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Available Work Rate of a Reversible System Bounded by Constant Thermal Resistances Linked to Isothermal Reservoirs

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AVAILABLE WORK RATE OF A SYSTEM BOUNDED BY THERMAL RESISTORS LINKED TO ISOTHERMAL RESERVOIRS

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KEY WORDS

exergy analysis, thermal reference environment, finite time thermodynamics, environmental reference temperature, exergy rate, available work rate

I. INTRODUCTION

I.1. Context and Objective

Exergy analysis is based on the concept of an idealized, all-enclosing reference environment that has infinite heat capacity and thermal conductivity, and is in equilibrium. The actual surroundings of a real plant such as a heat engine, a heat pump or a refrigerator may differ significantly from the ideal. The concepts of finite time thermodynamics are applied in an attempt to refine the concept of T_0, the environmental reference temperature, thereby making exergy analysis more reflective of reality.

I.2. Model

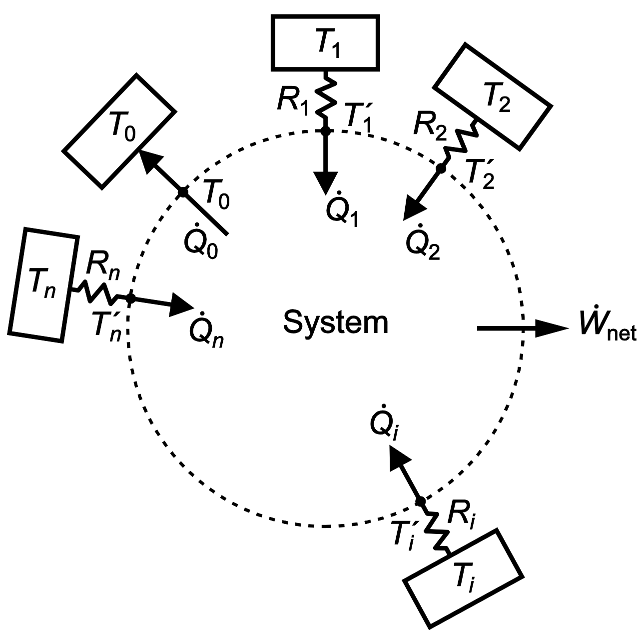

Figure 1: A system bounded by thermal resistors linked to isothermal reservoirs

As an actual ‘environment’ may not be in equilibrium, a real plant may interact with multiple thermal reservoirs, e.g. with the ground, with a large body of water, with the air, with the sun and with the sky as well as available sources of exergy such as a combustion region or an available source of waste heat. As a starting point, a system is considered that produces a net work output while acting with n isothermal reservoirs, each linked to the boundary of the system by a thermal resistance, Figure 1.

I.3. Approach

References from literature, such as [1] and [2], are considered in proposing an approach that may help make the choice of environmental reference state in exergy analysis somewhat more solid and method-based, recognizing the significance of finite time in real thermodynamic processes or occurrences.

II. CONCLUSION

Without going to extraordinary lengths, a methodical approach can be used to specify an appropriate thermal reference environment model as part of an overall exergy analysis approach respecting the principles of finite time thermodynamics.

REFERENCES

[1] R.A. Gaggioli, The Dead State, Inter. Journal of Thermodynamics, 15, 4 (2012).
[2] F.L. Curzon, B. Ahlborn, Efficiency of a Carnot Engine at Maximum Power Output, Am. Journal of Phy., 43, 22 (1975).

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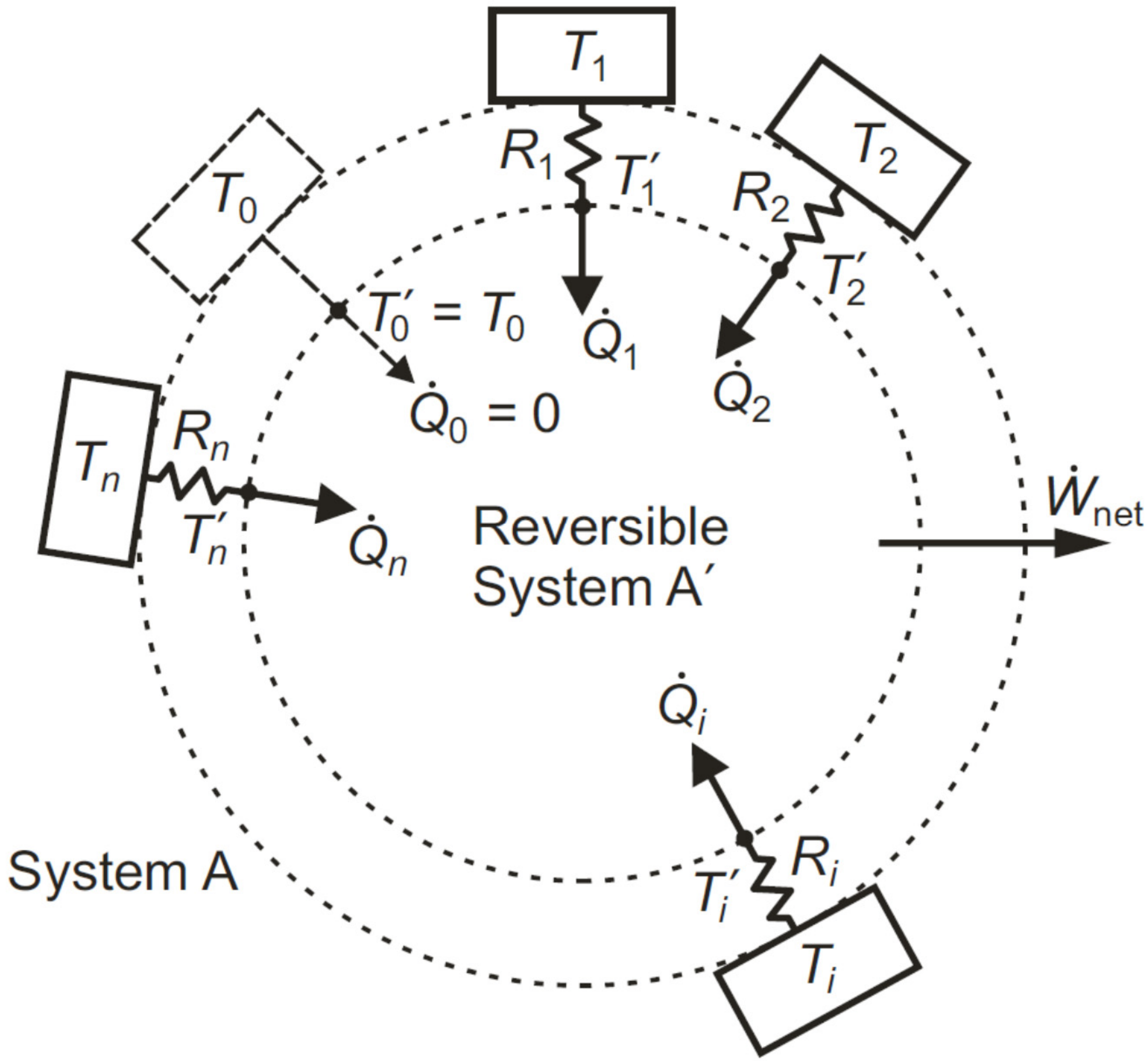
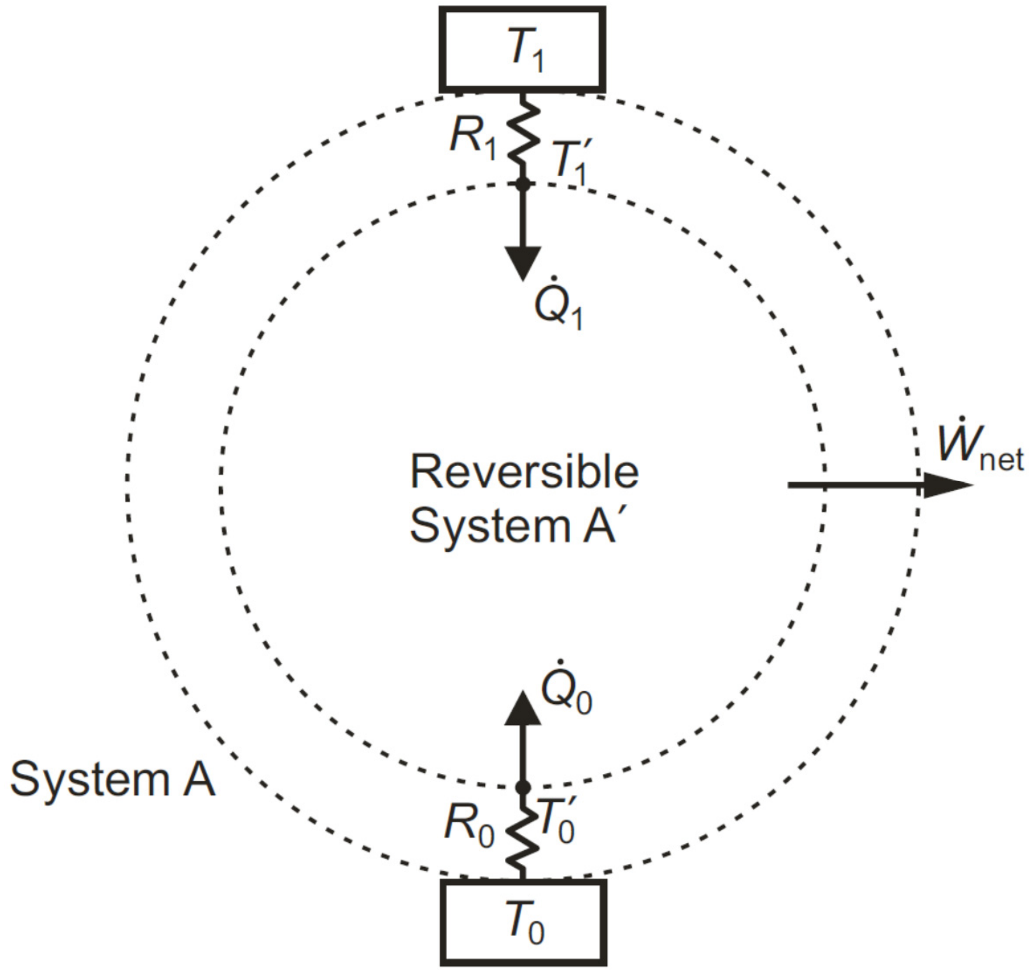


T_1
R_1
T_1'
T_2
R_2
T_2'
T_0
$T_0' = T_0$
$\dot{Q}_1$
$\dot{Q}_2$
$\dot{Q}_0 = 0$
T_n
R_n
T_n'
$\dot{Q}_n$
Reversible
System A′
$\dot{W}_{net}$
$\dot{Q}_i$
R_i
T_i'
T_i
System A

Aims

- Examine first law performance parameters and second law rational efficiency, e.g. component rational efficiencies
- Consider the meaning of the environmental reference temperature

Energy Analysis



$$\dot{W}_{\text{net}} = \dot{Q}_1 \left(1 - \frac{T_0}{T_1 - \dot{Q}_1 R_{\text{tot}}}\right)$$

$$E_{\text{th}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_1} = 1 - \frac{T_0}{T_1 - \dot{Q}_1 R_{\text{tot}}}$$

$$\dot{Q}_{1,\text{ max power}} = \frac{T_1 - \sqrt{T_1}\sqrt{T_0}}{R_{\text{tot}}}$$

$$E_{\text{th, max power}} = \frac{\dot{W}_{\text{net max}}}{\dot{Q}_{1\text{ max}}} = 1 - \frac{\sqrt{T_0}}{\sqrt{T_1}}$$

$$\mathrm{COP}_{\mathrm{hp}} = \frac{-\dot{Q}_1}{-\dot{W}_{\mathrm{net}}} = \frac{1}{E_{\mathrm{th}}} = \left(\frac{T_1 - \dot{Q}_1 R_{\mathrm{tot}}}{T_1 - T_0 - \dot{Q}_1 R_{\mathrm{tot}}}\right)$$

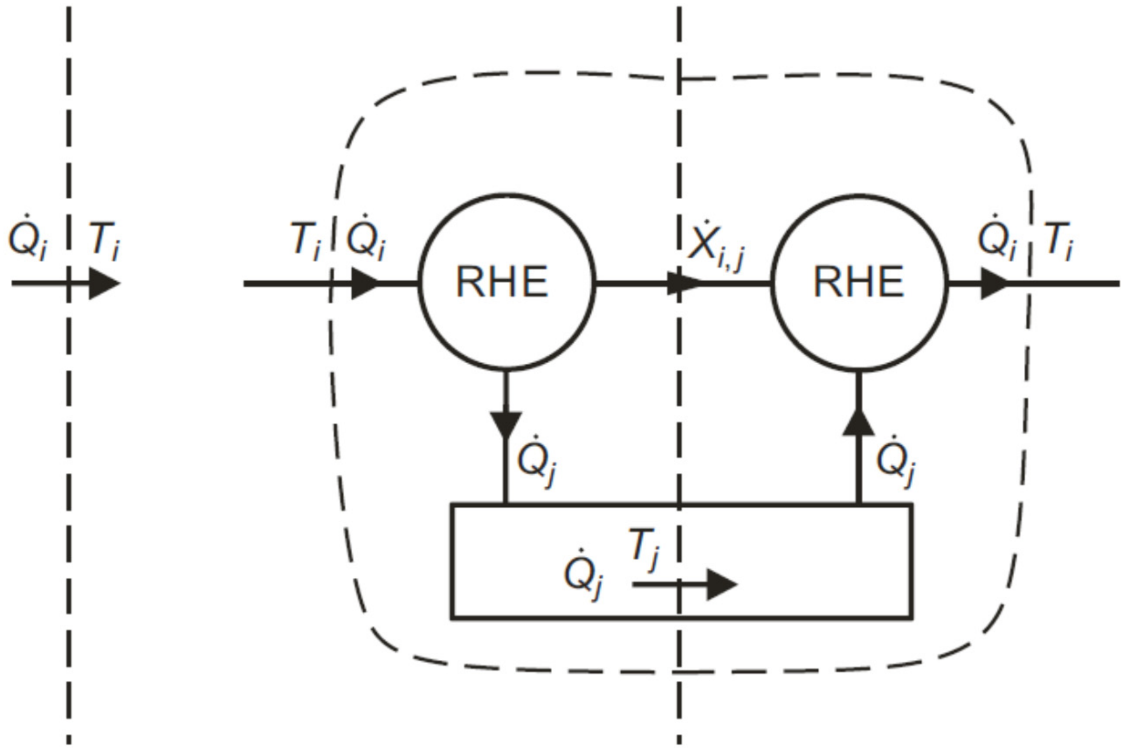
$$\mathrm{COP}_{\mathrm{refr}} = \frac{\dot{Q}_1}{-\dot{W}_{\mathrm{net}}} = -\frac{1}{E_{\mathrm{th}}} = \left(\frac{T_1 - \dot{Q}_1 R_{\mathrm{tot}}}{T_0 - T_1 + \dot{Q}_1 R_{\mathrm{tot}}}\right)$$

Exergy Analysis

$$\dot{W}_{\text{lost, }0} = T_0 \dot{S}_{\text{gen}}$$

$$\dot{S}_i = \frac{\dot{Q}_i}{T_i}$$

$$\dot{W}_{\text{lost, }j} = \frac{T_j}{T_0} \dot{W}_{\text{lost, }0}$$



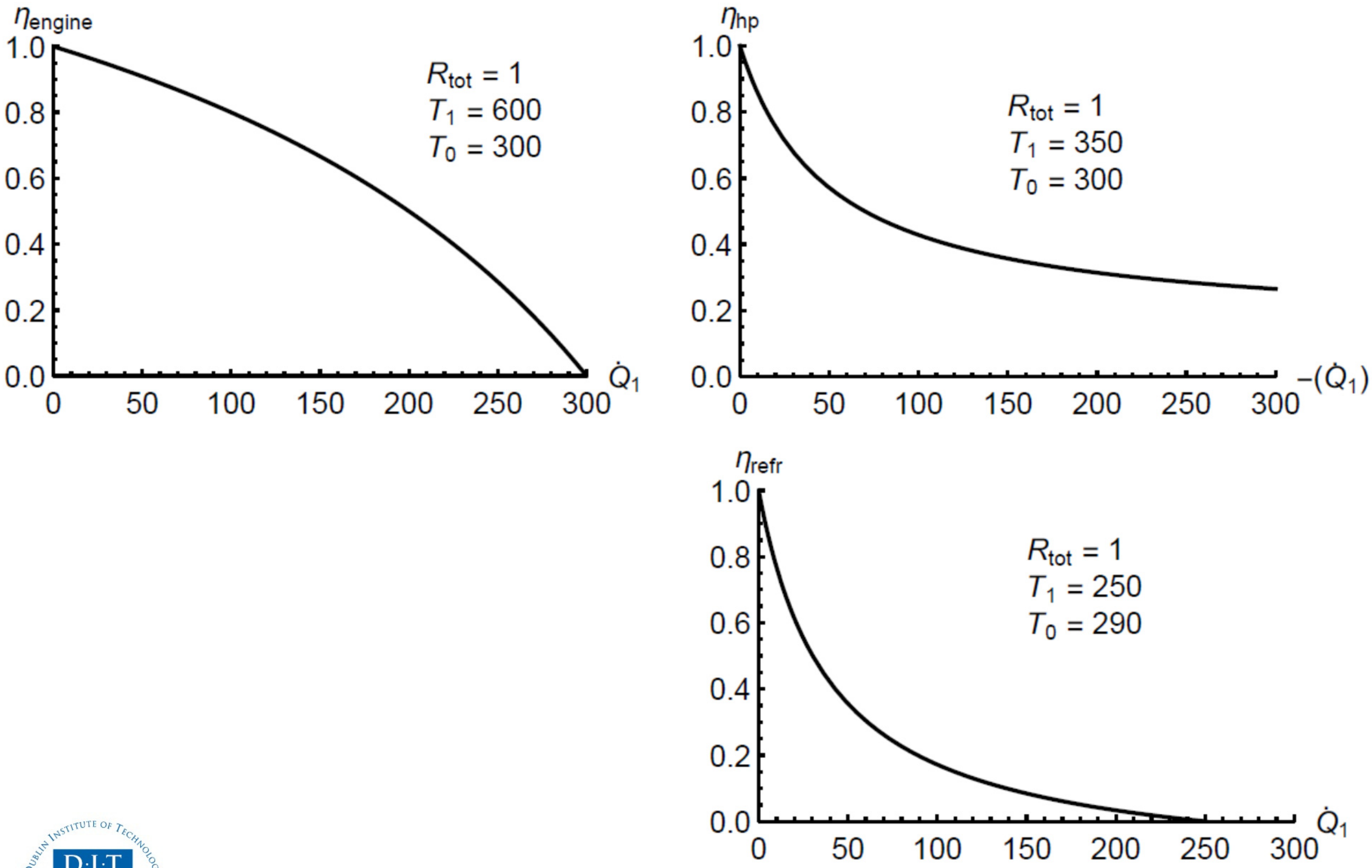
$$\dot{X}_{i,j} = \dot{Q}_i \left(1 - \frac{T_j}{T_i}\right)$$

Rational Efficiency

$$\eta_{\text{engine}} = \frac{\dot{W}_{\text{net}}}{\dot{X}_1} = \frac{T_1^2 - \left(\dot{Q}_1 R_{\text{tot}} + T_0\right) T_1}{(T_1 - T_0)(T_1 - \dot{Q}_1 R_{\text{tot}})}$$

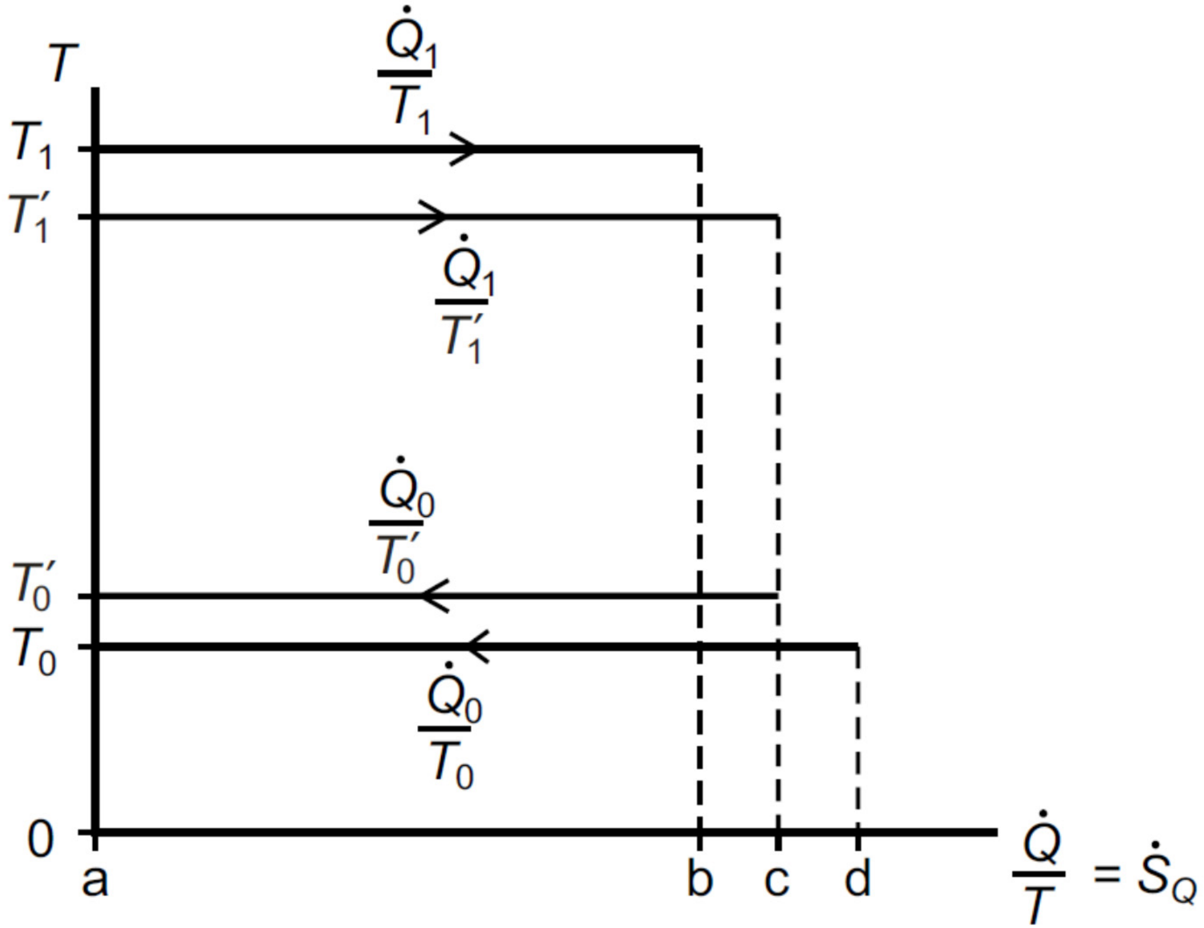
$$\eta_{\text{hp}} = \frac{-\dot{X}_1}{-\dot{W}_{\text{net}}} = \frac{1}{\eta_{\text{engine}}} = \frac{(T_1 - T_0)(T_1 - \dot{Q}_1 R_{\text{tot}})}{T_1^2 - \left(\dot{Q}_1 R_{\text{tot}} + T_0\right) T_1}$$

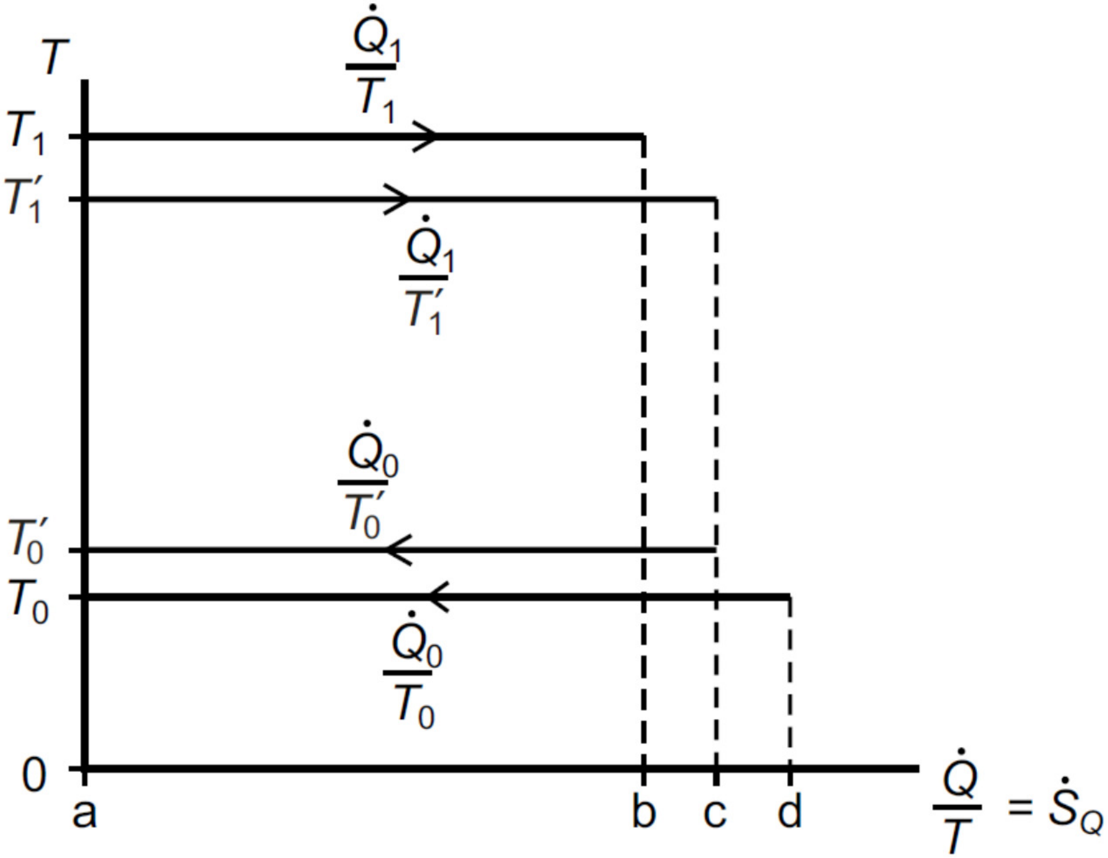
$$\eta_{\text{refr}} = \frac{-\dot{X}_1}{-\dot{W}_{\text{net}}} = \frac{1}{\eta_{\text{engine}}} = \frac{(T_1 - T_0)(T_1 - \dot{Q}_1 R_{\text{tot}})}{T_1^2 - \left(\dot{Q}_1 R_{\text{tot}} + T_0\right) T_1}$$



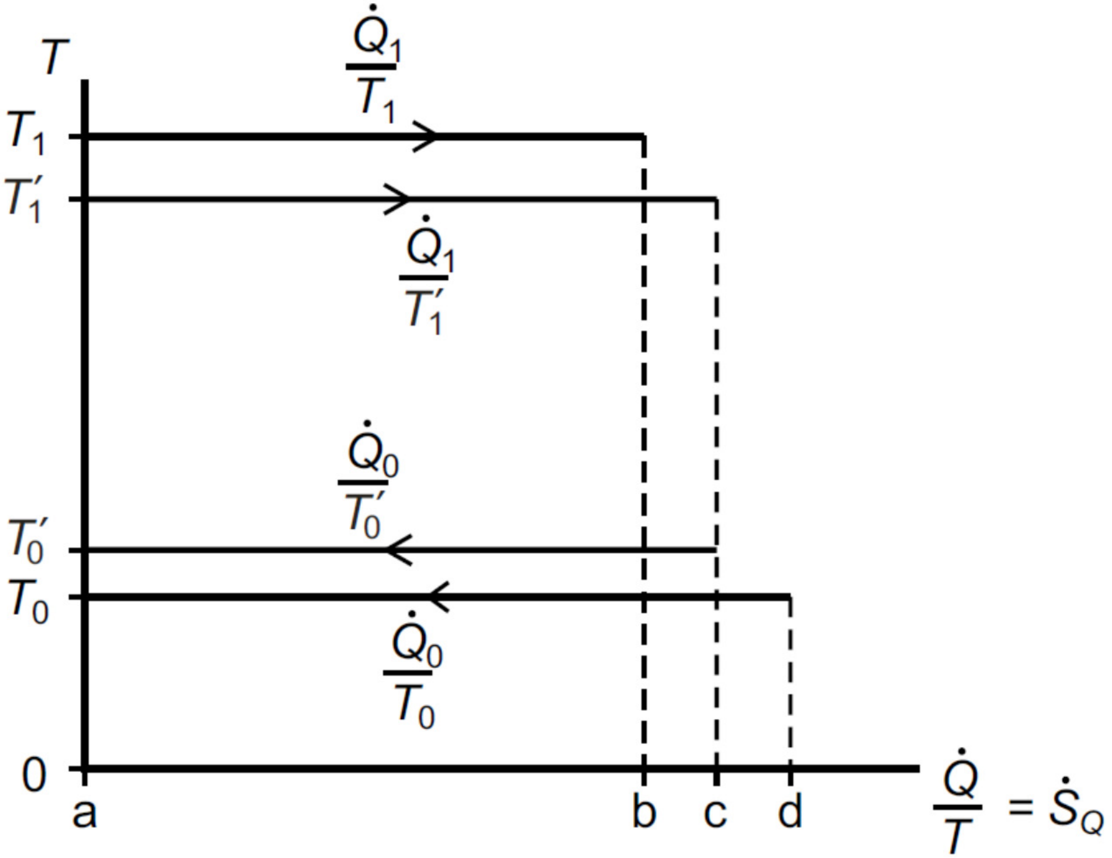
η_{engine}
$R_{tot} = 1$
$T_1 = 600$
$T_0 = 300$
$\dot{Q}_1$
η_{hp}
$R_{tot} = 1$
$T_1 = 350$
$T_0 = 300$
$-(\dot{Q}_1)$
η_{refr}
$R_{tot} = 1$
$T_1 = 250$
$T_0 = 290$
$\dot{Q}_1$

Temperature vs Entropy Transfer Rate Diagram





$$\eta_{\mathrm{A,\,0}} = \frac{\dot{X}_{\mathrm{in\ A}} - \dot{W}_{\mathrm{lost\ A}}}{\dot{X}_{\mathrm{in\ A}}} = \frac{\dot{S}_{\mathrm{ab}}\left(T_1 - T_0\right) - T_0\dot{S}_{\mathrm{bd}}}{\dot{S}_{\mathrm{ab}}\left(T_1 - T_0\right)}$$



$$\eta_{\mathrm{A}',\,0'} = \frac{\dot{S}_{\mathrm{ac}}\left(T_1' - T_0'\right) + T_0'\left(\dot{S}_{\mathrm{ac}} - \dot{S}_{\mathrm{ac}}\right)}{\dot{S}_{\mathrm{ac}}\left(T_1' - T_0'\right)} = 1$$

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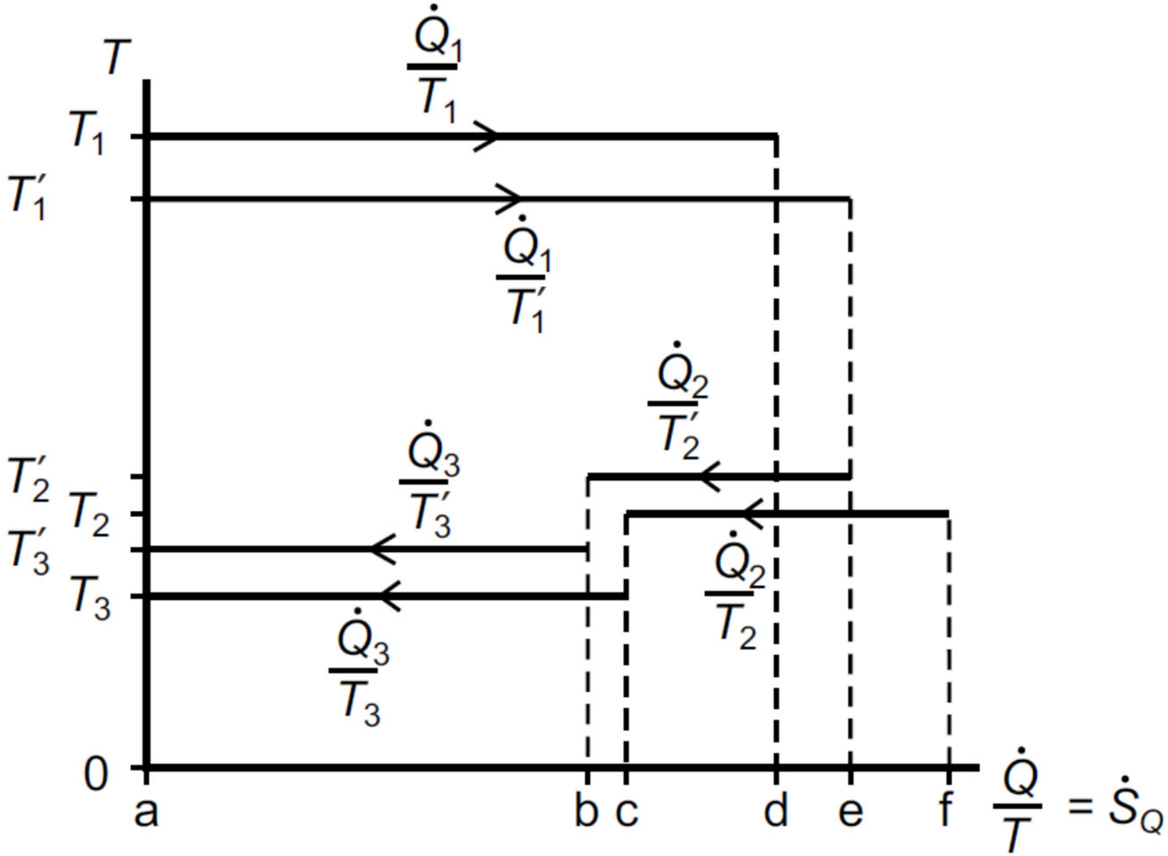
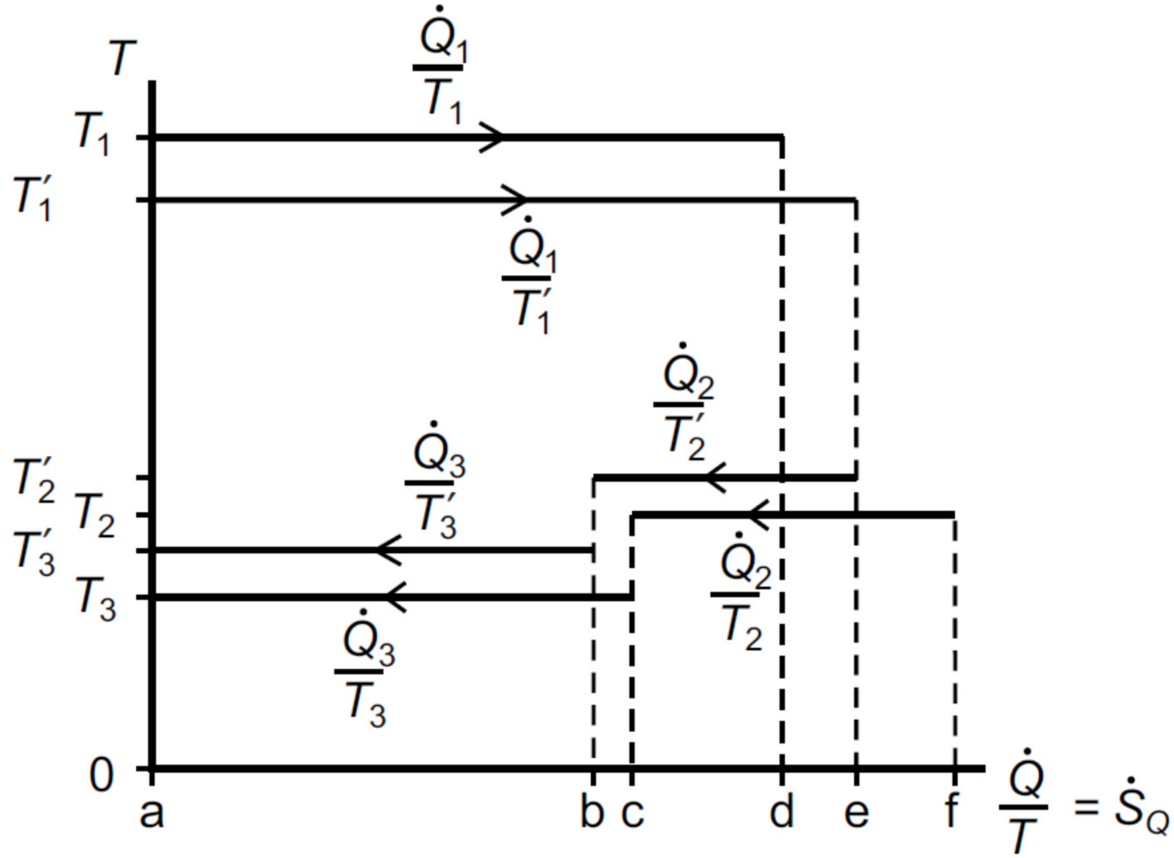
The Environmental Temperature





$$T_0 = \frac{T_3 \dot{S}_{\mathrm{ac}} + T_2 \dot{S}_{\mathrm{cf}}}{\dot{S}_{\mathrm{af}}} \qquad T_0 = \frac{\int T_{\mathrm{env}} \dot{S}_{\mathrm{env}}}{\int \dot{S}_{\mathrm{env}}}$$

Conclusions

- Model provides useful insights into finite time thermodynamics
- Temperature vs Entropy Transfer Rate method can localize exergy destruction rates and provide subsystem rational efficiencies
- Reference environment need not necessarily be at a single constant temperature

This document consists of the abstract that was submitted[1] for and the slides that were presented[2] at JETC 2015, the 13th Joint European Thermodynamics Conference, ENSIC, Nancy, May 20–22, 2015. Some minor adjustments and corrections have been made to the slides[3].

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[1] The title used on the submitted abstract was 'Available work rate of a system bounded by thermal resistors linked to isothermal reservoirs'.

[2] A revised title was used for the slides, as presented: 'Available work rate of a reversible system bounded by constant thermal resistances linked to isothermal reservoirs'.

[3] In the last equation on slide 5 the abbreviation 'max' has been added to the subscript of the *Q* term on the right hand side. The last equation on slide 7 has been corrected: the second subscript of the left hand side expression has been changed from *i* to *j*. The rational efficiency plots on slide 10 have been replaced by equivalent ones with a larger font size and the vertical axis label for the refrigeration plot has been corrected. The equation on slide 13 has been corrected: in the 'as presented' version the denominator in the equation was incorrect. In the right hand equation at the bottom of Slide 15 the subscript 'env' has been attached to the *T* term after the integration symbol. The date of this document is 29 May 2015.