% or 10 bar higher, whichever is the smaller. Hence the pressure relief valve would be set at about 132 bar. Note that 132 bar is less than the stated maximum allowable pressure rating.

(b) Size a suitable pump

The volume flow rate into the cylinder is the product of the velocity and the cross-sectional area, i.e. $v \times \pi D^2/4$. The velocity, v , is

$$v = \frac{0.3}{6} \text{ ms}^{-1} = 0.05 \text{ ms}^{-1}.$$

The volume flow rate is given by

$$Q = v \pi \frac{d^2}{4}.$$

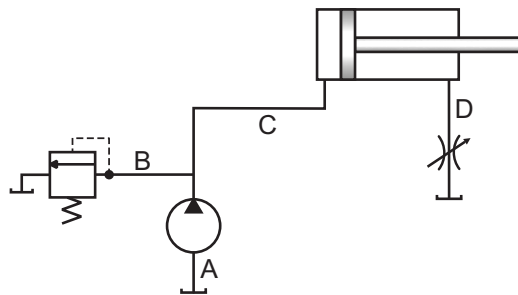
Therefore the volume rate in litres per minute is

$$\begin{aligned} Q &= 0.05 \pi \frac{0.16^2}{4} \left[\frac{\text{m}^3}{\text{s}} \right] \times 60 \left[\frac{\text{s}}{\text{min}} \right] \times 1,000 \left[\frac{\text{litre}}{\text{m}^3} \right] \\ &= 60.32 \text{ litre min}^{-1}. \end{aligned}$$

A suitable pump would be capable of operating at up to 132 bar and with a flow rate of about 61 litres per minute.

Problems

6-1



The diagram above shows a hydraulic power transmission system with meter-out flow control. The hydraulic cylinder has a bore of 75 mm, while the diameter of the cylinder rod is 30 mm. The hydraulic pump produces a flow rate of 0.754

m^3/hour . For a given load and setting of the flow restrictor, the volume flow rate out of the cylinder at D is $0.452 \text{ m}^3/\text{hour}$. Calculate the volume flow rate at positions A, B and C assuming the pump is ideal. Also, calculate the velocity of the piston.

- 6-2 A single-acting single-rod hydraulic cylinder is required for a pallet-lifter. The hydraulic ram is required to exert a force of 165 kN and must stroke through a distance of 1.4 m in 3 seconds. One of the following standard bores should be used: 25, 32, 40, 50, 63, 80, 100, 125, 160, 200, 250, 320 mm. The pressure relief valve should be set to a pressure that is 10% higher than that required by the design load. Select an appropriate cylinder bore. Specify the setting of the pressure relief valve and calculate the flow rate required from the hydraulic pump. Start with a trial pressure of 12 MPa. The running pressure should not exceed this value.

Chapter 7. Viscosity

Viscosity is a property of a fluid, which quantifies how strongly that fluid resists relative motion. A fluid that has a high resistance to relative motion, such as crude oil, would have a high viscosity and would be described as being viscous. A fluid that presented no resistance to relative motion would be described as inviscid, but under normal circumstances all fluids have a finite viscosity, however small.

Figure 7-1 illustrates a flat plate of negligible density sliding horizontally over a layer of liquid that covers a horizontal surface. Figure 7-2 illustrates how the velocity varies within the fluid layer. The fluid velocity below the plate is zero at the stationary surface and increases linearly with distance from the stationary surface to match the velocity, v , at the underside of the moving plate.

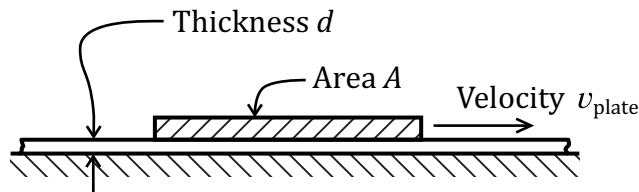


Figure 7-1 Plate sliding on a film of liquid

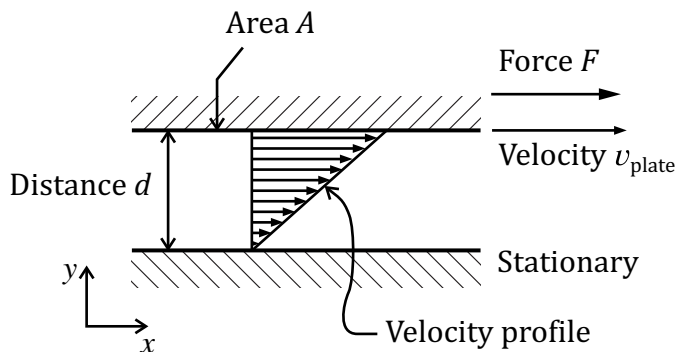


Figure 7-2 Velocity profile within a fluid undergoing shear motion

It is found that the force F required to maintain the velocity v_{plate} is proportional to the surface area A of the moving plate and to the velocity v_{plate} , while it is inversely proportional to the distance d between the stationary surface and the moving plate. This can be written as

$$F \propto \frac{Av_{\text{plate}}}{d}.$$

This relationship of proportionality can be made into an equation by including a constant of proportionality, μ , as follows:

$$F = \mu \frac{Av_{\text{plate}}}{d} \quad (7-1)$$

where

d	=	distance	m
F	=	force	N
A	=	area	m ²
μ	=	absolute viscosity	Pas (or Nm ⁻² s or kgm ⁻¹ s ⁻¹)
v_{plate}	=	fluid velocity at plate	ms ⁻¹
y	=	distance in direction normal to flow	m

Equation (7-1) is known as Newton's viscosity equation. The constant μ (the Greek letter mu) is a fluid property, which is known as the absolute or dynamic viscosity.

Newton's Viscosity Equation

Whereas in Figure 7-2 the velocity profile is a straight line, in general when a fluid flows over a stationary surface the velocity profile is likely to be represented by a curve, as shown in Figure 7-3. It is found that Newton's viscosity equation applies for each thin layer of fluid, e.g. for the layer shown in Figure 7-3 that has a thickness of Δy . Over this small thickness, the velocity of the fluid increases by an amount that is represented by Δv .

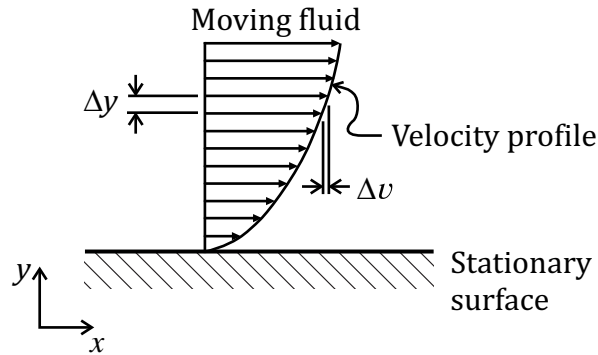


Figure 7-3 General velocity profile within a fluid

Newton's viscosity equation can be written in a more general form than Equation (7-2) as

$$F = \mu A \frac{\Delta v}{\Delta y} \quad (7-2)$$

where

$$\frac{\Delta v}{\Delta y} = \text{velocity gradient} \quad \text{ms}^{-1}\text{m}^{-1}, \text{ or } \text{s}^{-1}$$

At this point it is worth pausing to make sure that we understand what the force F in Equation (7-2) represents. It represents the shear force, owing to viscosity, between adjacent layers of the moving fluid. The layers can be regarded as sliding, one over another.

For the velocity profile shown in Figure 7-3, we can see that as we move away from the stationary surface, Δv becomes smaller for successive equal steps of y , the distance from the surface. Therefore $\Delta v/\Delta y$ becomes smaller as we move away from the surface. Hence, the shear force F between layers of the moving liquid decreases as we move further from the fixed surface where $y = 0$.

Newton's viscosity equation, Equation (7-2), gives the force that acts over a plane of area A within the fluid. We can easily modify it to give the force per unit area over the same plane, which is the shear stress over the plane. We do this by dividing both sides of the

equation by the area A . Hence, the shear stress τ (the Greek letter tau) is given by

$$\tau = \mu \frac{\Delta v}{\Delta y}. \quad (7-3)$$

where

$$\tau = \text{shear stress} \quad \text{Nm}^{-2}$$

The SI Units Involved in Newton's Viscosity Equation

The SI units for distance, velocity, area and force are relatively straightforward, as summarized here:

$$y = \text{distance in direction normal to displacement} \quad \text{m}$$

$$d = \text{distance between surfaces} \quad \text{m}$$

$$v = \text{fluid velocity} \quad \text{ms}^{-1}$$

$$F = \text{force} \quad \text{N}$$

We can note that N stands for the SI unit of force, the newton. We can also note that ms^{-1} is another way of writing m/s, which stands for 'metres per second'. As stress is defined as force divided by area (or force per unit area) its units are the units of force divided by the units of area. Hence

$$\tau = \text{shear stress} \quad \text{Nm}^{-2}$$

Even if we do not know or remember what the units of absolute viscosity are, we can determine the units easily from Newton's viscosity equation, Equation (7-2) or Equation (7-3). Rearranging Equation (7-2),

$$\mu = F \frac{\Delta y}{A \Delta v}. \quad (7-4)$$

The absolute viscosity μ must therefore have the same units as the right hand side of Equation (7-4). These units are

$$\text{N} \frac{\text{m}}{\text{m}^2 \text{m s}^{-1}} = \text{Nm}^{-2} \text{s}.$$

Therefore the correct SI unit for absolute viscosity is $\text{Nm}^{-2} \text{s}$ or Pas, which could also be written as Ns/m^2 . To summarize, based on what has been covered so far, the symbol and units for absolute viscosity are:

$$\mu = \text{absolute viscosity} \quad \text{Pas (or Nm}^{-2} \text{s)}$$

Newton's 2nd Law and the SI Units for Absolute Viscosity

The SI units for absolute viscosity can be expressed in a form that includes the unit for mass, the kilogram. Newton's second law, Equation (1-1), Page 4, is the key to the equivalence between forms that include mass and those that include force.

$$F = ma \quad (1-1) \text{ (repeated)}$$

Of the four units the newton, the kilogram, the metre and the second, any one can be expressed in terms of the other three. For example

$$1 \text{ N} = 1 \text{ kg m s}^{-2}.$$

This allows a version of the SI units for absolute viscosity to be used that does not include the newton, but does include the kilogram.

$$1 \text{ N m}^{-2} \text{s} = 1 \text{ kg m s}^{-2} \text{m}^{-2} \text{s} = 1 \text{ kg m}^{-1} \text{s}^{-1}.$$

Summarizing again, the symbol and units for absolute viscosity are:

$$\mu = \text{absolute viscosity} \quad \text{Pas or Nm}^{-2} \text{s or kg m}^{-1} \text{s}^{-1}$$

It is important to be aware of these different ways of writing the SI units for absolute viscosity. All three versions given above are in use.

Kinematic Viscosity

Kinematic viscosity ν (the Greek letter nu) is defined as

$$\nu = \frac{\mu}{\rho}$$

where

$$\nu = \text{kinematic viscosity} \quad \text{m}^2 \text{s}^{-1}$$

$$\rho = \text{density} \left(= \frac{\text{mass}}{\text{volume}} \right) \quad \text{kg/m}^3$$

We can note that the units for kinematic viscosity are got by dividing the units of absolute viscosity by the units of density. As the density units include kg, it is convenient to use the form of the absolute viscosity units that includes kg. Thus, the units of kinematic viscosity are represented as

$$\frac{\text{kg m}^{-1} \text{s}^{-1}}{\text{kg m}^{-3}} = \text{m}^2 \text{s}^{-1}.$$

More about Viscosity Units and Values

Absolute (or dynamic) viscosity values for real fluids, expressed in the fundamental SI units, are usually very small, e.g. 1×10^{-3} Pas for water. Kinematic viscosity values are even smaller, e.g. 1×10^{-6} $\text{m}^2 \text{s}^{-1}$ for water. Other viscosity units are also in use such as

$$1 \text{ poise} = 10^{-1} \text{ Pas (absolute viscosity)}$$

$$1 \text{ centipoise} = 10^{-2} \text{ poise} = 10^{-3} \text{ Pas (absolute viscosity)}$$

$$1 \text{ stoke} = 10^{-4} \text{ m}^2 \text{s}^{-1} \text{ (kinematic viscosity)}$$

$$1 \text{ centistoke} = 10^{-2} \text{ stoke} = 10^{-6} \text{ m}^2 \text{s}^{-1} \text{ (kinematic viscosity)}$$

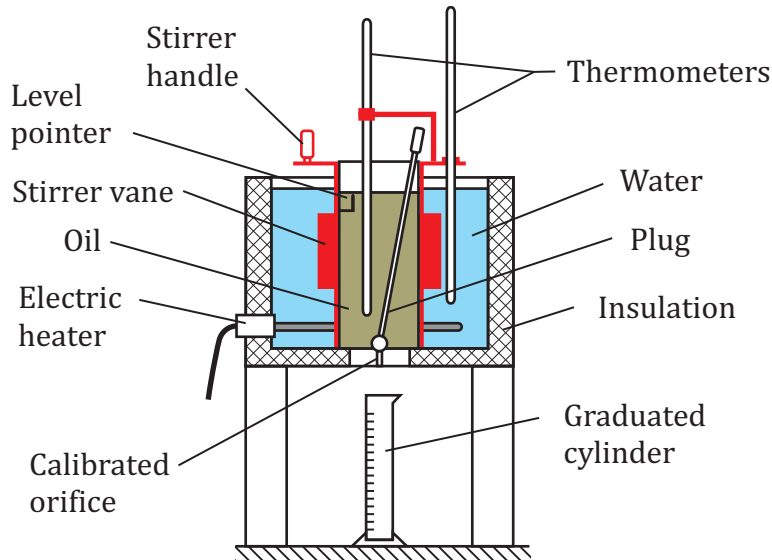


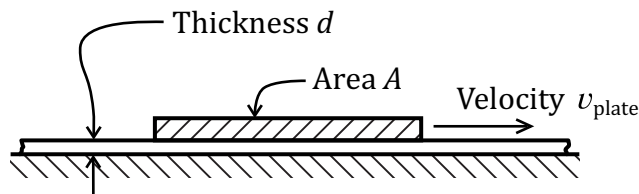
Figure 7-4 Schematic diagram of a Redwood viscometer

Another scale for viscosity is the Redwood scale in 'Redwood seconds'. The Redwood viscosity is determined by measuring how long it takes for a fixed amount of liquid to flow through a calibrated orifice, Figure 7-4. The Redwood scale is a practical scale for comparing viscosities of different liquids, which is essentially a kinematic viscosity scale. However, the relationship between the Redwood scale and a true kinematic viscosity scale is not linear. Therefore, values from the Redwood scale are not used directly for engineering calculations, but a calibration formula can be used to convert test readings from a Redwood viscometer to kinematic viscosity.

For liquids generally the viscosity decreases as the temperature rises. For gases, the opposite is generally the case: viscosity increases with increasing temperature.

Example 7-1 Newton's Viscosity Equation

Calculate the force required to cause relative motion at a rate of 1.7 m/s of a flat plate of area 0.36 m² with respect to a parallel flat surface where the plate is separated by a distance of 0.6 mm and where the space between the plate and the surface is filled with oil having an absolute viscosity of 64 centipoise. (1 centipoise = 10⁻² poise = 10⁻³ Pas). Include a sketch in your solution.

Solution

The shear force is given by

$$F = \tau A = \mu \frac{\Delta v}{\Delta y} A = \mu \frac{v}{d} A$$

$$F = 64 \times 10^{-3} [\text{N m}^{-2} \text{s}] \frac{1.7 [\text{m s}^{-1}]}{0.0006 [\text{m}]} 0.36 [\text{m}^2]$$

$$= 65.3 \text{ N}$$

Problems

7-1 Calculate the force required to cause relative motion at a rate of 2 m/s of a flat plate of area 3.6 m² with respect to a parallel flat surface where the plate is separated by a distance of 1 mm and where the space between the plate and the surface is filled with oil having an absolute viscosity of 70 centipoise. (1 centipoise = 10⁻² poise = 10⁻³ Pas). The following formula may be used

$$\tau = \frac{F}{A} = \mu \frac{\Delta v}{\Delta y}$$

7-2 A steel shaft of diameter 50 mm moves up and down within a sleeve of height 75 mm. The all-around clearance of 0.4 mm between the shaft and the sleeve is filled with oil having an absolute viscosity of 3.9 × 10⁻² Pas. Determine the force exerted on the shaft by the oil in the clearance gap and its direction at an instant when the shaft is moving upwards at the rate of 3.1 m/s.

7-3 Engine oil has an absolute viscosity of 4.86 × 10⁻³ Ns/m² and a density of 884.1 kg/m³ when it is at a temperature of 300 K. Calculate its kinematic viscosity at the same temperature.

7-4 Air at 300 °C and at atmospheric pressure has a kinematic viscosity of $15.89 \times 10^{-3} \text{ m}^2/\text{s}$ and a density of $1.1614 \text{ kg}/\text{m}^3$. Calculate its absolute viscosity.

Chapter 8. Pressure Losses in Pipes and Fittings

Laminar and Turbulent Flow

The easiest way to visualise the flow of a fluid within a pipe is to imagine that the velocity of the fluid is the same over the entire cross section of flow, as illustrated in Figure 8-1. This is known as uniform flow.

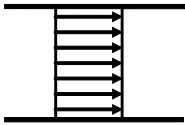


Figure 8-1 Uniform flow

For uniform flow, the velocity can be calculated by dividing the volume flow rate by the cross sectional area.

$$v = \frac{Q}{A} \quad (8-1)$$

In the case of a real fluid, with finite viscosity, the velocity is never perfectly uniform: the fluid in contact with the pipe wall will have zero velocity and, generally, the fluid in the middle of the pipe will have the highest velocity. For a fluid that has a relatively low velocity and a relatively high viscosity the variation of the velocity across the cross section, which is called the velocity profile, would be as shown in Figure 8-2. In this type of flow every particle of fluid moves with a velocity that is parallel to the centreline of the pipe. A lamina is a thin layer. Thin cylindrical laminae of fluid can be imagined to slide over one another, the laminae near the centre moving faster than those near the edges.

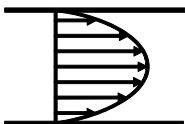


Figure 8-2 Laminar velocity profile

Where the velocity is relatively high and the fluid viscosity is relatively low, a different type of velocity profile, as shown in Figure

8-3, is produced. In addition to an average velocity in the direction of flow, each particle of fluid moves with a chaotic velocity that causes mixing over the cross section of the pipe. Local eddies occur and continue to change with time. The turbulent velocity flow profile comes closer to matching a uniform flow profile than the laminar profile does.

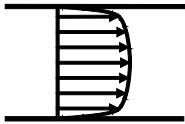


Figure 8-3 Turbulent velocity profile

Osborne Reynolds carried out an experiment in which he was able to visually compare laminar and turbulent flow through a glass pipe. This experiment is shown schematically in Figures 8-4 and 8-5. When dye is injected from a needle into laminar flow, a thin dye filament can be seen to continue over a long length within the flow, Figure 8-4.

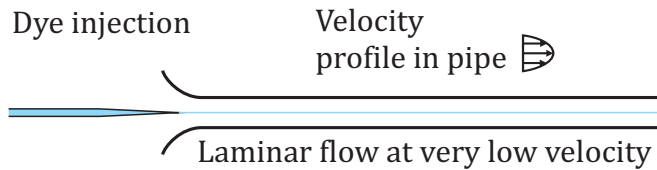


Figure 8-4 Reynolds' experiment with laminar flow

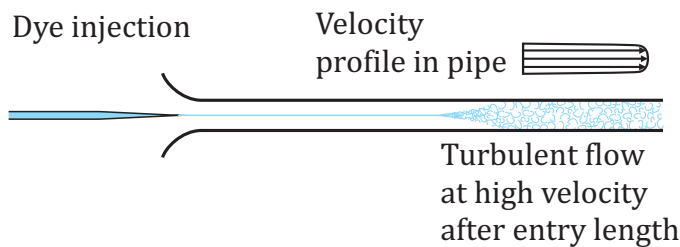


Figure 8-5 Reynolds' experiment with turbulent flow

When dye is injected into a high flow rate of fluid, Figure 8-5, the dye filament is maintained for a certain length from the pipe inlet (indicating laminar flow), but, further on, turbulent flow develops and causes mixing of the dye over the full bore of the pipe.

In laminar flow, Figure 8-4, viscous forces dominate, whereas in turbulent flow inertia forces (i.e. acceleration or deceleration

forces) dominate. In laminar flow there is no cross mixing between the layers. In turbulent flow a thorough cross-mixing of the fluid occurs and individual small volumes of fluid have chaotic motion superimposed on their average forward velocity.

It is worth noting at this point that, irrespective of the type of flow, the average fluid velocity is given by Equation (8-1).

Reynolds Number

Whether the flow in a pipe is laminar or turbulent depends on the density of the fluid, the velocity of flow, the diameter of the pipe and the absolute viscosity of the fluid. These parameters can be combined in a dimensionless number, called the Reynolds number. The value of this number can be used to predict the type of flow that will exist. The Reynolds number is defined as

$$\text{Re} = \frac{\rho v d}{\mu} = \frac{v d}{\nu} \quad (8-2)$$

where

Re	=	Reynolds number	
ρ	=	density	kg/m ³
v	=	average fluid velocity	m s ⁻¹
d	=	diameter	m
μ	=	absolute viscosity	Pa s (or Nm ⁻² s or kg m ⁻¹ s ⁻¹)
ν	=	kinematic viscosity	m ² s ⁻¹

Whether the flow is laminar or turbulent depends on the Reynolds number, as follows:

Laminar flow exists for Re below 2,000.

Turbulent flow exists for Re greater than 4,000.

Between these values laminar or turbulent flow is possible.

Example 8-1 Laminar or Turbulent Flow?

It takes 30 seconds to fill a kettle with two litres of water from a tap. The tap is supplied by a pipe that has an internal diameter of 12.7 mm. Determine whether the flow is laminar or turbulent in the supply pipe while the kettle is being filled. Take the density of water as $1,000 \text{ kg/m}^3$ and take its absolute viscosity as $1.002 \times 10^{-3} \text{ kg/ms}$.

Solution

Calculate the volume flow rate as

$$Q = \frac{V}{t} = \frac{2 \times 10^{-3} [\text{m}^3]}{30 [\text{s}]} = 66.67 \times 10^{-6} \text{ m}^3/\text{s}.$$

Calculate the cross sectional area as

$$A = \frac{\pi d^2}{4} = \frac{\pi 0.0127^2}{4} [\text{m}^2] = 126.7 \times 10^{-6} \text{ m}^2.$$

Calculate the mean velocity as

$$v = \frac{Q}{A} = \frac{66.67 \times 10^{-6} [\text{m}^3/\text{s}]}{126.7 \times 10^{-6} [\text{m}^2]} = 0.5262 \frac{\text{m}}{\text{s}}.$$

Calculate the Reynolds number as

$$\begin{aligned} \text{Re} &= \frac{\rho v d}{\mu} = \frac{1000 \left[\frac{\text{kg}}{\text{m}^3} \right] 0.5262 \left[\frac{\text{m}}{\text{s}} \right] 0.0127 [\text{m}]}{1.002 \times 10^{-3} \left[\frac{\text{kg}}{\text{ms}} \right]} \\ &= 6,669 \cong 6.7 \times 10^3 \end{aligned}$$

As $\text{Re} > 4,000$, flow is turbulent.

Head Loss Due to Friction

Different laws of fluid resistance apply for laminar and turbulent flows, although one common equation (the Darcy-Weisbach equation, which is sometimes just called the Darcy equation) can be used for fluid flow in pipes—it incorporates a ‘friction factor’ that depends on the nature of the flow.

The Darcy-Weisbach equation has the form

$$h_f = \frac{4fL}{d} \frac{v^2}{2g} \quad (8-3)$$

where the friction factor f is obtained as detailed below and where

h_f	=	head loss due to friction	m
f	=	friction factor	
L	=	length of pipe	m
d	=	diameter	m
v	=	mean fluid velocity	ms^{-1}
g	=	acceleration due to gravity	ms^{-2}

Note: The friction factor used here is known as the Fanning friction factor. In some textbooks and references the Darcy-Weisbach equation is written without the integer 4 in the numerator, but the friction factor used is four times the friction factor used here. That friction factor is known as the Darcy friction factor.

As might be expected, the internal roughness of a pipe, known as its surface roughness, can have an influence on the pressure drop that occurs in the pipe. In fact, it is the ratio of the surface roughness to the diameter that is of importance. This ratio, known as the relative roughness, influences the pressure drop for turbulent flow. However, for laminar flow the relative roughness does not influence the pressure drop.

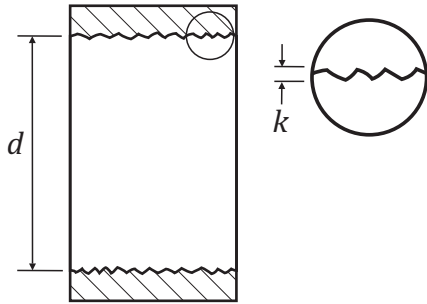


Figure 8-6 Diameter and surface roughness of a pipe

For *turbulent flow* the friction factor f is commonly obtained from a friction factor diagram, which is often referred to as a Moody diagram or chart, Figure 8-7. The friction factor depends on the Reynolds number and on the relative roughness of the pipe k/d where

k = Pipe surface roughness m

d = Pipe internal diameter m

The friction factor can also be found by using appropriate empirical formulae, e.g. within a spreadsheet. These formulae are too cumbersome for hand calculations; hence the friction factor diagram is useful.

For fully turbulent flow the head loss per unit length of pipe, h_f/L , is proportional to the mean velocity squared and depends on the relative roughness of the pipe. For flow that is not fully turbulent there is not a direct proportionality to velocity squared (roughly speaking, the head loss is proportional to velocity raised to a power that is close to and a little lower than 2).

For *laminar flow* the friction factor is given by

$$f = \frac{16}{\text{Re}}. \quad (8-4)$$

Note 1: Equation (8-3) is written on the friction factor diagram, so there is no need to remember it.

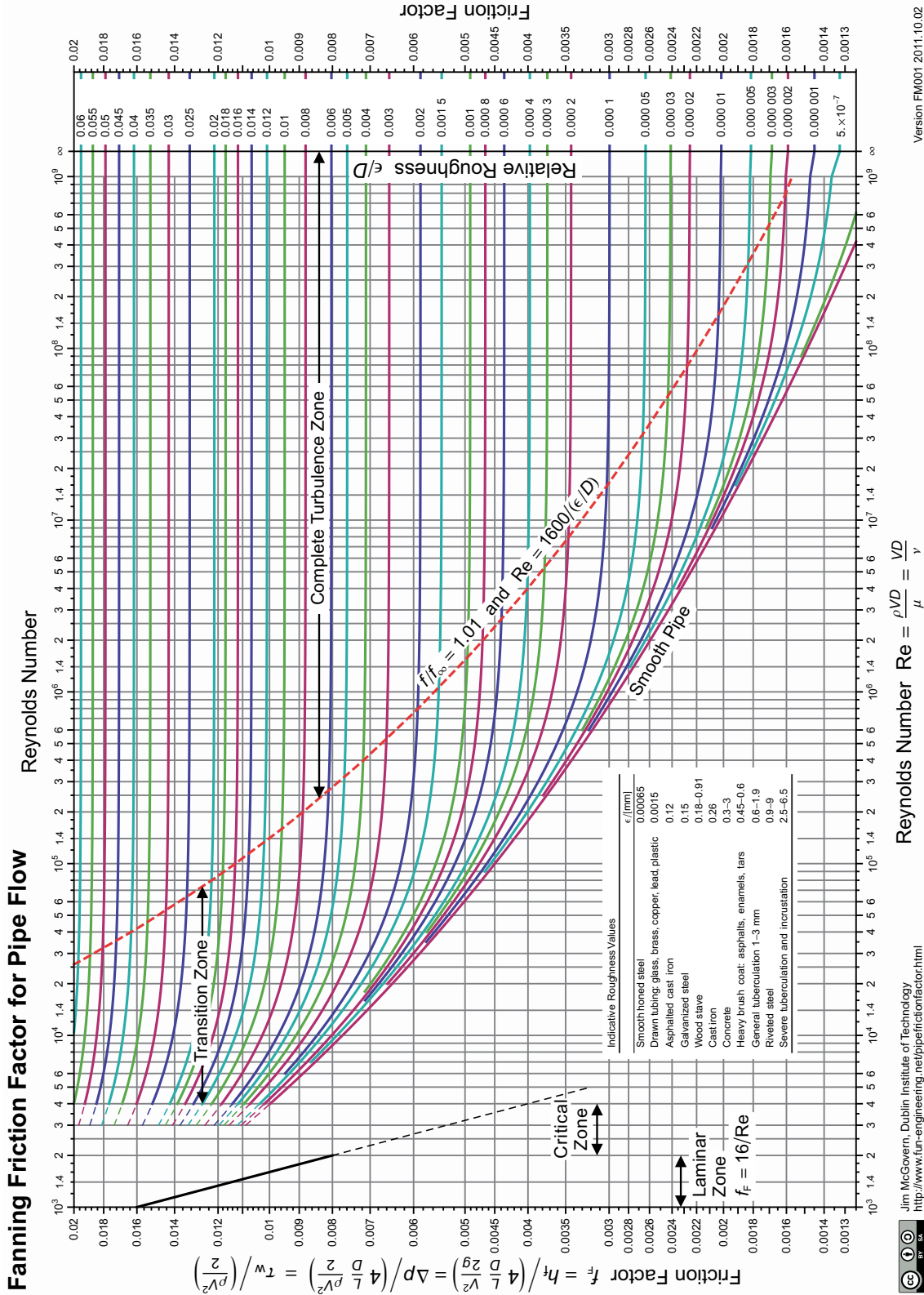


Figure 8-7 Friction factor diagram (Moody diagram).

Note 2: In references where the friction factor is four times the value used here, Equation (8-4) is written with 64 in the numerator instead of 16.

It can be shown that for laminar flow the head loss per unit length of pipe, h_f/L , is proportional to the mean velocity (or the mean flow rate) and independent of surface roughness. The friction factor for the laminar flow region can also be read from a friction factor diagram.

Flow through Valves and Fittings

The presence of a valve in a pipe, even when it is fully open, almost always causes a head loss owing to disruption of the flow and the formation of a vena contracta (reduced flow area). The same is true for many other types of fittings, e.g. a fitting that joins pipes of different diameters. In these cases a head loss coefficient is used, Equation (8-5).

$$h_{\text{loss}} = K \frac{v^2}{2g}. \quad (8-5)$$

In the above expression the velocity v is the mean velocity in the full bore pipe upstream.

K is a dimensionless constant for a particular configuration and will depend on the nature of the valve or fitting. Hence, if the head loss at a valve or fitting is measured for a given mean velocity of flow the head loss coefficient can be determined as

$$K = \frac{2gh_{\text{loss}}}{v^2}. \quad (8-6)$$

Flow Around Elbows and Bends

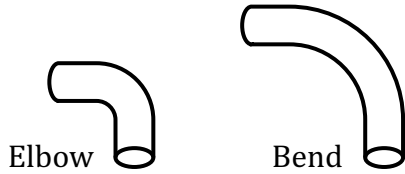


Figure 8-8 Elbow and bend

Whenever the direction of flow is abruptly changed at an elbow or bend (Figure 8-8), a loss of head occurs due to the formation of a vena contracta within the component and the rotational momentum that is imparted to the fluid, which tends to create turbulent eddies. The head loss can be characterized by a head loss coefficient, as in Equation (8-5). Some sample values are given in Table 8-1. The values are influenced by specific details of the components, e.g. whether there are any gaps, irregularities or sharp edges along the inside surfaces. The head loss calculated with the head loss coefficient should be added to the head loss calculated for the total length of the pipe.

Table 8-1 Head loss coefficients for elbows and bends²

	Diameter d /[mm]		
	10	32	100
Elbows			
Screw fitting	2.5	1.3	0.7
Smooth	1.10	0.75	0.7
Bends			
$(r/d > 1.5)$			
Screw fitting	-	0.65	0.24
Smooth	0.57	0.43	0.40

Equivalent Pipe Length

Commercially, the head loss caused by flow through fittings is sometimes presented as an 'equivalent length of straight pipe' to cause the same head loss. In this case the head loss for a pipe run that includes fittings (elbows, bends, reducers etc.) can be

² Data based on Chartered Institute of Building Services (CIBS), Guide C: Reference Data, 2007, London. The symbol r represents the mean radius of the bend.

calculated using the Darcy-Weisbach equation, Equation (8-3), but the pipe length used is the actual length of the pipe plus the equivalent lengths of any fittings used. Head losses associated with valves, fittings and bends are often described as minor pipe losses. Major losses are those associated with fluid friction over the full length of a pipe.

Pressure Loss and Pumping Power

The Darcy-Weisbach equation, Equation (8-3), allows us to calculate the head loss due to friction in a pipe. Head loss has the units of length, the metre. If we require the pressure loss, or pressure drop, over the length of pipe we can use Equation (2-4), $p = \rho gh$, to find it:

$$p_{\text{loss}} = \rho gh_f \quad (8-7)$$

where

p_{loss}	=	pressure loss due to friction	Pa
ρ	=	density	kg/m ³
g	=	acceleration due to gravity	m/s ²
h_f	=	head loss due to friction	m

Combining Equations (8-3) and (8-7)

$$p_{\text{loss}} = \rho g \frac{4fL}{d} \frac{v^2}{2g}$$

or

$$p_{\text{loss}} = \frac{4fL}{d} \frac{\rho v^2}{2} \quad (8-8)$$

Equation (8-8) is an alternative form of the Darcy-Weisbach equation, which gives the pressure loss rather than the head loss due to fluid friction.

In some cases flow through a pipe is caused directly by a difference in liquid levels in two tanks at either end of the pipe. The flow might also be caused directly by a pump, which would have to provide a pressure difference to match the pressure loss in the pipe. The pressure difference across the pump is often represented by Δp , Figure 8-9. If a pump is required to exactly meet a pressure drop due to friction in a pipe then $\Delta p = p_{\text{loss}}$.

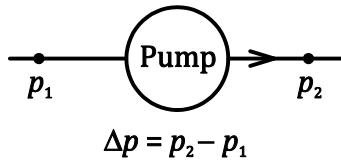


Figure 8-9 Pressure difference across a pump

The ideal pumping power equals the product of the pressure difference across the pump and the volume flow rate, Equation (8-9).

$$P = \Delta p Q \quad (8-9)$$

where

P	=	ideal pumping power	W
Δp	=	pressure difference	Pa
Q	=	volume flow rate	m^3/s

Summary of Key Formulae for Pipe Flow

The key formulae for pipe flow are summarized in Table 8-2.

Table 8-2 Summary of key formulae for pipe flow

Mean velocity of flow in a pipe:

$$v = \frac{Q}{A}$$

Reynolds Number:

$$\text{Re} = \frac{\rho v d}{\mu} = \frac{v d}{\nu}$$

$\text{Re} < 2,000$ Laminar Flow

$Re > 4,000$ Turbulent Flow

Head loss due to friction in a pipe (Darcy-Weisbach equation):

$$h_f = \frac{4fL}{d} \frac{v^2}{2g}$$

For laminar flow f can be calculated from

$$f = \frac{16}{Re}$$

For turbulent flow, f depends on Re and on the relative roughness k/d .

Head losses at bends or fittings:

$$h_{\text{loss}} = K \frac{v^2}{2g}$$

Pumping power:

$$P = \Delta p Q$$

Example 8-2 Head Losses due to Friction

A pipe of absolute roughness 0.0030 mm and bore 50 mm conveys water at a rate of 210 litres per minute. If the viscosity of the water is 0.0013 kg/ms determine

- (a) the friction factor
- (b) the head loss per metre run of pipe
- (c) the maximum allowable flow rate if the flow were to be laminar
- (d) the theoretical pumping power per metre length for the turbulent flow in the pipe.

Solution

The volume flow rate is given by

$$Q = \frac{210}{60 \times 1,000} \text{ m}^3/\text{s} = 0.0035 \text{ m}^3/\text{s}.$$

The cross-sectional area is

$$A = \frac{\pi 0.05^2}{4} \text{ m}^2 = 1.963 \times 10^{-3} \text{ m}^2.$$

The mean velocity of flow in the pipe is

$$v = \frac{Q}{A} = \frac{0.0035 \text{ [m}^3\text{/s]}}{1.963 \times 10^{-3} \text{ [m}^2\text{]}} = 1.783 \text{ m/s.}$$

The Reynolds number is

$$\begin{aligned} \text{Re} &= \frac{\rho v d}{\mu} = \frac{1,000 \times 1.783 \times 0.05 \text{ kg m}^{-3} \text{ m s}^{-1} \text{ m}}{0.0013 \text{ kg m}^{-1} \text{ s}^{-1}} \\ &= 68,577 \cong 6.9 \times 10^4 \end{aligned}$$

As $\text{Re} > 4,000$, the flow is turbulent.

The relative roughness is

$$\frac{k}{d} = \frac{0.003 \text{ mm}}{50 \text{ mm}} = 0.00006$$

For $\text{Re} = 68,600$ and $k/d = 0.00006$ the friction factor is found from the friction factor diagram to be 0.0049 (see diagram on page 68).

(a)

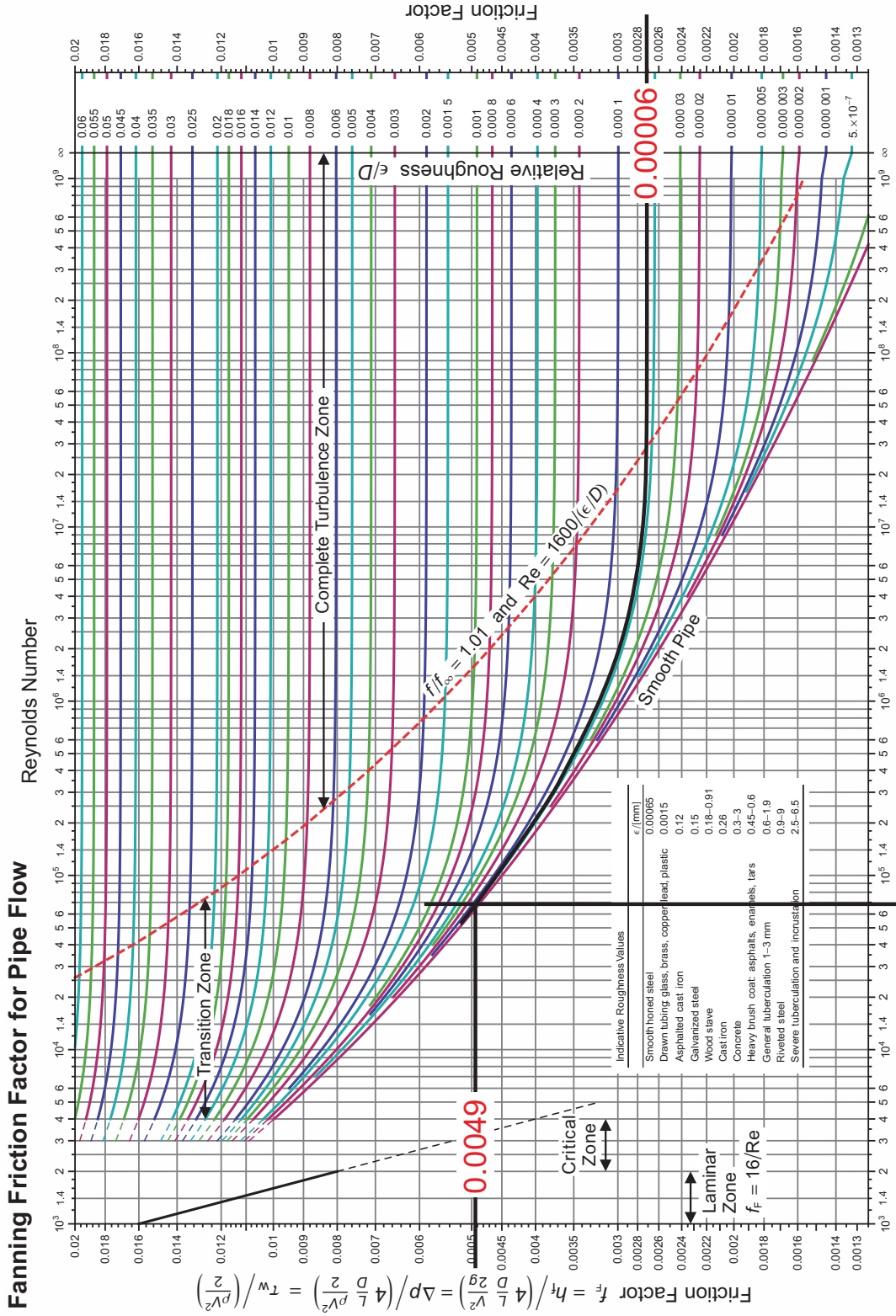
The head loss due to friction in a pipe (Darcy-Weisbach equation) is:

$$h_f = \frac{4fL}{d} \frac{v^2}{2g}$$

Hence

$$\begin{aligned} \frac{h_f}{L} &= \frac{4f}{d} \frac{v^2}{2g} = \frac{4 \times 0.0049}{0.050} \frac{1.783^2}{2 \times 9.81} \left[\frac{\text{m}^2 \text{s}^{-2}}{\text{m m s}^{-2}} \right] \\ &= 0.0635 \text{ m/m} \end{aligned}$$

That is, the head loss is 63.5 mm per metre of pipe length. (b)



Jim McGovern, Dublin Institute of Technology
<http://www.tun-engineering.net/pipfrictionfactor.html>

68,600

$Re = \frac{\rho V D}{\mu} = \frac{VD}{\nu}$

Version FM001 2011.10.02

The maximum allowable flow rate if the flow were to be laminar would require Re to equal 2,000. From the definition of the Reynolds number ($Re = \frac{\rho v d}{\mu}$) the maximum laminar flow velocity is

$$\begin{aligned} v &= \frac{Re\mu}{\rho d} = \frac{2,000 \times 0.0013}{1,000 \times 0.05} \left[\frac{\text{kg m}^{-1} \text{s}^{-1}}{\text{kg m}^{-3} \text{m}} \right] \\ &= 0.0520 \text{ m/s} \\ Q &= vA = 0.0520 \text{ [m/s]} \times 1.963 \times 10^{-3} \text{ [m}^2\text{]} \\ &= 0.1021 \times 10^{-3} \text{ m}^3/\text{s} \\ &= 0.1021 \times 60 \text{ L/min} = 6.13 \text{ L/min} \end{aligned} \tag{c}$$

The pumping power for the turbulent flow case is given by

$$P = Q\Delta p = Q\rho g h_f$$

Hence,

$$\begin{aligned} \frac{P}{L} &= Q\rho g \frac{h_f}{L} \\ \frac{P}{L} &= 0.0035 \text{ [m}^3/\text{s]} \times 1,000 \text{ [kg m}^{-3}\text{]} \times 9.81 \text{ [m s}^{-2}\text{]} \times 0.0635 \\ &= 2.18 \text{ kg m s}^{-3} = 2.18 \text{ N s}^{-1} = 2.18 \text{ N m s}^{-1} \text{ m}^{-1} \\ &= 2.18 \text{ W m}^{-1} \end{aligned}$$

That is, the pumping power required for each metre of pipe length is 2.18 watts.

(d)

Problems

- 8-1 If water flows in a pipe of 12 mm diameter at the rate of 8 litres per minute what is the Reynolds number?
- 8-2 What is the relative roughness of a concrete pipe of 1 m bore if the surface roughness is 1.2 mm?

8-3 Calculate the friction factor for water flow in a pipe where

a) $k/d = 0.01$ and $Re = 3,740,000$

b) the pipe is smooth and $Re = 10^6$

c) $Re = 55,000$ and $k/d = 0.00006$

d) $Re = 3,500$ and $k/d = 0.02$

e) $Re = 850$ and $k/d = 0.016$

8-4 Oil having a density of 840 kg/m^3 and a kinematic viscosity of 118 centistoke flows with an average velocity of 0.9 m/s in a 10 mm pipe with a length of 0.317 m. What is the pressure loss over the length of the pipe and what theoretical pumping power is required? (1 centistoke = $10^{-6} \text{ m}^2/\text{s}$).

Chapter 9. Valves and Heat Exchanger Flow Paths

This chapter describes valves and the flow paths of some heat exchangers. The basis for calculating pressure losses for these is explained.

Valves

Valves are used for isolation (i.e. shutting off flow in a pipe), for throttling (i.e. causing a pressure drop in the direction of flow) and for flow regulation (i.e. allowing the flow rate to be varied).

Most valves require a stem seal (or packing) to prevent fluid from leaking out. Valves that are to achieve a tight shut-off require a suitable seat (the area with which the plug element makes contact when the valve is closed) to maintain leak-tightness. This may involve precision-machined matching surfaces or one of the surfaces may be compliant, e.g. made from rubber.

Gate Valve

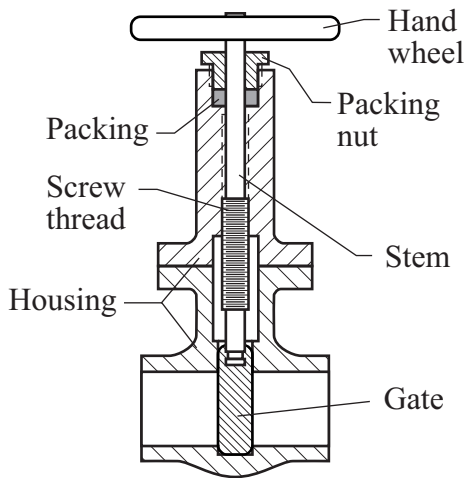


Figure 9-1 Gate valve diagram

This valve, Figure 9-1, contains a sliding gate that can be opened or closed by turning a hand wheel attached to a threaded stem for manual operation. For automation purposes, a rotary actuator may be used or a version without threads on the stem may be connected to a linear actuator. The gate slides into a recess in the housing,

allowing the fluid to flow. Opening or closing is relatively slow. A full bore opening can be provided but the housing must be very high to accommodate the sliding gate and the screw thread.

Applications: mainly shut-off but can be used for flow regulation or throttling.

Advantages:

Can provide tight shut-off (with suitable seat)

Minimal pressure drop when fully open

Disadvantages:

Slow operation (many turns of screw thread to open or close)

Large size

Globe Valve

In this valve, Figure 9-2, there is an orifice in the valve body through which the fluid flows, which can be shut by a plug. The plug is connected to a stem or spindle which has a screw mechanism for manual or rotary-actuator adjustment or shut-off. For automation purposes, a version without threads on the stem may be connected to a linear actuator.

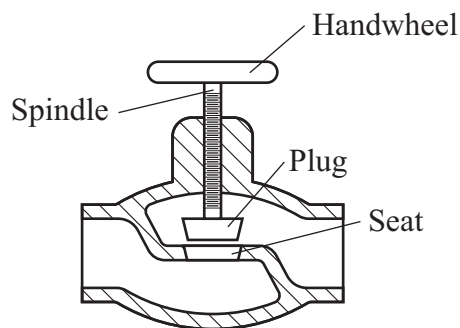


Figure 9-2 Globe valve diagram

Applications: shut-off, throttling, flow control.

Advantages:

Can provide tight shut-off (with suitable seat)

Can provide fine control of flow or throttling (with suitable design of plug and seat)

Moderately compact.

Disadvantages:

Significant pressure drop when fully open.

Ball Valve

Within this valve, Figure 9-3, there is a ball with a port through it. The ball is kept in place by a close-fitting spherical seating. A full-bore port through the ball is possible and shut-off is achieved by rotating the ball through a quarter turn. A handle or lever is attached by a short stem to the ball for actuation. The stem also located the ball and allows it to pivot. If there is only a stem to the handle the ball is said to be floating. Otherwise there may be two collinear stems and the ball is said to be trunnion mounted.

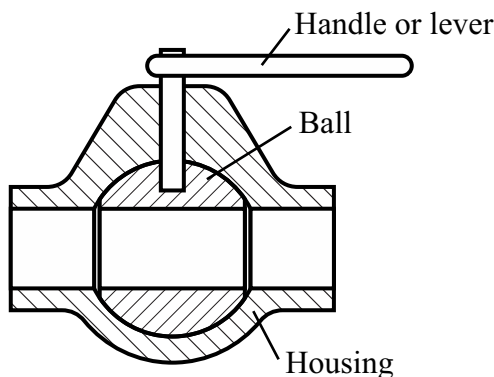


Figure 9-3 Ball valve diagram

Applications: mainly shut-off.

Advantages:

Can provide tight shut-off (with suitable seat).

Quick operation (quarter turn to open or close).

Minimal pressure drop when fully open.

Very compact.

Disadvantages:

Not good for flow regulation or throttling.

Precision spherical surfaces are required within the valve.

Head Loss Coefficients for Open Valves

Some indicative head loss coefficients for valves for use in Equation (8-5) are presented in Table 9-1.

Table 9-1 Indicative head loss coefficient values³ for fully open valves

	Head Loss Coefficient, K
Globe valve	4 to 8 depending on pipe diameter
Ball valve	very low
Gate valve	0.3

Example 9-1 Head or Pressure Loss Calculation

If a globe valve for a 40 mm pipe has a head loss coefficient of 4.9, calculate the head loss in metres when the mean velocity of the water flowing in the pipe is 1.3 m/s. Also calculate the pressure drop in pascal.

Solution

$$h_{\text{loss}} = K \frac{v^2}{2g} = 4.9 \frac{1.3^2}{2 \times 9.81} \frac{\text{m}^2 \text{s}^{-2}}{\text{m s}^{-2}} = 0.422 \text{ m}$$

The corresponding pressure loss is calculated as

$$\Delta p = \rho g h_{\text{loss}} = 1,000 \left[\frac{\text{kg}}{\text{m}^3} \right] \times 9.81 \left[\frac{\text{m}}{\text{s}^2} \right] \times 0.422 \text{ [m]} = 4,140 \text{ Pa.}$$

³ Figures for globe and gate valves from CIBSE, Guide C: Reference Data, 2007, London.

Heat Exchangers

A heat exchanger is a device in which heat transfer occurs from one fluid to another through a separating wall or partition. One fluid is heated as it flows through the heat exchanger while the other fluid is cooled.

Heat transfer is enhanced when the flow is highly turbulent and when the velocity is relatively high. However, there is a compromise between the pressure loss due to friction in the heat exchanger and the heat transfer performance.

Tube in Tube Heat Exchanger

A tube-in-tube heat exchanger, Figure 9-4, is made up of two concentric tubes. One fluid flows through the inner tube. The second fluid flows in the opposite direction in the space between the inner and outer tubes. Extended surfaces, e.g. spiral fins, may be used within both fluid spaces to enhance heat transfer.

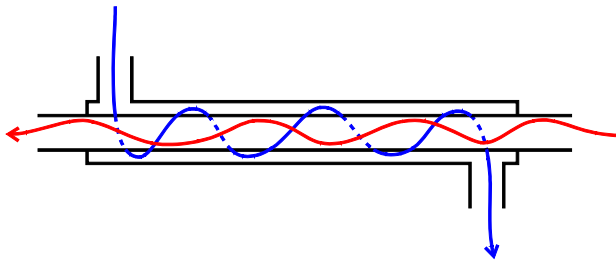


Figure 9-4 Tube in tube heat exchanger (straight)

Applications: Various fluids, moderate pressures.

Advantages:

Various configurations are available, saving space

Single unit with no removable parts

Lightweight

Disadvantages:

Cannot be dismantled for cleaning or repair

May take up more space than other types

Shell and Tube Heat Exchanger

In a shell and tube heat exchanger, Figure 9-5, a large number of tubes run between header plates at either end of a cylinder. One fluid passes through these tubes while the other fluid passes over the tubes, between the tubes and the shell. There may be multiple passes on the tube side, depending on the arrangement of the header boxes at the two ends. Usually there is just one pass on the shell side.

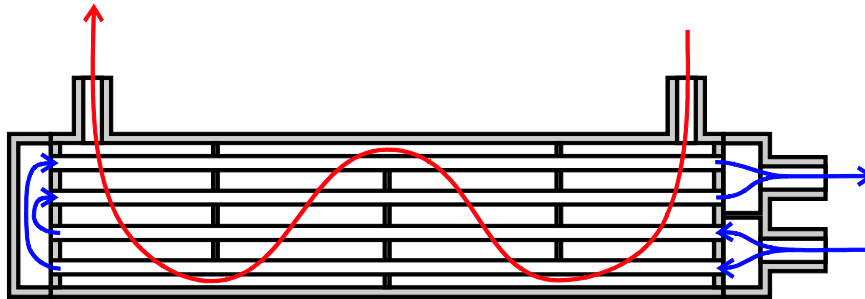


Figure 9-5 Shell and tube heat exchanger schematic

Applications: Various fluids, wide range of pressures including high pressures.

Advantages:

Compact

Can withstand high pressures

Can often be partially dismantled for inspection, cleaning or repair

Leaking tubes can be plugged or replaced (depending on design).

Can be custom built in very large sizes

Disadvantages:

Difficult to inspect the shell side of the heat transfer tubes

Heavy

Plate Heat Exchanger

A flat plate heat exchanger is constructed from multiple plates, as shown in Figure 9-6 and Figure 9-7. Between each pair of plates there is a flat region in which one of the two fluids flows. The inlet

port for the each fluid admits the fluid to every second space between the plates so the fluids are separated from each other but exchange heat through the plates. Special surface patterns, such as a chevron pattern, can be used to enhance heat transfer. In a frame and plate heat exchanger the plates are clamped together by tie rods and nuts, while in a brazed plate heat exchanger the plates are joined by brazing. Frame plate heat exchangers can be dismantled for cleaning or inspection, whereas brazed heat exchangers cannot. It should be noted that the end plates of the heat exchanger are not shown in Figure 9-6.

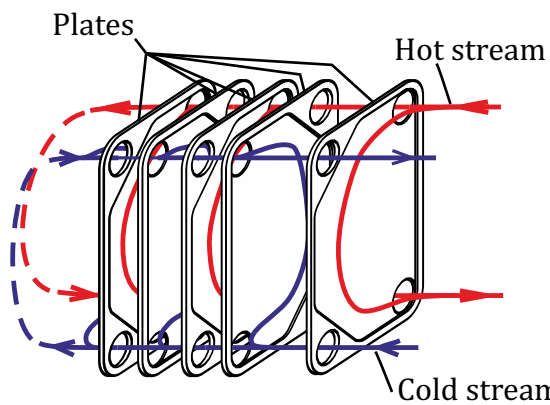


Figure 9-6 Plate heat exchanger assembly exploded view

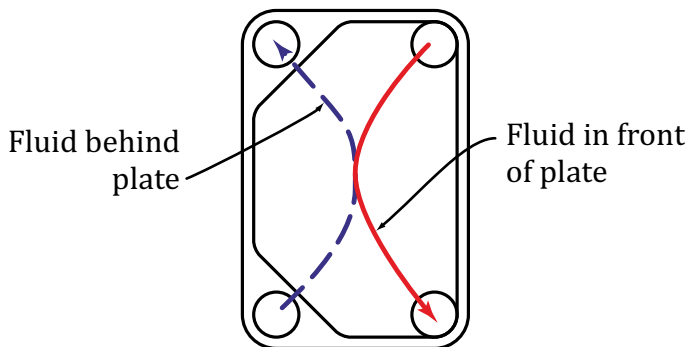


Figure 9-7 Plate heat exchanger flow paths

Applications: Medium to low pressure liquids.

Advantages:

Compact block shape and high heat transfer rates.

Plates can be added or removed to change capacity (frame type).

Disadvantages:

Cannot be dismantled for cleaning or repair (brazed types)

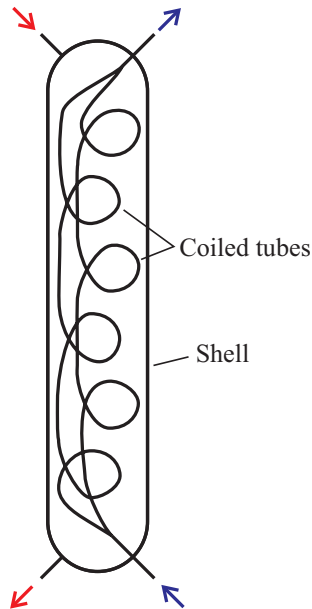
Shell and Coil Heat Exchanger

Figure 9-8 Shell and coil heat exchanger.

In reality the coils would be much more tightly packed than shown in this schematic representation.

In a shell and coil heat exchanger, Figure 9-8, a bundle of long tubes is formed into a helical shape to fit inside a tubular shell with hemispherical end caps. One fluid flows through the coil bundle while the other fluid flows over the coil bundle. The tubes may be corrugated to promote turbulence and enhance heat transfer.

Applications: Various fluids, wide range of pressures.

Advantages:

Compact long cylindrical shape

Can be installed vertically, saving space.

Single unit with no removable parts

Lightweight

Disadvantages:

Cannot be dismantled for cleaning or repair

Pressure Drops or Head Losses in Heat Exchangers

In general, separate head loss calculations are necessary for the two fluids that pass through a heat exchanger. In some cases the Darcy-Weisbach equation can be used directly, e.g. for flow through straight tubes in a shell and tube heat exchanger. In some other cases where the flow passages are not round like a pipe, an equivalent diameter can be used in the Darcy-Weisbach equation. If fins or roughened surfaces are used to increase heat transfer, there will inevitably be increased pressure loss.

Manufacturers of heat exchangers will normally provide details of the pressure drops. In general, the amount of turbulence in heat exchangers is likely to be high and so, as can be seen from the friction factor diagram for pipe flow, the friction factor will be invariant with Reynolds number or flow rate once the flow is in the fully turbulent region. Hence, from the Darcy-Weisbach equation, the pressure drop will be proportional to the square of the velocity or flow rate.

$$\Delta p_{\text{hx}} \propto Q^2 \quad (9-1)$$

where

Δp_{hx} = pressure drop Pa

Q = volume flow rate m^3s^{-1}

Example 9-2 Heat Exchanger Pressure Drop

Water flows through a heat exchanger at the rate of 126 litres per minute and the pressure drop is 28 kPa. Assuming turbulent flow, what would be the pressure drop through the heat exchanger if the flow rate were reduced to 86 litres per minute?

Solution

Let Δp_1 be the initial pressure drop and Δp_2 be the new pressure drop. Hence

$$\frac{\Delta p_2}{\Delta p_1} = \left(\frac{Q_2}{Q_1}\right)^2$$

Hence,

$$\Delta p_2 = \Delta p_1 \left(\frac{Q_2}{Q_1} \right)^2 = 28 \left(\frac{86}{126} \right)^2 \text{ kPa} = 13.0 \text{ kPa}.$$

Problems

- 9-1 Water is required to flow at a mean velocity of 1.2 m/s in a horizontal cast iron pipe that has a bore of 1.25 m and a length of 0.8 km. What pressure difference is required to provide this flow rate?
- 9-2 Water flows in a 50 mm bore pipe with an average velocity of 1.63 m/s. The pipeline includes a globe valve and the pressure drop across the valve only is measured as 5.94 kPa when it is fully open. What is the head loss coefficient of the fully open valve? The expression for pressure loss in terms of the head loss coefficient is

$$p_{\text{loss}} = \rho g h_{\text{loss}} = \rho g K \frac{v^2}{2g} = K \frac{\rho v^2}{2}.$$

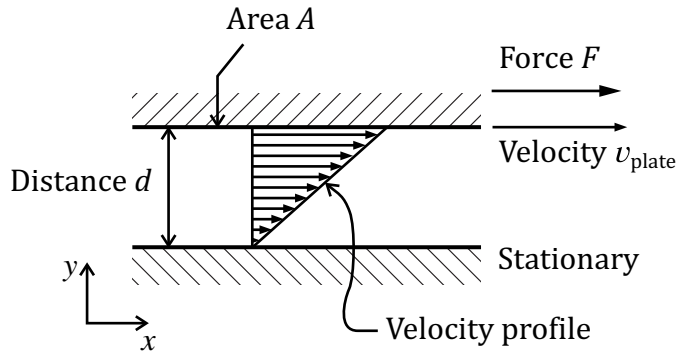
- 9-3 Water flows at the rate of 31.3 L/min in a pipe (drawn tubing) that has an internal diameter of 12.7 mm. The total length of straight pipe is 15 m and the pipe run includes a heat exchanger with a head loss coefficient of 11.2 and six right-angle bends that each have a head loss coefficient of 0.51. Calculate the overall head loss of the pipe run. Take the density of water as 1,000 kg/m³ and take its absolute viscosity as 1.002 × 10⁻³ kg/ms.

$$h_f = \frac{4fL}{d} \frac{v^2}{2g}$$

$$h_{\text{loss}} = K \frac{v^2}{2g}$$

Appendix A Formulae

Newton's Viscosity Equation



$$F \propto \frac{Av}{d}$$

$$\tau = \mu \frac{\Delta v}{\Delta y}$$

where

d	=	distance	m
F	=	force	N
A	=	area	m^2
τ	=	shear stress	Nm^{-2}
μ	=	absolute viscosity	Pas (or Nm^{-2}s or $\text{kgm}^{-1}\text{s}^{-1}$)
v	=	fluid velocity	ms^{-1}
y	=	distance in direction normal to flow	m

Kinematic viscosity

Kinematic viscosity, ν , is defined as

$$\nu = \frac{\mu}{\rho}$$

where

$$\rho = \text{density} \quad \text{kg/m}^3$$

$$\nu = \text{kinematic viscosity} \quad \text{m}^2\text{s}^{-1}$$

Newton's Second Law Applied to S.I. Units

$$F = ma$$

where

$$F = \text{force} \quad \text{N}$$

$$m = \text{mass} \quad \text{kg}$$

$$a = \text{acceleration} \quad \text{ms}^{-2}$$

Hence,

$$1 \text{ N} = 1 \text{ kg m s}^{-2}$$

Pressure

$$p = \frac{F}{A}$$

where

$$p = \text{pressure} \quad \text{Nm}^{-2} \text{ or Pa}$$

$$F = \text{Force} \quad \text{N}$$

$$A = \text{area} \quad \text{m}^2$$

Pressure and Head

$$p = \rho g h$$

where

$$\rho = \text{density} \quad \text{kg/m}^3$$

$$g = \text{acceleration due to gravity} \quad \text{ms}^{-2}$$

$$h = \text{height or head} \quad \text{m}$$

Compressibility and Bulk Modulus

$$\text{Compressibility, } \beta = \frac{\left(-\frac{\Delta V}{V}\right)}{\Delta p} = -\frac{1}{V} \frac{\Delta V}{\Delta p}$$

$$\text{Bulk modulus, } K = \frac{\Delta p}{\left(-\frac{\Delta V}{V}\right)} = -V \frac{\Delta p}{\Delta V}$$

where

β	=	compressibility	Pa^{-1}
K	=	bulk modulus of elasticity	Pa
V	=	volume	m^3
p	=	pressure	Nm^{-2} or Pa

Force Produced by a Hydraulic Cylinder

Force produced by the pressure acting on the piston at the cap end of a single-acting cylinder:

$$F = pA_{\text{cyl}}$$

where

F	=	force produced in the piston rod
p	=	pressure
A_{cyl}	=	cylinder cross-sectional area

Force produced by the pressure acting on the piston at the rod end of a single-acting cylinder:

$$F = p(A_{\text{cyl}} - A_{\text{rod}})$$

where

A_{rod}	=	rod cross-sectional area
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Displacement of a Hydraulic Piston Pump

With one cylinder:

$$V_{\text{displ}} = A_{\text{cyl}}L_{\text{stroke}}$$

where

V_{displ} = pump displacement per cycle

A_{cyl} = cross-sectional area of cylinder

L_{stroke} = pump stroke

With n cylinders:

$$V_{\text{displ}} = n_{\text{cyl}}A_{\text{cyl}}L_{\text{stroke}}$$

where

n_{cyl} = number of cylinders

Ideal Flow Rate of a Positive Displacement Pump

$$Q_{\text{ideal}} = V_{\text{displ}}N$$

where

V_{displ} = pump displacement per revolution (or cycle)

N = speed (rotations or cycles per unit time)

Efficiency of a Hydraulic Pump

Hydraulic power (for incompressible flow)

$$P = Q\Delta p$$

where

P = power Js^{-1} or W

Q = volume flow rate m^3/s

Δp = pressure difference Pa

$$E_{\text{pump, mech}} = \frac{Q_{\text{ideal}} \Delta p}{P_{\text{actual}}}$$

$$E_{\text{pump, vol}} = \frac{Q_{\text{actual}}}{Q_{\text{ideal}}}$$

$$E_{\text{pump, overall}} = E_{\text{pump, mech}} \times E_{\text{pump, vol}}$$

$$= \frac{Q_{\text{ideal}} \Delta p}{P_{\text{actual}}} \frac{Q_{\text{actual}}}{Q_{\text{ideal}}} = \frac{Q_{\text{actual}} \Delta p}{P_{\text{actual}}}$$

Area of a Circle

$$A = \pi r^2 = \frac{\pi d^2}{4}$$

where

$$A = \text{area} \quad \text{m}^2$$

$$r = \text{radius} \quad \text{m}$$

$$d = \text{diameter} \quad \text{m}$$

Mass Flow Rate

$$\dot{m} = \rho A v$$

where

$$v = \text{velocity} \quad \text{ms}^{-1}$$

$$\dot{m} = \text{mass flow rate} \quad \text{kg/s}$$

$$A = \text{area} \quad \text{m}^2$$

$$\rho = \text{density} \quad \text{kg/m}^3$$

Volume Flow Rate

$$Q = vA$$

$$v = \frac{Q}{A}$$

where

v = velocity ms^{-1}

Q = Volume flow rate m^3/s

A = area m^2

Reynolds Number

$$\text{Re} = \frac{\rho v d}{\mu} = \frac{v d}{\nu}$$

For laminar flow in a pipe: $\text{Re} < 2,000$

For turbulent flow in a pipe: $\text{Re} > 4,000$

where

ρ = density kg/m^3

v = fluid velocity ms^{-1}

d = diameter m

μ = absolute viscosity Pas (or Nm^{-2}s
or $\text{kgm}^{-1}\text{s}^{-1}$)

ν = kinematic viscosity m^2s^{-1}

The Darcy-Weisbach Equation

The head loss due to friction is given by

$$h_f = \frac{4fL}{d} \frac{v^2}{2g}$$

f depends on k/d and Re

where

h_f = head loss due to friction m

f = friction factor

L	=	length of pipe	m
d	=	diameter	m
v	=	mean fluid velocity	ms^{-1}
g	=	acceleration due to gravity	ms^{-2}
k	=	equivalent surface roughness	m
Re	=	Reynolds number	

The pressure drop due to friction is given by

$$\Delta p = \rho g h_f = \frac{4fL}{d} \frac{\rho v^2}{2}$$

where

Δp	=	pressure drop due to friction	Pa
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Friction Factor for Laminar Flow

$$f = \frac{16}{\text{Re}} = \frac{16 \mu}{\rho v d}$$

Head Loss Coefficient for Fittings and Features

(for reducers, bends, etc.)

$$h_{\text{loss}} = K \frac{v^2}{2g}$$

where

h_{loss}	=	head loss for fitting or feature	m
K	=	head loss coefficient	
v	=	mean velocity in pipe	ms^{-1}
g	=	acceleration due to gravity	ms^{-2}

Appendix B Glossary

Absolute Pressure

Absolute pressure is the true pressure. This term is used when it is necessary to emphasize that the pressure being referred to is not a relative pressure, such as gauge pressure, or a differential pressure, such as the pressure drop across a valve. The lowest possible value of absolute pressure is zero (in any pressure units).

Absolute Viscosity

Fluid property with SI units Pas. This is also known as dynamic viscosity. Equivalent SI units are Nm^{-2}s and $\text{kgm}^{-1}\text{s}^{-1}$.

Atmospheric Pressure

The pressure of the atmosphere, which can be measured by a barometer.

1 standard atmosphere = 1.01325×10^5 Pa

Bulk Modulus of Elasticity

The bulk modulus of elasticity of a fluid is the ratio of a change in pressure to the fractional volume change (or volumetric strain) it causes. It is the inverse of the compressibility. The units are Pa.

$$K = \frac{\Delta p}{\left(-\frac{\Delta V}{V}\right)} = -V \frac{\Delta p}{\Delta V}$$

K = bulk modulus Pa

V = volume m^3

Δp = pressure change Pa

ΔV = volume change m^3

Compressibility

The compressibility of a fluid is the ratio of a fractional volume change (or volumetric strain) to the pressure change that causes it. It is the inverse of the bulk modulus of elasticity. The units are Pa^{-1} .

$$\beta = \frac{\left(-\frac{\Delta V}{V}\right)}{\Delta p} = -\frac{1}{V} \frac{\Delta V}{\Delta p}$$

β = compressibility Pa^{-1}

Continuity Equation

The volume flow rate is the same at every transverse cross section of a pipe. If the velocity can be considered uniform over each transverse cross section then

$$Q = v_1 A_1 = v_2 A_2 = v_n A_n.$$

Q = volume flow rate m^3/s

v_n = mean velocity over cross section n m/s

A_n = area of cross section n m^2

Darcy-Weisbach Equation

This equation gives the head loss due to friction in a pipe. The friction factor depends on the nature of the flow and can be found from a friction factor diagram. The head loss also depends on the length of the pipe, its diameter, the square of the velocity and the acceleration due to gravity.

$$h_f = \frac{4fL}{d} \frac{v^2}{2g}$$

h_f = head loss due to friction m

f = friction factor

L = length of pipe m

d = diameter m

v = mean fluid velocity ms^{-1}

g = acceleration due to gravity ms^{-2}

Dynamic Viscosity

Fluid property with SI units Pas. This is also known as absolute viscosity. Equivalent SI units are Nm^{-2}s and $\text{kgm}^{-1}\text{s}^{-1}$.

Fixed Displacement Pump

A fixed displacement pump is a positive displacement pump that discharges fluid at a fixed flow rate and where the fixed flow rate cannot be varied by changing the setting of the pump.

Fluid

A fluid is a substance that cannot sustain a shear stress without undergoing relative movement. Liquids and gases are fluids.

Friction Factor

This is a dimensionless factor in the Darcy-Weisbach equation for the head loss associated with pipe flow. The form used in this work is known as the Fanning friction factor. Some references use the Darcy friction factor which equals four times the Fanning friction factor—in these cases the constant '4' is replaced with unity in the Darcy-Weisbach equation.

Friction Factor Diagram (Moody Diagram)

This is a diagram for looking up the friction factor for use in the calculation of pressure or head losses in pipes. Lines on the diagram represent the friction factor as a function of Reynolds number and relative roughness.

Gauge Pressure

Gauge pressure is a relative pressure: it is the amount by which the pressure exceeds the pressure of the atmosphere.

Head

The term 'head' refers to the height from a position within a liquid at rest to a connected free surface of the liquid. The concept of head difference can also be used to describe a pressure difference in terms of an equivalent head of liquid.

Incompressible

An incompressible fluid is one whose volume does not change with changes in pressure. In general, gases are not incompressible as a

moderate change in pressure can cause a significant change in volume. However, liquids have very low compressibility and, as a simplification, can be considered incompressible.

Kinematic Viscosity

Fluid property with SI units m^2s^{-1} .

Major and Minor Head or Pressure Losses in Pipes

Major losses are those associated with fluid friction over the full length of the pipe. Minor losses are those associated with fittings such as valves and connectors and flow changing features such as bends, elbows, sudden expansions and sudden contractions.

Mass Flow Rate

This is the amount of mass that flows through a specified system (such as a heat exchanger) or cross section (of a pipe or duct, for instance) per unit time.

Moody Diagram or Chart (Friction Factor Diagram)

This is a diagram for looking up the friction factor for use in the calculation of pressure or head losses in pipes. Lines on the diagram represent the friction factor as a function of Reynolds number and relative roughness.

Newton's Viscosity Equation

$$\tau = \mu \frac{\Delta v}{\Delta y}$$

τ = shear stress Nm^{-2}

μ = absolute viscosity Pas (or Nm^{-2}s or $\text{kgm}^{-1}\text{s}^{-1}$)

v = fluid velocity ms^{-1}

y = distance in direction
normal to flow m

Normal Stress

A normal stress is a stress where the force involved is normal to the area over which it acts.

Pascal's Law

Pressure at a point within a fluid at rest acts equally in all directions and at right angles to any surface on which it acts. Pascal's law also states that if a small pressure change is caused at one point, or position, within a fluid at rest then the same pressure change occurs at all points within the fluid.

Positive Displacement Pump

A positive displacement pump is a pump that discharges a fixed volume of pressurized fluid for each rotation of the input shaft or for each cycle in the case of a reciprocating pump.

Pressure

Pressure is force exerted per unit area in a direction that is normal to and towards the area. It is expressed as

$$p = \frac{F}{A}$$

Pressure and Head

The pressure owing to the weight of a column of incompressible fluid (such as a liquid) of height h is given by $p = \rho g h$

Head is a way of expressing the pressure of a fluid that has a constant density in terms of a vertical distance, equivalent to h in the equation above. The relationship between head h and pressure p is

$$h = p / (\rho g)$$

or

$$\text{Head} = \text{Pressure} / (\text{Density} \times \text{Acceleration due to gravity})$$

In the context of this term, the pressure is normally the gauge pressure. For a liquid at rest at a point within a pressurised system or pipe the head is the height above the point to which the liquid would rise in a vertical tube of sufficient length and open to the atmosphere that might be attached to the system at that position.

Pressure Relief Valve

A pressure relief valve is a spring-loaded valve that only allows flow when a certain pressure level is exceeded. The opening of the valve relieves the pressure, preventing it from going significantly higher.

Relative Roughness

The relative roughness of a pipe, k/d , is the ratio of the absolute roughness of its surface to its diameter (or bore). The concept of absolute roughness was originally based on roughness that was created artificially with grains of sand: the absolute roughness was equivalent to the height of the sand grains used to create the roughness. As both the numerator and the denominator have the units of length, the relative roughness is dimensionless.

Reynolds Number

A dimensionless number used to characterise the type of flow.

$$\text{Re} = \frac{\rho v d}{\mu} = \frac{v d}{\nu}$$

ρ	=	density	kg/m ³
v	=	fluid velocity	m s ⁻¹
d	=	diameter	m
μ	=	absolute viscosity	Pas (or Nm ⁻² s or kgm ⁻¹ s ⁻¹)
ν	=	kinematic viscosity	m ² s ⁻¹

Shear Stress

A shear stress is a stress where the force involved is in the same plane as the area over which it acts.

Steady Flow

Flow is said to be steady if the velocities and other parameters, e.g. density and pressure, to not vary with time. The velocity and other parameters may vary from point to point within the flow.

Stress

A stress is a force per unit area (S.I. units N/m^2).

Throttling

Throttling is said to occur wherever a flow restriction causes a pressure drop.

Uniform Flow

Flow is said to be uniform if the velocities and other parameters, e.g. density and pressure, do not vary over a region, usually a transverse cross-section, of a flow stream.

Vacuum and Vacuum Pressure

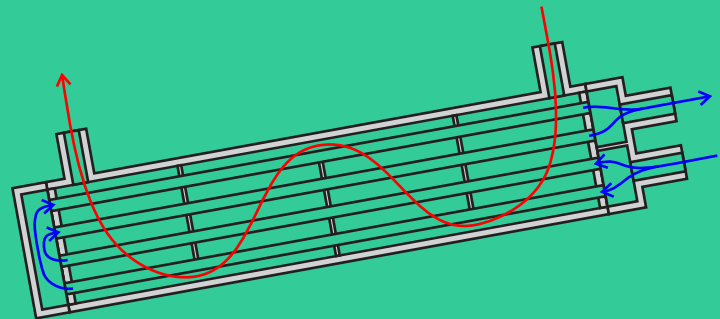
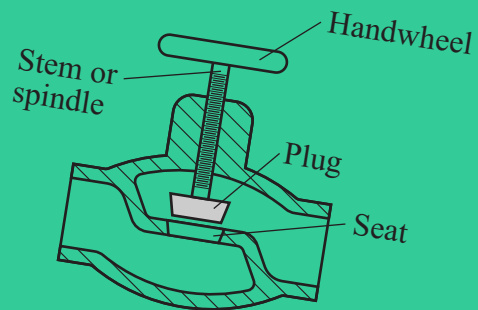
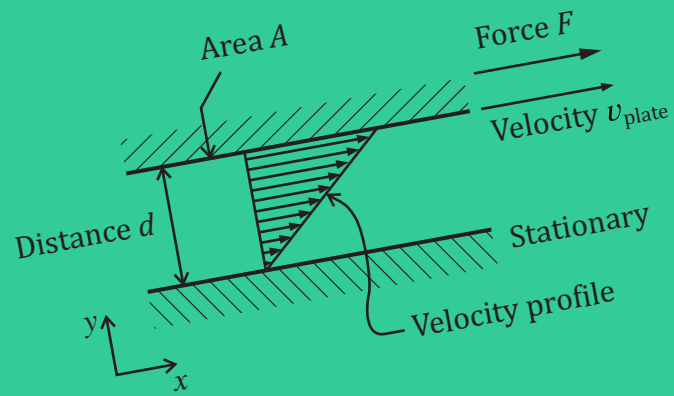
Vacuum pressure is the amount by which a pressure is less than atmospheric pressure. In a perfect vacuum the absolute pressure would be zero.

Viscosity

Viscosity is a measure of the resistance that a fluid provides to shear, i.e. the sliding of one fluid layer over another. Absolute viscosity has the units of Pas or $\text{kgm}^{-1}\text{s}^{-1}$. Kinematic viscosity has the units of m^2s^{-1} .

Volumetric Strain

Volumetric strain is defined as a change in the volume of a substance divided by the original volume. Volumetric strain can be caused in a gas or a liquid by increasing or decreasing the pressure to which it is subjected.



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