2008-09-01

Some Rather Mechanical Reflections on Symmetry: in Art, Science, Engineering, Mathematics, etc.

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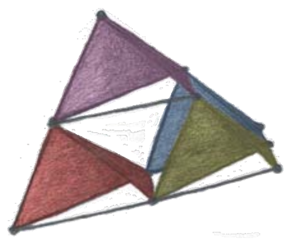
Some Rather Mechanical Reflections on Symmetry: in Art, Science, Engineering, Mathematics, etc.

Inaugural Lecture as a Professor of
Dublin Institute of Technology

by Jim McGovern



25th September 2008

Michael O'Donnell Lecture Theatre
Faculty of Engineering
Dublin Institute of Technology
Bolton Street, Dublin 1

This Inaugural Lecture consists of some of my rather mechanical, being an engineer, reflections on symmetry in diverse areas such as art, science, engineering, mathematics, etc. These reflections are not mine in the sense that I am their sole owner or want to keep them to myself. In fact I need to bounce them off others. They are yours too if any of them strike a chord with you. I have tried to include enough diversity that this might occur with some reflection or other.

I explain what symmetry is to me, giving examples with lots of images and mentioning or at least barely referencing art, science, architecture, engineering, heritage, cosmology, bicycles, flight, invention, ingenuity, history, wallpaper, mathematics, typography, structures, regular shapes, coordinate systems, spacetime, thermodynamics and suchlike.

Symmetry

Symmetry is sameness when objects at different positions are viewed from one position or when one object is viewed from different positions. Symmetry is closely associated with concepts or expressions such as 'the same again', familiarity, repetition, patterns, building blocks and order. We are immersed in symmetry and we are comfortable with it.

Bilateral Symmetry

The sculpture in Figure 1 by Joan Miró illustrates bilateral symmetry very well and has beauty because of that. If there is a feature on the left then there is also one on the right. If there is something in the middle, like the neck, there may be just one of that.

Figure 1 Personage, 1970, Bronze, Joan Miró, 1893–1983 (Fundació Joan Miró, Barcelona)
Photo: J. McGovern

The Symmetry of Repetition

In Figure 2 'same again' symmetry or the symmetry of repetition is used to striking effect in architecture. The symmetry of replicated Roman arches is a feature of the Pont du Gard, Figure 3. This was part of a 50 km aqueduct that supplied water to the Roman city of Nimes, in southern France, for about eight centuries.

Figure 2 Centro Vasco de Gama, Lisbon, 1999. Architects: BDP
Photo: J. McGovern

Figure 3 Pont du Gard, 1st century AD, southern France
Photo: J. McGovern

In engineering, the symmetry of repetition can be a very powerful tool: effects that could not be achieved by one instance of a device or process can often be realised by multiple instances. Figure 4, which is also from southern France, shows seven connected locks of the Canal du Midi, which was constructed in the seventeenth century and provided a transportation route between the Atlantic and the Mediterranean.

Figure 4 Locks of the Canal du Midi at Fonséranes near Béziers, southern France
Photo: J. McGovern

In music too symmetry is of the essence. The four-tap motif of Beethoven's 5th symphony is an example. The repeated motif of taps is a musical framework of a sort and has character in itself. Each tap is a point in time that belongs to a pattern that recurs in time.

Rotational Symmetry

The flowers in Figure 5 possess a high degree of symmetry. Each of the eight petals of a flower has the same characteristics and they are arranged symmetrically. The unopened buds possess the same symmetry. On examining the flower to the forefront it can be noticed there is bilateral symmetry, rather than strict rotational symmetry. There is a topmost petal, a bottommost petal and there are three pairs in between.

Figure 5 Flowers
Photo: J. McGovern

The flower of Figure 6 and the starfish of Figure 7 share five-fold rotational symmetry.

Figure 6 A pentagon in a flower
Photo: J. McGovern
(illuminated by flash)

Figure 7 A starfish (Oceánario de Lisboa, Portugal)
Photo: J. McGovern

Broken or Altered Symmetry

Figure 8 is a rough sketch based on Vincent van Gogh's 1888 painting of sunflowers. The shapes of the sunflowers are non-symmetrical and to me the emotion seems to be one of distress.

Figure 8 Rough pencil sketch based on Sunflowers, Vincent van Gogh, 1888 (Neue Pinakothek, Munich)
Sketch: J. McGovern (colour added with Corel PhotoPaint)

Rhyme in poetry is another example of symmetry. In the following verse William Blake chose to break the symmetry of rhyming couplets.

Tiger, tiger, burning bright
In the forests of the night,
What immortal hand or eye
Could frame thy fearful symmetry?

The Tiger, William Blake, 1757–1827

An appreciation of symmetry is important for any artist or designer. For example, the symmetry of three is significant in the design of electric shavers with rotating cutters. There is a need to maximise contact area and accommodate convex or concave contours and, perhaps, for all of the cutters to be driven via gears from a single motor.

Figure 9 shows an electric shaver and a transparent plastic cover that can be used to protect the delicate cutting heads. The cover comes close, visually, to being symmetrical, but only fits in one position. As the user, I find this aspect to be poor design.

Figure 9 An electric shaver with its plastic head-protecting cover
Photo: J. McGovern

Perspective, Mechanical Drawing, Art and a Grid

The original painting on which the drawing shown in Figure 10 is based is in the Sistine Chapel in the Vatican. The fresco is remarkable for its symmetry in a number of respects:

- The square tessellation of the plane is a feature of the work.
- The central temple is octagonal and symmetrical.
- There are *two* symmetrical triumphal Arches of Constantine (Rome 315 AD)
- Perspective is implemented geometrically and mechanically (as well as by tone).
- Figures in standard profiles are scaled according to their position on the flat plane.
- Perspective is a mapping (a mathematical term) from a higher to a lower dimension (3-D to 2-D).

Figure 10 Drawing with superimposed silhouettes based on 'Christ handing over the keys to St. Peter', Pietro Perugino, 1482, fresco, Sistine Chapel, the Vatican. Illustration: J. McGovern (pencil drawing; colour added with Corel PhotoPaint)

The fresco contains considerably more persons than are shown in Figure 10. It contains two gospel episodes in addition to the main foreground scene—the soldier in the middle ground is from one of these. Events at different times and places are thus situated on the same regular grid, marked out on the ground. To me this is a spacetime grid of sorts.

Wallpaper Symmetry and Group Theory

Wallpapers and ceramic tiles are examples of the use of symmetry for decorative purposes. Figure 11 is an example of wallpaper on a *ceiling*! Mathematicians look at such things abstractly under the heading of 'wallpaper groups'. In fact the mathematical theoretical basis for understanding symmetry is group theory. This developed from the work of Évariste Galois (1811–1832), who greatly advanced understanding of the solutions of polynomial equations by correctly identifying the underlying symmetries. Who would ever have expected that the solution to a quadratic equation could be related to wallpaper?

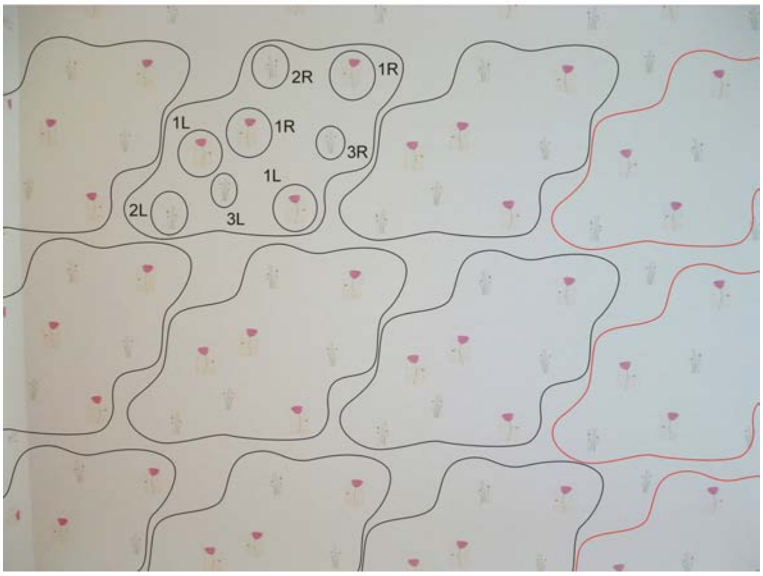


Figure 11 Wallpaper on a ceiling. The superimposed black lines and labels identify reflected and repeated patterns. The red lines enclose a different pattern, resulting from a (possibly unintentional) glide transformation (to use the mathematical terminology).
Photo: J. McGovern

Figure 12 Paving tiles in Barcelona to the design of Antoni Gaudí (1852–1926)
Photo: J. McGovern (at night with flash)

The Spanish architect Antoni Gaudí had a very deep appreciation of symmetry. He designed the hexagonal tiles shown under my feet in Figure 12. There is three-fold rotational symmetry of the entire pattern about three of the vertices of each tile. Notice also the triple spiral at one vertex of each tile.

Pythagoras and the Symmetry of the Counting Numbers

Still in the area know as group theory I would like to mention something that left me flabbergasted when I came across it. Most people have heard of the theorem of Pythagoras, which is connected with geometry and symmetry. The fact that right-angled triangles can be constructed having certain combinations of whole numbers as the lengths of their sides is very useful for constructing right angles in two dimensions:

$$a^2 + b^2 = c^2 \tag{1}$$

for example

$$3^2 + 4^2 = 5^2 \text{ or } 20^2 + 21^2 = 29^2.$$

The following equations illustrate the further generality of the concept when applied to certain combinations of whole numbers (counting numbers).

$$a^2 = b^2 \tag{2}$$

for example

$$1^2 = 1^2 \text{ or } 723^2 = 723^2.$$

Equation 2 only works if $a = b$, which is rather trivial. It is a one-dimensional equivalent of the theorem of Pythagoras. There is also a three-dimensional equivalent:

$$a^2 + b^2 + c^2 = d^2 \tag{3}$$

for example

$$1^2 + 2^2 + 2^2 = 3^2 \text{ or } 2^2 + 3^2 + 6^2 = 7^2.$$

Therefore certain cuboids can be constructed with whole-number side lengths *and* a body diagonal that also has a whole-number length. What about four dimensions? There is a version:

$$a^2 + b^2 + c^2 + d^2 = e^2 \tag{4}$$

for example

$$1^2 + 2^2 + 2^2 + 4^2 = 5^2 \text{ or } 3^2 + 4^2 + 12^2 + 84^2 = 85^2.$$

The following equation is the one that flabbergasted me when I came across it in the book 'Symmetry and the Monster' by Mark Ronan[1]:

$$\begin{aligned} &a^2+b^2+c^2+d^2+e^2+f^2 \\ &+g^2+h^2+i^2+j^2+k^2+l^2 \\ &+m^2+n^2+o^2+p^2+q^2+r^2 \\ &+s^2+t^2+u^2+v^2+w^2+x^2 = y^2 \end{aligned} \tag{5}$$

with the following solution

$$\begin{aligned} &1^2+2^2+3^2+4^2+5^2+6^2 \\ &+7^2+8^2+9^2+10^2+11^2+12^2 \\ &+13^2+14^2+15^2+16^2+17^2+18^2 \\ &+19^2+20^2+21^2+22^2+23^2+24^2 = 70^2. \end{aligned}$$

Helical Symmetry

Figure 13 is from the Nobel Lecture of Maurice H. F. Wilkins in connection with the 1962 Nobel Prize in Physiology or Medicine. This was awarded to Francis Crick, James Watson and Maurice Wilkins for their discovery of the double-helical structure of DNA. Rosalind Franklin, who had made a significant experimental contribution, died in 1958 and would not therefore have been eligible to be a joint awardee in 1962. There is a certain asymmetry here, in addition to that of gender imbalance; generally one should contrive and make provision to live for as long as possible after making any major discovery, or make one's most important discoveries towards the beginning of one's life.

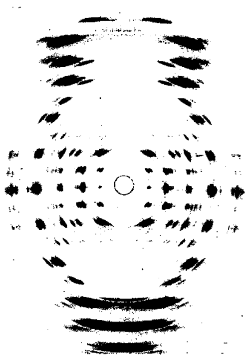


Figure 13 X-ray diffraction pattern of microcrystalline fibres of DNA[2]. Photograph: M. H. F. Wilkins with H. R. Wilson[2]

The 2-D diffraction pattern of Figure 13 indicated the double-helical symmetry of the DNA molecule. The diffraction pattern technique was not new as it had already been used to determine the structures of crystals. In his 1952 book[3], Hermann Weyl, the German mathematician and philosopher of science, clearly stated the already-established underlying principle:

> *If conditions which uniquely determine their effect possess certain symmetries, then the effect will exhibit the same symmetry.*

It is not know who invented or discovered the first helix, or double helix. Some examples are presented in Figures 14 to 16.

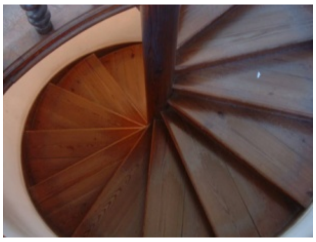


Figure 14 A helical staircase, Palácio Nacional de Sintra, Lisbon
Photo: J. McGovern

Figure 15 A double-helical staircase, the Vatican
Photo: J. McGovern

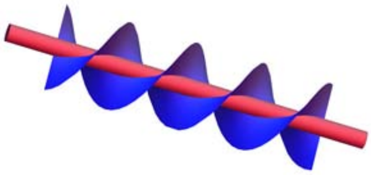


Figure 16 Schematic of a double-helical Archimedean screw. If enclosed in a cylinder and rotated the screw can lift water. This device has been used since ancient times as a water pump.
Illustration: J. McGovern (rendered in Mathematica as a cylinder and a helical

Figure 17 Spiral galaxy NGC 4414
Image: Hubble Heritage Team[5] (AURA/STScI/NASA), 1999
(Distance about 60 million light-years).

Spiral Symmetry

Spiral symmetry, as illustrated on a vast scale in Figure 17, is a recurrent theme in nature at *every* scale. Certain sub-atomic particles move in spiral trajectories, as first observed in classical cloud chamber experiments. Stephen Hawking's popular 2001 book[4] 'The Universe in a Nutshell' contains a schematic of two spiralling compact neutron stars as well as an explanation of the significance of astronomical observations of such a binary pulsar. Ancient carvings at Newgrange, Figure 18, indicate that spiral patterns have been part of human consciousness for a long time.

Figure 18 Carved spiral patterns at Newgrange[6], circa 3200 BC
Photo: J. McGovern

From Bikes to Flight

Innovation requires a certain freedom of thought, not overly constrained by experience or the status quo. The bicycle shown in Figure 19 is a good example of what I mean, as well as being a direct practical application of the concept of symmetry. It is a bicycle made for two equal partners in more than one sense. The technology, from 1889, is still recognisably current. This type of technology was familiar to Orville and Wilbur Wright, but they pushed it to new heights, aided by their sense of symmetry.

Figure 19 Sociable bicycle, 1898, Graz. Národní Technical Museum, Prague
Photo: J. McGovern

In developing the first successful heavier-than-air engine-powered aircraft the Wright brothers, who were bicycle designers and manufacturers, built on the research and experience of Otto Lilienthal (1848–1896), who lived and worked in Berlin, Germany. Just as Leonardo da Vinci (1452–1519) had done centuries earlier, Lilienthal studied bird flight for inspiration. He built and flew many gliders, such as that illustrated in Figure 20.

Figure 20 Display model of Otto Lilienthal flying one of his gliders (in about 1894) Deutsches Museum, Munich
Photo: J. McGovern

The Wright Flyer, Figure 21, was the result of much further, careful, research and experimentation by the Wright brothers.

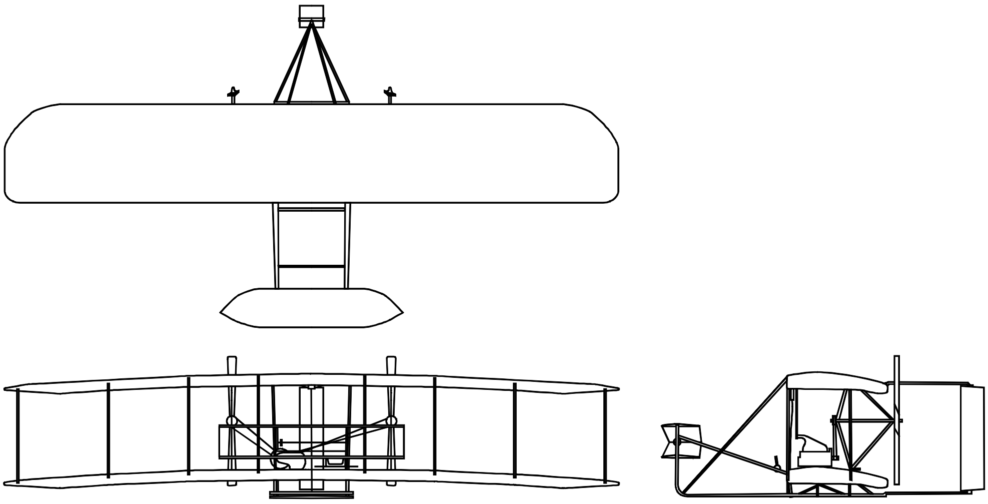


Figure 21 The Wright Flyer, 1903[7]. Simplified line diagram: J. McGovern

The Flyer incorporated many aspects of symmetry:

- For a low airspeed and low propulsion power the two geometrically identical, vertically stacked wings maximised lift.
- The biplane wings also provided strength and rigidity while being light in weight.
- The front elevator also had two geometrically identical aerofoils, as had the rear rudder.
- Bilateral symmetry was essentially maintained. The petrol engine, which was offset to one side, was counterbalanced by the pilot who was offset to the other side.
- Control of yaw and roll was by distorting the bilateral symmetry of the wings in unison with rudder operation. The Wright brothers devised a system of wing warping, no doubt based on their knowledge of bird flight, which they checked-out experimentally using kites.
- The Flyer employed two pusher propellers, or air screws; one on each side.
- The Wright brothers were concerned that two propellers driven from the same engine shaft and turning in the same sense might cause the aircraft to roll in the opposite sense. They addressed this by making the propellers counter-rotate by twisting the drive chain of one, Figures 22 and 23.

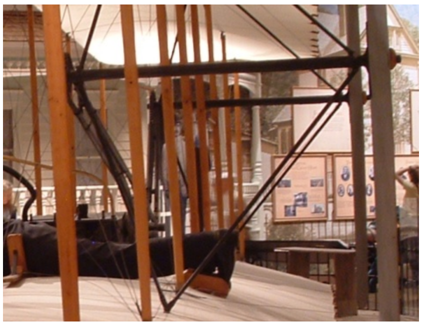


Figure 22 View showing the two propellers and propeller shafts of the Wright Flyer.
The Smithsonian National Air and Space Museum, Washington.
Photo: J. McGovern

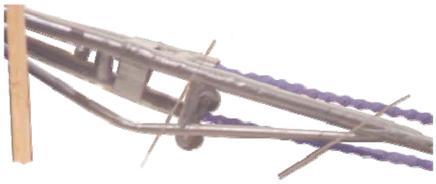


Figure 23 Tubular sleeves and struts for left propeller chain, Wright Flyer, 1903.
The Smithsonian National Air and Space Museum, Washington.
Photo and detail enhancement: J. McGovern

The clever trick employed by the Wright brothers of twisting one of the drive chains of the Flyer involved the symmetry of the figure-of-eight, as illustrated in Figure 24.

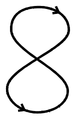


Figure 24 A figure-of-eight drawn with a continuous motion, which involves both clockwise and counter clockwise rotation.

Flying a course in a figure-of-eight was part of the requirements for the Kramer Prize for sustained manoeuvrable human-powered flight. The competition was won in 1977 by Paul B. McCready Jr's Gossamer Condor, Figure 25. The strut seen to the front supported a front elevator. In many ways this was a 'mono' version of the Wright Flyer: front mono elevator, mono wing, mono pusher propeller and the pilot and engine were fully integrated. Bilateral symmetry was generally maintained, although propeller rotation in one direction only was evidently not a significant issue.

Figure 25 Gossamer Condor, Paul B. McCready Jr., The Smithsonian National Air and Space Museum, Washington.
Photo: J. McGovern

From Abl to Zeebl (or Zedbl)

A number of years ago I came up with the idea of a symbol alphabet based on symmetrically doubled roman letters. In the meantime I have spent far too much time quietly developing the idea further.

Mathematicians, scientists and engineers often like to use a single symbol to represent something, but sometimes it can be hard to find a symbol that is not already associated with other *things* in the same area. For a mathematician the *thing* might be as totally abstract as a tree, which is a graph and should not to be confused with something that can be blown over in a storm or can provide firewood. A physicist might want a symbol to represent something as elementary as a beta particle. For a while the number of elementary particles seemed to become unmanageably large. Thankfully it now seems to have come down again to around forty. An engineer might want a symbol to represent something meaningful… to an engineer, like a tensile stress. But engineers would rather not have to learn the names of any more funny letters like Ξ (Xi) or Ψ (Psi) or to remember which is which is which. I thought I could lessen some of these problems with my idea.

Figure 26 illustrates some of the characters of the AblBeebl alphabet, which can also be described as an ablbeeblbet (abl-beebl-bet). Figure 27 illustrates a full set of upper case roman letters and a full set of lower case roman italic letters. Equation 6 shows how the symbols might be used in a formula.

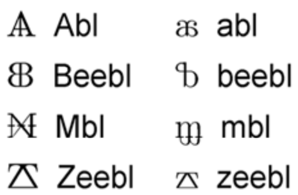


Figure 26 Some letters from AblBeebl, a symbol alphabet concept by J. McGovern, based on the letter forms of Donald Knuth's Computer Modern font[8]. Zeebl can of course also be called Zedbl.

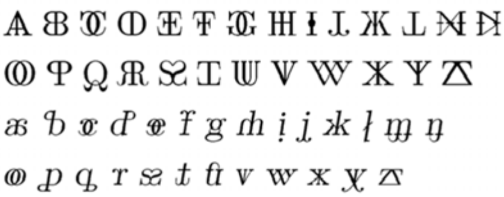
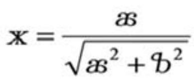


Figure 27 AblBeebl in normal uppercase and italic lower case.



(6)

Symmetrical 2-D Point Patterns

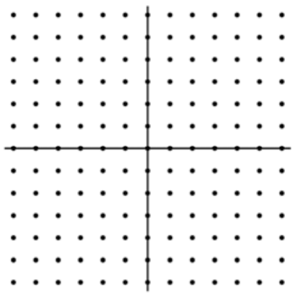


Figure 28 Square pattern[9]

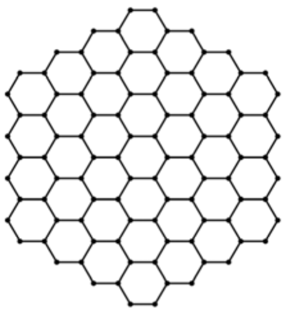


Figure 29 Hexagonal pattern[9]

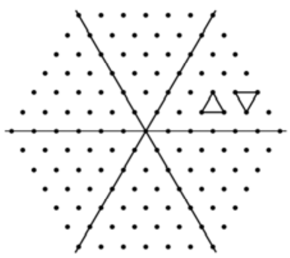


Figure 30 Equilateral triangular pattern[9]

In two dimensions there are only three fully regular arrangements of points or nodes that can create a repeating pattern: square, hexagonal and triangular, as shown in Figures 28 to 30.

Figure 31 illustrates a direct engineering application of one of these patterns: geogrid technology provides soil stabilisation in the construction of motorways and paved areas. The symmetry of the pattern allows this particular geogrid fabric to withstand external stresses and distribute them within its links.

Figure 31 Equilateral triangles of a geogrid product
Illustration: Advertisement for TriAx geogrid technology, Tensar International, The Engineers Journal, 2008[10]

Symmetrical Grids that Form Closed Surfaces or Polyhedra

In 1996 Robert Curl, Harold Kroto and Richard Smalley were awarded the Nobel Prize in Chemistry for the discovery of a new class of carbon compounds, for example the carbon 60 molecule, C_{60}. The approximation of a sphere by a structure of network links had already been popularised by the American architect Richard Buckminster Fuller (1895–1983). Hence the C_{60} molecule is known as a buckyball and the class of compound is called fullerene. Many soccer balls use the same arrangement of pentagonal and hexagonal faces. Who would have thought that soccer balls exist at the nanometre scale?

I made the model shown in Figure 32 using 60 spheres and 90 links of the Geomag construction set toy. I used an inflated balloon to provide necessary outward force on the links and nodes.

The buckyball is not a fully regular polyhedron. In fact it is a truncated form of the regular icosahedron, Figure 33, which is one of the five regular solids (tetrahedron, cube, octahedron, dodecahedron, icosahedron).

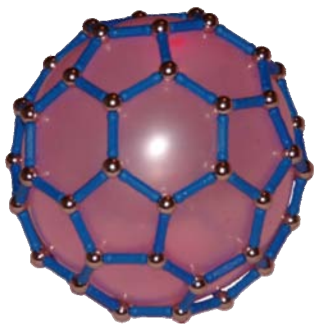


Figure 32 Model of a buckyball made with Geomag and an inflated balloon
Photo: J. McGovern

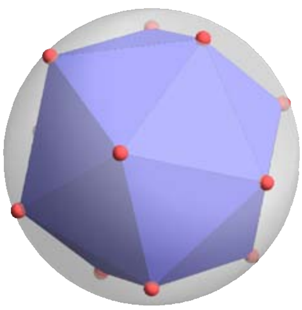


Figure 33 A regular icosahedron within a sphere with spherical nodes at the vertices
Diagram: J. McGovern using Mathematica

Euclid's Elements (300 BC approx.) contains all the necessary theory for the geometrical construction of regular polygons and polyhedra (3-D solids with flat faces) using a compass (or 3-D equivalent) and a straight edge. Figure 34 is another example: the regular tetrahedron.

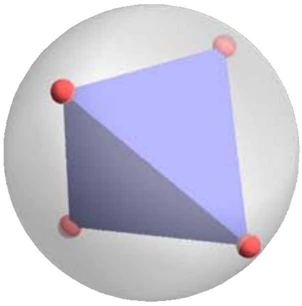


Figure 34 A regular tetrahedron within a sphere with spherical nodes at the vertices
Diagram: J. McGovern using Mathematica

Lattices in Three Dimensions

Many people are aware that various crystal lattices exist in nature, but Figure 35 illustrates a lattice as an engineering structure for a kite. Alexander Graham Bell and Richard Buckminster Fuller were both fascinated by this lattice, which has amazing symmetry properties. I like to call it the Bell Fuller lattice. In crystallography it would be called the cubic close-packed lattice.

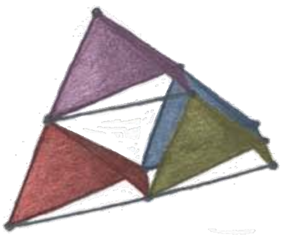


Figure 35 Tetrahedral kite structure, of the general type devised by Alexander Graham Bell and built by him in a great variety of overall shapes and sizes[11]
Sketch: J. McGovern

In three dimensions there are only a limited number of arrangements of points or nodes that can create a repeating pattern and that allow translations, rotations and reflections. Figure 36 illustrates the arrangement of links around any interior node within the Bell Fuller lattice and Figure 37 shows a Bell Fuller lattice that forms a cube.

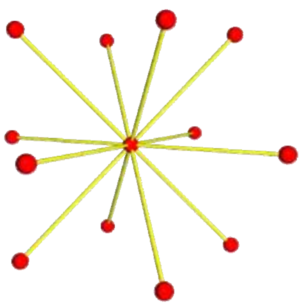


Figure 36 Bell Fuller lattice nodes and links[9]—each interior node has twelve nearest neighbours.

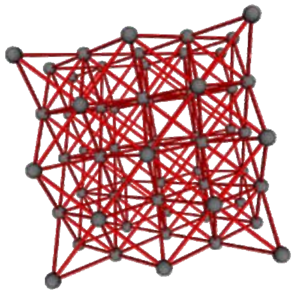


Figure 37 Bell Fuller lattice in the form of a cube[9]

In investigating the characteristics of the Bell Fuller lattice I found that four of these lattices can be interlaced in a symmetrical way, giving a composite lattice, as shown in Figures 38 and 39. I have used the colours red, yellow, blue and violet to distinguish between the four interlaced lattices. Figure 38 shows a line of nodes of the composite lattice.

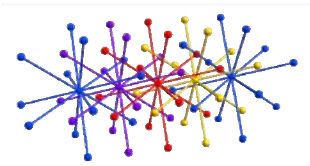


Figure 38 Node gussets (or joints) from four Bell Fuller lattices forming a line of nodes of the composite 4-colour lattice[12]. The two blue gussets are from the same Bell Fuller lattice.

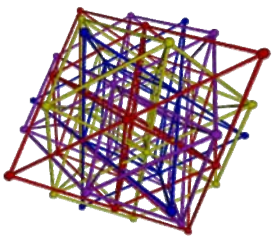


Figure 39 Four interlaced Bell Fuller lattices in the form of a rhombic dodecahedron[9]

The shortest links between the nodes of the 4-colour lattice are illustrated in Figures 40 and 41. I have described this lattice as the 4-colour rhombohedral lattice.

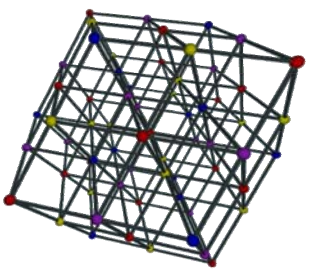


Figure 40 Shortest links of the 4-colour rhombohedral lattice in the form of a rhombic dodecahedron[9]

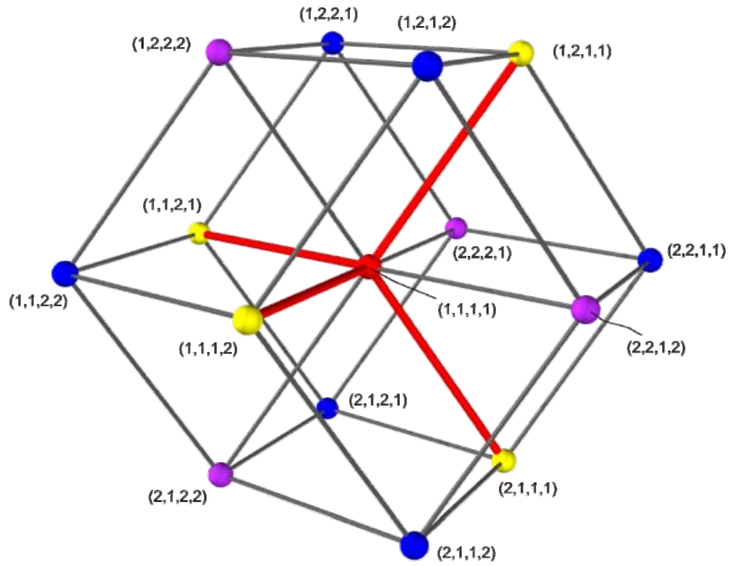


Figure 41 4-tuple coordinate system for the 4-colour rhombohedral lattice
Diagram: J. McGovern

This discrete, or point, lattice is a viable alternative reference frame for undertaking calculations and analyses in what is often called 3-D space. It may also be viable for what is often called 4-D spacetime. Every point on the lattice can be represented as an integer 4-tuple, e.g. (7, 3, 4, 11) or perhaps (1 378 957 386, 857 603 625 485, 86 042, 98 567 895 289).

I believe the 4-colour rhombohedral lattice and the 4-tuple coordinate system illustrated to have many potential applications in Engineering, Physics, Nanotechnology, Cosmology etc. For instance, the 4-colour rhombohedral lattice could be a candidate for being the fundamental lattice of spacetime.

In a recent paper[12] I have provided an explanation of the relationship between the 4-colour rhombohedral lattice and a discrete cubic (or cartesian) lattice as shown in Figure 42.

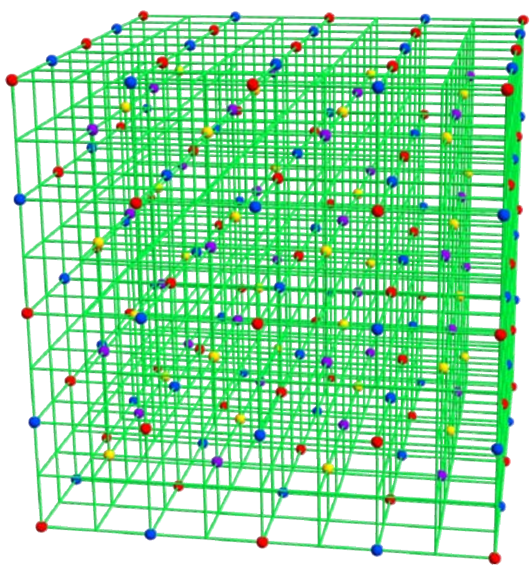


Figure 42 Points of the 4-colour rhombohedral lattice superimposed on a discrete cubic lattice[12]

Symmetry and Time

Figure 43 Frames from a video clip in which the author unstrikes a match.
Video: J. McGovern

The laws of physics allow time to proceed in reverse… except for one little law. The match in Figure 43 can be unstruck, exactly as in the video, except for the second law of thermodynamics, which in effect defines what the future can and cannot contain. It seems to me, however, that the fundamental symmetry of displacement and time is not violated. The apparent asymmetry results from the fact that we are on the arrow of time!

Symmetry and a Discrete Spacetime Hypothesis

What follows is rather speculative. My general line of thought is that perhaps spacetime is made up of discrete points rather than being a continuum. If so, then intuition suggests that the points of spacetime would be arranged in an ordered way: they would comprise a lattice. It turns out that there are not many ways that the points of discrete spacetime could be arranged. A cubic lattice is one possibility. However, my own investigations suggest that the 4-colour rhombohedral lattice would be a strong candidate to be the most fundamental lattice shape as it seems to have greater symmetry than a cubic lattice and in fact contains sixteen interleaved cubic lattices[12].

I was particularly struck by the notion of quantum mechanics being present on every scale on reading a comment in the book '*The Fabric of the Cosmos*' by Brian Greene[13]. He stated:

> *According to inflation, the more than 100 billion galaxies, sparkling throughout space like heavenly diamonds, are nothing but quantum mechanics writ large across the sky.*

The Relationship Between Discrete Time and Discrete Distance

In 1879 Albert Michelson[14] carried out a classical experiment of Physics in which he made an accurate determination of the 'velocity of light'. An alternative interpretation of this experiment might be[9]:

> *Michelson measured the 'distance equivalent of time', just as Joule measured the 'mechanical equivalent of heat'.*

Distance in metres and time in seconds would then be related by the following defining equation

$$299\,792\,458\ [\mathrm{m}] = 1\ [\mathrm{s}].$$

In coherent units, the 'velocity of light' has a value of unity. This is not as outlandish as it might seem, as particle physicists sometimes find it convenient to work with such units.

Change, Motion and Action in Discrete Spacetime

- Anything that one might consider a 'particle' exists not just at one point in spacetime, but as a repeating pattern (of perhaps very many points) along a lattice path (or string) in spacetime.
- Change is a morphing transition between patterns in spacetime, Figure 44.
- Motion and action can be explained in terms of patterns, as in Figure 45.

Figure 44 Change: the hat in state (a) has morphed into the hat in state (b). Photos: J. McGovern

Figure 45 Motion or action[9]: for an observer on pattern 1, pattern 2 is moving away.

In Figure 45 the second pattern has a constant velocity with respect to the first. Relative acceleration would imply that the distance between instances of one of the patterns changes.

Thermodynamics and Discrete Spacetime

Thermodynamics is the science of energy transformations involving heat, work and the properties of substances or systems. Properties are the attributes of a substance or system when it is in a particular state. In the context of discrete spacetime, a state would be an instance of a pattern.

If spacetime were a regular point lattice then all instances (or states) of a fixed or morphing pattern could be ordered (put into sequence). Two applied mathematicians, Elliott Lieb and Jakob Yngvason[15] (1999), have presented a well founded thermodynamics theory based on the ordering of sets. It includes the concept of entropy and that of temperature derived from it.

In relativity and cosmology the Hawking temperature[16] (the temperature at which a black hole radiates energy) and the Unruh temperature[17] (the temperature to which an accelerated observer is excited while travelling through the vacuum) are both proportional to acceleration. Therefore it would seem these temperature effects too can be related to patterns in discrete spacetime.

Conclusion

At the risk of stating what may be obvious, it seems to me that symmetry underlies the observed structure of the universe at all scales, including the following, rather technical, areas:

- mechanics
- quantum mechanics
- the Standard Model of particle physics
- general and special relativity
- cosmology.

Reflections on symmetry
Photo: J. McGovern

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About the Author

Jim McGovern has worked for Adtec Teoranta, University College Galway, Carlow Institute of Technology, Trinity College, Simtherg Limited (which he established) and the Irish Government's Department of Transport. He has also worked, on contract from Simtherg Limited, with SMCI Software of London as a simulation specialist and manager. He has undertaken research and/or published work in the following areas: heat pumps; energy use in buildings; refrigeration systems; positive displacement compressors; advanced power or combined cycles; engineering thermodynamics; exergy (ex ergon, meaning 'from work') analysis; zero emissions technologies and concepts; and transportation. Jim McGovern is currently the Head of the School of Mechanical and Transport Engineering at the Dublin Institute of Technology.

About the School of Mechanical and Transport Engineering

The School has two Departments: Mechanical Engineering and Transport Engineering. The activities of the School include taught degrees in Mechanical Engineering; Automotive Management and Technology; and Transport Operations and Technology. Apprentice training is provided for Light Vehicle Mechanics, Heavy Vehicle Mechanics, Vehicle Body Repairers and Aeronautical Mechanics. The National Institute for Transport and Logistics (NITL) is within the School and this provides research and consultancy as well as postgraduate education in Supply Chain Management. Various part-time or short programmes are also delivered within the School. Programmes are accredited by a range of external bodies, for example Engineers Ireland or the European Aviation Safety Agency. Research is undertaken that supports the teaching and training activities of the School and provides the context for education to Masters, PhD and post-doctoral levels.

