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Probalistic analysis of highway bridge traffic loading

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PROBABILISTIC ANALYSIS OF HIGHWAY BRIDGE TRAFFIC LOADING

by

COLIN C. CAPRANI

OCTOBER 2005

Thesis submitted to the National University of Ireland, University College Dublin, School of Architecture, Landscape and Civil Engineering, in fulfilment of the requirements for the degree of Doctor of Philosophy

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Project Supervisor

DEDICATION

To my mother, Joan Caprani.

DECLARATION

The author hereby declares that this thesis, in whole or part, has not been used to obtain any degree in this, or any other, university. Except where reference has been given in the text, it is entirely the author's own work.

The author confirms that the library may lend or copy this thesis upon request, for academic purposes.

Colin C. Caprani

October 2005

ABSTRACT

Many bridges of the world's highway networks have been in service for decades and are subject to escalating volumes of traffic. Consequently, there is a growing need for the rehabilitation or replacement of bridges due to deterioration and increased loading. The assessment of the strength of an existing bridge is relatively well understood, whereas the traffic loading it is subject to, is not as well understood. Accurate assessment of the loading to which bridges may be subject, can result in significant savings for highway maintenance budgets internationally. In recent years, a general approach has emerged in the research literature: the characteristics of the traffic at a site are measured and used to investigate the load effects to which the bridge may be subject in its remaining lifetime.

This research has the broad objective of developing better methods of statistical analysis of highway bridge traffic loading. The work focuses on short- to medium-length (approximately 15 to 50 m), single- or two-span bridges with two opposing lanes of traffic. Dynamic interaction of the trucks on the bridge is generally not included.

Intuitively, it can be accepted that the gap between successive trucks has important implications for the amount of load that may be applied to any given bridge length. This work describes, in quantitative terms, the implications for various bridge lengths and load effects. A new method of modelling headway for this critical time-frame is presented.

When daily maximum load effects (for example) are considered as the basis for an extreme value statistical analysis of the simulation results, it is shown that although this data is independent, it is not identically distributed. Physically,

this is manifest as the difference in load effect between 2- and 3-truck crossing events. A method termed composite distribution statistics is presented which accounts for the different distributions of load effect caused by different event types. Exact equations are derived, as well as asymptotic expressions which facilitate the application of the method.

Due to sampling variability, the estimate of lifetime load effect varies for each sample of load effect taken. In this work, the method of predictive likelihood is used to calculate the variability of the predicted extreme for a given sample. In this manner, sources of uncertainty can be taken into account and the resulting lifetime load effect is shown to be calculated with reasonable assurance.

To calculate the total lifetime load effect (static load effect plus that due to dynamic interaction), the results of dynamic simulations based on 10-years of static results are used in a multivariate extreme value analysis. This form of analysis allows for the inherent correlation between the total and static load effect that results from loading events. A distribution of dynamic amplification factor and estimates for a site dynamic allowance factor are made using parametric bootstrapping techniques. It is shown that the influence of dynamic interaction decreases with increasing static load effect.

ACKNOWLEDGEMENTS

In the course of any work such as this, it is inevitable that an author comes to rely on many people; the work that results is partly theirs: this is no exception. It is because of these people that these years have proved the best of my life.

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To all of my family, here with us and gone before us, it simply wouldn't have happened without you. To Mam and Maurice; Anna, Connor, Jack and Carla; Stephen, Tracy and Joey; thank you. To the memories of Dad; Nanna and Grandad; and Honey; I know your support in the early years has me here.

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To you the reader, whether you make it all the way through or fail in a valiant attempt, thank you.

Finally, I would like to thank all those people from whom, with or without their knowledge, I have learned.

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Chapter 1

INTRODUCTION

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"One should aim not at being possible to understand, but at being impossible to misunderstand" – Marcus Fabius Quintilian

Chapter 1 - INTRODUCTION

1.1 Background

The developed economies of the world have, as a prerequisite, a transport infrastructure that is efficient in the movement of goods and people. In such economies, the highway transport infrastructure was, in many cases, built in the decades following World War II. Hence, the bridges built for these highway networks have been in existence for a significant proportion of their design lifetime. Deterioration of these older bridges has been found in many countries; yet with economic growth, their importance has increased, as has the cost of their replacement or refurbishment.

Throughout the last century, as scientific knowledge broadened, more accurate standards for highway bridge design developed. Indeed the in-situ strength of bridges is now well understood relative to the in-situ loads to which bridges are subject. The highway bridge load models in bridge design codes are consequently quite conservative. Whilst acceptable for the majority of new bridges, where the cost of providing additional strength is minimal, the loading standards are conservative when applied to bridges in operation. In the past, when bridges were fewer in number, more lightly trafficked, and cheaper to repair or replace, the overall economic cost of conservative loading codes was small. Today, the rehabilitation of existing bridges to conservative code load requirements is therefore known to be an area in which savings can be made.

The factors just outlined have combined in recent years to significantly increase the value of accurate assessment of the loads to which a bridge may be subject. A general solution of the problem is emerging in which the characteristics of the traffic at a given site are measured and used to investigate the load effects to which the bridge may be subject in its remaining lifetime.

Static weigh-station sites, typical of those used in law enforcement efforts, are known to produce biased measurements of traffic, due to the avoidance of grossly overloaded vehicles. Lately, unbiased measurement of real traffic is obtained by Weigh-In-Motion (WIM) systems. These systems have acknowledged measurement inaccuracies but produce unbiased data because the installations are not readily visible to traffic.

Even with modern WIM systems, the quantity of traffic data is usually limited: such data is generally expensive to obtain and measurement periods are consequently limited. To extend the amount of traffic data, synthetic traffic data can be generated, based upon the measured traffic characteristics, through the use of Monte-Carlo simulation. Such extended traffic records are then used for estimation of rare extreme load effects which may result from the traffic at the measurement site in the bridge lifetime. Even with this form of simulation, it is necessary to have some form of statistical extrapolation technique, based on the load effect history, to estimate a lifetime value of load effect.

1.2 Objectives and Scope

1.2.1 Objectives

The research described in this exposition has the broad objective of critically examining the statistical analysis of highway bridge traffic loading. It had been recognized that the contemporary literature on the subject included areas of subjectivity that can affect the results of an analysis. Thus the main thesis of this work is to further the level of knowledge regarding the calculation of the bridge loading that may be expected to occur with an acceptably low level of probability in the remaining lifetime of the bridge.

More specifically, with reference to previous work in the area, the objectives are:

1. to maximize the information gained from a limited amount of measured traffic data;
2. to develop appropriate software tools to produce robust information for further analysis;
3. to improve the statistical analyses performed on load effect histories such that robust and realistic estimates of lifetime maximum load effect are determined;
4. to introduce further statistical techniques through which introduced inaccuracies may be accounted for in the lifetime maximum load effect estimate.

1.2.2 Scope of work

This work focuses on short- to medium-length bridges (approximately 15 to 50 m) of two opposing lanes of traffic. While it is acknowledged that congested traffic may govern for bridges in the upper part of this length range, only free-flowing traffic is considered in this work. Vehicles of Gross Vehicle Weight

(GVW) greater than 3.5 tonnes are considered: lighter vehicles do not contribute significantly to the loading to which a bridge is subject, but their role in the spatial arrangement of traffic is acknowledged. Dynamic interaction of the trucks on the bridge is generally not considered. Only single and two-span bridges are examined and the load effects are limited to bending moment, shear force, stress and/or strain as appropriate to the problem under study.

1.3 Outline of the Research

1.3.1 Traffic modelling and simulation

The research presented herein is heavily reliant on the software tools developed as part of this work. The efficacy and power of the software has implications for the manner in which bridge load research is carried out: for example, larger sample sizes generally result in more accurate load effect prediction. Accordingly, in this work, the object-orientated approach to programming is used. An explanation of this method, and the programs based upon it, is given in Chapter 4. As a result of these developments, it is now possible to simulate 5 years of traffic for a typical heavily trafficked European trunk motorway on a typical high-specification desktop personal computer. This traffic may be used to assess load effects from any form of influence line or slices of an influence surface. The statistical analysis outlined later may then be applied to the complex of results gathered.

1.3.2 Headway modelling

Intuitively, the gap between successive trucks has important implications for the quantity of load that may be applied to a bridge: this work describes, in quantitative terms, the implications for various lengths and load effects. It is found that existing headway (gap plus the lead truck length) models do not focus on the small headways that are critical for bridge loading events. A new method of modelling headway for this critical range is presented: it exhibits less variability in load effect estimation; conforms to the physical requirements of traffic; and preserves measured headway distributions. This method is described in Chapter 5, along with comparisons to existing methods.

1.3.3 Composite distribution statistics

The load effect output from the process of measurement, modelling, and traffic simulation, requires a statistical analysis to permit estimations of future load effect values. Extreme value analysis assumes that the data to be analysed is independent (or, at most, has minor dependence) and identically distributed. When daily maxima (for example) are considered as the basis for further statistical analysis, it is shown here that although this data is independent, it is not identically distributed. Physically, this is manifest as the difference in load effect between 2- and 3-truck crossing events, for example. Intuitively, such events are not identically distributed, and as such, should not be mixed as a single distribution in an extreme value statistical analysis. A method termed composite distribution statistics is presented which accounts for the different distributions of load effect caused by different event types. Exact equations are derived, as well as asymptotic expressions which facilitate the application of the method. The method is checked against results derived from the exact distribution, and compares favourably. Also, the method is applied to the output from the simulation process and compared with the traditional approach. It is shown that the composite distribution statistics method can give significantly different results.

1.3.4 Prediction of extreme load effects

The ***raison d'être*** of the bridge loading model, and subsequent statistical analysis, is the prediction of extreme, or maximum lifetime, load effects. Basic prediction techniques are outlined in Chapter 3, but more advanced methods are required to reflect the complexity of the underlying process and its model, such as the method of composite distribution statistics developed as part of this work. Such extrapolation methods, are subject to substantial variability:

different samples give different estimates of lifetime load effect. To allow for this variability, the method of predictive likelihood is used in this work. This is a relatively new area of Frequentist statistics and is not yet adopted in many practical fields of research. Predictive likelihood yields many benefits for the bridge loading problem. Most importantly, the variability of the predicted extreme can be calculated. Further, sources of uncertainty, such as the random variation of the data and of the parameter fits to the data, can be taken into account. Therefore the result of a predictive likelihood analysis gives a measure of the uncertainty inherent in the bridge loading problem, and enables this uncertainty to be taken into account.

1.3.5 Multivariate extreme value analysis

The full spectrum of bridge traffic load modelling must account for the effect of dynamic interaction between the traffic and the bridge during crossing events. The modelling and simulation described in this work are strictly static analyses. To allow for the effects of dynamics at the return period of bridge loading, 10 years of traffic were simulated for a bridge which has been tested and modelled extensively by other authors. These results are used as a basis for dynamic models of crossing events. Both of these data sets form the basis of a multivariate extreme value analysis which allows for the correlation between the static and dynamic aspects of a crossing event. Using re-sampling techniques, estimates for a site dynamic allowance factor are made. It is shown that, while dynamic amplification may be large (around 30%) for some individual events, the allowance that should be made for dynamics to obtain an appropriate overall lifetime load effect value is much less (around 5%).

1.4 Layout of the Thesis

The process under study is described in this chapter and illustrated in Figure 1.1, where its integration into the chapters of this dissertation is shown.

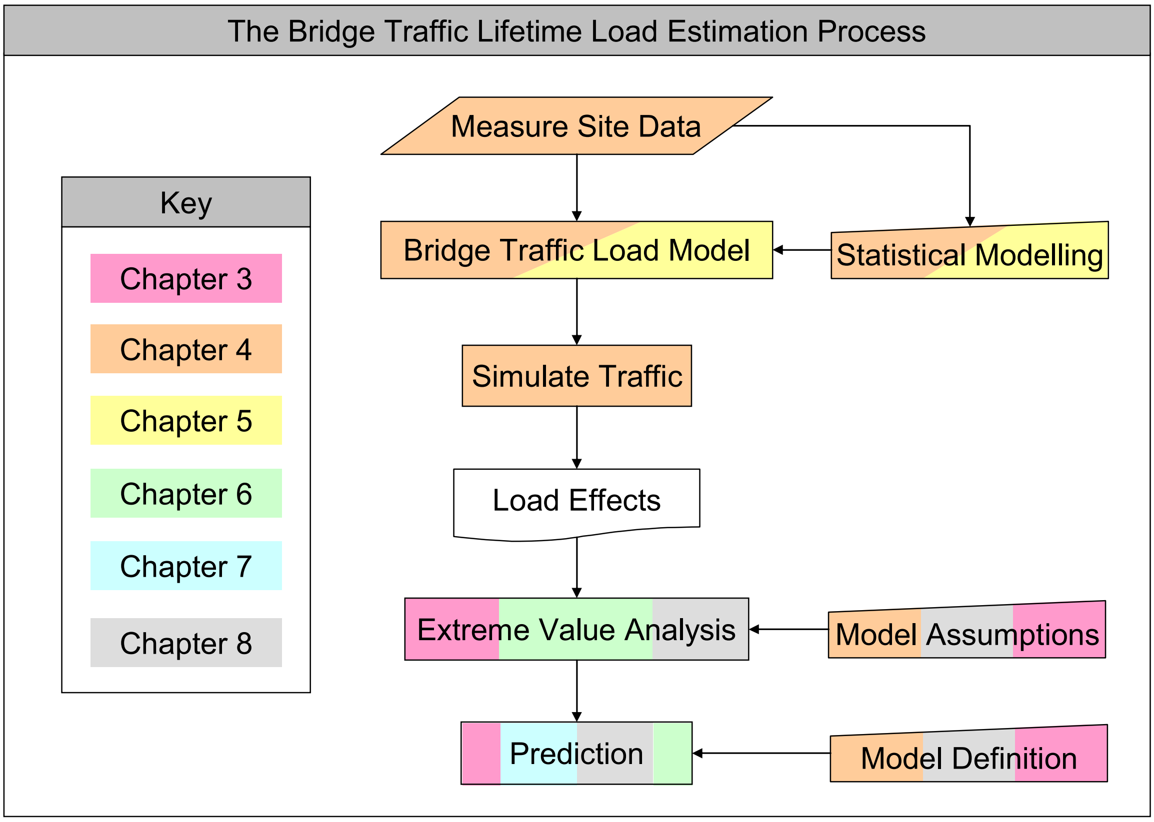

Figure 1.1: Load estimation process and chapter layout.

Chapter 2 gives more detailed information on the background to this work by surveying the scientific literature in the field. The areas of particular importance to this project are highlighted.

An introduction to the fundamental probability methods used in this work is given in Chapter 3. Particular attention is given to the areas of statistical analysis that are built upon in other parts of the work.

The bridge load models used are described in Chapter 4. Measurements of real traffic, taken from various sites, are described along with the development of a sympathetic bridge traffic load model. The final part of this chapter describes the implementation of the traffic model, to generate data for further analysis.

It is shown in Chapter 5 that the headway model used is important to the types of event and values of load effect that result. A novel headway model is described, based on Headway Distributions Statistics of a particular site, termed HeDS. A comparison of HeDS with other headway models of the literature is made, and differences to existing models described

In Chapter 6 it is shown that the existing methods of fitting and extrapolating load effects do not reflect the underlying statistical phenomena. A method termed composite distributions statistics is proposed and shown to give good predictions when compared to known return levels. It is applied to the bridge loading problem and compared to the conventional means of extrapolation.

Chapter 7 presents the application of predictive likelihood theory to the bridge loading problem. It is shown that this method accounts for the variability of the data and parameter values in the composite distribution statistics model and a probabilistic assessment of future load effect is found.

A multivariate extreme value statistical analysis is presented in Chapter 8 in the context of relating lifetime static to total (the combination of the static and dynamic components of a bridge crossing event) load effect. A dynamic factor is derived which relates lifetime static load effect to lifetime total load effect and it is shown that required dynamic allowance decreases with increasing lifetime.

The conclusions reached by this work are presented in Chapter 9 along with areas in which further research may be directed.

Chapter 2

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"In literature as in love, we are astonished at what is chosen by others" – *Andre Maurois*

Chapter 2 - REVIEW OF THE LITERATURE

2.1 Introduction

That the work herein attempts to improve and extend the work of other authors is testament to the importance to be placed upon those works. In particular, the work of Grave (2001) is to be noted as a basis for this research.

Initially, existing traffic models for the purposes of bridge load estimation, which are based on measurements, are discussed. Headway modelling in the literature is then reviewed as this has been a large focus of this research. Following this, the statistics used thus far in the analysis of bridge loading is examined. Also covered are the areas of the statistical literature which are relevant to this work.

It is the statistics currently used in the bridge load estimation research that is most relevant to this work. Indeed, the main area of progress in this research has been the adoption of extreme value theory for the estimation of bridge loads.

2.2 Bridge Traffic Load Estimation

2.2.1 Background

Of all the loads that a bridge may be subject to, traffic loading is probably the most difficult to predict. In the general reliability problem, traffic loading remains one of the most difficult variables to predict and incorporate. The assessment of load-carrying capacity is more readily understood and has been well researched (Melchers 1999, Bailey 1996).

Bridge Code Calibration

The development of recent bridge loading standards for the design and assessment of highway bridges has been predominantly based on the use of measured data and statistical extrapolations. Indeed, O'Connor (2001) outlines the development of codes such as the Ontario Highway Bridge Design Code (OHBDC), the Canadian Highway Bridge Design Code (CHBDC), the American Association of State Highway and Transportation Officials (AASHTO) *tandard e i i ation or eri an i a rid es*, the United Kingdom bridge design code, BD37/88 and the Eurocode for bridge traffic loading, Eurocode 1: Part 3, *ra i tions on rid es*. All of these codes are calibrated for load effects that have been obtained from statistical analyses of the load effects that result from various forms of traffic model.

O'Connor and Shaw (2000) and Ryall et al (2000) provide other outlines of highway bridge loading codes and their development.

Weigh-In-Motion

The advent of Weigh-In-Motion (WIM) technology (Moses 1979) allowed the use of measured unbiased traffic streams for bridge load modelling. Before that, traffic studies involved estimating the properties of traffic or sampling the population though the use of static weigh stations (Agarwal and Wolkowicz 1976), which are known to give biased results. It had been recognized (Agarwal and Wolkowicz 1976, Dorton and Csagoly 1977, OHBDC 1979) that measurement of the traffic characteristic at a site (or sites) is essential to any solution (O'Connor 2001).

Since the development of WIM, unbiased statistics of traffic characteristics have become available and this has resulted in more accurate traffic models as may be seen from the following section.

2.2.2 Simulation of traffic loading

Crespo-Minguillón and Casas (1997) and O'Connor (2001) note that there are three main types of traffic models for bridge load effect, split as follows:

- *eoreti a statisti a ode s* – stochastic process theory and distributions representing traffic characteristics are used in statistical convolution to determine the distribution of traffic loads that result. O'Connor et al (2002); Fu and Hag-Elsafi (1995); Ghosn and Moses (1985); Ditlevsen (1994); and, Ditlevsen and Madsen (1994) are examples.

- *tati tra i on i rations* – measured (or set) traffic data is used to calculate the load effects that result. Variation in the traffic stream is not allowed, therefore the quantity of traffic used is therefore of prime importance. This represents a significant drawback to this approach.

- *i ation o rea tra i o* – measured traffic is used as the basis of statistical distributions of traffic characteristics. Monte-Carlo simulation is used to generate synthetic, yet representative traffic which is then used to calculate load effects. In this way, unobserved traffic is allowed for.

Theoretical statistical models are not directly relevant to this work and as such are not considered further (refer to Grave 2001 for further reference). Use of measured or static traffic configurations is only relevant to two aspects of this work; the calculation of load effect from a given traffic stream and the subsequent statistical analysis for lifetime load effect. O'Connor (2001) provides a literature review of those authors dealing with static configurations and their associated extrapolations (Cooper 1995, 1997; Nowak 1991, 1993; for example).

It is the development of traffic models, based on measured traffic, which is directly relevant to this work – Grave (2001) and O'Connor (2001) provide thorough backgrounds on the research in this area. By basing traffic models – defined by statistical distributions for each of the traffic characteristics – on a set of measured traffic, the traffic model can be claimed to represent real traffic. The advantage offered by this approach is that unobserved traffic is allowed to occur randomly in computer simulations, whilst the overall characteristics remain those of the measured traffic. O'Connor (2001), Nowak (1993) and Crespo-Minguillón and Casas (1997) identify problems with the load effects that result when this process is not undertaken.

Bailey (1996)

Bailey (1996) develops a detailed statistical traffic load model for medium- to long-length bridges and allows for different types of traffic flow. The model is based on WIM measurements taken at various sites in Switzerland. The headway model used by Bailey is considered in Section 2.3.2 and Chapter 5.

In Bailey's model, the traffic composition is comprised of 14 different types of vehicle which make up 99% of Swiss truck traffic. The observed frequency of each vehicle type is used in the simulations.

Bailey considers axle groups as having a single weight, as the weight is generally evenly distributed between closely-spaced axles. A generalized bi-modal beta distribution is used to fit the observed axle group weights, shown in Figure 2.1. Correlation of this weight with the GVW is allowed for though generation of the other axle weights based on the axle group weight. Therefore random variation about perfect correlation (as assumed in Vrouwenvelder and Waarts, 1992) is allowed for. The procedure adopted for calculating axle weights is shown in Figure 2.2. The vehicles' geometries are modelled by a beta distribution for each of the axle spacings and overhangs of each type of truck in the classification. The flow rates used in this study are specified, rather than being based on the measured flow rates.

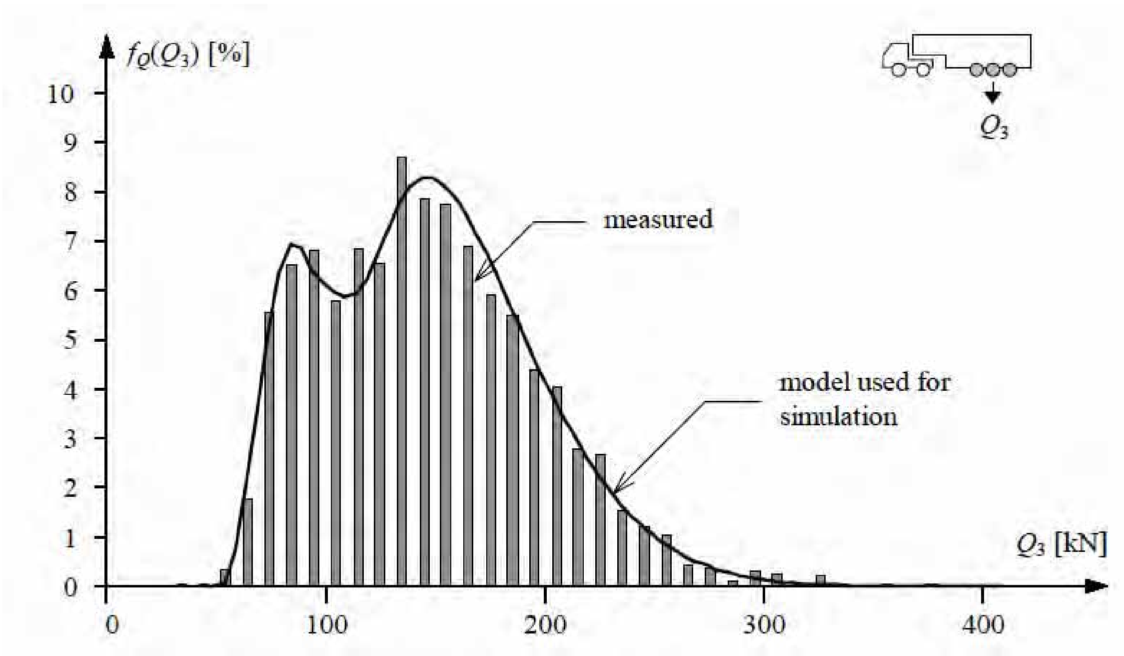


Figure 2.1: Axle-group weight distribution (after Bailey 1996).

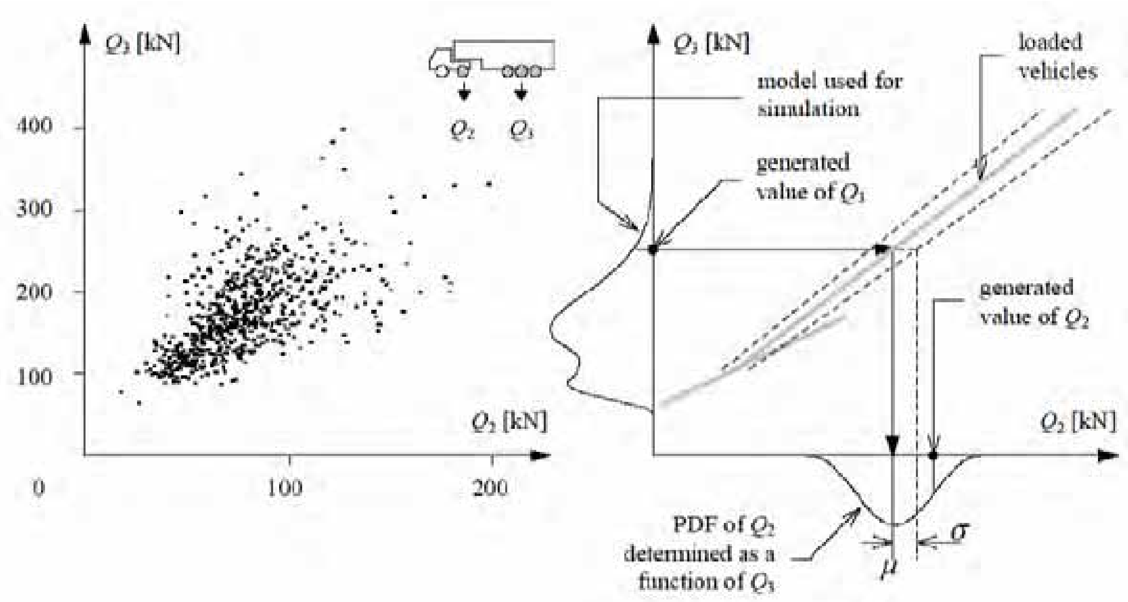


Figure 2.2: Modelling the axle weight relationships (after Bailey 1996).

Crespo-Minguillón and Casas (1997)

These authors present a substantial effort to develop a general and comprehensive traffic model for bridge loading. The generation of traffic and the modelling of each of the traffic characteristics, are explained in the following sequence:

1. The yearly mean daily flow is selected for the site under analysis.
2. Calibration curves for the flow (or traffic intensity) during the day of the week and the hourly variation are then used (shown in Figure 2.3).
3. A binomial decision making process is used to determine whether the traffic state will be jammed or free-flowing – the parameters of this process are not given by the authors, yet stated to be dependent on the hour. In this way then, the increased probability of traffic jams during rush hour is included.
4. Given the state of the traffic and its intensity, the traffic density can then be determined from measured intensity-density curves shown in Figure 2.4.

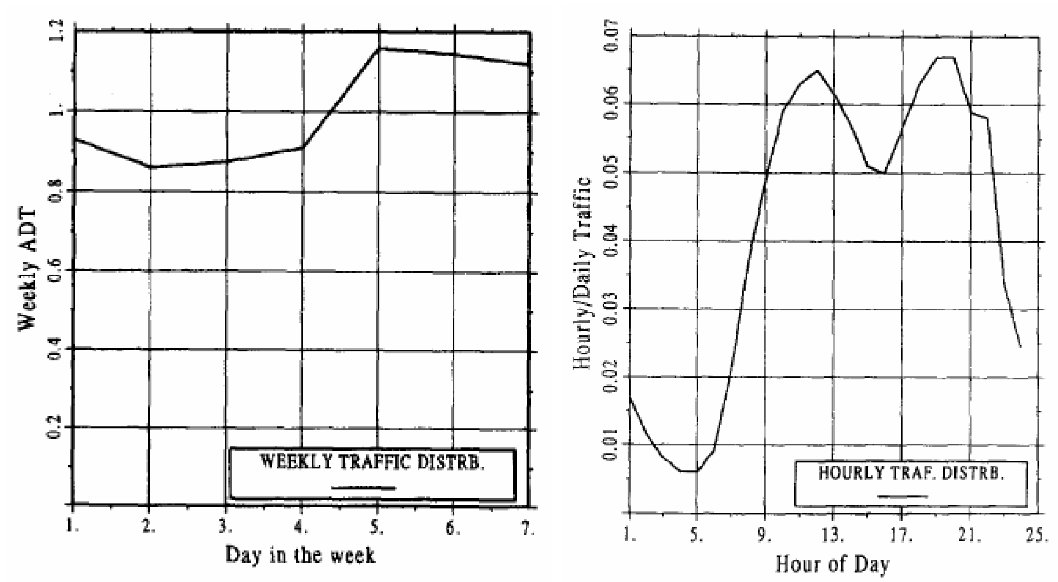


Figure 2.3: Calibration curves for traffic intensity

(after Crespo-Minguillón and Casas 1997).

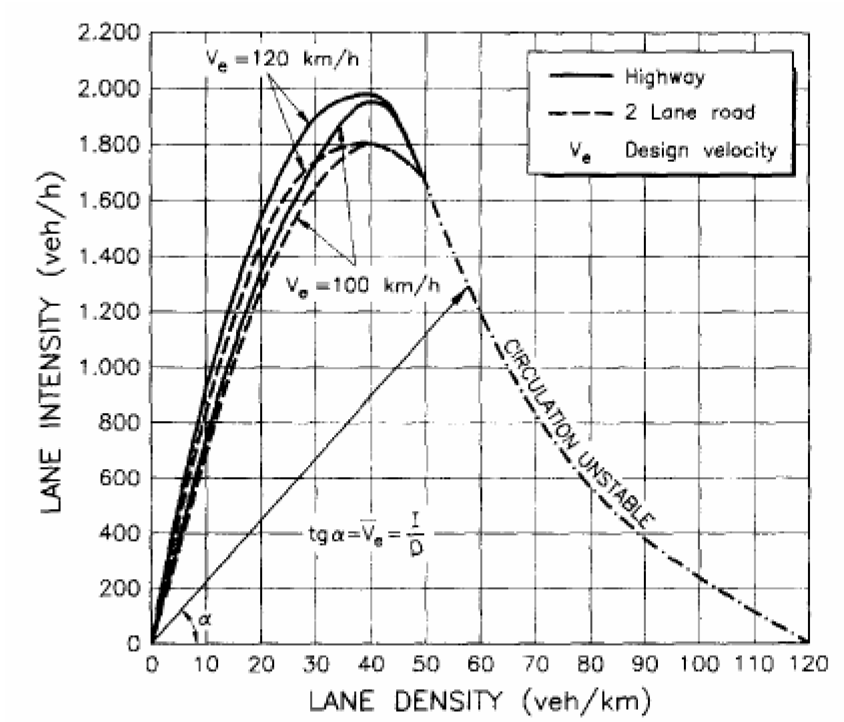


Figure 2.4: Intensity-density curves for traffic condition

(after Crespo-Minguillón and Casas 1997).

5. The traffic compositions are taken from measured WIM data at the site. The vehicle type, for the next vehicle arriving on the bridge, is calculated

using a Markov-chain method, with transition matrices based on those of the measured WIM data.

6. Velocities are then allocated to each vehicle based on a normalized velocity function (similar to that of headway, explained next) which can then be related to the intensity-density graph for the current flow condition.
7. Headway is assigned using the normalized headway model (Section 2.3 and Chapter 5). Different such models are specified for different forms of driver behaviour, heavy and light vehicles and lanes.
8. Weights and geometries are then allocated. Axle weights and GVW are allocated based on measured correlations (Table 2.1) between GVW and axle weights. Geometries are based on measured correlation coefficients for axle spacings. The GVW and axle weight distributions are defined numerically from measured cumulative distribution functions derived from the histograms of Figure 2.5.

In running this model across a bridge, the authors allow for interaction between the vehicles; that is, overtaking events, and changes in speed are modelled. Invariability this added complexity increases the number of design decisions that must be made.

Correlation matrix of weights for vehicles type 8. W_i = axle weight i; W_G = gross weight

	W_1	W_2	W_3	W_G
W_1	1.00	0.40	−0.74	−0.04
W_2	0.40	1.00	−0.89	0.11
W_3	−0.74	−0.89	1.00	−0.02
W_G	−0.04	0.11	−0.02	1.00

Table 2.1: Correlation values between axle weights and GVW

(after Crespo-Minguillón and Casas 1997).

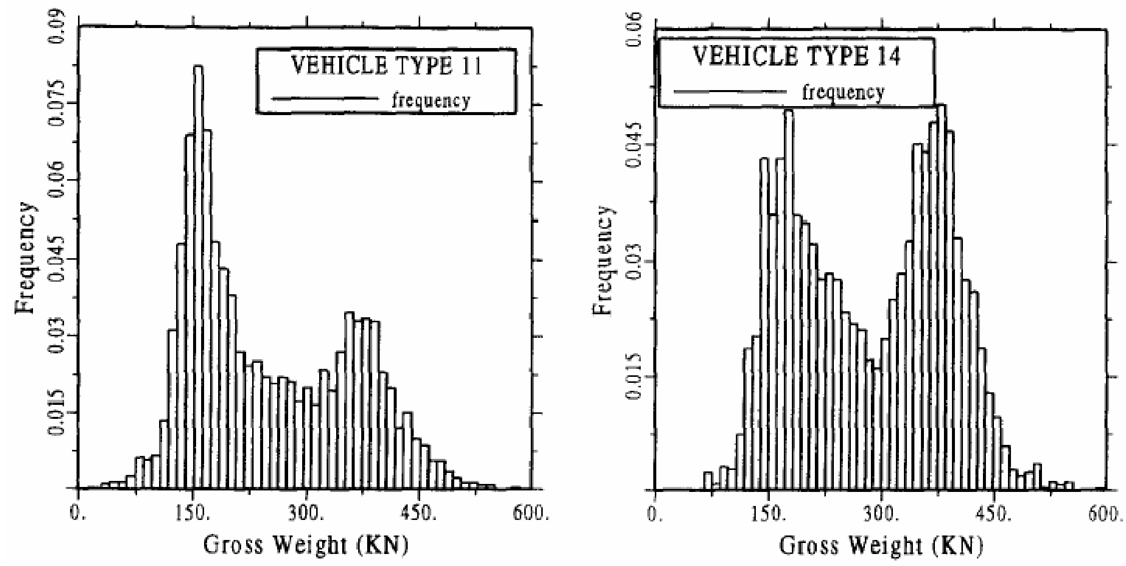


Figure 2.5: GVW histograms for two vehicle types
(after Crespo-Minguillón and Casas 1997).

Grave (2001)

Grave also develops a comprehensive traffic load model for use on short- to medium-length bridges. The traffic model in this research is largely based on the model developed by Grave. This model is therefore described in detail in Chapter 4. Some of the main aspects are discussed here, however.

Most of the traffic characteristics have been modelled statistically by Grave. Only traffic composition percentages and flow rates are deterministic. The headway model used by Grave is the same as that of Crespo-Minguillón and Casas (1997). The number of vehicle types is more limited than that of the other studies mentioned here, though Grave points out that the added complexity is not required for the WIM data under study (Chapter 4).

Other studies

The study by Harman and Davenport (1979), based on a survey of Canadian trucks by Agarwal and Wolkowicz (1976), is one of the first papers to use

Monte Carlo simulation of vehicles and headways to obtain load effects for further statistical analysis. However, the study is quite limited: it does not model real traffic flow; rather, a form of importance sampling of critical loading events on single lanes up to 90 m long is used. The authors use a mixed normal distribution with three modes to fit to the gross-weight ratio – defined as a truck's weight, divided by the legal weight limit – measured from several truck surveys. The geometries of the measured traffic and random GVWs (derived from the gross-weight ratio distribution) are used to generate a truck sample. Axle weights, as a proportion of GVW, are kept constant. Headways are randomly assigned based on a uniform distribution (see 2.3 for more information) and velocities are not required for this model.

Vrouwenvelder and Waarts (1992) describe a study in which a simplified traffic model for the estimation of lane loads (not bridge load effects) is developed. The main statistic of use is the distribution of gross vehicle weight (GVW). Axle weights, as a proportion of GVW, are kept constant. Different types of flow are considered, and deterministic headways are used. The observed frequencies of many different truck configurations are used in the model.

Other bridge loading traffic models are described but without the details being given, such as O'Connor (2001); Bruls et al (1996), and; Flint and Jacob (1996).

Discussion

Bailey (1996) uses the beta distribution for each of the traffic characteristics. This is a good distribution for such use: it is sufficiently flexible, and has upper and lower limits. It is difficult to compare this model to full site-specific models, as there appears to be no mechanism to incorporate hourly flow variation.

The model of Crespo-Minguillón and Casas (1997) is the most complex reviewed here. There may, however, be errors introduced through the use of numerical cumulative distribution functions to represent GVW histograms, for the reasons given in Section 2.4.4. Indeed, a substantial quantity of WIM data would be required to overcome these limitations. The complexity of the operations developed for passing the traffic across the bridge mean that subjective design decisions must be made, and this is a potential source of inaccuracy.

The model described by Grave (2001) is described and criticized in Chapter 4. Vrouwenvelder and Waarts's (1992) model does not claim to represent a full bridge load traffic model whilst that of Harman and Davenport (1979) is also simplistic, yet thorough for its use.

2.3 Headway Models

2.3.1 Introduction

Headway, or the distance from the front of one vehicle to the front of the next, is of great importance to bridge loading events. As is shown in Chapter 5, the types of loading events, and the values of the resulting load effects, are greatly influenced by the headway model adopted. Various methods of modelling the headway have been used by authors writing on Monte Carlo simulation for the analysis of the load effects induced on a bridge by the passage of trucks. Also, headway is of significance to the traffic engineering community. The headway models developed by both sets of researchers are reviewed next.

2.3.2 Headway modelling for bridge traffic loading

Poisson Process-Based Models

Traffic is often seen as a Poisson process and Grave (2001) gives a review of the literature on this subject. As a consequence of the Poisson process, the Exponential Distribution is used to model headway (Grave 2001, Bailey and Bez 1994, Bailey 1996). Often, this distribution is shifted to the right to allow for a minimum headway and is known as the Shifted Exponential Distribution:

$$F(t) = 1 - \exp\left[-\gamma(t - t_0)\right] \quad \text{where } \gamma = \frac{\lambda}{(\lambda t_0 - 1)} \tag{2.1}$$

and where t_0 is the minimum headway and λ is the flowrate (in trucks per hour). It may be seen from Figure 2.6 that this formulation gives inordinately high probabilities to values of headway close to the minimum allowed (Bailey 1996). Further, the minimum headway allowed is a subjective element in the process and this has been the subject of study in this research (see Chapter 5). However, this formulation allows for the effect of flowrate upon the distribution of headways: a higher flowrate requires vehicles to travel closer together. This relationship is known to hold until the capacity of the highway is reached when flow breaks down and congestion results (see Figure 2.4 and Haight, 1963).

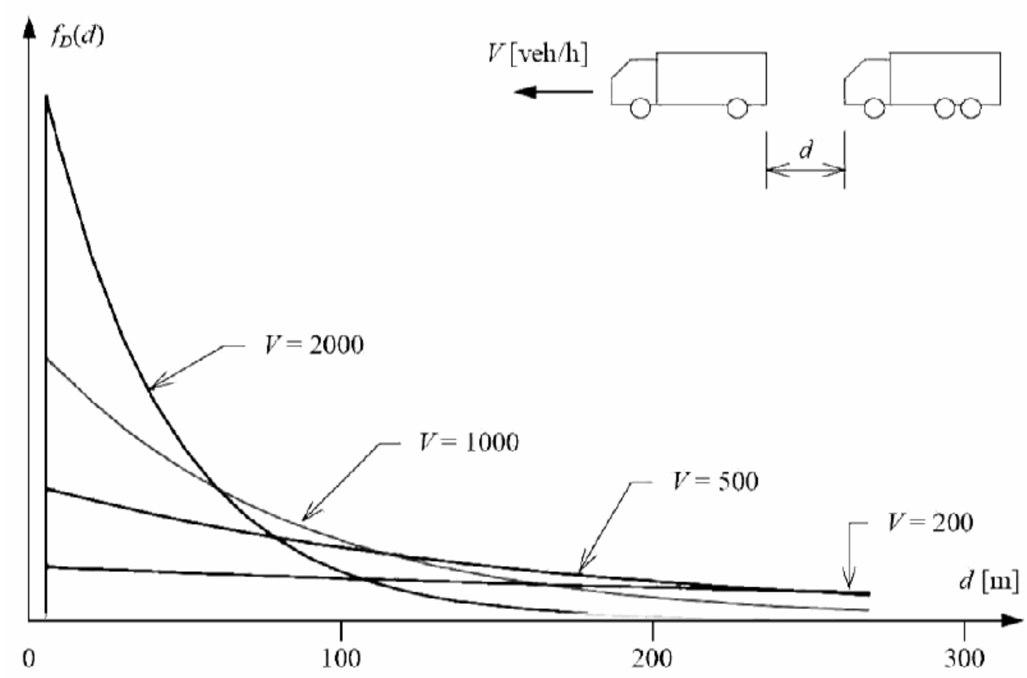


Figure 2.6: Headway (***d*** plus lead truck length) PDF model (after Bailey 1996).

Harman and Davenport (1979) recognize that the usual Poisson process assumptions of traffic engineering are not wholly applicable to bridges as only short headways (0 to 97.5 m in their study) are of interest. Based on a study by Goble et al (1976), they assume that the probability density function (PDF) of short headway is a constant equal to the average number of trucks per unit time. Relative to the negative exponential distribution, this is expressed as:

$$F(t) = 1 - \mathrm{e}^{-\lambda t} \approx \lambda t \tag{2.2}$$

where the symbols have their previous meaning. Also, Harman and Davenport limit the headway to be greater than 7.32 m, which allows for the front and rear overhangs of the truck bodies beyond the axles.

Gamma Distribution Model

The Gamma distribution function is an extension of the exponential distribution and passes through the origin – ensuring small probabilities for small headways:

$$F(t) = \frac{\Gamma(k, \gamma t)}{\Gamma(k)} \tag{2.3}$$

where $\Gamma(k, \gamma t)$ is the incomplete Gamma function, and the parameters γ and are analogous to the scale and location parameters but have physical interpretations of mean recurrence rate and the th arrival from a Poisson process. This distribution is used extensively in the background studies for the Eurocode for traffic loads on bridges (Bruls et al 1996, Flint and Jacob 1996, O'Connor et al 2002) and in other studies (O'Connor 2001, Getachew 2003). O'Connor (2001) finds that the parameters are dependent on the volume of flow, similar to the negative exponential distribution. His study also examines the effect of various periods for which the volume is obtained (be it 1, 3, 6 or 24 hours) on the characteristic extreme derived therefrom; concluding that flow periods based upon 1 hour give minimum variation of the extreme on average. The Gamma distribution does not, in its left tail, take account of the driver behaviour or other factors that must feature in very small headways. Further, this distribution passes through the origin; a check must therefore be performed such that the physical limitations of the process are not infringed. Bruls et al (1996), Flint and Jacob (1996) and O'Connor (2001) use the Gamma distribution but assume a minimum gap of 5 m, representing the distance from the back axle of the lead truck to the front axle of the following truck.

Driver Behaviour Models

Some authors have adopted headway models based on considerations of driver behaviour. Buckland et al (1980) proposed a simple method of calculating the headway based upon speed and a minimum distance:

$$h = 1.5 + \frac{v}{16} \cdot L \qquad (m) \tag{2.4}$$

where is the velocity (km/hr) and is the truck length (m). It must be recognized that their study is confined to long-span bridges, but their model is worthy of consideration nonetheless. Such a model only accounts for flow indirectly though the velocity, does not account for driver behaviour, and has no facility for site-specific modelling.

The study by Vrouwenvelder and Waarts (1993) uses different headway models for different traffic conditions. They assume that, in free flowing conditions, the headway randomly lies in the range:

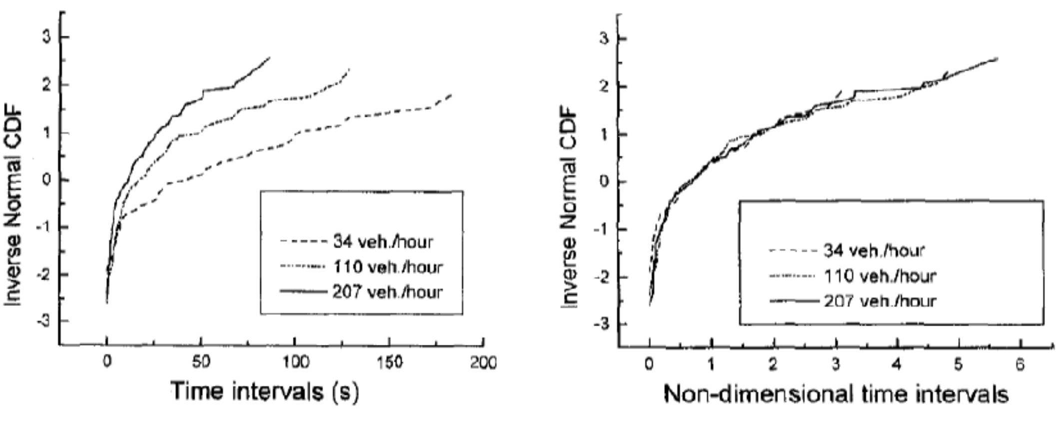
$$L \le h \le 30 - L \qquad (m) \tag{2.5}$$

where the symbols have their previous meanings. Following a similar approach, for lengths up to 60 m, Nowak considers that gaps (headway minus the length of the lead vehicle) may be 4.5 or 9 m (Nowak at al 1991); may be 5 m conservatively, that is, bumper-to-bumper traffic (Nowak 1994); or, can vary between 5 and 30 m (Nowak 1993). These models may be reasonably realistic in terms of their acknowledgment of driver behaviour, but allow no facility for site-specific modelling and are subjective. Furthermore, there is no facility for modelling long headways which have an effect on the occurrence of trucks in another lane.

Normalized Headway Model

It is important to recognize that different headway distributions result from different truck flows – Figure 2.7(a) – and this has been noted in the literature (Bailey 1996, Crespo-Minguillón and Casas 1997, Grave 2001). Rather than fitting individual distributions for each flow, Crespo-Minguillón and Casas note that a single distribution resulted from consideration of a 'normalized headway', defined as the vehicles' headway divided by the average headway for a given flowrate – Figure 2.7(b). This distribution may be subsequently altered for the particular flow of the period of interest and where γ is the mean normalized headway and is the flow (trucks/hour), is:

$$F(t) = \frac{Q}{3600}\left[1 - \mathrm{e}^{-\gamma t}\right] \tag{2.6}$$



(a) (b)

Figure 2.7: (a) Different headway distributions and, (b) Normalized headway variable, for different flows (after Crespo-Minguillón and Casas 1997).

For the sites mainly used in this study, Grave (2001) shows that, for the same flowrate, the distribution of headways is very similar. Further, Grave shows the effect of flowrate upon the headway distribution for the same sites and that in using the normalized headway distribution it is necessary to perform checks on the resulting trucks so that they do not overlap or come within 5 m.

2.3.3 Headways in traffic engineering

The traffic engineering community has been studying the headway of vehicles for many years (Haight 1963, Banks 2003). The general models are as described above, along with other more complex models which allow mixing of constrained and free flowing traffic (Grave 2001, Anon. 2003). Thamizh-Arasan and Koshy (2003), acknowledge that different flows and different traffic types follow different headway patterns. Also, at low flow rates, interaction between vehicles takes place at longer headways than at higher flow rates (Gazis 1974). Banks (2003) notes that drivers' different expectations of the traffic they are to face results in different headway distributions: in morning peak traffic there was no evident relationship between headway and speed, for speeds under 100 km/hr.

Many authors (Lieberman and Rathi 1992, Jensen 2003, Gazis 1974, HRB 1965) discuss the motivational aspect to the headway distribution: the ratio of drivers' actual- to desired-speed, and their aggression level, will affect how closely they are willing to drive to the vehicle in front. These factors affect the likelihood of overtaking, which in turn is controlled by the vehicle's positioning relative to vehicles in target lanes, further affecting the headway distribution. Drivers are also willing to operate at the mechanical limit of their vehicles, resulting in modified headways which allow for potential rapid deceleration of the driver's vehicle and the vehicle in front. Specifically of interest to this work, truck drivers exhibit different characteristics than other drivers: good route planning, commercial pressures, specialised training, high route familiarity and fatigue are factors affecting truck drivers. Also, the mechanical performances of trucks are known to be different – they are less able than other vehicles on the highway to respond quickly. All of these factors affect the headway distribution of trucks.

2.4 Determination of Extreme Load Effect

2.4.1 Introduction

The previous sections have examined the traffic models that are used in the literature to estimate bridge traffic load effect. The use of such models to determine lifetime load effect for a bridge, invariably involves some form of statistical analysis. It may be seen from Chapter 1 that the main objective of this work is the improvement of such statistical analyses. In this section, the statistical methods used in the literature are examined. Attention is given to areas of weakness in current practice that are addressed by this research.

The methods of statistical extrapolation used in the literature are quite varied. A general observation is that European authors, in recent years, are agreed on the adoption of some form of extreme value analysis. Conversely, American publications on the topic (Moses 2001, Ghosn et al 2003) are greatly influenced by the work of Nowak who generally uses a form of normal probability paper extrapolation. There are of course exceptions to these observations in both continents.

The following critique of the literature is broken into two sections: those dealing with extreme-value methods, and those using other methods. Such a layout reflects the importance of the extreme-value approach (Chapter 3) in this work.

2.4.2 General statistical methods

Harman and Davenport (1979)

Harman and Davenport consider single to five-truck events separately and then combine the results. The histograms for load effects caused by the five different types of loading event are shown in Figure 2.8 and may be seen to be considerably different. Also shown for each type of loading event, are the histograms from the measured traffic configuration and from simulated traffic.

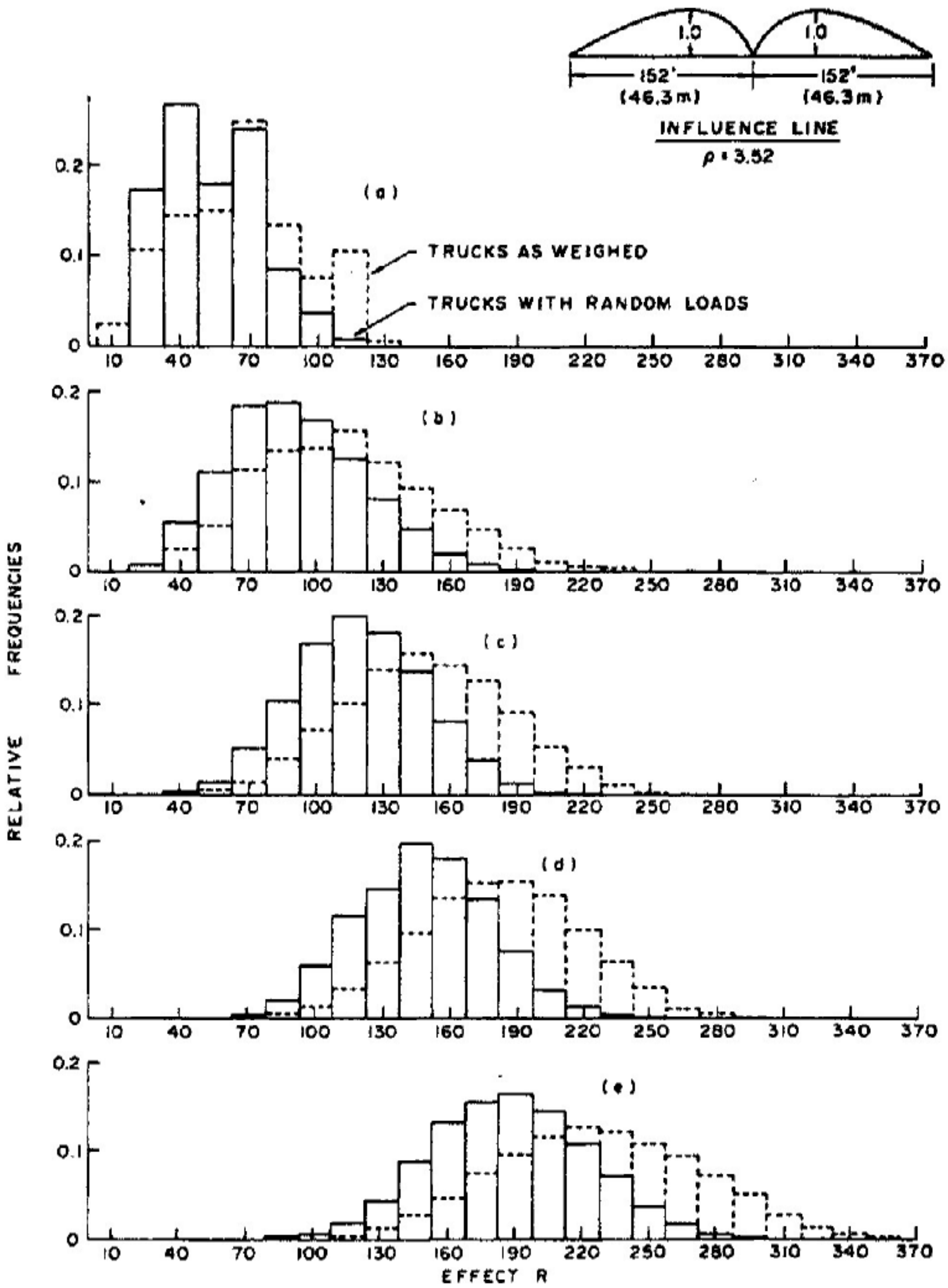


Figure 2.8: Histograms of load effect for different loading events, (a) – (e) represent 1- to 5-truck events (after Harman and Davenport 1979).

In Section 2.5.1, the method used by Harman and Davenport is explained in more detail, in the context of the statistical background to this work. Briefly however, the statistical analysis used is as follows. Harman and Davenport note that each mechanism may be represented by a negative exponential function and fit straight lines on log-scale paper to data points from the upper tail of the parent histogram – the plotting position method is not described nor is the arbitrary cut off level for the upper tail (see Figure 2.9). These functions are compared with Gaussian (normal distribution) functions fitted to the whole distribution but especially weighted to best fit the mean.

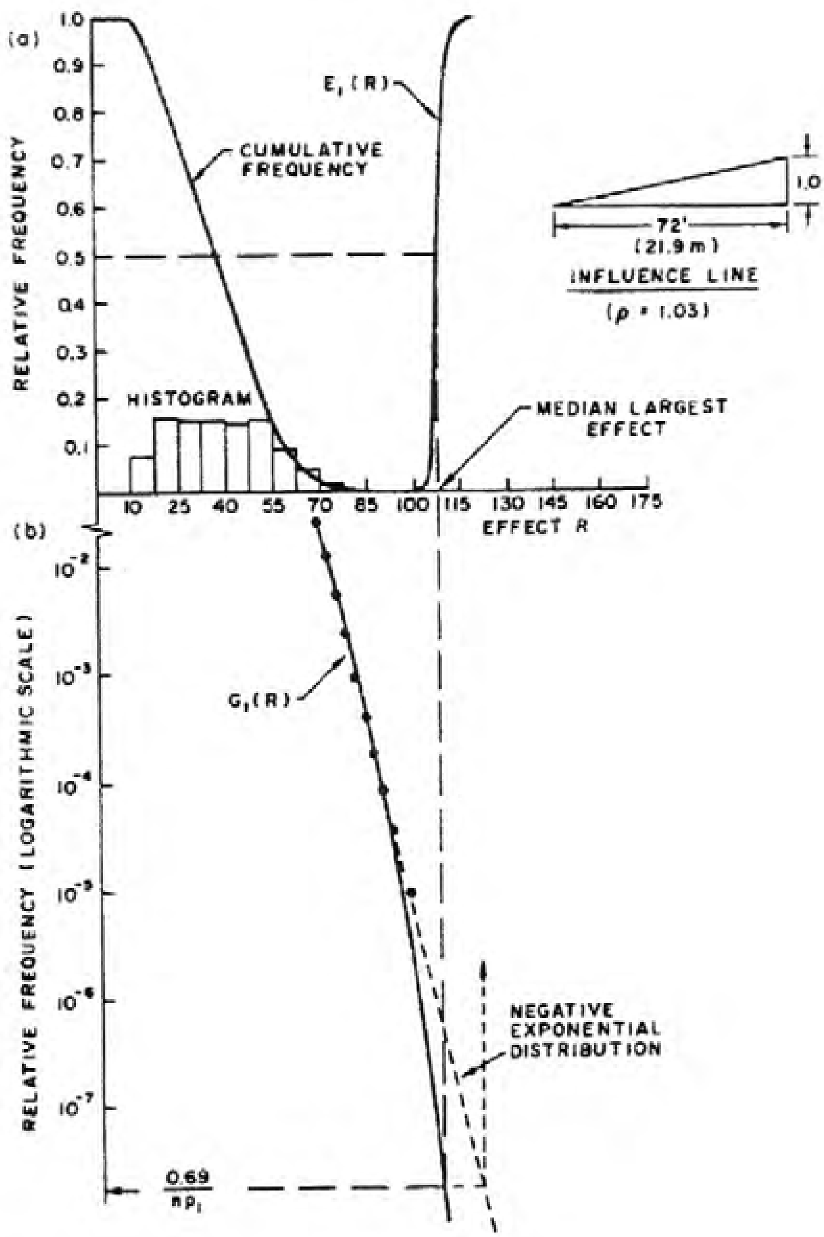


Figure 2.9: Extrapolation method (after Harman and Davenport 1979).

Nowak

Nowak has published widely on the subject of bridge load modelling. The truck survey carried out by Agarwal and Wolkowicz (1976) for the bridge load model of the Ontario Highway Bridge Design Code (OHBDC 1979) is used as the basis of most of the papers surveyed here: Nowak (1989), Heywood and Nowak (1989), Nowak et al (1991), Nowak and Hong (1991), and Nowak (1993). A contemporary truck survey is compared to the OHBDC survey in Nowak (1994). The OHBDC survey consists of 9250 trucks, especially selected as they appeared to be heavily loaded. This is assumed to correspond with a two-week period of traffic for a busy highway (see Nowak 1993 for example). Therefore, for a design lifetime of 75 years (used in most of the cited papers), the number of two-week periods is reported as 1500 (Nowak and Hong 1991) and 2000 (Nowak 1993). Based on these figures, the corresponding probabilities are reported as an inverse standard normal deviate as = 5.26 (Nowak and Hong 1991) and = 5.33 (Nowak 1993). The reason for the difference is due to the differing estimates of the number of weeks in the 75 year bridge lifetime.

Single- and two-lane shear force and bending moments are calculated for the trucks in the survey noted, taken individually. However, in the single-lane case, this is only done for spans up to 30 m as it is assumed that multiple trucks begin to feature thereafter (Heywood and Nowak 1989, Nowak and Hong 1991), though in later studies, the effect of headway is studied (Nowak 1993).

Based on the truck survey data, the results for the load effects are plotted on normal probability paper (see Chapter 3) for different spans. In the papers Nowak (1989), Heywood and Nowak (1989), Nowak (1991), and Nowak and Hong (1991) it appears that straight lines, superimposed on the tails of the distributions plotted, are used to extrapolate the load effects. This is specifically stated as the case in Heywood and Nowak (1989). However, in the same paper it is recognized that towards the tail of the distributions, curvature is evident. The authors suggest that an exponential distribution may provide a reasonable fit in this case. In the remaining papers, Nowak (1993) and Nowak (1994), it appears that curved lines on normal probability paper are used to extrapolate for the load effects of various return periods. This can be seen in Figure 2.10, for example.

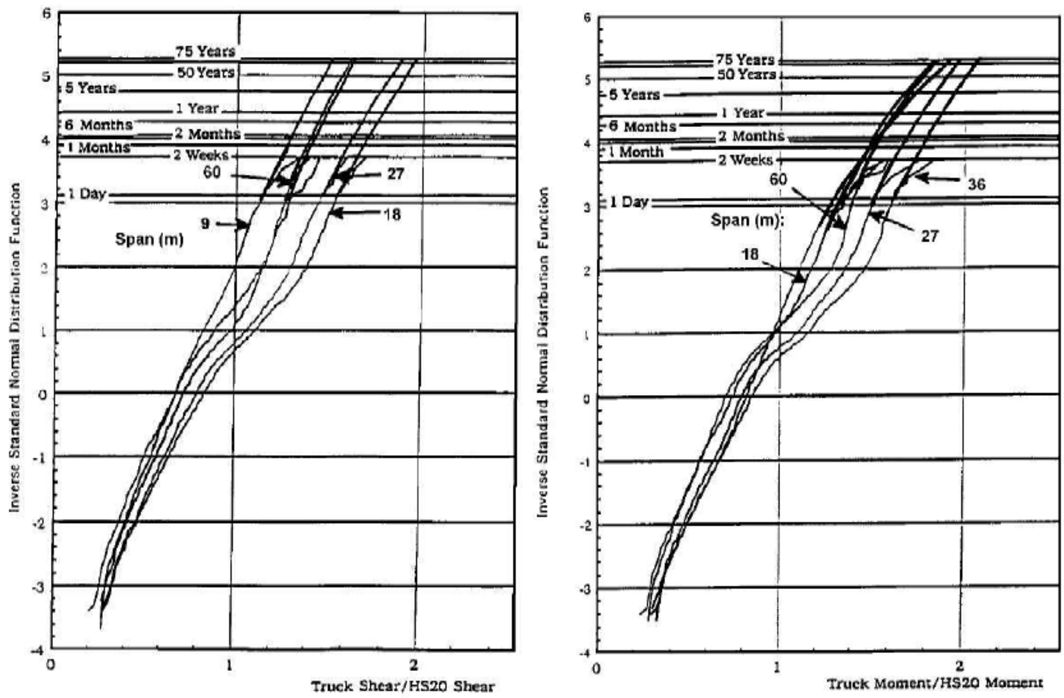
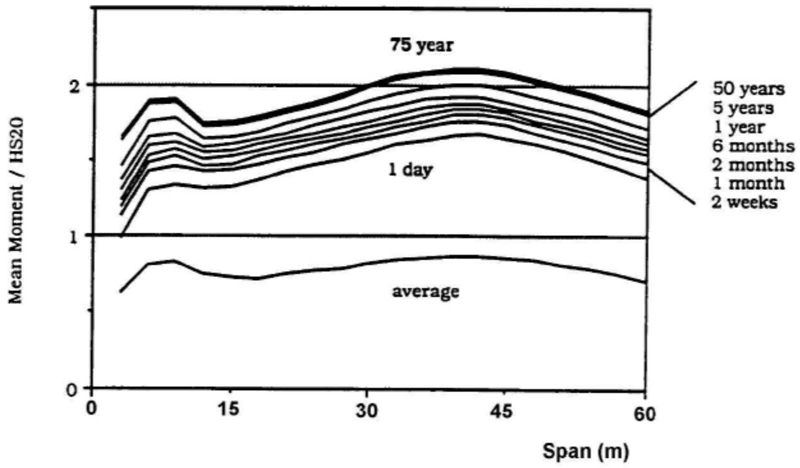
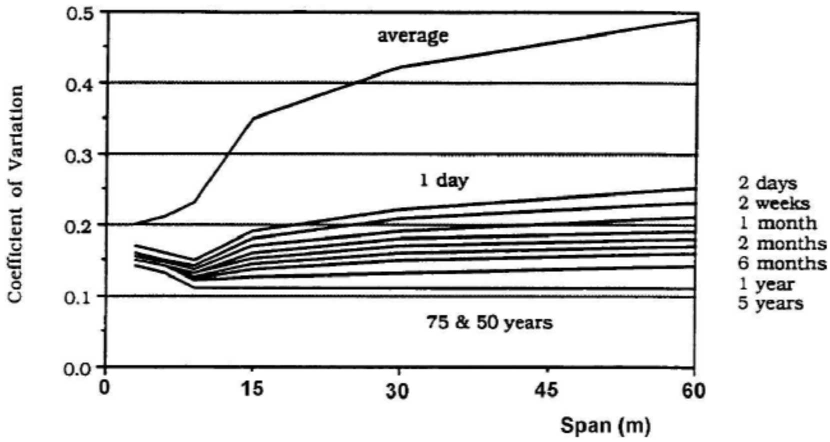


Figure 2.10: Load effect extrapolation for a range of spans (after Nowak 1993).

Nowak (1993) states that the cumulative distribution functions of load effect are raised to a power to obtain the mean and coefficient of variation of the maximum load effect – as shown in Figure 2.11. It is possible that it is this method that is used to extrapolate on the normal probability paper, though it is not explicitly stated. In a reply to a discussion about the extrapolation methods used in Nowak (1994), Nowak (1995) states that extrapolations based on the normal distribution are not used; rather, the power transform is used.



(a) Mean maximum moment;



(b) Coefficient of variation;

Figure 2.11: Estimation of lifetime mid-span maximum moment (after Nowak 1993).

Eurocode Background Studies

The background studies carried out for Eurocode 1: Part 3, *ra i oads on rid es* (EC 1: Part 3: 1994) generated significant interest in bridge traffic load modelling in Europe. The important papers are described here.

Based on measured traffic samples, Bruls et al (1996) consider and compare several methods of extrapolation of the basic histogram of load effect:

- a half-normal curve fitted to the end of the histogram;
- a Gumbel distribution fit to the tail of the histogram;
- Monte-Carlo simulation of artificial traffic and Gumbel extrapolation.

Flint and Jacob (1996) consider various methods also, some of which are applied to the loading on the bridge, rather than the load effects resulting. The methods considered are:

- a half-normal curve fitted to the end of the histogram;
- Rice's formula for a stationary Gaussian process;
- Monte-Carlo simulation of artificial traffic and Gumbel extrapolation.

This last method, in the lists for both papers, amounts to an extreme value approach and will be considered further in Section 2.4.3. The half-normal and Gumbel distribution fits to the histograms suffer from some drawbacks as discussed in Section 2.4.4.

Rice's formula has been used extensively in the literature (Flint and Jacob 1996, O'Connor 2001, Cremona 2001, Getachew 2003). One of the problems involves the choice of a threshold (see Figure 2.12), above which data will be recorded. Given the histogram of the recorded data (see Figure 2.13), Cremona (2001) develops an optimal level (x_0) at which to set the threshold, based on minimization of the Kolmogorov-Smirnov statistic (see Figure 2.14). Getachew (2003) and Cremona (2001) describe the method in full. For the current purposes, it suffices to recognize that the fits depend on the threshold, the optimal level calculated, and the number and width of the histogram intervals.

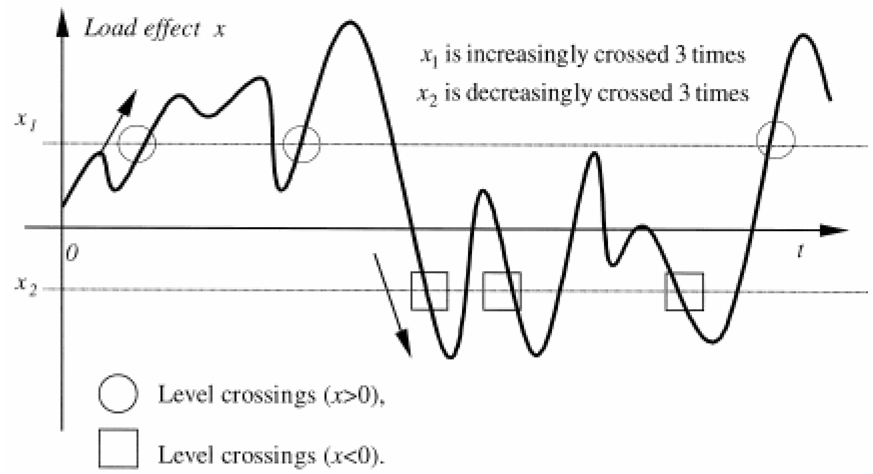


Figure 2.12: Basis for Rice's formula (after Cremona 2001).

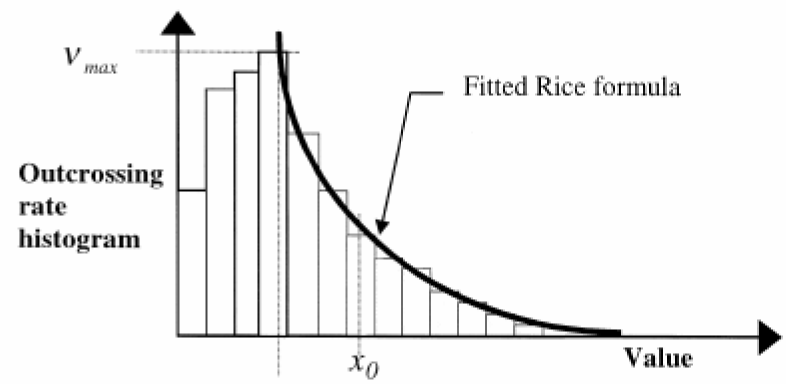


Figure 2.13: Histogram of out-crossings (after Cremona 2001).

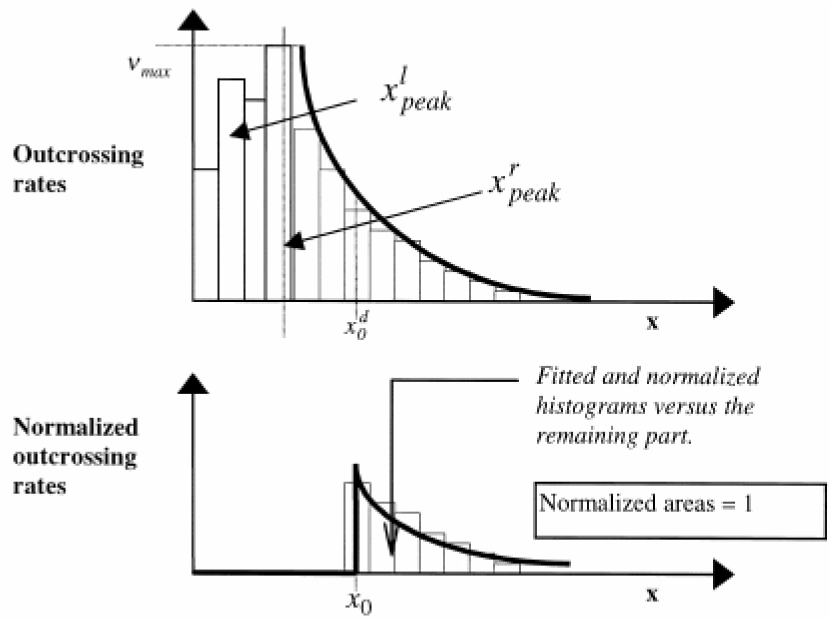


Figure 2.14: Basis of optimal fitting (after Cremona 2001).

Other Studies

Vrouwenvelder and Waarts (1993) do not attempt to estimate the load effects that result from their derived simplified lane load model. The extrapolations carried out are for truck weights and presences. Considering the truck weights alone, the authors use a truncated (at the lower tail) Weibull distribution on the upper mode of the gross vehicle weight (GVW) histogram. It is this fit that is used to extrapolate for the truck weight at the return period.

Even when sampling variability is removed, as in the case of the convolution methods noted earlier (Section 2.2.2), authors do not agree on the extrapolation method. Two such studies are described next.

Fu and Hag-Elsafi (1995) describe a probabilistic convolution method to obtain bending moments for single truck events. These authors obtain the distribution of moment for 2 years of traffic with an annual average daily truck-flow of 2000 vehicles. This is done by raising the original distribution to the power of $2\times365\times2000 = 1.46\times10^6$.

Ghosn and Moses (1985) describe a Markov-Renewel process to convolute for the bridge load effect distribution. The authors adopt a 0.1 (2.4 hour) daily maximum as their extreme data which is then fitted using a normal distribution on normal probability paper. The distribution thus estimated is raised to the power of $10\times365\times50$ to obtain the distribution of 50-year load effect.

Raising distributions to a power to obtain an 'exact' distribution of maxima is normally a cause for concern (Section 2.4.4), but as the authors use a convolution method, it may be presumed that the tail of the parent distribution has been calculated carefully. Therefore there should be little inaccuracy introduced in the distribution of maximum load effects.

2.4.3 Extreme value theory based methods

Introduction

After the simulation and modelling of loading events, a statistical analysis is required to estimate the lifetime load effect. Extreme value theory provides a theoretical and practical framework to carry out this analysis and prediction.

The extreme value theory utilized in extrapolating data to the return period required is well established. However it was not until recently that these theories were applied to the modelling of traffic loading on bridges. Many authors approach the problem by identifying the maximum load effect recorded during a loading event or in a reference period such as a day or a week, and then fit these maxima to an extreme value distribution. In all cases, the fitted distributions are used to extrapolate to obtain an estimate of the lifetime maximum load effect. This approach is based on the assumption that individual loading events are independent and identically distributed.

Irish-Based Literature

To determine the characteristic deflection of the Foyle Bridge, OBrien et al (1995) used 8 minute periods of measurements taken for each 4-hour rush hour period of a day. Each day of measurement is then represented by a 48 minute sample. The authors then consider the daily maximum deflection as an extreme value population. The Gumbel distribution is used to fit the data graphically on Gumbel probability paper. The extrapolation for the 1000-year return period is based on this distribution (shown in Figure 2.15). Interestingly, the authors establish the variance of the predicted load effect through the use of an empirical formula (Goda 1992).

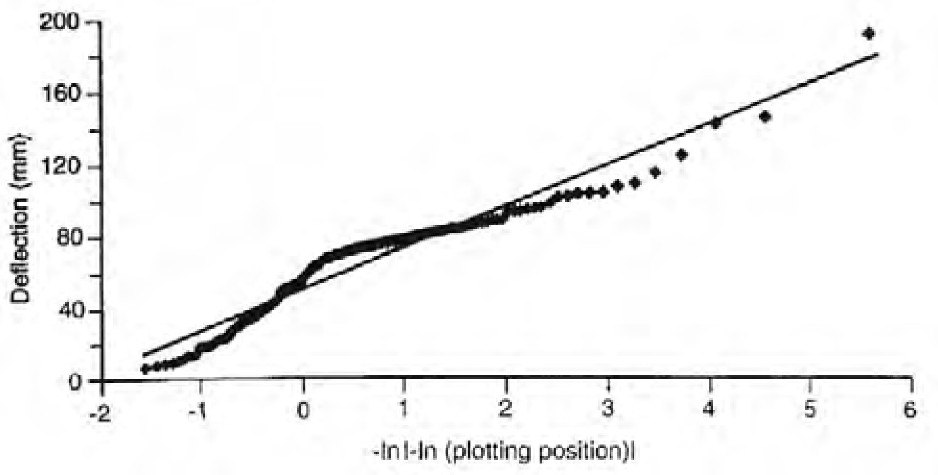


Figure 2.15: Gumbel extrapolation for the Foyle Bridge

(after OBrien et al 1995).

Grave et al (2000) describe the population of extreme values as the load effects caused by the 'critical' loading events, though critical is not qualified. A weighted least-squares approach is used to fit Weibull distributions to these critical load effects. This process is repeated to give an estimate of the distribution of characteristics values, though this distribution is not given. It is possible that the critical events are determined in a manner similar to that of Grave (2001). In this work, the 100 worst load effects noted during a 5-day simulation period are assumed to form an extreme value population. The data is plotted on Gumbel probability paper and straight lines are fitted. Such distributions form the basis for the extrapolation. The author uses the upper $2\sqrt{n}$ data points as recommended by Castillo (1988) for data that may not be convergent to an extreme value population.

In the simulations carried out as part of his work, O'Connor (2001) fits Gumbel and Weibull distributions to a population of 'extreme' load effects. The author does not specify the manner in which the 'extremes' are determined. Maximum likelihood fitting is carried out on a censored population. O'Connor (2001) censors for the upper $\sqrt{n}$, $2\sqrt{n}$ and $3\sqrt{n}$ data points (Castillo 1988), and notes that different estimates of lifetime load effect result from different censoring.

In OBrien et al (2003), hourly maximum strain values are plotted on Gumbel probability paper. A least-squares, straight-line, fit is made to the upper $2\sqrt{n}$ data points similar to O'Connor (2001) and Grave (2001). Also, González et al (2003) use the Gumbel and Weibull distributions to extrapolate bridge load effect. The population upon which the distributions are fit is not described.

Getachew and OBrien (2005) fit the Generalized Extreme Value (GEV) distribution (Chapter 3) to the distribution of load effects from a number of simulated 2-truck meeting events representing two weeks of traffic. The fitting method is not identified, but is compared with histograms of load effect.

Bailey

Bailey has published widely on the estimation of traffic load on bridges. Most of the publications are based on his doctoral dissertation (Bailey 1996). The general approach is to use traffic models to derive load effects which are then statistically analysed. Bailey (1996) describes the use of plots of the mean and standard deviation of the load effects, as they change with the number of loading events, to estimate the appropriate extreme value distribution. Based on Bailey (1996), Bailey and Bez (1994 and 1999) describe a qualitative analysis of 500 simulated upper tails of mean maximum load effects plotted against the number of events contributing (see Figure 2.16). They determine that the Weibull distribution is most appropriate to model these tails and used maximum likelihood estimation. They report that the Fréchet distribution has been used by other authors and that, in comparison to the Weibull distribution, this approach leads to an overestimation of the load effects.

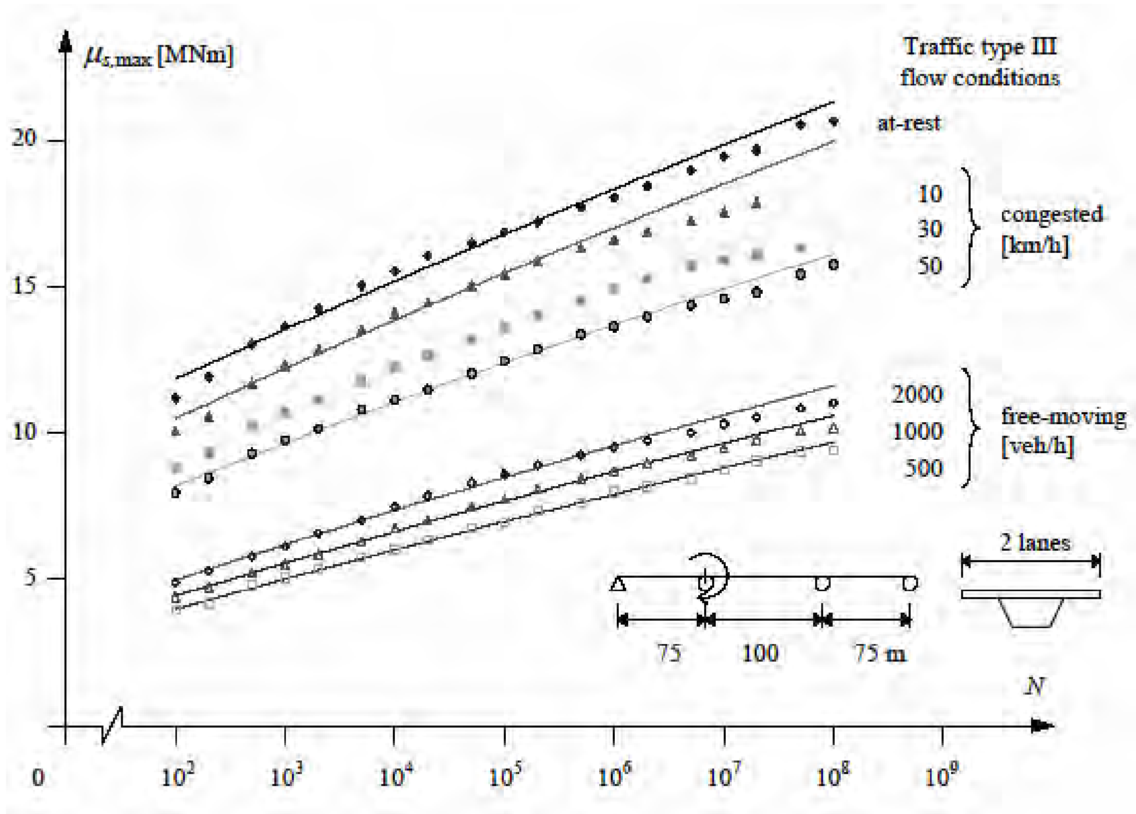


Figure 2.16: Mean maximum moment from **N** load events (after Bailey 1996).

Distributions are determined from the tail of the load effect histograms (though the tail region is not specified) by using fits based on a nonlinear least-squares technique – the Levenberg-Marqhuart method, described in Press et al (1993). Minimization of the chi-square statistic is used as the basis of the fit. The distributions thus determined are then raised to a power, as appropriate, to determine the distribution of maximum load effect (Bailey 1996, Bailey and Bez 1994, Bailey and Bez 1999) for a given number of loading events. Bailey and Bez (1994) also describe a weighted sum technique to allow for different traffic conditions.

Bailey and Bez (1999) and Bailey (1996) provide a parametric study of the parameters of the load effect distributions for many simulations. The results are used to express the parameters in terms of the traffic characteristics at the site.

Cooper

Cooper has published widely on the bridge loading problem as it relates to the United Kingdom. In Cooper (1995), a traffic model of about 81 000 measured truck events, which represents one year of traffic, is used to determine the distribution of load effects due to a 'single event'. The author raises this distribution to powers to determine the distribution of load effect for 1, 4, 16, 256 and 1024 such events – where 1024 events is stated to roughly correspond with 4.5 days of traffic. A Gumbel distribution is then fitted to this 1024-event distribution and used to extrapolate to a 2400 year return period.

Figure 2.17 shows this process, and the distribution of events obtained by raising the initial distribution to various powers is presented. It may be seen that two sharp peaks are progressively amplified as the power is increased (resulting in two sharp peaks in the distribution of 1024-event load effect). This is caused by sparse data in the tail of the initial distribution, amplified by the large power applied. This is an important limitation of the power method, and is returned to in Section 2.4.4.

In Cooper (1997), histograms of two-week traffic load effects are obtained from measured WIM data. The histograms are converted into cumulative distribution functions (CDFs), which are then raised to a power equal to the number of daily trucks, to give the distribution of daily maxima (Figure 2.18). The points of the CDF are then plotted on Gumbel paper and a straight line is fitted (Figure 2.19).

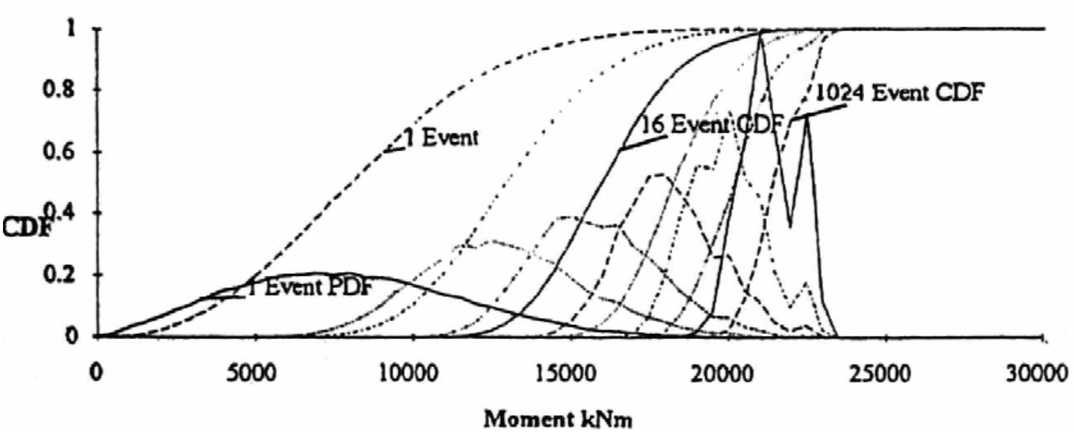


Figure 2.17: Densities and CDFs of extreme effects (after Cooper 1995).

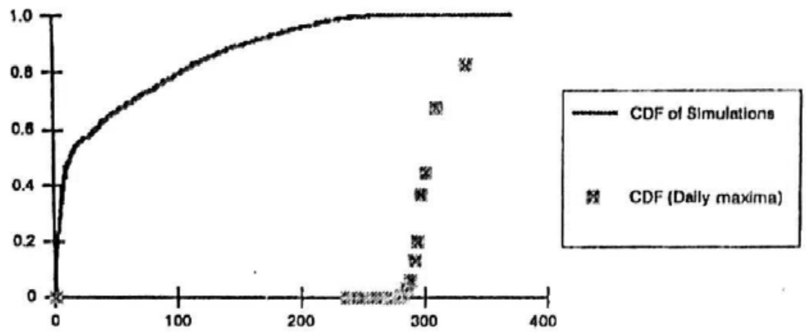


Figure 2.18: Individual event CDF and daily maxima CDF (after Cooper 1997).

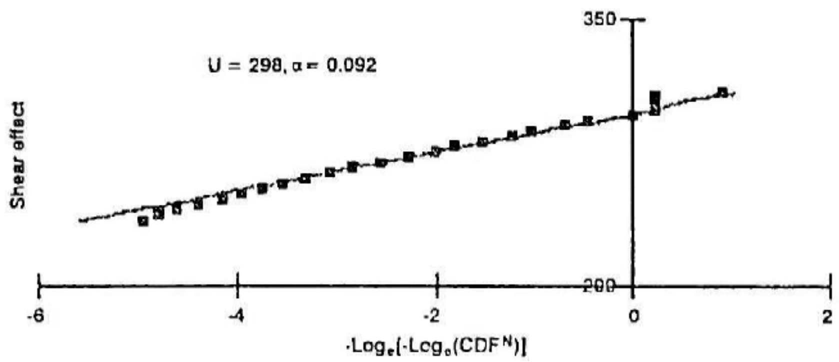


Figure 2.19: Daily maxima CDF fitted to Gumbel distribution

(after Cooper 1997).

Other Work

Crespo-Minguillón and Casas (1997) acknowledge the uncertainties involved in the extrapolation techniques of their contemporary literature. The authors plot the CDF of monthly maximum load effect on Gumbel probability paper and note that it is not linear (Figure 2.20) – the plotting position method used is not stated and is not that of Chapter 3. The authors then adopt a peaks-over-threshold (POT) approach and use the Generalized Pareto Distribution (GPD) to model the exceedances of weekly maximum traffic effects over a certain threshold. The fitting method adopted is a least squares approach, minimized on the empirical distribution estimate. An optimal threshold is selected based on the overall minimum least-squares value and it is the distribution that corresponds to this threshold that is used as the basis for extrapolation.

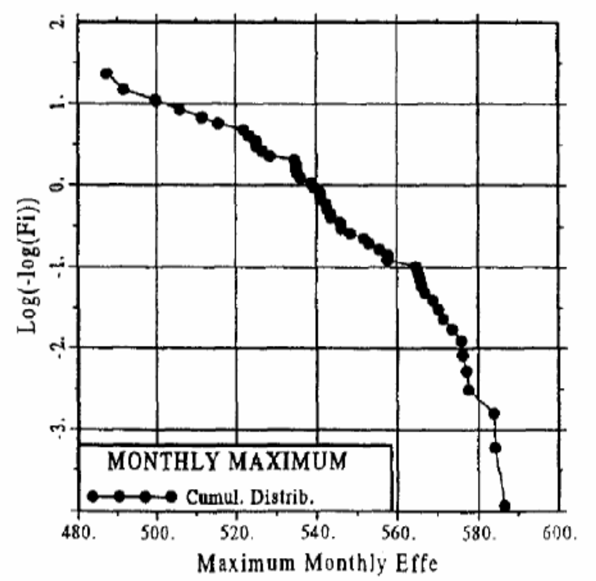


Figure 2.20: Monthly maxima plotted on Gumbel paper

(after Crespo-Minguillón and Casas 1997).

In Moyo et al (2002) the authors record strain measurements on a bridge. The daily maximum strain values are plotted on Gumbel probability paper and a least-squares fit is used to determine the parameters of the daily maxima Gumbel distribution. The authors also employ a method for deriving improved plotting positions taken from wind loading literature (Cook 1982).

Buckland et al (1980) use a Gumbel distribution to fit the 3-monthly maximum load effects and this is then used to extrapolate to any return period.

Getachew (2003) uses methods similar to those of O'Connor (2001). Gumbel and Weibull distributions are used to fit the "extreme" data (extreme is not qualified), and the results compared. The method given by Cremona (2001) is also used by Getachew (2003).

2.4.4 Discussion

It is clear that there are varying degrees of subjectivity in the literature. It does not induce confidence in the estimated lifetime load effect, when it is known that different decisions yield different results. It is one of the main objectives of this research to eliminate such subjective decisions in the statistical analysis of load effect data. It must also be recognized however, that subjectivity sometimes forms an essential part of any engineering solution to a problem, and Bardsley (1994) argues for this in the case of statistical extrapolation.

Choice of Population

It is important to choose a population that is in keeping with the limitations of the statistical model to be applied. In the works reviewed, Crespo-Minguillón and Casas (1997), Moyo et al (2002) and OBrien et al (1995) adhere to the recommendations of Gumbel (1958) for example. In these works, the form of the parent distribution is not established, and an extreme value distribution is fitted to the (presumed) population of maxima. Other authors surveyed describe an undefined 'extreme' population (O'Connor 2001, González et al 2003, Grave 2001, Grave et al 2000, and Getachew 2003) which may or may not meet the requirements of the theory. OBrien et al (2003) use the hourly maximum, whilst Ghosn and Moses (1985) use 2.4 hourly maxima, to form the extreme population. In the light of the hourly variation of traffic this does not meet the requirements of the extreme value theory; the initial population cannot be considered as identically distributed.

Other authors surveyed do not adopt the asymptotic extreme value theory and estimate the initial, or parent, distribution. They then estimate the theoretical exact distribution of maxima (Chapter 3) by raising the parent distribution to an appropriate power. Such authors include: Bailey (1996); Bailey and Bez (1994 and 1999); Cooper (1995 and 1997), and; Getachew and OBrien (2005).

The data upon which Nowak's and Harman and Davenport's result are based, represents a biased survey of trucks from 1976 and both sets of authors correctly identify this as a source of significant uncertainty (see, for example, Nowak 1993).

Distribution of Extreme Load Effects

Those authors that chose a sample of extreme values are faced with the problem of choosing a form of extreme value distribution. It is generally not acknowledged that, through use of the GEV distribution, such a decision is not required. Though Getachew and OBrien (2005) do use the GEV distribution, they use it to model the parent distribution of load effect, and not as an asymptotic approximation to the distribution extreme values. Therefore, the authors surveyed have introduced possible error by the adoption of different forms of extreme value distribution. It is recognized however (Bailey 1996, O'Connor 2001, for example), that traffic load effects normally exhibit Weibull-type behaviour and the authors that use this model are probably more accurate. This is not the general case however.

Other authors surveyed attempted to calculate the exact distribution of extreme load effect, based on a fit to the parent distribution (Bailey 1996; Bailey and Bez 1994 and 1999; Cooper 1995 and 1997; Getachew and OBrien 2005; Ghosn and Moses 1985; Nowak and Hong 1991; Nowak 1993). This is done by raising the initial distribution to an appropriate power. It is to be noted that Getachew and OBrien (2005) do not do this but estimate the characteristic value directly from the parent distribution. The procedure followed by these authors is problematic in the light of the arguments of Coles (2001b) and Castillo (1988) which state that fitting parent distributions and raising them to a power to obtain an 'exact' distribution of maxima is inaccurate in most situations. This is so because the extreme tail may be of different form to the overall parent and consequent tail-fitting errors are raised to the same power. Therefore the resulting distribution may be significantly erroneous. Such problems can be seen in the work by Cooper (1995), reproduced in Figure 2.17. In this figure, it can be seen that a slight undulation in the 1-event distribution tail (more clearly observable in the tail of the 16-event PDF) becomes two sharp peaks in the distribution of the 1024-event load effect. Sparsity of data in the tail of the initial distribution (by definition) is the cause of this. Indeed, Cooper avoided compounding this error by raising the original distributions to a power, rather than using fitted distributions.

Nowak and Hong (1991) and Nowak (1993) also raise the distributions to the power of the number of repetitions of the survey: 1500 and 2000 respectively even though both studies are based on the same data and are estimating load effects for the same return period (75 years).

Estimation

The methods used in the examined literature to estimate, or fit, the parameters of the chosen distribution(s) to the data, are considerably varied. This is surprising as the statistical literature recognizes that the method of maximum likelihood gives minimum-variance estimates in general (Chapter 3). Only O'Connor (2001) appears to use maximum likelihood estimation.

Many of the authors use 'graphical' (but not necessarily graphed) methods to fit the data – that is, a vector of $(x,\hat{y})$ pairs representing the distribution is fitted to the $(x,\tilde{y})$ pairs representing the data. However, it is clear that data only provides the x-ordinates of its pairs – the y-ordinates are established through various plotting position formulae. Gumbel (1958) and Castillo (1988) discuss the choice of plotting position. Therefore, regardless of the actual fitting algorithm, subjectivity has been introduced. This is the case for OBrien et al (1995), Grave et al (2000), Grave (2001), OBrien et al (2003), Cooper (1995 and 1997), Moyo et al (2002), and Crespo-Minguillón and Casas (1997). The fitting algorithms used by these authors are all based on a form of least-squares fitting.

Some authors introduce subjectivity by basing their fits on 'binned' data; data grouped according to arbitrary (though regular once chosen) intervals of some value – the bin width. The application of Sturge's Rule (Benjamin and Cornell 1970) may reduce the effect, but it remains an area of subjectivity. Bruls et al (1996), Cooper (1995 and 1997), Cremona (2001), Flint and Jacob (1996), Getachew (2003), Getachew and OBrien (2005), O'Connor (2001), and Vrouwenvelder and Waarts (1993) fit distributions directly to histograms. Grave (2001) notes correctly that the form of distribution which results is greatly influenced by the number of intervals chosen, and O'Connor (2001) notes sensitivity of predicted extremes to the number of intervals. Further, as these distributions are fit to all, or a significant part, of a histogram of interest, the fit to the extreme values is not emphasized – by the very nature of extreme values. Therefore, such fits do not represent the extreme values well. Also, by raising such fits to a power amplifies the errors, as discussed earlier. The chi-squared fitting used by Bailey (1996) and Bailey and Bez (1994 and 1999) also requires the data to be 'binned' and the same problems therefore apply.

Choice of Thresholds

Many of the authors reviewed make decisions (and therefore introduce subjectivity) regarding various threshold choices. For instance, O'Connor (2001), Grave (2001), Grave et al (2000), OBrien et al (2003), Bailey (1996), and Bailey and Bez (1994 and 1999) fit the distributions to 'tail' data only. In some cases the decisions as to what constitutes tail data is not stated; in others the decision is based on Castillo's suggestion (Castillo 1988). Crespo-Minguillón and Casas (1997) are an exception to this as their model inherently requires the selection of a threshold, and their choice is rationally based on the overall least-squares value for all the thresholds considered.

Nowak also relies on extrapolating from the tails of load effect distributions. The level at which the tail (upon which the extrapolation is to be based) starts is not stated. The normal distribution-based extrapolations of the earlier papers (Heywood and Nowak 1989, for example) are therefore subjective to implement.

Summary

It can be seen that most authors exhibit sources of error under several of the categories and the errors in such works are therefore compounded. This has an effect on the characteristic load effect estimated from such methods. Also, it is clear that many authors describe subjective choices in their analyses.

2.5 Statistical Background

2.5.1 Composite Distribution Statistics

Introduction

Load effects can be the result of any number of loading events involving different numbers of trucks. In general, a load effect due to the passage of a single vehicle has a different distribution to that induced by the occurrence of multiple vehicles (see Figure 2.8 for example). Multiple truck presence events usually yield critical load effects. Normally, it is the maximum per day load effect that is used as the basis for the extreme value analysis which assumes independent and identically distributed (iid) data. Therefore, to mix load effects from different types of loading events violates the iid assumption used in extreme value analysis.

The problem of mixing different statistical generating mechanisms in an extreme value analysis has been examined by previous authors in different fields and their work is examined in this section.

Gumbel (1958)

In his summary, Gumbel (1958) states that "the initial distribution […] must be the same for each sample". Gumbel gives an example of the "the two sample problem" – a study of river discharges, where one series of floods is due to the melting of snow in the spring, and the other to autumnal rainfalls. Gumbel's approach to the problem is described as: take the largest value of each of two large samples, thus forming a couple. By repetition, obtain many such couples and then, for each couple, take the largest value. It is the distribution of this final variable that is of interest. Gumbel notes that this distribution is the product of the two initial distributions of largest values.

Gumbel's reasoning is based on the following development. The basic results of probability (described in Chapter 3) state that for a value **x** and **N** random variables, $X_1, \ldots, X_N$:

$$P\left[x \geq X_1, \ldots, x \geq X_N\right] = \prod_{i=1}^{N} P\left[x \geq X_i\right] \tag{2.7}$$

$$P\left[x \leq X_1, \ldots, x \leq X_N\right] = \prod_{i=1}^{N} P\left[x \leq X_i\right] \tag{2.8}$$

This is so, regardless of the 'type' of random variable, X_i. That is, it is irrelevant whether X_i represents an extreme population or a parent population. Equation (2.8) is more useful, due to its relationship with the cumulative probability function (Chapter 3). Therefore,

$$F_C\left(x\right) = P\left[x \leq X_1, \ldots, x \leq X_N\right] = \prod_{i=1}^{N} F_{X_i}\left(x\right) \tag{2.9}$$

where $F_{X_i}\left(x\right)$ represents the distribution of load effect resulting from different types of truck loading events, and so $F_C\left(\cdot\right)$ is the composite distribution of load effect. The load effect considered can be extreme or parent.

Wind Speed Analysis

The analysis of maximum wind speed is complex due to its nature. There are some similarities, though, with the bridge loading problem. Through study of the approaches taken in the wind speed literature, methods for analysing bridge loading can be adapted. Two of the more important papers are described next.

Gomes and Vickery (1978)

The work of Gomes and Vickery (1978) provides a direct analogy between the wind speed and bridge loading problems. They describe the problem of estimating the distribution of extreme wind speeds in mixed wind climates – climates in which wind may be caused by extensive pressure system storms, thunderstorms, hurricanes or tornados. They use the Gumbel distribution (Chapter 3) to model the extreme wind speeds from each of the mechanisms that occur at a particular site, and combine, without proof, as follows (in their notation):

$$P\left[V_M \le v\right] = \prod_{q=1}^{Q} P\left[V_q \le v\right] \tag{2.10}$$

where V_q is the annual maximum gust speed of the qth meteorological phenomenon and V_M is the annual maximum gust speed, regardless of the source. $P\left[V_X \le v\right]$ is the cumulative distribution function of the variable X.

Also of importance in their paper, Gomes and Vickery consider the annual maximum gust speed from thunderstorms, with an unknown number of thunderstorms in any given year. Adapted slightly here, they derive the distribution of annual maximum gust speed from thunderstorms as:

$$G_T\left(v\right) = \int_0^\infty \left[F_T\left(v\right)\right]^n f_N\left(n\right) dn \tag{2.11}$$

where $F_T\left(\cdot\right)$ is the parent distribution of thunderstorm gust speed and $f_N\left(\cdot\right)$ is the probability density function of the number of thunderstorms per year, N. Clearly a functional form of (2.11) may be difficult to obtain and include in (2.10). Gomes and Vickery (1978) report a study which shows that approximating the distribution of N by its mean value does not result in significant inaccuracy. Hence (2.11) may be written as:

$$G_T(v) \approx \left[F_T(v)\right]^{\bar{N}} \tag{2.12}$$

where $\bar{N}$ is the mean value of N. Gumbel (1958) describes a similar formulation to (2.11) for the exact distribution of maxima when the sample size itself is a random variable.

Cook et al (2003)

The paper by Gomes and Vickery (1978) was considered in detail by Cook et al (2003) in the light of more recent developments in statistics. Of note in this work, is their proof of (2.10), described next.

The authors consider two mechanisms; A and B which give values V_A and V_B and in general, for a given period, a pair of events $\{V_A, V_B\}$ can occur. Thus, there are four possible outcomes. Representing $\varnothing$ as the null set, the events are:

1. No events from either mechanism, $\{\varnothing, \varnothing\}$;
2. An event from both mechanism, $\{V_A, V_B\}$;
3. An event from A only, $\{V_A, \varnothing\}$;
4. An event from B only, $\{\varnothing, V_B\}$.

Given that the duration of the sampling period will be long enough such that an event from both mechanism occurs, the authors show that:

$$P\left[\hat{V} \le v\right] = P\left[\hat{V}_A \le v\right] \times P\left[\hat{V}_B \le v\right] \tag{2.13}$$

They extend this to the general case by induction:

$$P\left[\hat{V} \le v\right] = \prod_{i=1}^{n} P\left[\hat{V}_i \le v\right] \tag{2.14}$$

This proof includes acknowledgement of the temporal aspect of the sample space considered. However, it is approximate as it considers the relative frequency of each of the possible outcomes, rather than the relative frequency of each of the mechanisms themselves. The authors solve this by modifying the contribution to (2.14) of a mechanism by considering its occurrence as a Poisson process.

Harman and Davenport (1979)

The study by Harman and Davenport noted earlier, also recognizes the composite nature of the bridge loading problem. In revised terminology, the load effect caused by the i-truck event has cumulative distribution function $F_i(\cdot)$ and the event has probability of occurrence, f_i. The distribution of load effect greater than a value, r, is then $\bar{F}_i(r) \equiv 1 - F_i(r)$. Therefore, the 'complete' distribution of load effect greater than r is given by Harman and Davenport as:

$$\bar{F}_C(r) = \sum_{i=1}^{5} f_i \cdot \bar{F}_i(r) \tag{2.15}$$

which is an application of the theorem of total probability (Chapter 3). The cumulative distribution function of the largest load effect from a sample of size n is then given by:

$$G(r) = \left[1 - \bar{F}_C(r)\right]^n \cong \exp\left[-n\bar{F}_C(r)\right] \tag{2.16}$$

which is reasonable for large n. Substitution of (2.15) into (2.16) yields:

$$G(r) = \prod_{i=1}^{5} \exp\left[-nf_i\bar{F}_i(r)\right] \tag{2.17}$$

Section 2.4.2 describes the log-scale paper fitting procedure used by Harman and Davenport, based on (2.17).

2.5.2 Predictive Likelihood

Introduction

The relatively new theory of frequentist predictive likelihood can be used to estimate the variability of the predicted value, or predictand. Applications of predictive likelihood to real-world problems are sparse. Davison (1986) presents one in the context of his revised form of predictive likelihood. Lorén and Lundström (2005) present the only full paper (obtained for this work) on the application of predictive likelihood techniques; in their case, to the prediction of fatigue limit distributions for metals.

Fisher (1956) is the first clear reference to the use of likelihood as a basis for prediction in a frequentist setting. A value of the predictand ($\mathbf{z}$) is postulated and the maximized joint likelihood of the observed data ($\mathbf{y}$) and the predictand is determined, based on a model with parameter vector θ. The graph of the likelihoods thus obtained for a range of values of the predictand yields a predictive distribution. Such a predictive likelihood is known as the profile predictive likelihood. Denoting a normed likelihood by $\bar{L}(\theta;x)$ this is given by:

$$L_P(z \mid y) = \sup_{\theta} \bar{L}_y(\theta;y)\,\bar{L}_z(\theta;z) \tag{2.18}$$

It is to be noted that likelihood is not a probability and so the usual conditional probability rule does not apply. Mathiasen (1979) appears to be the first to study Fisher's predictive likelihood and notes some of its problems. Foremost for this work is the problem that it does not take into account the parameter variability for each of the maximizations of the joint likelihood function required (Lindsey 1996, Bjørnstad 1990). Lejeune and Faulkenberry (1982) propose a similar predictive likelihood, but include a normalizing function.

Predictive likelihood is a general concept (see Berger and Wolpert 1988) and in the literature many versions have been proposed. The paper by Bjørnstad (1990) is seminal in predictive likelihood for it collects all of the literature and examines each of the predictive likelihoods proposed. Bjørnstad notes that the Fisherian predictive likelihood of (2.18) "plays a central role in prediction". The other predictive likelihoods considered by Bjørnstad are those based on sufficiency principles put forth by Lauritzen (1974), Hinkley (1979) and Butler (1986). Based on the Lauritzen-Hinkley definition, Cooley and Parke put forward a number of papers dealing with the prediction issue (Coole and Parke 1987, Cooley et al 1989, Cooley and Parke 1990). However, their method relies on the assumption that the parameters are normally distributed, and they use Monte-Carlo simulation as a result. Leonard (1982) suggests a similar approach.

Davison (1986) provides a relevant example of the application of predictive likelihood methods to river discharges and wave heights. Though he uses a different form of predictive likelihood, the explanation of his approach with the GEV distribution (Chapter 3) is important to this work.

2.5.3 Multivariate Extreme Value Analysis

Allowing for the effect of the dynamic interaction between the bridge and the trucks which form a loading event is essential to determine the total load effect to which the bridge is be subject. As part of a study described in Chapter 8, dynamic interaction simulations are described for 10 years of monthly maximum events. To determine the lifetime total load effect for the bridge, the correlation between static and total load must be accounted for. As extreme values of two correlated variables are required, multivariate extreme value analysis is adopted.

The study of multivariate extreme value theory began in the 1950s (Galambos 1987). Coles (2001a) and Galambos (1987) agree that the work of Tiago de Oliveira was essential to its development – refer to Coles (2001a) and Galambos (1987) for references to his work.

An approach to the modelling of bivariate extreme value distributions, including consideration and estimation of several dependence structures, is presented by Tawn (1988). Several general models of extreme value distributions are examined by Tawn (1990) who also presents an application – the modelling of tri-variate extreme sea level data. Large dimensional problems in multivariate extreme value modelling are considered by Embrechts et al (2000). In this paper, the authors also present an application in the field of sea level analysis for flood protection. Coles and Tawn (1991) present a generalization of the peaks-over-threshold (POT) approach to the modelling of multivariate extreme values.

Capéraà et al (1997) present the modelling and estimation of extremal dependence functions. Klüppelberg and May (1998) also discuss the bivariate dependence functions and state that the only possible models are the mixed and logistic classes. Coles et al (1999) also discuss the dependence functions used in multivariate extreme value analysis. A thorough presentation of multivariate extreme value analysis and the modelling of dependence through the use of dependence structures and copulas is given by Demarta (2002). Segers (2004) also discusses the estimators of use for the bivariate extreme value dependence function of Pickands (1981) whilst Hefferenan (2005) gives a review of the dependence measures used in multivariate statistical modelling in recent years.

Literature on the statistical computational aspects of multivariate extreme value statistics is sparse. Stephenson (2004) presents a user guide to R (R

Development Core Team 2005) software for the analysis of multi- and uni-variate extreme analysis. The guide gives several applications of the theory and serves well as a collection of examples, and an introduction to the theory. Nadarajah (1997) and Stephenson (2003) both describe procedures to simulate multivariate extreme value distributions. This is important for the application of bootstrapping methods to the problem

There have been several applications of the theory, mostly in the statistical literature. Hawkes et al (2002) discuss the use of multivariate extreme value theory in estimating coastal flood risk due to combinations of high tides and wave surges. An application of bivariate extreme value analysis to the wave height and sea level problem of coastal flood defence is presented by Draisma and de Haan (2004). Zachary et al (1998) use the theory to estimate the loads caused on offshore structures by combinations of wave height, wave period and wind speed. An application of multivariate extreme value theory to structural design problems is considered by Coles and Tawn (1994); a detailed application to coastal engineering is presented. Also, Gupta and Manohar (2005) use multivariate extreme value theory in the analysis of random vibration problems. Specifically, a two span bridge subject to earthquake support motions is examined.

The multivariate extreme value analysis used in this work is based mainly on the work of Stephenson (2003 and 2004). The software developed as part of Stephenson's work has been used here – the `evd` library for the **R** (R Development Core Team 2005) language. Stephenson's work is, in turn, based on that of the many authors mentioned previously, most notably the work of Coles and Tawn.

2.6 Summary

This chapter presents the background literature to the various aspects of this research project. Initially, the contemporary work in the field of bridge traffic load models is presented, followed by some discussion. One area of significant development of such models is presented in detail as it forms a substantial part of the current research: that of headway modelling. The literature for the main theme of this work is then presented – methods of statistically analysing the results of bridge traffic load simulations. An extensive discussion is provided, in which various problems with the current methods are outlined. Following this, a section outlining the background statistical literature of this work is presented. General statistical literature is not presented, rather, the literature specific to the main areas of use in this work. General statistical literature is discussed in Chapter 3 instead.

Chapter 3

FUNDAMENTAL PROBABILISTIC METHODS

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"Statistics in the hands of an engineer are like a lamppost to a drunk—they're used more for support than illumination" *- AE Housman*

Chapter 3 - FUNDAMENTAL PROBABILISTIC METHODS

3.1 Introduction

Karl Pearson (1920) posed "the fundamental problem of statistics" as follows:

> *An 'event' has occurred p times out of p + q = n trials, where we have no a priori knowledge of the frequency of the event in the total population of occurrences. What is the probability of its occurring r times in a further r + s = m trials?*

That Pearson's 'problem' applies to the bridge loading problem is immediately apparent. Note also that prediction is an integral part to this "fundamental problem" – just as it is to the bridge loading problem. This chapter presents the background material necessary for the development and presentation of the statistical analyses used to solve Pearson's "fundamental problem".

Initially, the fundamental definitions of any random experiment are given, followed by the mathematical tools need to operate on random experiments. Inference from the outcomes of a statistical experiment is then considered: the method of maximum likelihood, which is of central importance to this work, is presented here. Following this, the statistics of extreme values is introduced and the basic definitions and limitations of the theories outlined. Finally, the problem of predicting future outcomes of a statistical experiment is addressed. The material introduced herein forms the background to the analyses carried out by many other authors in this field, as may be seen from Chapter 2.

3.2 Basic Results

The fundamentals presented in this section are required for further developments in this work as a whole. Standard texts that may be referred to for more information on these basic results are Mood et al (1974) and Ang and Tang (1975). Other highly relevant texts are Castillo (1988), Lindsey (1996), Coles (2001a), Cox and Hinkley (1974), Feller (1968), and Azzalini (1996).

3.2.1 Probability, events and sample spaces

The classical, or frequency **definition of probability** is:

> If a random experiment can result in n mutually exclusive and equally likely outcomes and if n_A of these outcomes have attribute A, then the probability of A is the fraction n_A/n.

The **sample space** is the collection of all possible outcomes of an experiment. Considering an experiment with a single die, the sample space would the integers 1 to 6, representing the six possible faces of the die. Sample spaces may be finite with discrete points, or infinite with continuous 'points'.

The terminology '**event** A' is used to represent an outcome of a statistical experiment that has attribute A. The **event space**, A, is defined as the collection of all permutations of events, or the collection of all subsets of the sample space. The sample space itself is a subset of the event space.

A **probability function**, $P[\cdot]$, is a set function with a domain of the event space and counterdomain the interval [0,1] on the real number line. $P[A]$ represents the probability of event A. Where Ω represents the sample space of an experiment, $P[\Omega]=1$, by definition. A **probability space**, denoted $(\Omega, A, P[\cdot])$, describes the sample space, event space and probability function, respectively, for a given random experiment.

Given two events, A and B in $\mathcal{A}$, the **conditional probability** of event A given that event B has occurred is defined as:

$$P[A \mid B] = \frac{P[AB]}{P[B]} \tag{3.1}$$

The division by $P[B]$ is equivalent to a re-scaling of the sample space for A. Conditional probabilities appear when an outcome is dependant on another outcome.

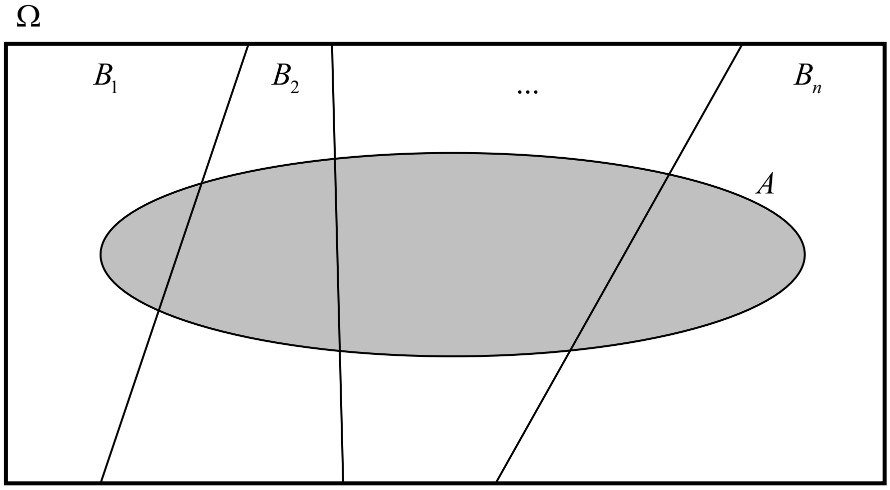

Figure 3.1: Illustration of the theorem of total probability

The **theorem of total probability**, illustrated in Figure 3.1, is defined as:

> For a given probability space $(\Omega, \mathcal{A}, P[\cdot])$, if $B_1, B_2, \ldots, B_n$ is a collection of mutually exclusive events in $\mathcal{A}$, satisfying $\Omega = \bigcup_{i=1}^{n} B_i$ and $P[B_i] > 0$ for $i = 1, 2, \ldots, n$, then for every $A \in \mathcal{A}$,
>
> $$P[A] = \sum_{i=1}^{n} P[A \mid B_i] \cdot P[B_i] \tag{3.2}$$

In the cases where A does not depend on B, $P[A \mid B] = P[A]$, and the events A and B are therefore **independent**. For several events, and using (3.1), independence is defined as:

> For a given probability space $(\Omega, \mathcal{A}, P[\cdot])$, if $A_1, A_2, \ldots, A_n$ is a number of events in $\mathcal{A}$, then these events are said to be independent if and only if:

$$\begin{aligned} P\left[A_i A_j\right] &= P\left[A_i\right] \cdot P\left[A_j\right] \\ P\left[A_i A_j A_k\right] &= P\left[A_i\right] \cdot P\left[A_j\right] \cdot P\left[A_k\right] \\ &\vdots \\ P\left[\bigcup_{i=1}^{n} A_i\right] &= \prod_{i=1}^{n} P\left[A_i\right] \end{aligned} \tag{3.3}$$

Independence of events features largely in this research and the above definition is of central importance.

3.2.2 Random variables and distribution functions

Often it is not the occurrence of a particular event that is of interest, but rather, the value of an attribute realised by the event:

> For a given probability space, $(\Omega, \mathcal{A}, P[\cdot])$, a **random variable**, denoted X or $X(\cdot)$, is a function with domain Ω and counterdomain the real number line.

A random variable links the sample space with a unique real number; consequently all outcomes are described numerically. Another function is required to relate the realized value of the random variable to a probability:

The **cumulative distribution function** of a random variable **X**, denoted $F_X(\cdot)$, is that function with domain the real line and counterdomain the interval [0,1] which satisfies $F_X(x) = P[X \le x] = P[\{\omega : X(\omega) \le x\}]$.

$\{\omega : X(\omega) \le x\}$ is read as the set of all points ω for which $X(\omega) \le x$. The cumulative distribution function will normally be abbreviated to CDF. It is the cumulative aspect of this function (the '$\le$') that urges another definition:

The **probability density function** of a random variable **X**, denoted $f_X(\cdot)$, is that function defined by:

$$\begin{aligned} f_X(x) &= \lim_{\Delta \to 0} \left(P[X \le x + \Delta] - P[X \le x] \right) \\ &= \frac{dF_X(x)}{dx} \end{aligned} \tag{3.4}$$

The probability density function is abbreviated as PDF. It is to be noted that the above definitions relate to continuous random variables. The relationship between CDF and PDF is thus defined as:

$$F_X(x) = \int_{-\infty}^{x} f_X(u)\, du \tag{3.5}$$

There are many forms of distributions and any of the textbooks given at the start of this section may be referred to for further information.

3.2.3 Probability paper

Graphical methods for the analysis of statistical data have a long history and an important place even in modern techniques; the histogram being the most prevalent – see Coles (2001a) for example. In this work, data and their corresponding statistical models are usually graphed on probability paper; a graph in which the x-axis is in arithmetic scale, and the data is plotted at its value. The y-axis is modified to give the standard variate of the distribution under study, such that, when the data is plotted, a straight line reveals adherence to the distribution.

The plotting position of the data on probability paper is governed by the empirical distribution function: the CDF of a data set, $x_1, \ldots, x_n$. When the data is arranged in increasing order, for any one of the x_i exactly i of the n observations have a value less than or equal to x_i, therefore the cumulative probability is given by:

$$P\left[X \le x_i\right] = \tilde{F}\left(x_i\right) = \frac{i}{n} \approx \frac{i}{n+1} \tag{3.6}$$

The adjustment is made such that $\tilde{F}\left(x_n\right) \neq 1$. The right hand side of (3.6) is the empirical probability. It is this probability that is used to identify the plotting position. Gumbel (1954) and Castillo (1988) discuss many other plotting positions. The choice of plotting position is not as important as it once was, as most inference is now done numerically rather than graphically.

Gumbel probability paper will be mostly used in this work and the Gumbel distribution is given by:

$$G_I\left(x\right) = \exp\left[\exp\left(\frac{x-\mu}{\sigma}\right)\right] \tag{3.7}$$

The standard Gumbel (or extremal) variate is:

$$s = \frac{x-\mu}{\sigma} \tag{3.8}$$

Therefore, the standard extremal variate, corresponding to the probability from the Gumbel distribution, s, and the empirical distribution, $\tilde{s}$, for a given data point, x, may be plotted on the y-axis once the following inversions are applied:

$$\begin{aligned} s &= -\ln\left[-\ln\left(G_I(x)\right)\right] \\ \tilde{s} &= -\ln\left[-\ln\left(\tilde{F}(x)\right)\right] \end{aligned} \tag{3.9}$$

Should the extremal variates correspond for each of the data points, a straight line results. Thus, the comparison of the fitted data may be got by drawing a straight line through the data points. Figure 3.2 illustrates the concept: a straight line is fitted through the data points (in this case by maximum likelihood – see section 3.3.2). The left y-axis gives the standard extremal variate whilst the right y-axis gives the cumulative probability. The x-axis corresponds to the data values.

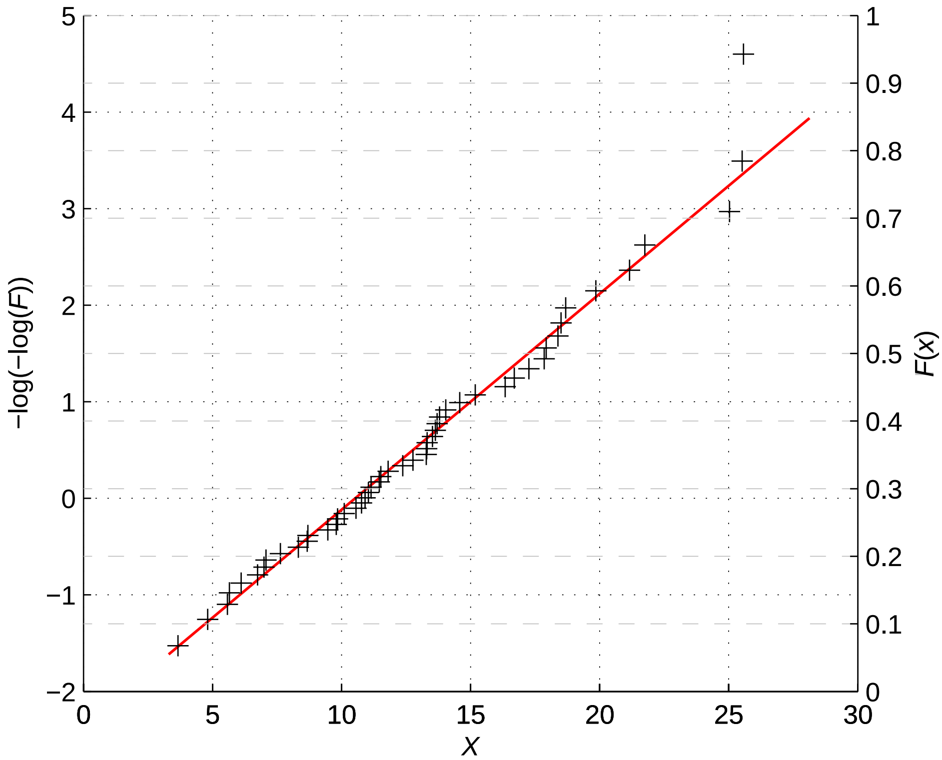

Figure 3.2: Gumbel paper probability plot.

It was previously stated that Gumbel paper is used, almost exclusively, in this research. However, it is not usually the Gumbel distribution being fit – rather the Generalized Extreme Value (GEV) distribution. This is a more flexible distribution that may exhibit curvature on Gumbel paper (or probability plot). Upward curvature reveals an asymptote to an x-axis value – corresponding to a physical limit on the statistical process. A curve asymptotic to a y-axis value (as well as a straight line) corresponds to a statistical mechanism with no physical limitation. Figure 3.3 gives two examples of GEV distributions plotted on Gumbel paper.

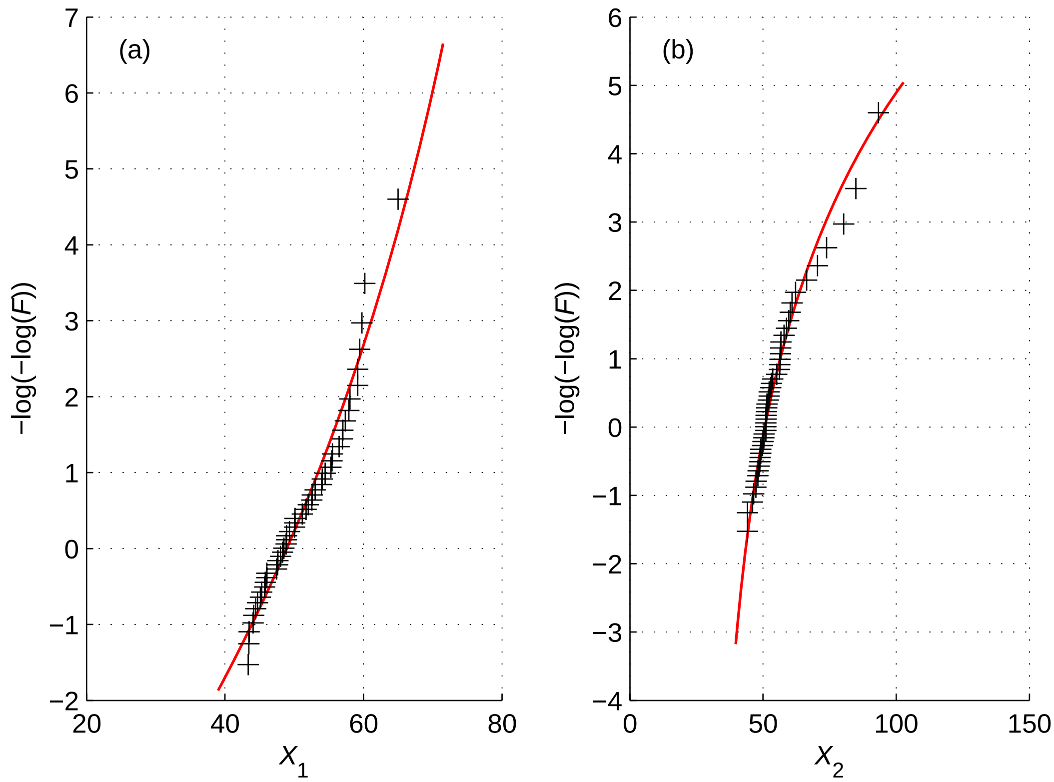

Figure 3.3: GEV distributions plotted on Gumbel probability paper: (a) bounded, and; (b) unbounded.

3.3 Statistical Inference

Azzalini (1996) defines statistical inference as the operation through which information provided by a sample of a population is used to draw conclusions about the characteristics of that population. The population is defined as the totality of elements about which information is desired, and the sample is defined as a collection of observed random variables taken from the population.

The following example will be developed through the following sections: consider a container holding 5000 small balls which are either black or white, of which the proportion of black balls is required. Rather than examining each of the 5000 balls, a sample could be taken at random from the container. Azzalini (1996) describes the reasons why this is often preferable. Suppose that 50 balls are drawn at random, of which 4 are found to be black. The proportion of black balls is $\hat{\theta} = 4/50$, in which the 'hat' notation shows that this is only an estimate of the true parameter value. It is reasonable to think that drawing another sample of 50 balls may not result in the same value for θ. However as of yet, it is the best estimate of the proportion of black balls in the population. Another issue is the sample size, and the amount of information it holds about the population: should 100 balls have been drawn and 8 found to be black, it is intuitive to expect extra 'information' about the estimate of θ from this larger sample.

Approximating the hypergeometric distribution with the binomial distribution (valid for the size of the sample), the probability that the random variable **Y** yields the observed number of black balls, y, is:

$$P[Y = y] = \binom{50}{y} \theta^{y} (1-\theta)^{50-y} \tag{3.10}$$

Approximating the set of possible values for θ as the interval $[0,1]$, it can be seen that (3.10) represents a family of probability distributions for each value of the parameter θ. Inference is identifying the true distribution of Y through estimation of the parameter θ (Silvey 1970, Lindsey 1996).

The Likelihood method of inference is mainly used in this work. As will be shown, it is a robust, accurate estimator with excellent asymptotic properties. It is also a minimal sufficient statistic (Zacks 1971) – it contains as much information about the distribution of the data as the data itself (Mood et al 1974). There are some known cases in which likelihood can give anomalous results (see for example, Zacks 1981), but these do not affect the work herein.

3.3.1 Likelihood

Edwards (1992) gives the first example of a likelihood argument and attributes it to Daniel Bernoulli, who states: "…one should choose the one which has the highest degree of probability for the complex of observations as a whole". Edwards (1992) himself also defines likelihood informally: "Our problem is to assess the relative merits of rival hypotheses in the light of observational or experimental data that bear upon them". Fisher first defined mathematical likelihood in 1912 in an undergraduate essay and continued to advance it, culminating in his paper "On the Mathematical Foundations of Theoretical Statistics" in 1922 (Fisher 1922, Alrich 1997). Fisher's idea is to examine the probability of having observed the data that was observed, given the proposed probability model. For a probability density $f_X(x;\theta)$ – where the notation indicates that the density is a function of the parameter (or vector of parameters) of the model – the likelihood of having observed a particular realization x is defined as:

$$L(\theta) = L(\theta; x) = c(\theta) \cdot f(x; \theta) \tag{3.11}$$

where the notation emphasizes the dependence of the density upon the parameter, and similarly for the likelihood upon the data. The multiplicative constant is required to make the probability density a probability for each data point. For the set of n sample values the probability of having observed the observed values is:

$$\begin{aligned} L(\theta; x) &= c(\theta) \cdot \left[f(x_1; \theta) \cdot f(x_2; \theta) \cdot \dots \cdot f(x_n; \theta) \right] \\ L(\theta; x) &= c(\theta) \cdot \prod_{i=1}^{n} f(x_i; \theta) \end{aligned} \tag{3.12}$$

In practice it is more convenient to work with the log-likelihood to avoid the multiplicative nature of the likelihood function:

$$l(\theta; x) = \log L(\theta; x) = c(\theta) + \sum_{i=1}^{n} \log f(x_i; \theta) \tag{3.13}$$

Generally the constant $c(\theta)$ is not involved in any calculations using likelihood as one seeks knowledge of relative likelihoods and c is thus not relevant.

Returning to the example of the 5000 balls, it can be seen that for the single observed value $y = 4$, equation (3.10) corresponds to (3.11) and is thus the likelihood function for the parameter θ. This is graphed in Figure 3.4(a) which shows an increased likelihood for a parameter value around 0.05 to 0.10, relative to other possible values of the parameter. Also shown is the likelihood function for the case when the number of samples is 100 and the number of observed black balls in this sample is 8, as are the graphs of the likelihood ratio, which is the likelihood function, normalized on its maximum value, and the log-likelihood, for comparison.

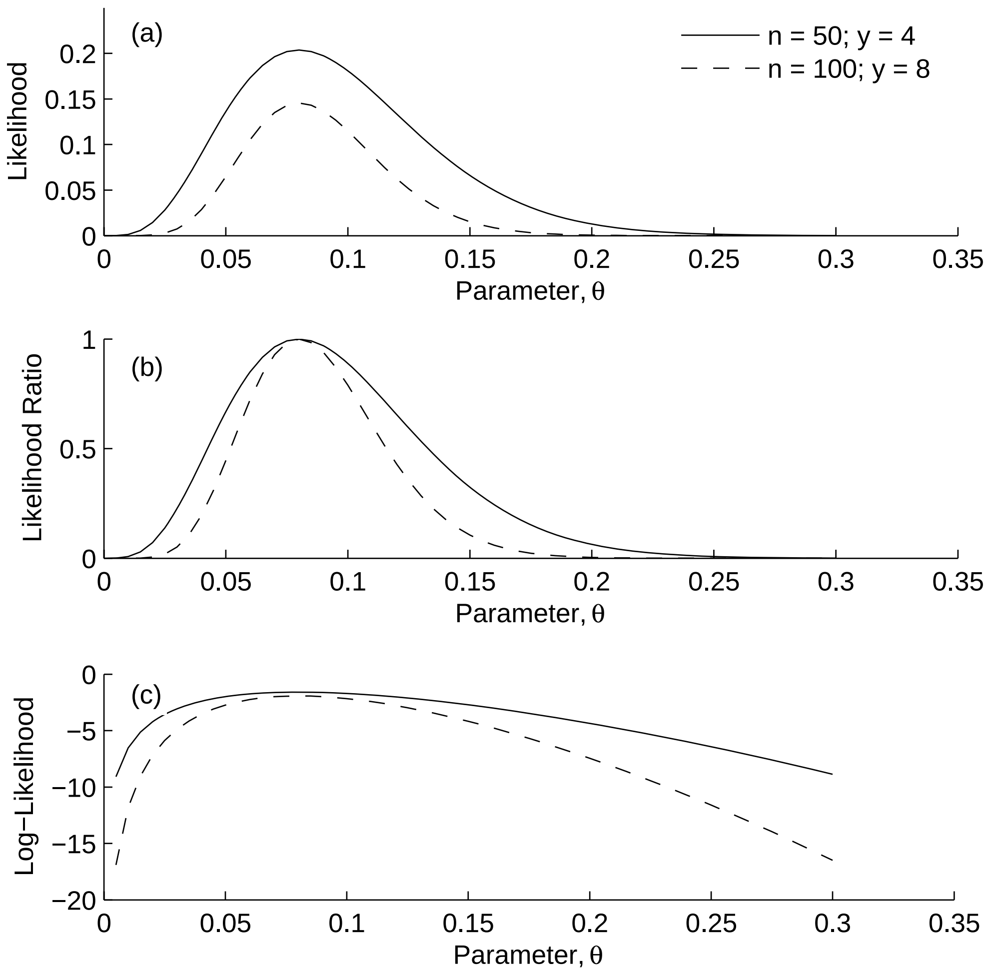

Figure 3.4: Likelihood functions for the 'ball' example: (a) absolute likelihood; (b) relative likelihoods, and; (c) log-likelihoods.

The question regarding the amount of 'information' held in the data was raised previously: more information regarding the 'true' value of θ should surely be available from a larger sample. Trivially, if the sample is the total population, then the amount of information about θ is at a maximum. This increase of information may be seen in the likelihood ratio and log-likelihood graphs – the width of the $n=100$ curve is less than that of the $n=50$ curve. This means a smaller range of likely parameter values, at any level of relative likelihood, results from the larger sample size, than for the smaller. Thus the $n=100$ curve holds more information about the true parameter value, as expected.

3.3.2 Maximum likelihood and Fisher information

A mathematical definition of the information contained in the sample may be obtained by considering the log-likelihood function: the value of the parameter that maximizes the likelihood function is most likely to be the 'true' parameter value. This is the Method of Maximum Likelihood. A parameter value found in this way is denoted $\hat{\theta}$ to emphasize that it is an estimate; the notional 'true' value of the parameter is denoted θ_0. Using the log-likelihood function, the maximum likelihood estimate (MLE) of a parameter is the value that satisfies:

$$\frac{d\, l(\theta;x)}{d\theta}=0 \tag{3.14}$$

Geometrically this is the slope of the tangent to the log-likelihood curve at its maximum (Figure 3.4). Using a Taylor series approximation about the MLE, the log-likelihood function is approximated as:

$$l(\theta)=l(\hat{\theta})+(\theta-\hat{\theta})\cdot\frac{d\,l(\hat{\theta})}{d\theta}+\frac{1}{2}\cdot(\theta-\hat{\theta})^2\cdot\frac{d^2 l(\hat{\theta})}{d\theta^2}+\cdots \tag{3.15}$$

having dropped the dependency notation for brevity. Then approximately, incorporating (3.14) and dropping third-order and higher terms:

$$l(\theta)=l(\hat{\theta})+\frac{1}{2}\cdot(\theta-\hat{\theta})^2\cdot\frac{d^2 l(\hat{\theta})}{d\theta^2} \tag{3.16}$$

Empirically, equation (3.16) measures how informative the data is about the MLE. It states that the support offered by the data to $\hat{\theta}$, and some other value θ, differs by an amount proportional to the second derivative of the log-likelihood function about $\hat{\theta}$. Hence, the observed (or Fisher) information (Cox and Hinkley 1974) is defined as:

$$I\ (\theta) = -l''\left(\hat{\theta}\right) = -\frac{d^2 l\left(\hat{\theta}\right)}{d\theta^2} \tag{3.17}$$

Referring to Figure 3.4(c), it is apparent that the curve for the larger sample size (n = 100) is narrower than that for the smaller sample size (n = 50) and is therefore more curved near the MLE than the log-likelihood function of the smaller sample. Hence, (3.17) may be perceived as the spherical curvature of the log-likelihood function at the estimate: its reciprocal is the radius of curvature at the estimate. The reciprocal is also the value of the Cramér-Rao lower bound for the variance of an unbiased estimator (Azzalini 1996, Mood et al 1974, Zacks 1971) – the smallest possible variability a parameter estimator can have.

Though the above has been presented relating to one-dimensional parameters, the theory is extendable to multi-dimensional parameters. In such cases the reciprocal of the information may be thought of as related to the volume under the likelihood surface. The square root of the determinant of the information matrix may be seen as a measure of the width of the likelihood surface (Edwards 1992). Also, the diagonal entries of $I\ (\theta)$ represent the variance of a parameter with respect to itself. Hence, the square root of the diagonal term corresponding to a parameter represents the standard error of that parameter.

Figure 3.5 shows two log-likelihood surfaces for the normal distribution. The flatter surface is derived from 50 random deviates of $N\left(100,5^2\right)$; the more curved surface is found from 200 random $N\left(100,5^2\right)$ deviates. This figure clearly shows that 'support' for differing values of μ and σ drops away much quicker for the larger data set. Put another way, the volume under the curve at its maximum is less; its reciprocal is the information, which is thus greater.

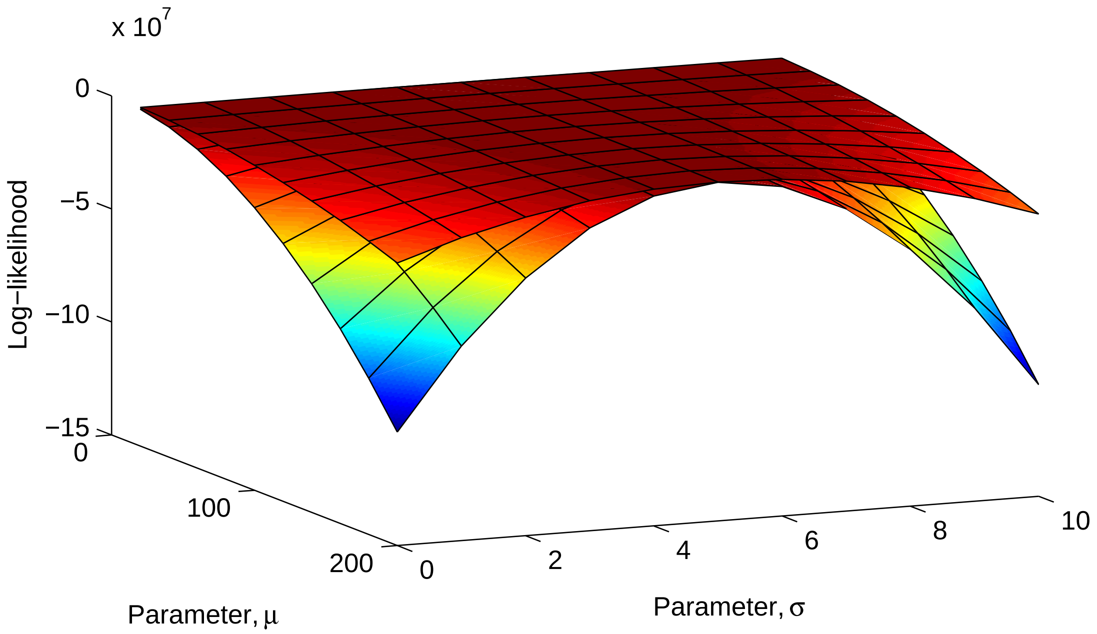

Figure 3.5: Log-likelihood surfaces of $N\left(100,5^2\right)$, for n = 50 and 200.

3.3.3 Asymptotic normality of an MLE

The maximum likelihood estimator has many properties desired of an estimator – refer to Azzalini (1996), Edwards (1992), and Mood et al (1974) for further information. Of direct relevance is that it is a Best Asymptotic Normal (BAN) estimator. An estimator (for example, maximum likelihood), $T(X)$, such that

$$\sqrt{n}T(X) \xrightarrow{d} N\left(\theta, \operatorname{var}(\theta)\right) \tag{3.18}$$

where n is the sample size, is said to be a BAN estimator if $\operatorname{var}(\theta) = I\ (\theta)^{-1}$ which indicates that the estimator is asymptotically normally distributed. In the multi-dimensional case, the reciprocal of the observed information is the usual variance-covariance matrix of the parameter estimates. Therefore, parameters of distributions estimated using maximum likelihood estimation may be taken to be asymtotically normally distributed; the accuracy of the approximation improves with increasing sample size due to the central limit theorem.

3.3.4 Profile likelihood and deviance

The previous section described the asymptotic distribution of maximum likelihood parameter estimates. Often, it is more useful to obtain an estimate of the actual distribution of a parameter (Barndorff-Nielsen 1983). In the uni-dimensional case this does not pose a problem: Figure 3.4 illustrates how the parameter estimate varies. As the likelihood function cannot provide an absolute statement of the suitability of a parameter estimate, the likelihood ratio graph of Figure 3.4(b) is particularly important in aiding estimates of parameter distributions. Having evaluated the log-likelihood, the likelihood ratio is given by the difference of two log-likelihoods. The **deviance function** is defined as:

$$D(\theta) = 2\left[l(\hat{\theta}) - l(\theta) \right] \tag{3.19}$$

As the log-likelihood is usually a negative quantity, the deviance is positive. The likelihood ratio is multiplied by 2 for reasons outlined by Lindsey (1996). The deviance, as defined in (3.19), is approximately chi-squared distributed with the number of degrees of freedom equal to the number of parameters in the model (Coles 2001a). With such knowledge, it is possible to work backwards from a pre-specified probability (such as 95%) to find the value of $l(\theta)$ that defines the confidence region. Figure 3.6 illustrates this for the ball example. It can be seen that the 95% confidence interval narrows for the larger sample size, reflecting the increase in information available. Also, it is of note that the confidence intervals are not symmetric about the MLE (corresponding to zero deviance). Thus the distribution of the likelihood estimate is skewed which is not compatible with the assumption of normality.

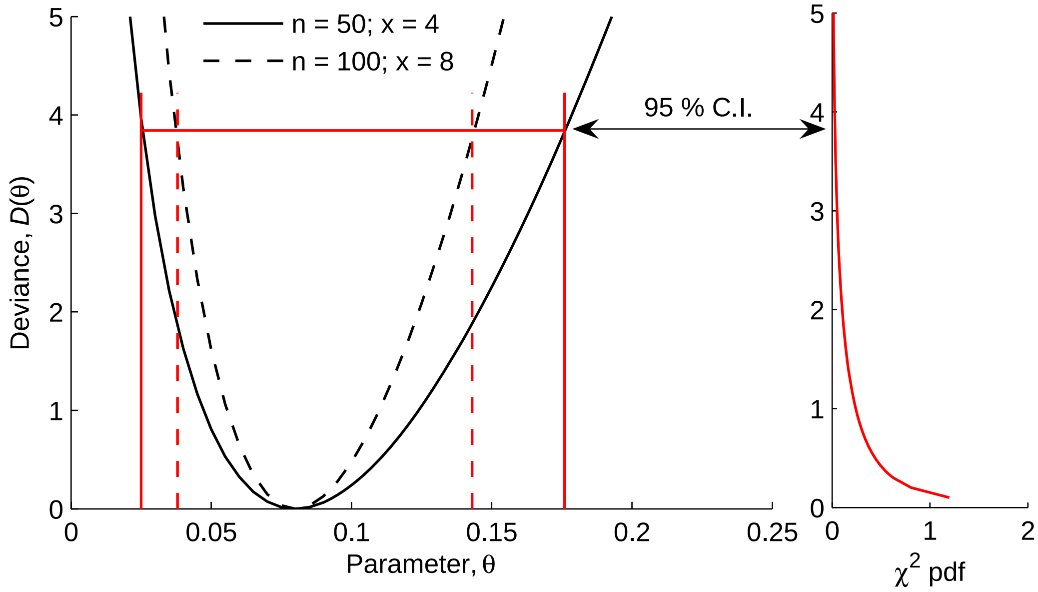

Figure 3.6: Deviance function and confidence intervals for the ball example samples (note that the χ^2 PDF graph is rotated 90°).

In multi-parameter cases, the application of the preceding method is more difficult. It may be seen from Figure 3.5 that the parameters are orthogonal in multi-dimensional space, though not independent. To estimate the distribution of a parameter, a notional 'slice' through the likelihood surface is made parallel to the axis of the parameter of interest – approximately, the resulting cross section is the **profile (log-)likelihood** of the parameter of interest. However, the 'slice' is in fact a point, as it must be taken at the MLEs of the other parameters, conditional on the current value of the parameter of interest. The profile log-likelihood of a parameter, θ_i, is defined as:

$$l_p(\theta_i) = \sup_{\bar{\theta}} l(\theta_i, \bar{\theta}) \tag{3.20}$$

where $\bar{\theta}$ denotes the restricted parameter vector which is θ without θ_i and $\sup$ may be read as 'the maximum of'. Thus, for each value of the parameter of interest, the profile log-likelihood is the maximized log-likelihood with respect to all of the other parameters. In the case of the example of Figure 3.5, the profile likelihood for the μ parameter is shown in Figure 3.7. The 95% confidence intervals derived from the $\chi^2(0.95,2)$ distribution – where the number of degrees of freedom is 2, corresponding to the number of parameters in the model – is also shown in Figure 3.7. Note also that each unit of the χ^2 distribution corresponds to two units of log-likelihood due to the deviance function. Further, it may be seen that the confidence intervals are close to symmetric about the MLE of the mean; the normal approximation in this case would be quite reasonable.

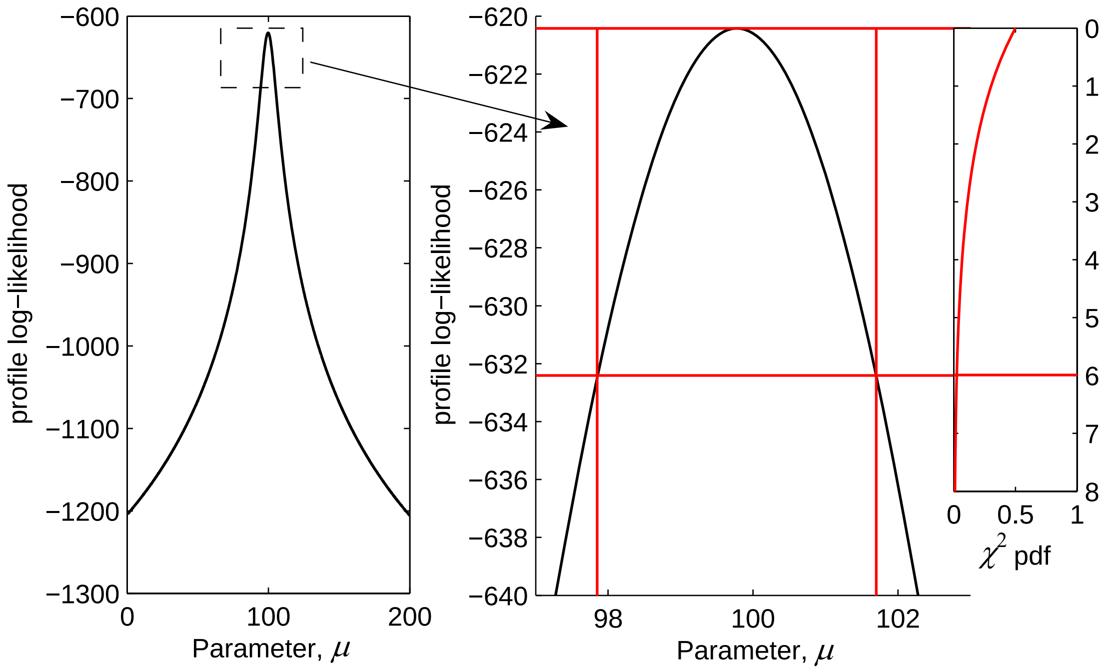

Figure 3.7: Profile likelihood for μ of $N(100,5^2)$, with confidence intervals got from $\chi^2(0.95,2)$.

Though only parameters have been examined here, the method of profile likelihood can be extended to cover any functional combination of the parameters. As will be shown in Chapter 7, this extension of profile likelihood has considerable benefit for the prediction of extreme values.

3.4 Statistics of Extremes

The statistics of extremes, or extreme value theory, is concerned with identifying trends in the extreme (maximum or minimum) values obtained from a set of samples. The theory has found extensive use in the practical sciences where decisions have to be made and not postponed until a better theory, or more data emerges (Castillo 1988, Coles 2001a). Bardsley (1994) argues that the theory has reached its zenith and that the results of an elaborate objective analysis are not significantly better than a subjective analysis by an experienced investigator. This view is certainly not as widespread as its counterpart. The statistical analyses used in this work employ extreme value theory throughout.

3.4.1 Basic formulation

Only the distribution of the maximum of a sample is considered here, though that of the minimum follows a similar formulation – refer to Castillo (1988), Ang and Tang (1984) and Galambos (1978) for more details on what follows.

Consider a set of n random variables, $X_1,\ldots,X_n$ and allow $Y = \max\left[X_1,\ldots,X_n\right]$. Given a set of observations, $x_1,\ldots,x_n$ for which $y = \max\left[x_1,\ldots,x_n\right]$. When the X_is are independent, the distribution function, $F_Y(\cdot)$, of y is:

$$F_Y(y) = P\left[Y \le y\right] = P\left[x_1 \le y;\ldots;x_n \le y\right] \tag{3.21}$$

which results because the largest of the x_is is less than or equal to y if, and only if, all of the x_is are less than or equal to y. If the X_is are independent and identically distributed (iid), then, similar to (3.3):

$$P\left[x_1 \le y;\ldots;x_n \le y\right] = \prod_{i=1}^{n} P\left[X_i \le y\right] \tag{3.22}$$

and therefore, where $F_X(\cdot)$ is the distribution function of $X_1,\ldots,X_n$:

$$F_Y(y) = \left[F_X(y)\right]^n \tag{3.23}$$

The distribution $F_X(\cdot)$ is known as the parent distribution. As the parent CDF is raised to the power of n, it is important that the parent distribution is both known and closely models the data – especially in the upper tail of the distribution (Coles 2001b, Castillo 1988). Any deviations of the model from the true distribution are raised to the power of n and can therefore distort the analysis. Also, explicit expressions for the distribution of the maxima are difficult to obtain from (3.23). These problems with this formulation have resulted in the development of the asymptotic theory of extreme order statistics – most notably associated with Fisher and Tippett (1928), though other authors were writing on this subject around the same time (Gumbel 1958).

3.4.2 Fisher-Tippett and Gnedenko

The asymptotic theory of extreme order statistics attempts to identify possible limiting forms of the distribution of the extreme as n tends to infinity, avoiding the degenerate results; 0 when $F_X(y) < 1$, and 1 when $F_X(y) = 1$. Fisher and Tippett (1928) recognized that the maximum of N sets of observations of n values of X, must also be the maximum of n values of X. Therefore any non-degenerate distribution must be of the same form, but linearly transformed by location and scale parameters (a_n and b_n respectively) that depend only on n:

$$G^n(y) = G(a_n + b_n y) \tag{3.24}$$

where $G(\cdot)$ indicates an extreme value distribution representing a limiting asymptotic form of the distribution of maxima. This equation is known as the stability postulate and any distribution that meets this equation is said to be max-stable. With this as a basis, the limiting form of the distribution of the maximum from a parent distribution is:

$$\left[F_X(y)\right]^n = F_X(a_n + b_n y) \tag{3.25}$$

Fisher and Tippett gave three solutions to this equation, based on different values for a_n and b_n: the Type I, II and III limiting forms. Gnedenko (1943) established the strict mathematical conditions under which the Type I, II or III distributions form the limiting distribution for various forms of parent distributions – known as the domain of attraction of the parent distribution (Castillo and Sarabia 1992).

3.4.3 Jenkinson and von Mises

The three forms of limiting distributions, to which almost all distributions converge, are the Gumbel, Frechet and Weibull distributions (Gumbel 1958). Jenkinson (1955) and von Mises (1936) independently solved expression (3.25) for a single form: the Generalized Extreme Value distribution (GEV), given by:

$$G(y) = \exp\left\{-\left[1-\xi\left(\frac{y-\mu}{\sigma}\right)\right]_+^{1/\xi}\right\} \tag{3.26}$$

where $\left[x\right]_+ = \max(x,0)$ and where the parameters satisfy $-\infty < \mu < \infty$, $\sigma > 0$ and $-\infty < \xi < \infty$. The model has three parameters: location, μ; scale, σ; and shape, ξ. The Type II and III families correspond to the cases $\xi > 0$ and $\xi < 0$ respectively. The Type I family is the limit of $G(y)$ as $\xi \to 0$. The major benefit of using the GEV distribution is that, through inference on ξ, the data itself determines the correct tail model, avoiding the need to make a subjective a priori judgment on which of the Fisher-Tippett limiting forms to adopt.

The power of the concept of asymptotic limiting forms is that the actual form of the parent CDF $F_X(y)$ is not required for fitting the GEV (or indeed any of the extreme value distributions). It is worthy of note, however, that the speed of convergence with n repetitions of the parent distribution to the GEV varies: the normal distribution is notoriously slow, whist the exponential distribution converges rapidly (Cramér 1946). Figure 3.8 illustrates the exact and asymptotic (Gumbel) distributions from these two parent distributions – based on the constants a_n and b_n given by Galambos (1978) and Cramér (1946) and the methodology of Gumbel (1958).

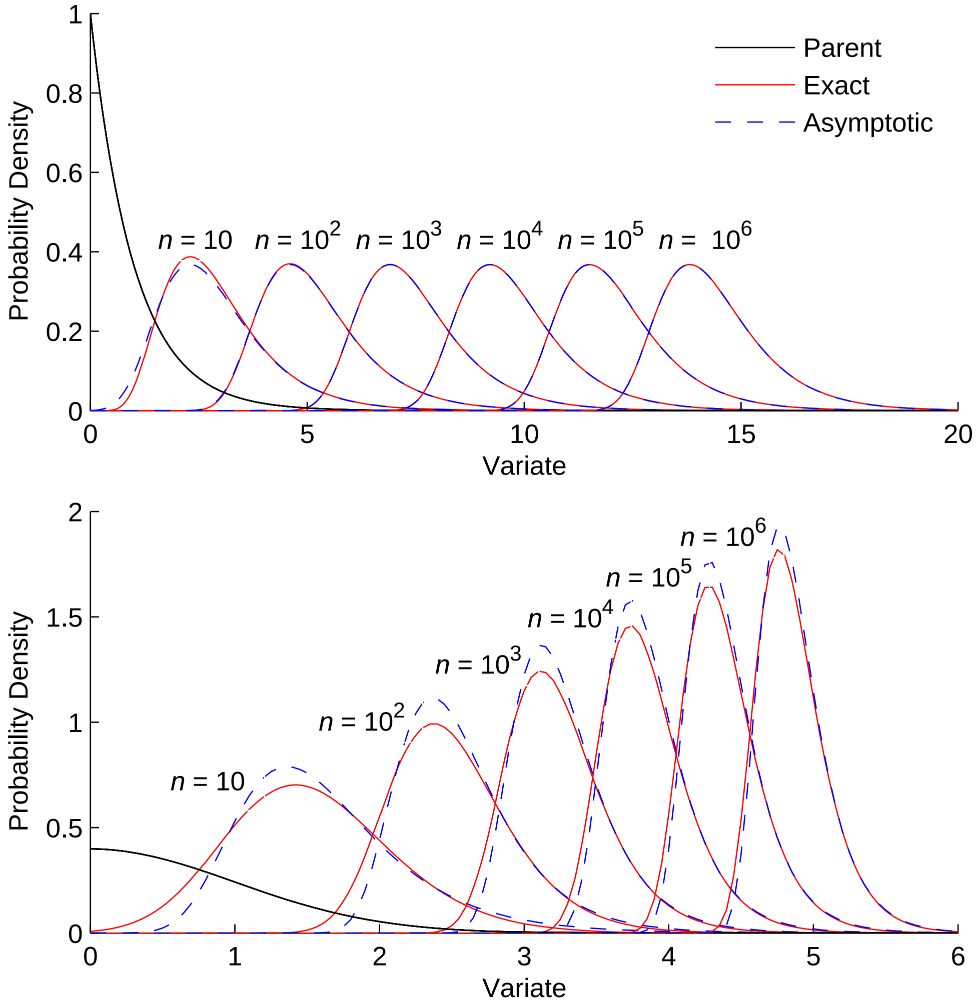

Figure 3.8: Asymptotic and exact distribution of maxima: (a) standard exponential distribution, and; (b) standard normal distribution.

From Figure 3.8, the difference in the speed of convergence for these two distributions is readily apparent. Castillo (1988) and Gomes (1984) discuss penultimate forms of asymptotic distributions; for suitable parameters the Weibull distribution (a particular case of the GEV distribution) can offer a better approximation of the distribution of maxima from a normal parent than its true asymptotic distribution (Gumbel). When it is necessary to check on the form of the parent, due to small sample sizes, speed of convergence tests may be used and are detailed in Galambos (1978).

3.4.4 Estimation

The method of maximum likelihood requires the maximization of the log-likelihood function. Optimization techniques often deal with minimizing functions. Hence minimization of the negative log-likelihood is usually performed. In this work, the GEV distribution is mostly used and Jenkinson (1969) gives the log-likelihood function for the GEV distribution:

$$l(\mu,\sigma,\xi;y) = -n\log\sigma - \left(1-\frac{1}{\xi}\right)\sum_{i=1}^{n}\log y_i - \sum_{i=1}^{n} y_i^{1/\xi} \tag{3.27}$$

$$\text{where } y_i = 1-\xi\left(\frac{x_i-\mu}{\sigma}\right) > 0 \quad \text{for } i = 1,\ldots,n \tag{3.28}$$

For parameter combinations where $y_i < 0$ (which occurs when a data point x_i has fallen beyond the range of the distribution) the likelihood is zero and the log-likelihood will be numerically ill-defined. Solution of (3.27) is done by numerical means – there is no analytical solution. Jenkinson (1969) describes an approximate iteration technique for solving the equation which uses the expected information matrix (the matrix of second derivates of (3.27) with respect to each of the parameters). However Jenkinson only derived approximate values for this. Prescott and Walden (1980) detailed the elements of the observed information matrix, $I(\theta)$, for the GEV distribution. They furthered this work (Prescott and Walden 1983) by proposing a Newton-Raphson technique which uses $I(\theta)$ and is found to converge quickly. Hosking (1985) presents an algorithm for the estimation of the parameters of the GEV distribution based on Prescott and Walden's proposal.

Good starting values for the minimization of the negative log-likelihood function of the GEV distribution are obtained from the method of probability weighted moments (PWMs) described by Hosking et al (1985). The exact solution requires iterative methods, but within the range usually encountered in practice, $\{-0.5 \le \xi \le 0.5\}$, Hosking et al (1985) have proposed an estimator, b_r, which uses the data, x_j, and is then used to solve for the other parameters in the sequence:

$$b_r = n^{-1}\sum_{j=1}^{n}\frac{(j-1)(j-2)\cdots(j-r)}{(n-1)(n-2)\cdots(n-r)}x_j \tag{3.29}$$

$$c = \frac{2b_1 - b_0}{3b_2 - b_0} - \frac{\log 2}{\log 3} \tag{3.30}$$

$$\hat{\xi} = 7.8590c + 2.9554c^2 \tag{3.31}$$

$$\hat{\sigma} = \frac{\hat{\xi}(2b_1 - b_0)}{\Gamma(1+\hat{\xi})(1-2^{-\hat{\xi}})} \tag{3.32}$$

$$\hat{\mu} = b_0 + \frac{\hat{\sigma}}{\hat{\xi}}\left[\Gamma(1+\hat{\xi}) - 1\right] \tag{3.33}$$

The PWM approach is written in C++ and used to initiate a C++ version of Hosking's (1985) algorithm. Data sets from Coles (2001a) are used to verify the output against published results. It is found, however, that there are cases in which Hosking's algorithm does not converge, or does not achieve the same minimum function value as the `WAFO MATLAB` toolbox (Brodtkorb et al 2000) which was used occasionally to verify output. As a result, a more robust optimization method is implemented.

The Nelder-Mead (NM) optimization algorithm (Nelder and Mead 1965) is also known as the amoeba algorithm (**Numerical Recipes in C** – Press et al 1992) because of its slow robust movement across the k-dimensional surface of a function, where k is the dimension of the optimization problem. The NM algorithm is based on a simplex – a geometric shape with $k+1$ corners. Lagarias et al (1997) describe, in detail, the operations of the algorithm.

In the processing undertaken in the `AnalyseEvents` program (Chapter 4), the PWM method is used to initiate both the Hosking and NM algorithms – processing time is not substantial in any case. The program checks to see if the Hosking algorithm has a smaller negative log-likelihood than that of the NM algorithm. If not, the results of the NM algorithm are used. While good results can be obtained with manual re-injection of the Hosking algorithm, in general this is not possible for this research – the number of individual GEV fits is substantial for each run. Checks have been performed both against published results and other algorithms such as `WAFO` (Brodtkorb et al 2000) and `EVD` (Stephenson 2004) for the R-language (R Development Core Team 2005).

3.5 Prediction

Some of the solutions to Pearson's "fundamental problem of statistics" are described in this section. Initially the traditional extrapolation procedure – which uses a fitted distribution – is described. However, the variability of both the parameters and the data itself intuitively produce uncertainty in the estimate found in this manner. The delta method uses the asymptotic normality principle to estimate this variability, whilst the bootstrap method uses computational means to establish variability. Both methods are briefly described here.

3.5.1 The characteristic value and return period

The characteristic value is that value of a random variable that is expected to be exceeded once in a given return period. Given a random variable X, with distribution function $F_X(\cdot)$, the probability of exceeding a value, u, is:

$$P\left[X > u\right] = 1 - F_X(u) \tag{3.34}$$

For a given return period, R, consider n repetitions of the sampling period, T_X, from which X was determined, such that:

$$n = \frac{R}{T_X} \tag{3.35}$$

In n such repetitions, the probability that the characteristic value, u, will be exceeded is:

$$P\left[X > u \text{ in } n \text{ repetitions}\right] = n\left(1 - F_X(u)\right) \tag{3.36}$$

From the definition of a characteristic value, this probability must be equal to unity, that is, is expected to occur at least once in n repetitions, hence,

$$n\left[1-F_X(u)\right]=1$$
$$\Rightarrow F_X(u)=1-\frac{1}{n} \tag{3.37}$$

The characteristic value may therefore be determined by:

$$u=F_X^{-1}\left(1-\frac{1}{n}\right) \tag{3.38}$$

3.5.2 Extrapolation

In the work that follows, it will be usual to have a return period of 1000 years with each year comprising 50 working weeks of data with 5 working days per week, for reasons outlined in Chapter 4. The distribution obtained from the simulations is usually that of the maximum per day: the number of repetitions of the sampling period is then:

$$n_{1000}=1000\times 50\times 5=250\,000$$
$$\Rightarrow F_X(u_{1000})=1-\frac{1}{n_{1000}}=0.999996 \tag{3.39}$$
$$\Rightarrow u_{1000}=F_X^{-1}(0.999996)$$

As described previously, this will correspond with a standard extremal variate derived from the Gumbel distribution as:

$$G_I^{-1}\left(0.999996\right)=-\log\left(-\log\left(0.999996\right)\right)$$
$$=12.429 \tag{3.40}$$

An example of such extrapolation is shown in Figure 3.9 on Gumbel probability scale (Ang and Tang 1975, Section 3.2.3). Also, as the sampling period approaches the return period, the extrapolation distance decreases, intuitively resulting in an better estimate – though this needs to be proved using other methods.

There is inherent variability in the extrapolation process described: parameter estimates vary due to estimator uncertainty; the data varies; and different investigators may use different estimation techniques, which may or may not be biased. Prediction of a single number does not reflect the statistical nature of the underlying problem. Various methods for estimating the variability of the characteristic extreme are available; two are described next. Another method is preferred and described in Chapter 7 in relation to this research.

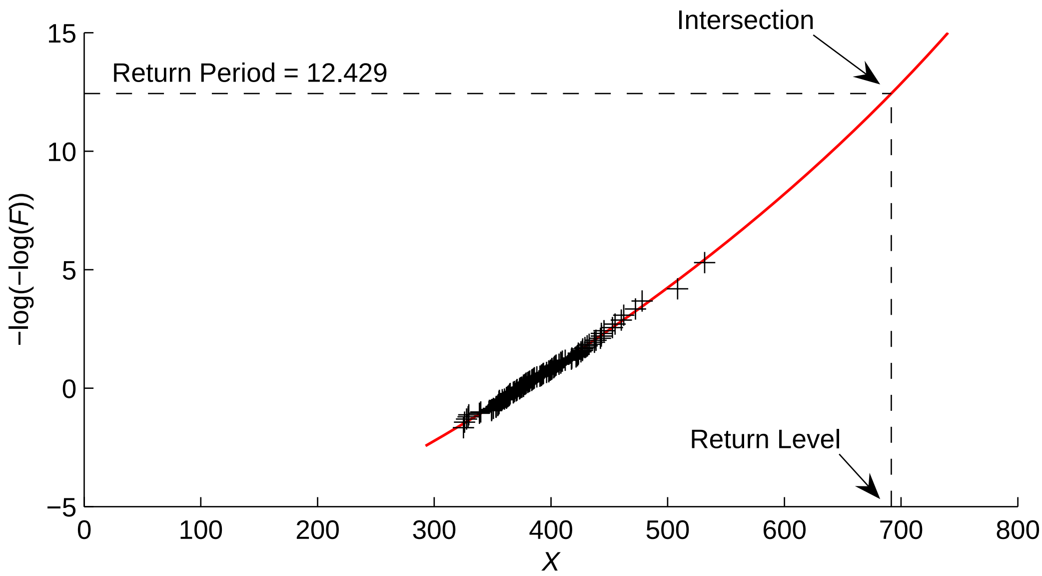

Figure 3.9: Sample extrapolation procedure.

3.5.3 The Delta Method and the normality assumption

The delta method for the approximation of moments of functions of random variables is usually based on a first-order Taylor series expansion of the function about the point of interest (Rice 1995, Oehlert 1992). Given a random variable **X** and a one-to-one function, **Y** = g(**X**), the first-order Taylor approximation about the mean is:

$$Y = g(X) \approx g(\mu_X) + (X - \mu_X)\frac{dg(\mu_X)}{dx} \tag{3.41}$$

Noting that this is a linear function of **X**, the linear transformation of variables rule (Mood et al 1974) gives:

$$\mathrm{Var}(Y) = \left(\frac{dg(\mu_X)}{dx}\right)^2 \cdot \mathrm{Var}(X) \tag{3.42}$$

Use of the matrix form of the Taylor series expansion (Beck and Arnold 1977) enables this to be extended to the case of several variables:

$$\mathrm{Var}(Y) = \nabla g(\mathbf{X}) \cdot \mathbf{V}_X \cdot \nabla g(\mathbf{X})^T \tag{3.43}$$

where **Y** is a scalar value of the function $g(\cdot)$ with parameter vector $\mathbf{X}$. $\mathbf{V}_X$ is the variance-covariance matrix of the parameter vector and $\nabla g(\mathbf{X})$ is the gradient vector of the function (Coles 2001a, Azzalini 1996, Efron and Tibshirani 1998, Lindsy 1996, Zacks 1971).

In (3.41) when **X** represents the (asymptotically) normally-distributed parameter(s) of a distribution, and as **Y** is a linear transformation of **X**, then:

$$Y \xrightarrow{d} N\left(g(\mu_X), \mathrm{Var}(Y)\right) \tag{3.44}$$

where $\mathrm{Var}(Y)$ may be given by (3.42) for a single parameter function or (3.43) for a multi-parameter function.

For large sample sizes the delta method approximations give good results as a result of the central limit theorem (Mood et al 1974). However, for smaller sample sizes and where the linear approximation of the function in the region of interest is not good, the delta method can give inaccurate results (Rice 1995).

With respect to the GEV distribution used in this work, for **R** sampling periods, the maximum likelihood estimate of the characteristic value is got by rearranging the equality (3.37):

$$\hat{z}_R = g(R) = \mu + \frac{\sigma}{\xi} \cdot \left[1 - y_R^{\xi}\right] \tag{3.45}$$

where, $y_R = \log\left(1 - \frac{1}{R}\right)$. From (3.43), letting $\theta \equiv \mathbf{X} = (\mu, \sigma, \xi)^T$ and:

$$\begin{aligned} \nabla g(\theta) &= \nabla \hat{z}_R^T \\ &= \left[\frac{\partial z_R}{\partial \mu}; \quad \frac{\partial z_R}{\partial \sigma}; \quad \frac{\partial z_R}{\partial \xi} \right] \\ &= \left[1; \quad \xi^{-1}\left(1 - y_R^{\xi}\right); \quad \sigma\xi^{-2}\left(1 - y_R^{\xi}\right) - \sigma\xi^{-1} y_R^{\xi} \log y_R \right] \end{aligned} \tag{3.46}$$

From (3.43) with the substitutions of (3.45) and (3.46) the distribution of $\hat{z}_R$ may be got from (3.44). The estimate notation on the parameters of the GEV was dropped for clarity: the expressions are evaluated at the estimates.

Implicit in methods like the delta method, is the central limit theorem and the assumption of asymptotic normality. Often it is not the case that the sample size is sufficient to converge to normality and the distribution may, in fact, be skewed. It is shown in Chapter 7 that the distribution of the bridge traffic load effect return level estimate is generally highly skewed and therefore highly non-normal. Therefore confidence limits, or variance estimates, based on the assumption of normality can give misleading results and should be avoided where possible.

3.5.4 Bootstrapping

The bootstrap has emerged as a fundamental tool in statistical analysis since its introduction (Efron 1979). This is, in part, due to the ready availability of computing power and the intuitive nature of its application. Efron and Tibshirani (1993) and Davison and Hinkley (1997) both give thorough accounts of bootstrapping and its flexibility of use. Boos (2003) exemplifies the power of the bootstrap applied in a civil engineering, extreme value analysis, setting.

The bootstrap process (Figure 3.10) consists of re-sampling the original data (non-parametric bootstrap) or a model fitted to it (parametric bootstrap) and estimation of the statistic, $s(\cdot)$, of interest for the model. This process is repeated many times (bootstrap replications) and a distribution of the statistic of interest is found.

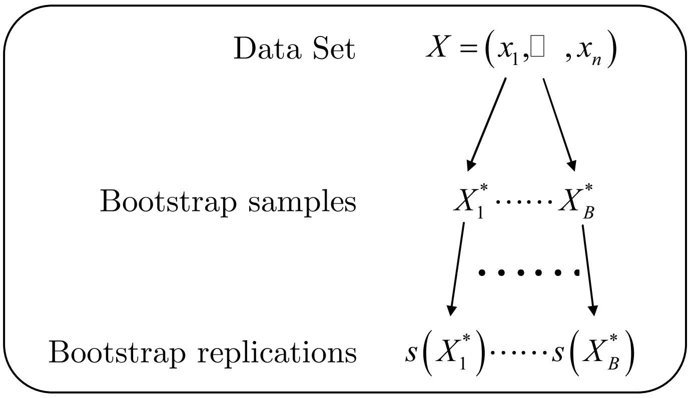

Figure 3.10: Illustration of the bootstrap process.

Extreme values are of particular importance to this work. Efron and Tibshirani (1993) describe a case where the bootstrap fails to give reasonable answers due to the sparsity of data in the tail, and the associated poor estimate of the true distribution by the empirical distribution (3.6).

As an illustration of this problem, and the non-parametric bootstrap process in general, consider a data set $x_1,\ldots,x_n$ randomly taken from a uniform distribution of bounds $[0,\theta]$. The maximum likelihood estimator for θ is:

$$\hat{\theta} = \max_{i=1,\ldots,n} x_i \tag{3.47}$$

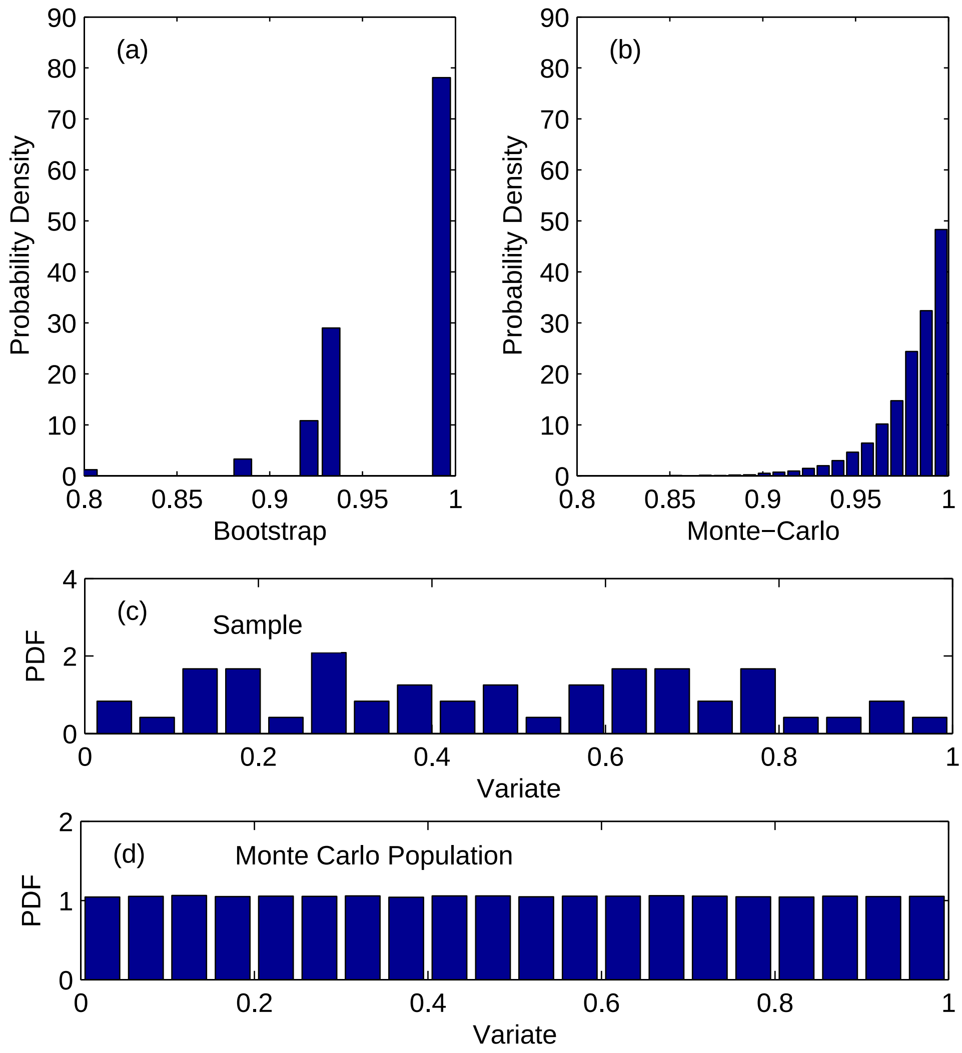

Figure 3.11: Problem noted by Efron and Tibshirani (1993): (a) non-parametric bootstrap estimate; (b) parametric bootstrap estimate; (c) sample histogram, and; (d) histogram of the parametric bootstrap populations.

For the interval $[0,1]$, $\theta_0 = 1$ and a sample of n = 50 is generated on this interval from which $\hat{\theta} = 0.9858$; the histogram for the sample is shown in Figure 3.11(c). For each bootstrap replication, the data is sampled, with replacement, to provide a bootstrap sample from which an estimate of $\hat{\theta}_i^*$ is made. Such estimates are made for B = 1000 bootstrap replications. The histogram of these estimates is shown in Figure 3.11(a). Further, to obtain an estimate of the actual distribution of $\hat{\theta}$, 1000 further samples of size n = 50 were randomly generated on the interval $[0,1]$. The resulting distribution of $\hat{\theta}$ is shown in Figure 3.11(b).

It can be seen from Figure 3.11 that the bootstrap distribution does not match that of the Monte-Carlo estimated distribution. Efron and Tibshirani (1993) refer the reader to Beran and Ducharme (1991) for further information on this problem. The example presented is a non-parametric bootstrap method; the parametric bootstrap method does not fail in this setting Figure 3.11(b). It is to be noted that the variability of the parameters of the parametric bootstrap cannot be taken into account (Efron and Tibshirani 1993); when compared to the method of Chapter 7, this becomes significant.

3.6 Summary

The basic statistical methods, essential to the work that follows, have been presented either in detail or by introduction and reference. Basic tools that will be used throughout this work, such as the method of maximum likelihood, probability paper, characteristic values, return periods and extrapolation have also been presented. More advanced tools that will be used further have also been presented, for example: profile log-likelihood, the bootstrap, Nelder-Mead solution of the GEV likelihood function, Fisher information, probability weighted moments, and the speed of convergence of the asymptotic extreme value distributions. Basic methods of prediction analysis such as the delta method and the bootstrap approach have also been presented.

Chapter 4

SIMULATION OF BRIDGE TRAFFIC LOADING

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"Anyone who attempts to generate random numbers by deterministic means is, of course, living in a state of sin" *– John Von Neumann*

Chapter 4 - SIMULATION OF BRIDGE TRAFFIC LOADING

4.1 Introduction

This chapter describes the measure, model and simulate phases of the bridge traffic load model used in this research. This approach has become more prevalent in recent years as more accurate unbiased measurements of real traffic have become available due to progress in Weigh-In-Motion (WIM) technology.

WIM measurement, and its accuracy, is investigation by many authors (Jacob and OBrien 2005, OBrien et al 2005). The implication of the accuracy of site measurements on resultant characteristic load effects has been studied by O'Connor (2001) and O'Connor et al (2002). The objectives of this research focus on the efficient use of expensive site data such that sufficiently accurate predictions of future load effect are made by further statistical analysis.

Basic statistical distributions of measured traffic characteristics form the input for the traffic model. Such an approach enables site-specific traffic characteristics to be generated which, even though not necessarily measured, represent those of the site. This chapter describes the modelling process undertaken for this research.

The software tools developed for this research are described in Section 4.5. The adoption of object-oriented programming techniques is shown to have significant benefit for traffic load simulation. Substantially increased periods of simulation are possible, increasing the amount of information available to the statistical analysis, which reduces uncertainty in the extreme.

4.2 Measurement of Highway Traffic

This work is based on traffic data from a number of European sites. The development of simulation methodologies is generally independent of the accuracy and amount of traffic data obtained from these sites. However, progress in the overall process does depend on having a sufficient quantity of data upon which reasonably general methods may be based.

WIM technology is the method through which the measured traffic data is obtained, and it is explained briefly in the following section. The work of Grave (2001) formed the early basis of this research programme and the sites analysed in his research are mainly used in this work. Those sites, and other sites also used, are described later in this section.

4.2.1 Weigh-In-Motion measurement

As outlined previously, highway traffic is essential to the bridge traffic load simulation process. Static weigh stations are generally not suitable for this purpose: it is known that traffic measured with such installations is often biased (Laman and Nowak 1997) as drivers of overweight trucks become aware of the installation and avoid the site. Therefore, for bridge traffic loading purposes, the measurement system must be unobtrusive so that unbiased data is gathered. Data should be recorded continuously for the duration of the recording period. Also, measurements of the traffic in free-flow are required to obtain headway, speed and overlapping data. WIM technology has been developed to meet these requirements. Pavement-based WIM systems use sensors located in the road to detect and weigh each of the axles. Alternatively, Bridge-WIM systems effectively use the bridge as a form of weighing scales. Either system can be used to collect traffic data that may be used in bridge loading studies. Recent advances in the accuracy and durability of WIM technology have improved the accuracy of the measured truck and axle weight statistics (Jacob et al. 2002). O'Connor et al. (2002) have looked at the important issue of sensitivity of bridge loading to the accuracy of the original weight measurements.

Figure 4.1 illustrates the Bridge-WIM process; an example of a Bridge-WIM installation is shown in Figure 4.2 and a typical layout of the detectors is shown in Figure 4.3. For this layout, an example of the voltages realised by the passage of a truck is shown in Figure 4.4. A passing vehicle induces voltages in the axle detectors which give its speed, transverse position, number of axles, axle spacing and, importantly for flow and headway, the time stamp of arrival. The voltages induced in the strain transducers are processed with the axle detector information through a Bridge-WIM algorithm (OBrien et al 2005) to give the axle weights and GVW (Gross Vehicle Weight) for the vehicle.

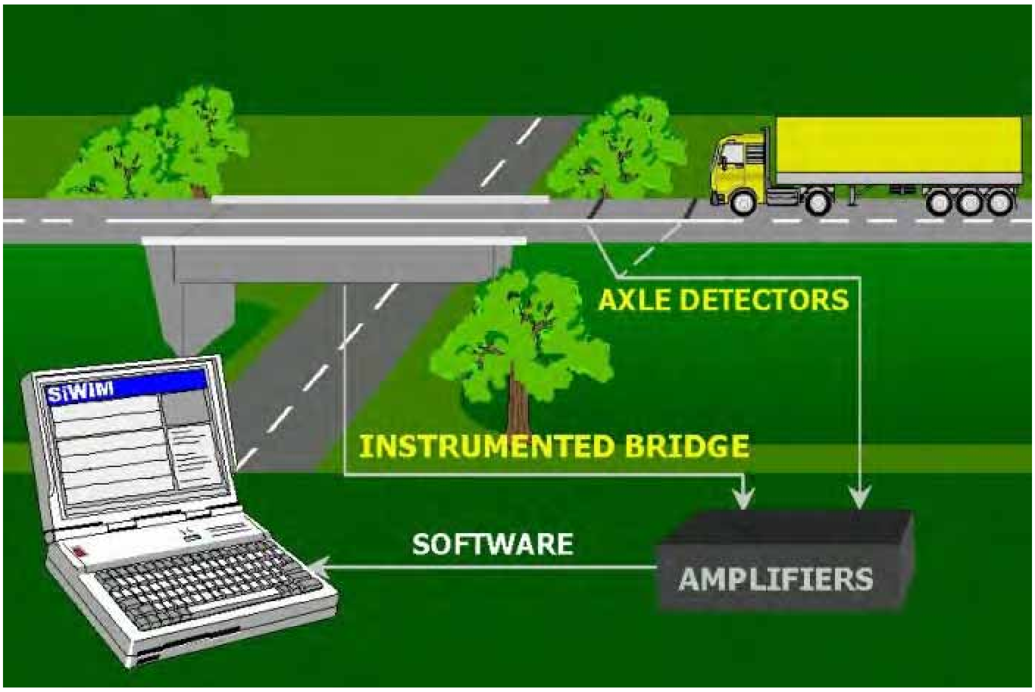

Figure 4.1: Bridge Weigh-In-Motion overview (courtesy of ZAG, Slovenia).

(a) (b)

Figure 4.2: WIM installations: (a) road surface axles detectors; (b) bridge soffit strain transducers.

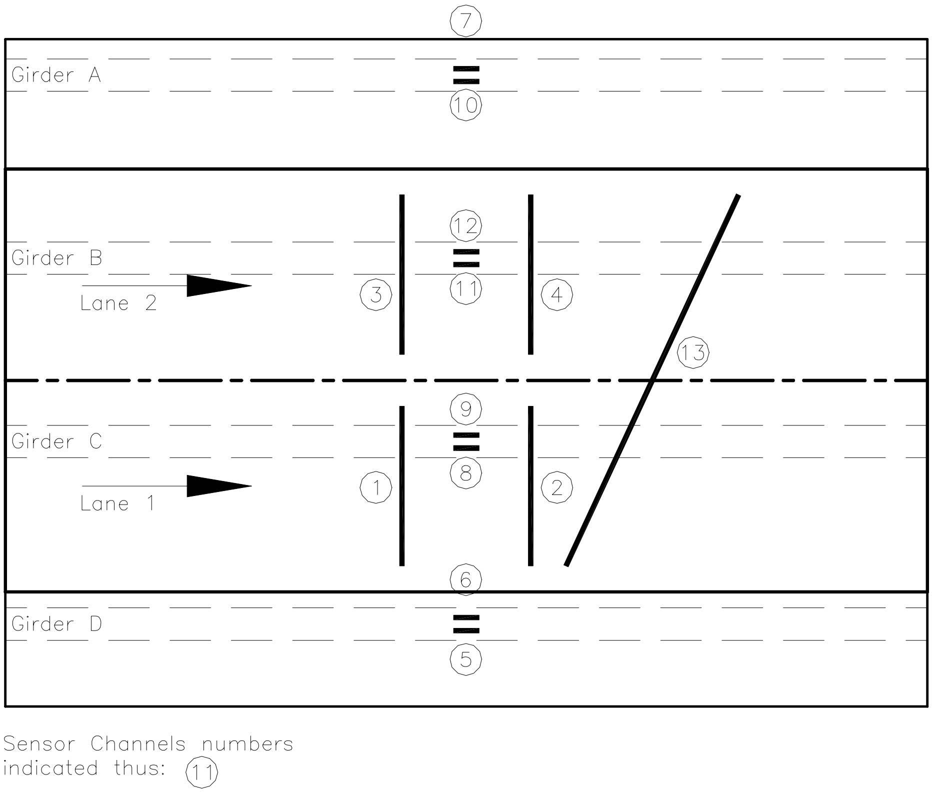

Figure 4.3: Typical Bridge-WIM installation showing the locations of axle detectors and strain transducers along with their channel numbers.